Semiparametric Analysis of Polygenic Gene-Environment Interactions in Case-Control Studies with caseControlGE

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Abstract

Gene-environment interactions can be efficiently estimated in case-control data by exploiting the assumption of gene-environment independence in the source population, but until recently such techniques required parametric modelling of the genetic variables. The **caseControlGE** package implements the methods of Stalder, Asher, Liang, Carroll, Ma, and Chatterjee (2017, *Biometrika*, **104**, 801-812) and Wang, Asher, and Carroll (2018, unpublished), which exploit the assumption of gene-environment independence without placing any assumptions on the marginal distributions of the genetic and environmental variables. These methods are ideally suited for analysis of complex polygenic data for which parametric distributional models are not feasible. In addition to the two estimators, the package also supplies a function to simulate case-control data and several helper functions for use on model objects. Use of this package is illustrated using simulated data from a case-control study of breast cancer.

Keywords: case-control study; gene-environment interaction; genetic epidemiology; retrospective method; semiparametric analysis; pseudolikelihood; polygenic analysis.

1. Introduction

1.1. caseControlGE package

The **caseControlGE** package (Asher 2018) contains tools for the analysis of case-control data using R (R Core Team 2018). It implements the methods of Stalder *et al.* (2017) and Wang *et al.*, both of which fall under the class of semiparametric retrospective profile likelihood estimators. These methods are the first available to exploit the assumption of gene-environment independence while treating the genetic component nonparametrically. As such, they are well suited to replace logistic regression as the preferred method in situations where parametric distributional models are not feasible, such as in the analysis of complex polygenic data.

caseControlGE contains three main functions: `simulateCC`, `spmle`, and `spmleCombo`, as well as several helper functions. Section 2 of this paper introduces `simulateCC` in the context of simulating case-control data analogous to the data analyzed in Wang *et al.*. Section 3 introduces `spmle` as a tool to analyze the simulated data, and section 4 introduces `spmleCombo` to conduct a more efficient analysis of the simulated data.

1.2. Background

Case-control studies are retrospective observational studies in which the sample consists of a group of healthy subjects and a group of diseased subjects. A crucial aspect of the case-control design is that the outcome, disease status, is known *before* sampling. The ability to deliberately oversample diseased subjects makes the case-control design cost effective, which is why it is widely popular in studies of gene-environment interactions.

Given the genetic and environmental covariates G and E, we assume the risk of disease D in the underlying population follows the model

$$\mathrm{pr}(D = 1 \mid G, X) = H\{\alpha_0 + m(G, X, \boldsymbol{\beta})\},$$

where $H(x) = \{1 + \exp(-x)\}^{-1}$ is the logistic distribution function and $m(G, X, \boldsymbol{\beta})$ is a function that describes the joint effect of G and X and is known up to the unspecified parameters of interest $\boldsymbol{\beta}$.

Given the retrospective nature of case-control sampling, it is surprising that standard prospective logistic regression can be used to obtain unbiased estimates of $\boldsymbol{\beta}$ (Prentice and Pyke 1979). Logistic regression requires no assumptions about the joint distribution of G and E, but it suffers from low power when estimating $G{*}E$ interaction effects. To gain efficiency, Chatterjee and Carroll (2005) exploited the assumption of gene-environment independence in the source population to maximize the retrospective likelihood while profiling out the distribution of E. Their method is available as the function `snp.logistic` in the *Bioconductor* package **CGEN** (Bhattacharjee, Chatterjee, Han, Song, and Wheeler 2012).

The method of Chatterjee and Carroll, and subsequent methods utilizing the same retrospective profile likelihood framework, require a parametric model for the distribution of G given E. This becomes difficult as the number and complexity of genetic variables in the model grows. Capitalizing on advances in high-throughput genomics, genome-wide association studies have identified scores of SNPs associated with complex diseases such as cancers and diabetes. Modern case-control studies of gene-environment interactions need efficient methodology that allows for a flexible and arbitrarily complex genetic component, such as multiple correlated SNPs and/or continuous polygenic risk scores (PRSs).

The method of Stalder *et al.* (2017) extends the retrospective profile likelihood framework of Chatterjee and Carroll, dispensing with the need to model G parametrically. When the population disease rate π_1 is known, the retrospective profile loglikelihood can be estimated (up to an additive constant) using just the case-control sample and without modeling the distribution of G. When π_1 is unknown but the disease is rare, estimates can be obtained using the *rare disease approximation* that $\pi_1 \approx 0$, which typically introduces negligible bias (Stalder *et al.* 2017).

Wang *et al.* proposed an improvement to the method of Stalder *et al.* (2017) that increases the efficiency of the estimates with no additional assumptions. This development relies on the observation that the method of Stalder *et al.* removes dependence on the distribution of the genetic and environmental variables in two different fashions; by treating the genetic and environmental variables symmetrically Wang *et al.* generate two sets of parameter estimates that are combined to generate a more efficient estimate.

1.3. Implementation

The semiparametric method of Stalder *et al.* (2017) is implemented as the function `spmle` in **caseControlGE**, detailed in section 3. Estimating the semiparametric profile likelihood is a computationally intensive process, and significant effort was invested in speeding up calculations. Estimation functions, including the analytic gradient and hessian, are written in C++ and compiled using **Rcpp** (Eddelbuettel 2013), providing a tremendous speedup over native R code. Extensive benchmarking and code profiling was conducted, and estimation functions were written to apply matrix operations to contiguous memory locations whenever possible, reducing memory latency and allowing modern processors to exploit data level parallelism and perform the same operation on multiple data points simultaneously.

The estimated semiparametric likelihood is maximized using the quasi-Newton optimizer **ucminf** (Nielsen and Mortensen 2016) using starting values from logistic regression. `ucminf` is particularly well suited for this application because it allows us to precondition the optimization with the analytic hessian, and it evaluates the gradient after each call to the objective function. Calculating the gradient along with the likelihood adds negligable computational complexity, so we call a single C++ function to compute them both, then return them separately to `ucminf`. This leads `ucminf` to converge in roughly half the time of the next-fastest optimizers (several of the various R implementations of the BFGS algorithm tie for second place). The unmatched speed of `ucminf` means we are willing to tolerate its bugs, which include occasionally declaring convergence before actually converging. To address this, `spmle` checks the gradient at the reported optimum and restarts the optimization if necessary (with different starting values).

Computational complexity of the asymptotic covariance estimation, which contains a sum of the form $\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \partial\mathcal{L}_{ijk}(\Omega)/\partial\Omega$, was reduced from $O(n^3)$ to $O(n^2)$ by storing intermediate values in a three-dimensional array. This increases speed at the cost of memory usage, which climbs from $O(n)$ to $O(n^2)$, setting a practical limit on sample size in the low tens of thousands for average personal computers. This is sufficient to analyze all but the largest case-control studies; covariance estimates for larger studies should be computed using the bootstrap.

Asymptotic covariance estimates for the Symmetric Combination Estimator of Wang *et al.* converge slowly and unreliable in practice, often providing poor coverage. Wang *et al.* recommend a balanced bootstrap, with cases and controls resampled separately, to estimate covariance. **caseControlGE** offers users with multicore computers the option to speed up computation by using multiple processors. Parallelization is implemented using the R base package **parallel**, which is installed by default on all operating systems. Parallelization on computers running Linux or macOS is done by forking the active R session, saving time and memory. This option is unavailable in Windows, so parallelization is fractionally slower because a PSOCK cluster is created with a new instance of R running on each core.

2. Simulating case-control data with simulateCC

Wang *et al.* demonstrate the utility of their method

```
R> 1
```

```
[1] 1
```

3. Analyzing case-control data with spmle

```
R> 1
```

```
[1] 1
```

4. Analyzing case-control data with spmleCombo

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