Quantitative Macro HW 5

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Q1

First data for the "population" is created and actual output for each firm is calculated as described in the problem set.

Next in order to get the efficient allocation of capital given firm specific productivity I solve the optimization problem. This yields the FOCs

$$\gamma k_i^{\gamma-1} z_i = \lambda \quad \forall i. \tag{1}$$

Plugging this into the BC k_1 is calculated as

$$k_1 = K / \left(1 + \left(\frac{z_1}{z_2}\right)^{\frac{1}{\gamma-1}} + ...\right) \tag{2}$$

and using this I can calculate all efficient k. Comparing the efficient to the random allocation of ks it is obvious that there is a missallocation with radnom ks, as more productive firms often do not get enough capital and less productive firms use more then optimal capital.

Now computing the possible output gains of reallocatoin for all given specifications of the model yields

	$cov = 0$	$cov = 0.5$	$cov = -0.5$
$\gamma = 0.6$	138	76	223
$\gamma = 0.8$	743	427	1273

So if the population data is created with a positive covariance between z and k the inefficiency decreases. This is the case because more productive firms now tend to have a higher capital stock assigned. The opposite holds for a negative relationship between the draw of z and k.

Increasing gamma increases the inefficiency sizeable for all covariance specifications. A higher gamma means more weight in the production function on

the (inefficiently) assigned capital and increases therefroe the gain to be made by reallocating capital.

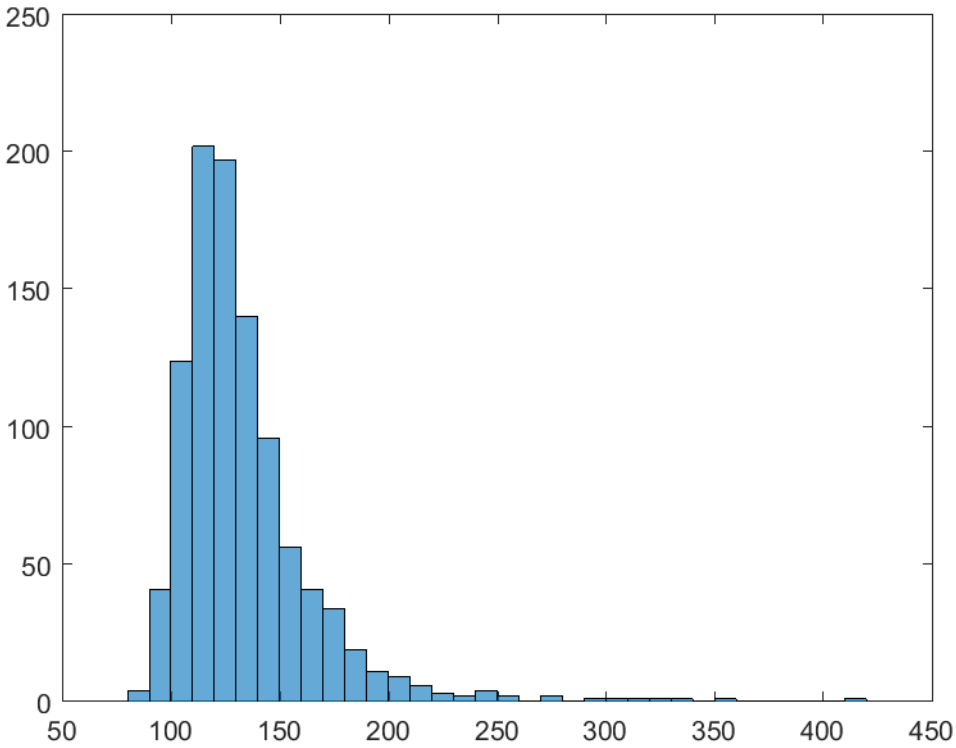
Q2

The results for part 2 of the problem set for all sample sizes are summarized in the following table.

	100	1000	$10,000$	$100,000$
var	0.96	0.98	1.005	0.99
cov	0.13	0.02	0.001	0
Median	107	126	135	137.5
Pr	0.17	0.36	0.776	0.99

using a sample seems to systematically underscore the true possible gains of reallocation as the median for all simulations is below the true possible gains of 138. Using sample sizes of 10,000 or 100,000 the outcomes are fairly close to the real ones, although even with 10,000 samples the probability of not being within 10% of the true gains is 15%.

As an example we can also plot a histogram for the cases of 1000 and 10,000 observations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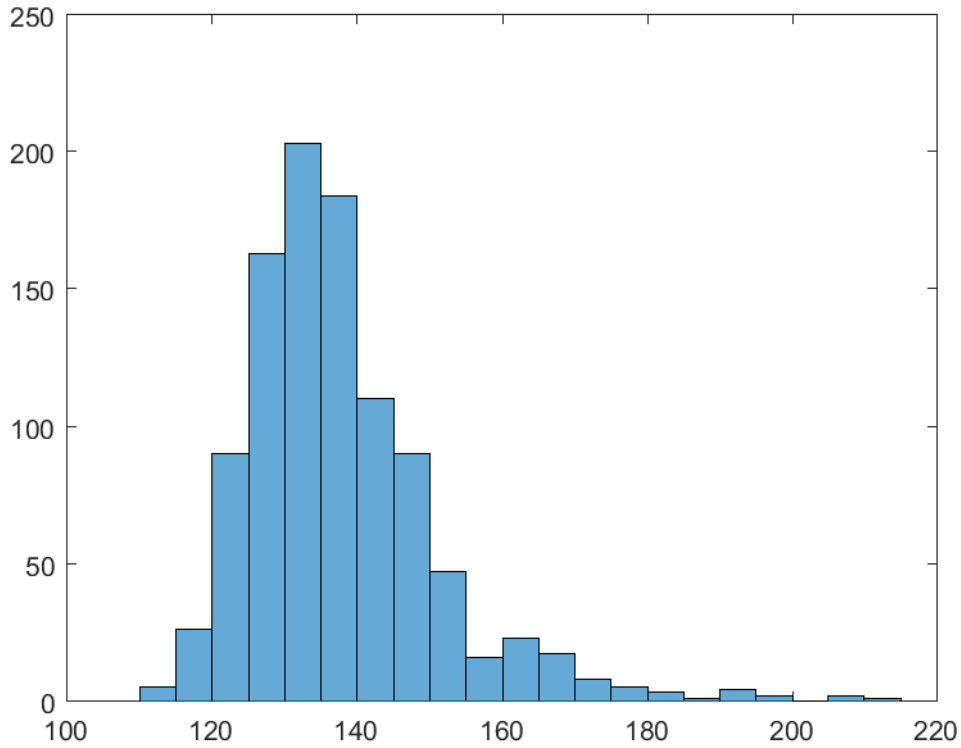

For 1000 observation, for which usually the law of large numbers applies, outcomes are quite far from the real one.

I redid the exercise for the two different specifications of the covariance matrix. But as results are qualitatively unchanged, they are omitted here.