Quantitative Macro HW 3

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Q1

a)

In order to calculate the steady state I normalize output to 1. From there it follows that capital in steady state is 4. By the production function we then know that $z = 1.6298$. Finally we can use the Euler Equation

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} [(1 - \delta) + (1 - \theta) k_{t+1}^{-\theta} (zh)^{\theta}] \tag{1}$$

to get the definition of capital in steady state

$$k^{ss} = \left(\frac{\beta(1 - \theta}{1 - \beta(1 - \theta)} \right)^{\frac{1}{\theta}} zh. \tag{2}$$

I use this to derive $\beta = 0.98$. All other steady state variables can now be calculated. In summery:

$k^{ss} = 4$

$c^{ss} = 0.75$

$y^{ss} = 1$

$i^{ss} = 0.25$

b)

The new steady state is defined by:

$k^{ss} = 8$

$c^{ss} = 1.5$

$y^{ss} = 2$

$i^{ss} = 0.5$

c)

To determine the transition path to the new steady state one has to solve a system of equations consisting of the Euler Equations and Budget Constraints for each period begging in t=1 till T, where the new steady state is reached. As this might be computational costly I use following algorithm to solve for the transition path:

1. Guess initial consumption value in $t = 1$ (i.e. after the shock realizes)

2. Use this and the budget constraint to pin down k_{t+1}.

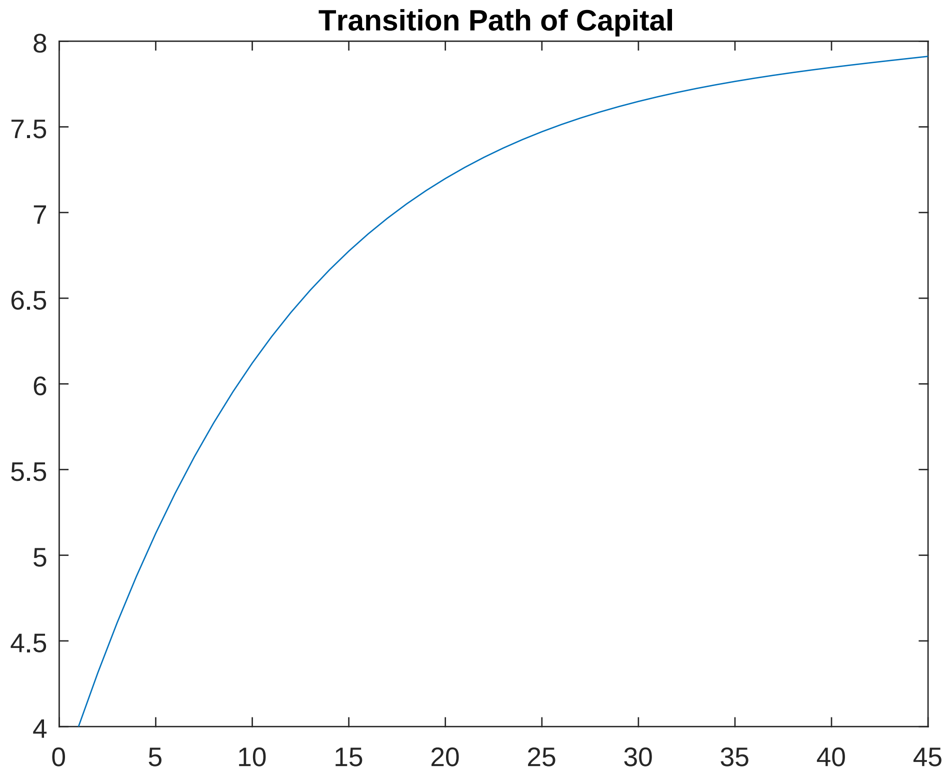
3. Plug both into the EE and solve for c_{t+1}

4. Iterate this process forward until $|k^{ss} - k_t| < \epsilon$

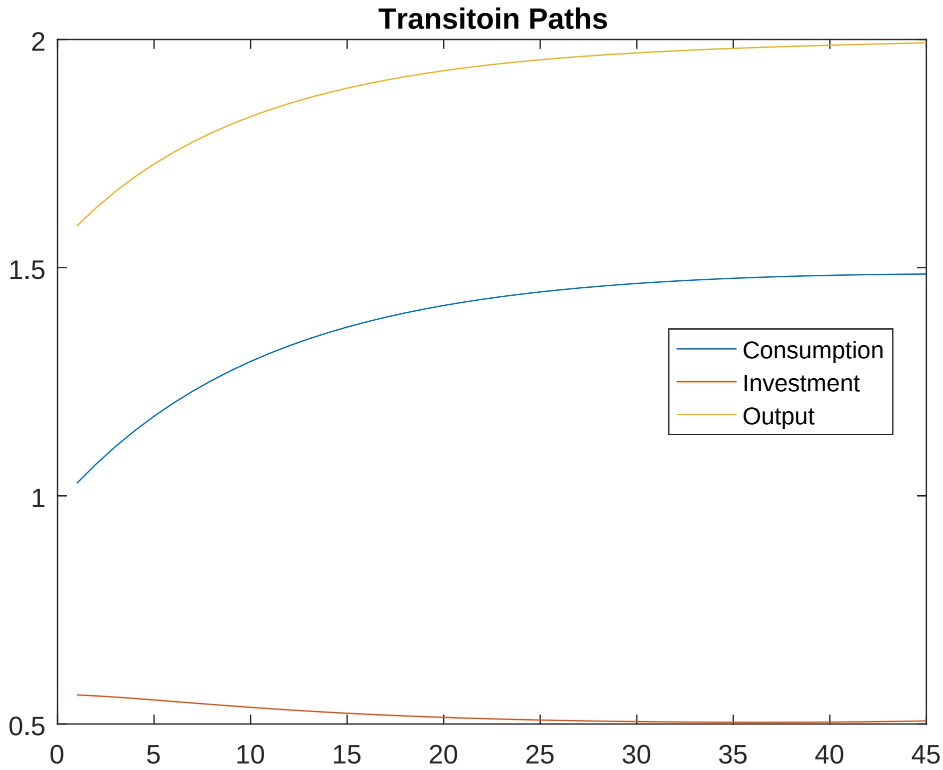
5. Check whether $|c^{ss} - c_t| < \epsilon$. If not slightly increase the first guess.

6. Repeat steps 1-5 until k and c reach steady state in the same period.

Results

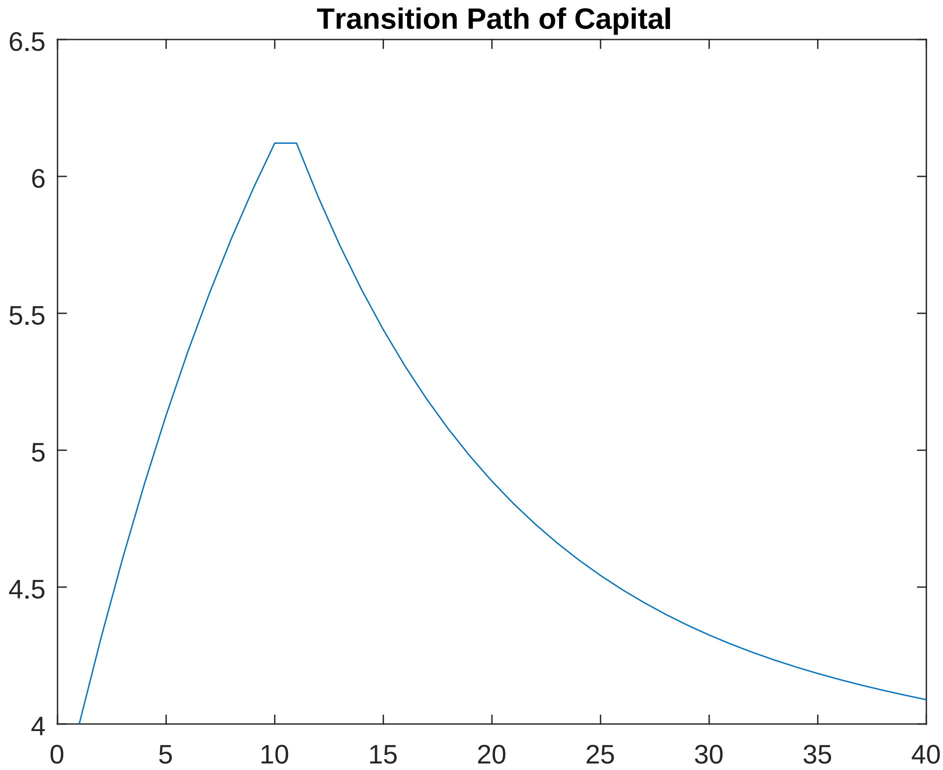


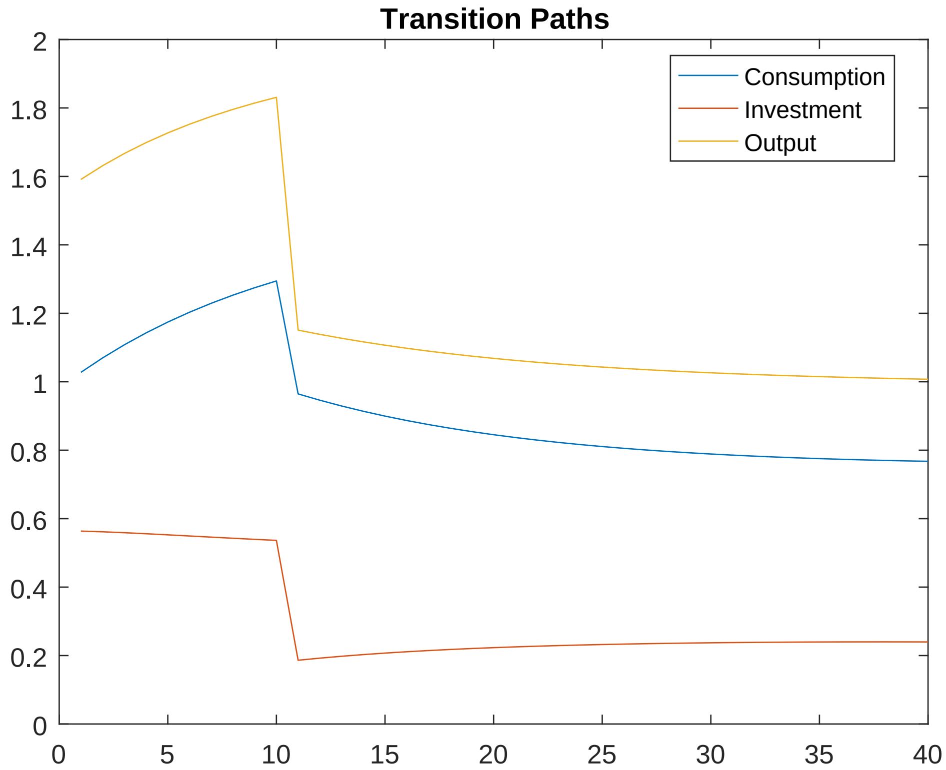
Transition Path of Capital
8
7.5
7
6.5
6
5.5
5
4.5
4
0
5
10
15
20
25
30
35
40
45



d)

Now the transition follows the same path as before for the first 10 periods and then converges back to the old steady state after the new shock hits. The derivation of the new new transition path follows the algorithm in c), with the initial guess being in $t = 11$.


Transition Path of Capital
6.5
6
5.5
5
4.5
4
0
5
10
15
20
25
30
35
40



e)

I introduced a permanent consumption and capital tax. EE is now

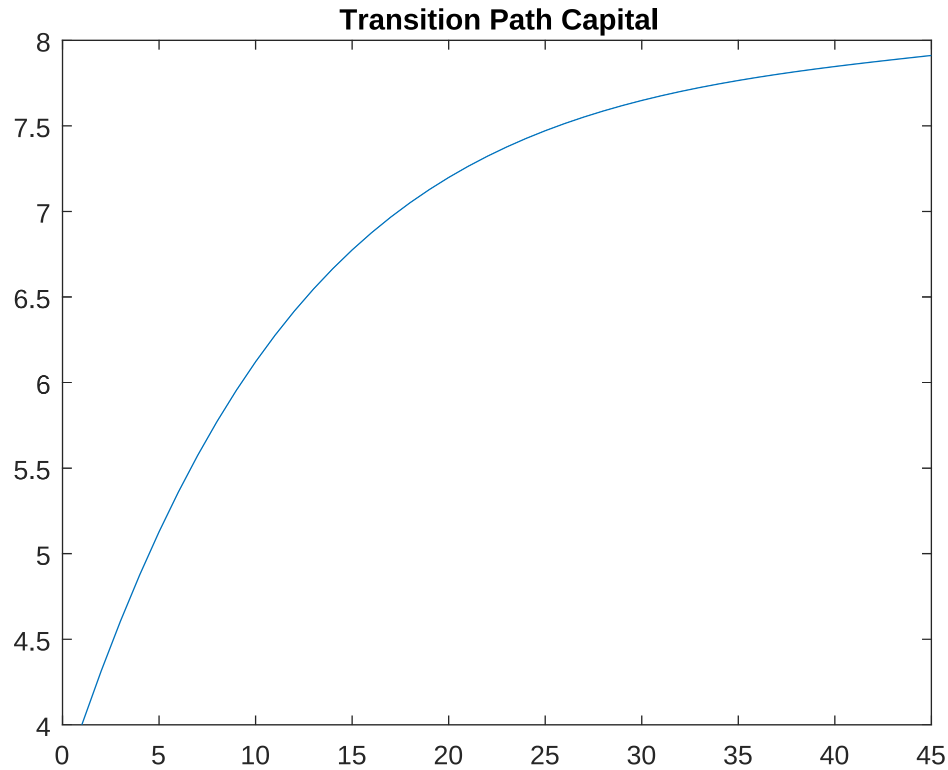
$$c_{t+1} = \beta c_t[(1 - \delta - \tau^k) + (1 - \theta)k_{t+1}^{\theta}(zh)^{\theta} \tag{3}$$

while the BC is now

$$(1 + \tau^c)c_t + k_{t+1} - (1 - \delta - \tau^k)k_t = y \tag{4}$$

Setting $\tau^k = 0$ I arrive at following transition path for capital. The transition path for capital stays unchanged and again it takes 45 periods to reach the new steady state (consumption stays on a lower level, i.e. $c_{new}^{ss} = c_{old}^{ss}/(1 + \tau^c)$.

This result is robust to different values of τ.



Q 2

For the whole Question I assume an equal mass of low type and high type agents in the economy. To simplify things I therefore model a two agent economy. Furthermore throughout the whole exercise I assume that the aggregate capital stock in each economy is evenly distributed among the two agents.

a)closed economy

An equilibrium of the given economy is defined as:

1. Given w, r and a tax policy λ, ϕ households of type low and high maximize the given utility function s.t. to the Budget Constraint (as this part deals with a closed economy, foreign investment is zero).

2. Given w, r firms maximize their profit function.

3. Markets clear

Solving algorithm

A short summary of the solving algorithm applied in the following is given by:

1. Obtain all optimality conditions (household an firm problem)

2. Plug the market clearing conditions into the FOC of the firm problem.

3. Use this together with the budget constraints to solve the system of equations for the exogenous variables of the model.

Description and results

To solve the model I derive the FOC of the household and firm problem analytically. As the problem is static and the economy is closed, capital supply is inelastic and equal to the respective capital stock of each agent. Maximizing over c and h in the household problem delivers the Euler Equation

$$(1-\phi)c^{-\sigma}\lambda(wh\eta)^{-\phi}w\eta = \kappa h^{1/v} \tag{5}$$

which holds for both agents.

The firms problem pins down prices as

$$r = (1-\theta)ZK^{-\theta}H^{\theta} \tag{6}$$

$$w = \theta ZK^{1\theta}H^{\theta-1} \tag{7}$$

By market clearing we know that $K = k_l + k_h$ and $H = \eta_l h_l + \eta_h h_h$, which we can plug into the FOC of the firm. The goods marke clears by Walra's Law, so we don't have to worry about it.

In total we have now two Euler Equations (one for each type) two FOC of the firm and two Budget constraints (again one for each agent). This is a system of 6 equations which will be used to solve for the six unknowns c_l, c_h, h_l, h_h, r, w.

This is solved numerically by the computer for both economies and yields following results

	A	B
c_l	0.48	0.82
c_h	1.475	1
h_l	0.151	0.288
h_h	0.355	0.315
r	0.4	0.38
w	0.597	0.623

Cross checking market clearing in the goods market yields for economy A and for B

b) open economy

For the next part it is assumed that we have capital mobility. That is in addition to the decision problem of the closed economy agents also choose how much capital to supply to the domestic firm and how much to the foreign firm. For this reason another optimality condition is added to for each household problem

$$\eta_l r_l k_l = r_{-l} \tag{8}$$

Since capital can move across borders the domestic capital market clears by

$$K_l = k_{ll} + k_{lh} + \bar{k}_{-ll} - k_{-ll} + \bar{k}_{-lh} + k_{-lh} \tag{9}$$

So now we have four optimality conditions from the household problem in each country, as well as two optimality conditions from the firms problem and two budget constraints. Since there is a union in the capital market we have to solve both economies at the same time, giving us 16 equations in 16 unknowns. This is again done numerically by the computer. Reporting all results is omitted here.

The solution shows however that the interest rate in roughly the same (0.4) in both countries, which was to be expected by a no arbitrage argument. Every agent in the economy supplies to both foreign and domestic firms, but aggregate supply of capital is roughly the same for both economies. As both economies had quit similar values for the interest rate and wage in the closed economy set up, this is not too surprising.

c) Optimal progressive tax

To solve for the optimal tax I adopt the usual Ramsey problem. That is maximizing social welfare given the optimality conditions given by the decentralized solution. Basically the social planner checks which tax scheme generates the socially preferred (or optimal given the welfare function) decentralized outcome. The social planner faces the problem:

$$max \quad \psi \left(\frac{c_l^{1-\sigma}}{1-\sigma} - \kappa \frac{h_l^{1+\frac{1}{v}}}{1+v} \right) + (1-\psi) \left(\frac{c_h^{1-\sigma}}{1-\sigma} - \kappa \frac{h_h^{1+\frac{1}{v}}}{1+v} \right) \tag{10}$$

s.t. all Euler Equations

all Budget Constraints

Firms marginal Products

Market Clearing

Balanced Government Budget

I therefore assume a utilitarian welfare function with arbitrary pareto weights. The balanced government Budget is to be set such that the new, optimal tax scheme yields the same revenue as the old one. The problem can then be solved following the same algorithm as before.