Quantitative Macro Project 2

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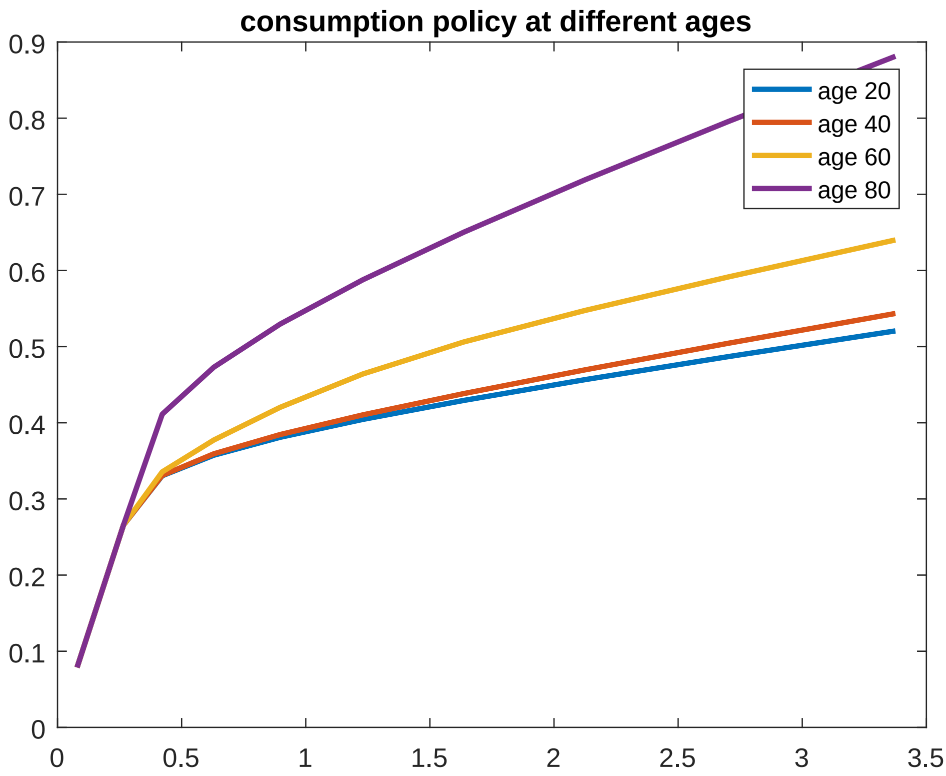
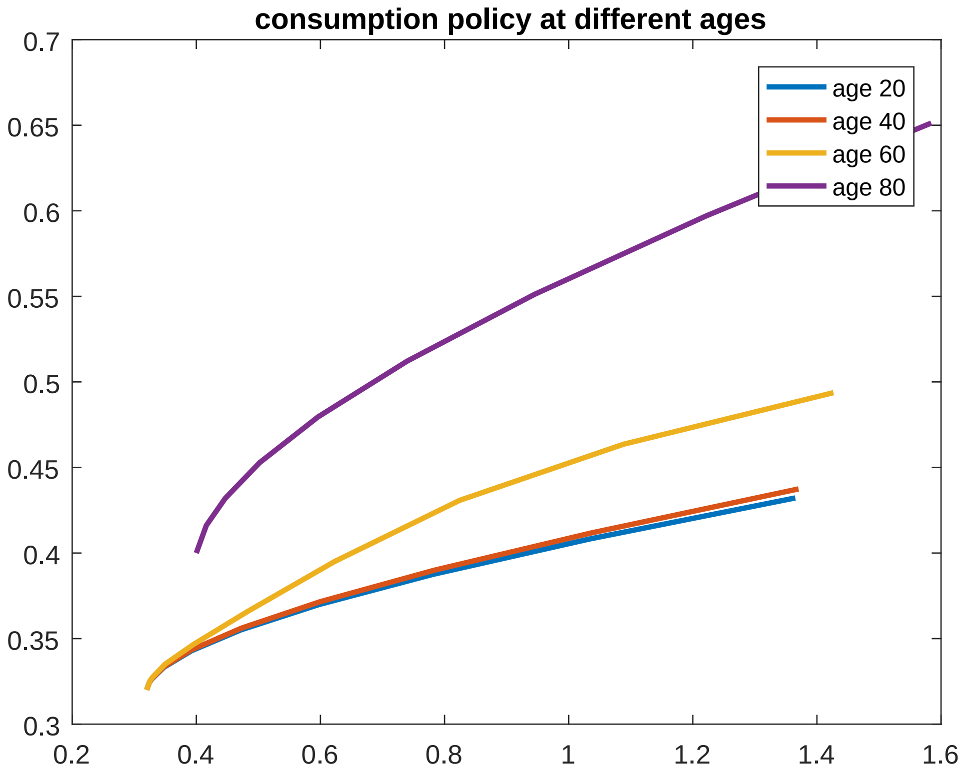
Problem B.1

Introducing an exogenous grid method reduces speed severely. From 7.5 seconds to 33.23 seconds. Reason for this is of course again the need to use root finding for a non linear function, i.e. find consumption from the EE evaluated on the grid on cash on hand.

Plotting the policy function for consumption where the first graph is from my own algorithm with an exogenous grid and the second is from the original code with an endogenous grid. Comparing the two graphs they yield the same policy functions.





Increasing speed

Getting rid of the unnecessary loops and adding the break in the aggregation increases speed and time elapsed decreases by each 0.2 seconds in my implementation.

Problem 2

Different Pension Systems

Embedding the partial equilibrium into a general equilibrium framework: First i leave the retirement income at the exogenous value of 0.04 from the exercise before, in order to compare the partial to the general equilibrium. Afterwards I will implement the pension system. I start with an initial guess for the interest rate of 0.04. After 30 iterations I arrive at an equilibrium interest rate of 0.022.

Now changing to a PAYG pension system: First the replacement rate is zero. This leads to an equilibrium interest rate of only 0.0078. Wage will be 1.58 and capital to output ratio 5.7.

Introducing a replacement rate of 0.6 changes results sizeable. As agents don't have to safe that much any more, interest rates increase to 0.037 in equilibrium. Wages are at 1.29 and capital to output ratio drops 3.7. The latter two outcomes are also in line with theory. Less savings means less aggregate capital and by that a lower marginal product of labour (through the cobb douglas production function) and therefore a lower wage.

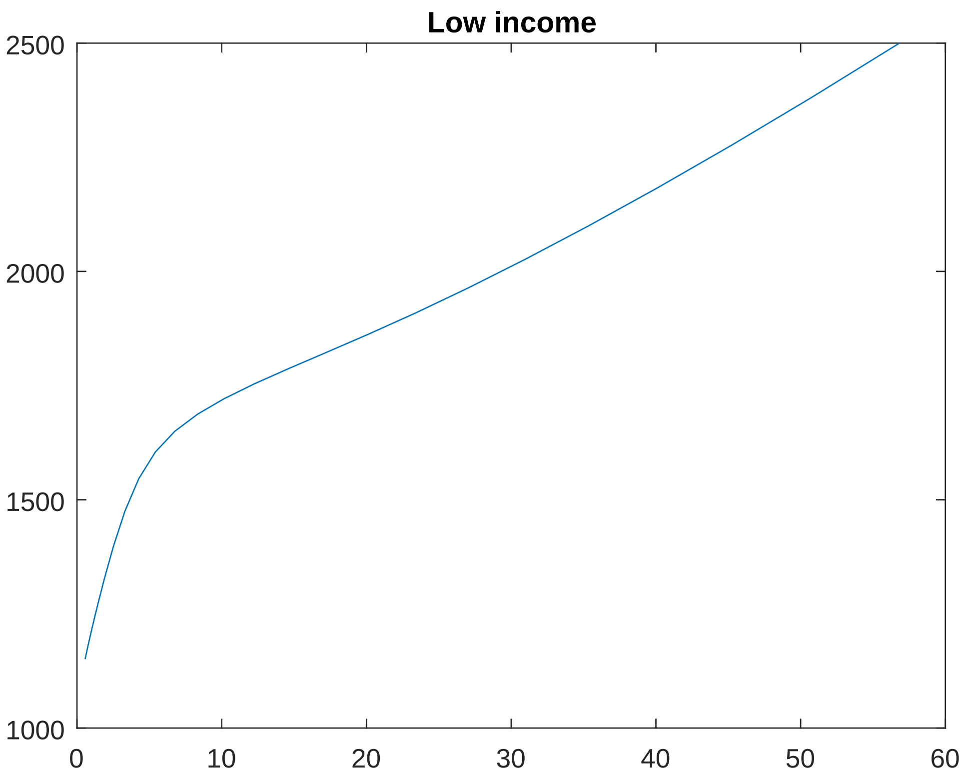
Inefficiency

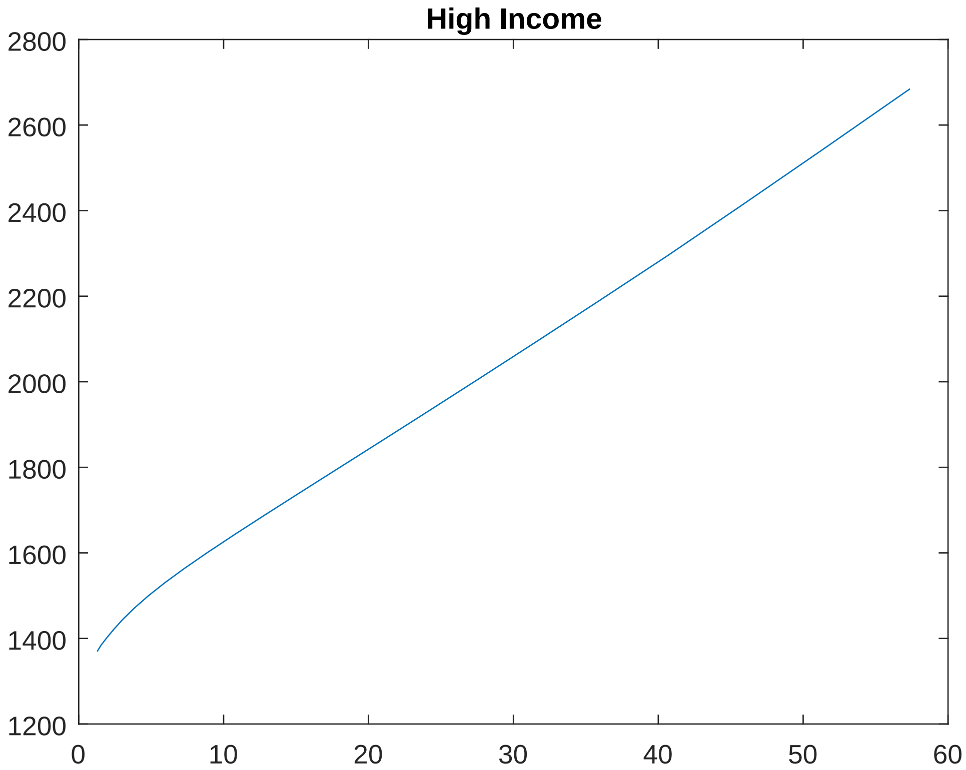
As we have an incomplete markets problem, our outcome will be inefficient since agents over save in comparison to the complete markets equilibrium. This can also be seen since $r \neq \rho$ ($\rho = 0.03 and r = 0.0078$ with no pension). Create pareto efficiency by taxing assets ?? We can all ready suspect that a replacement rate of 0.6 might improve welfare as the inefficiency seems to smaller as the interest rate is closer to the complete markets interest rate of 0.03.

Welfare Comparison

Important to note here is, that we are doing a Steady state comparison. The consumption equivalent variation $g(x,y) = \left[\frac{v_T(x,y)}{v_0(x,y)}\right]^{\frac{1}{1-\theta}}$ between replacement rates 0 and 0.6 is positive for all income states and all values of cash on hand. It is further more increase in income and in cash on hand. So richer agents profit more from the change in policy. This should be the case since the new pension system increases the interest rate and therefore making the assets of the richer households more valuable.

g is between roughly 1500 and 2600, so extremely high. When doing the comparison practically it is important to note that v_0 and v_T live on different grids of cash on hand(since they are endogenous), so I interpolate over v_0 and both grids to make them comparable! g can be plotted as follows:

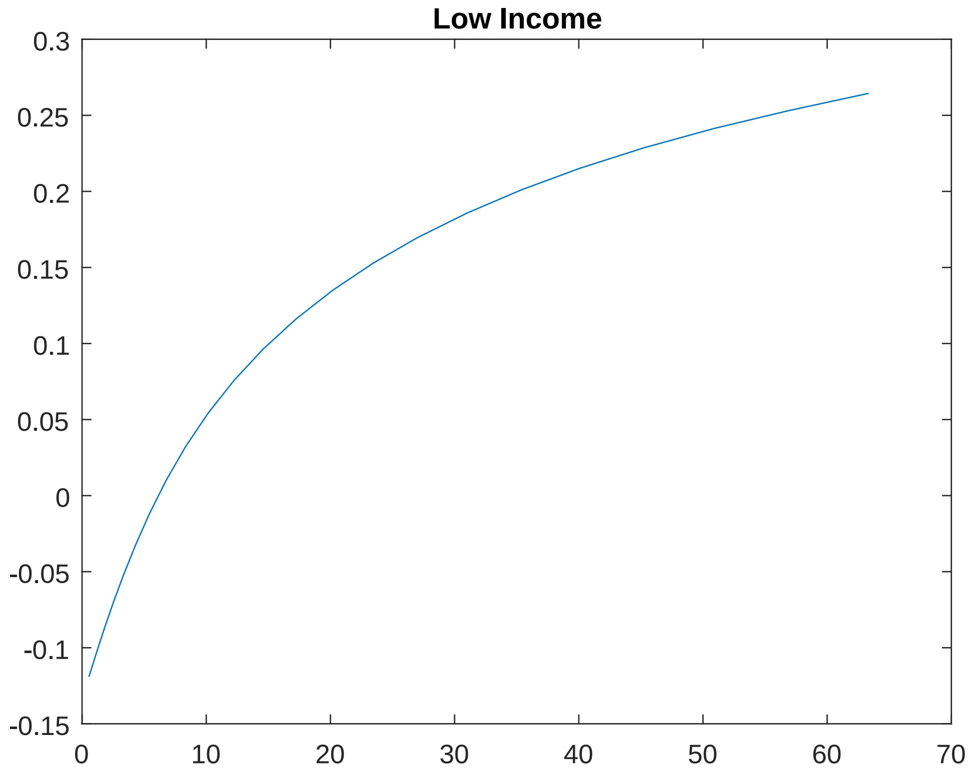


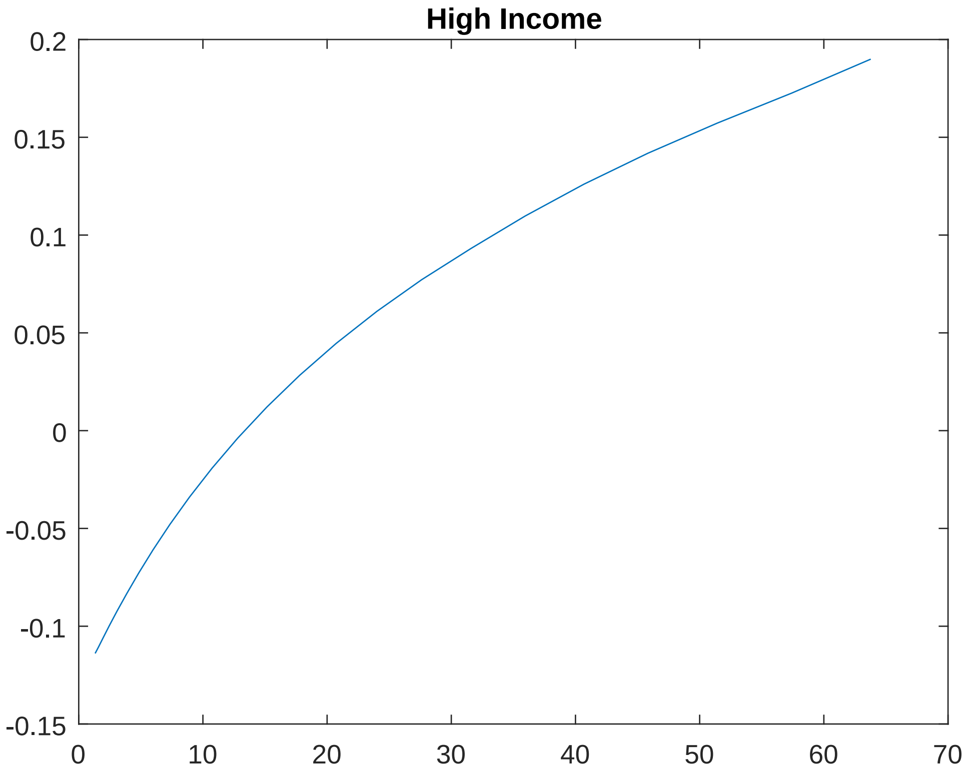


Changes in θ

If I choose $\theta = 1.1$ (almost log utility), equilibrium interest rate in the case of $rr = 0$ increases to $r = 0.025$. In the case of $rr = 0.6$ the interest rate increases to $r = 0.044$. The results make economically sense. As risk aversion decreases precautionary and life cycle savings motives decrease and by that there is less savings and a lower interest rate.

The effect on the consumption equivalent variation is manifold. For both income states and low levels of cash on hand the effect of the policy change on life time utility is actually negative. So households would prefer to save by them self than being "forced" to save through the pension system to this degree. Taxes are simply too high, given the low risk aversion. But again the gains of the new policy increase in cash on hand, so richer households prefer the new policy over the old one. In comparison to $\theta = 2$ the CEV is small, as it is below between -1 and 1 in comparison to more than 1000 before.

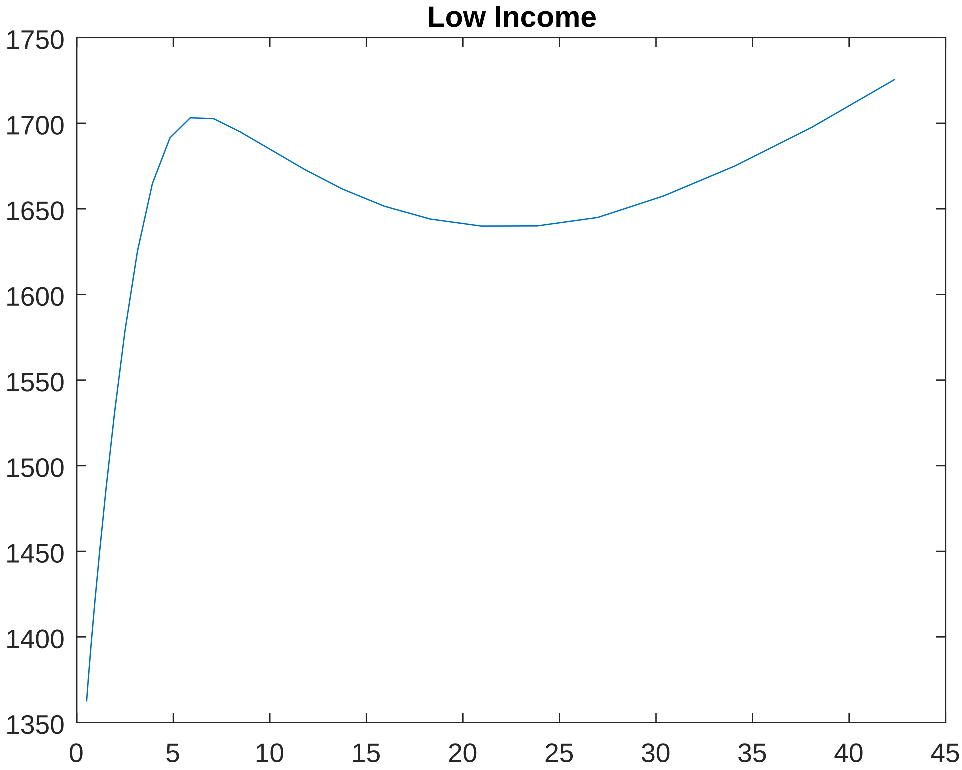


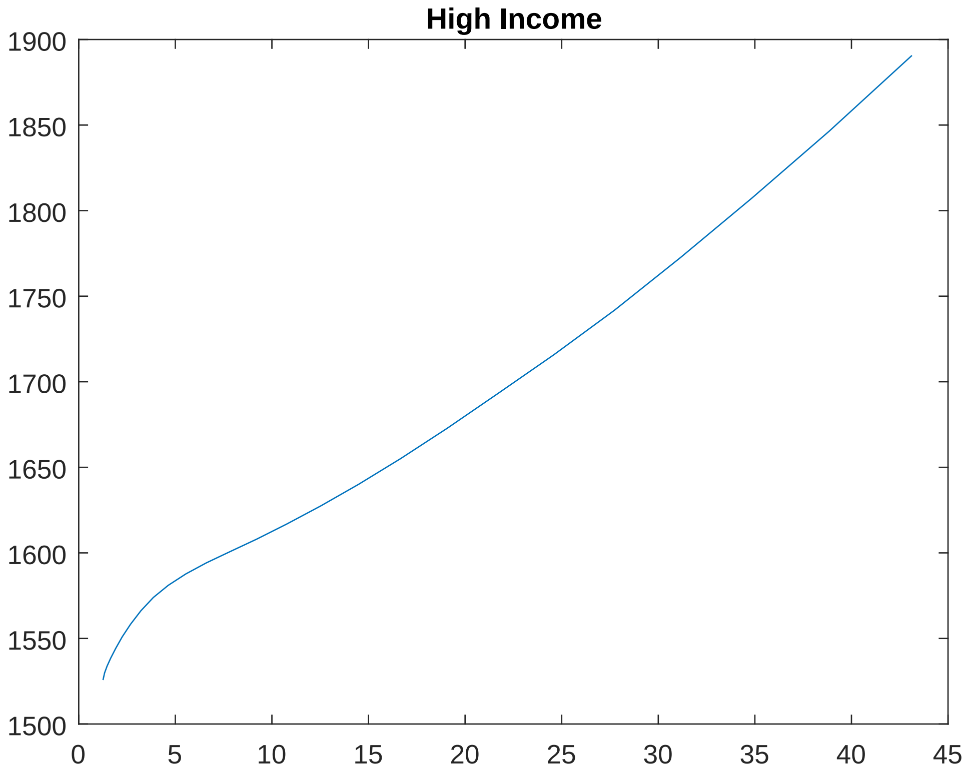


Decomposition

Setting θ back to 2. In order to decompose the effects, I go back to the Partial Equilibrium OLG and implement the new pension system, i.e. $rr = 0.6$ and $\tau = 0.176$. The tau being the tau that balances the budged in GE with $rr = 0.6$. But now the interest rate and the wage are kept constant at the values of the GE with $rr = 0$, so $r = 0.0078$ and $w = 1.58$. The following plots show the results. For poor households the insurance effect is bigger than the overall effect, meaning a negative general equilibrium effect, while for rich households the insurance effect is smaller than the overall effect, meaning a positive general equilibrium effect. This holds for both income states.

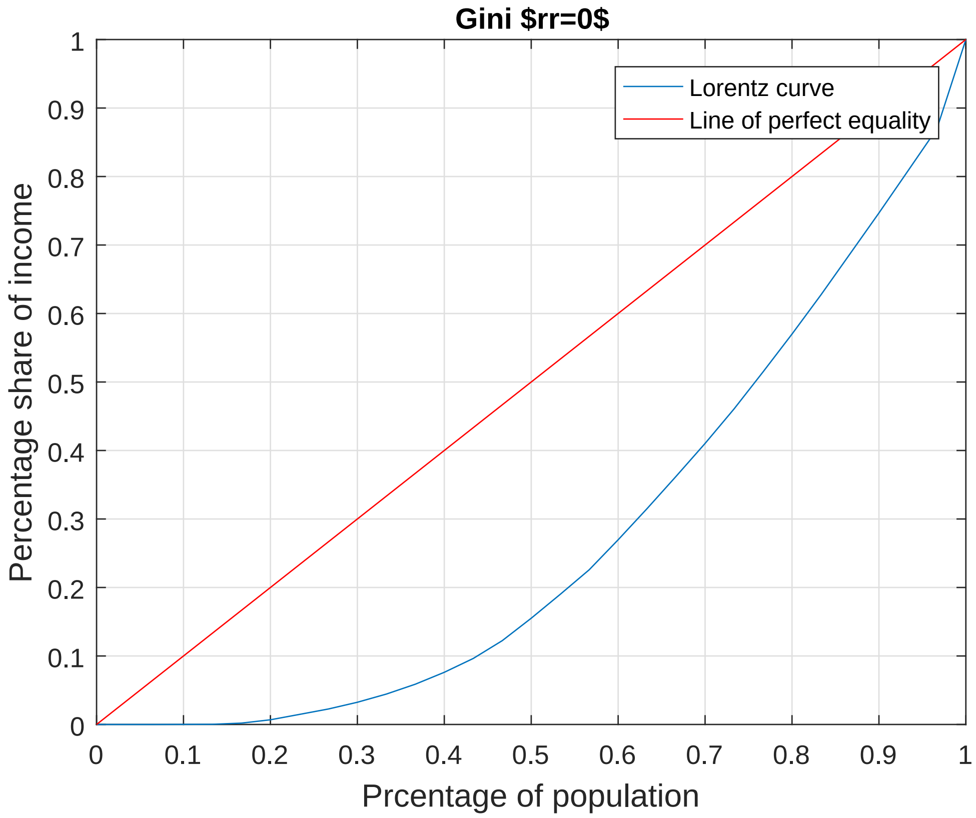
This goes in the same direction as previous findings, poor households seem to suffer from lower wages and do not gain much from a higher interest rate, such that the general equilibrium effect is negative. The insurance effect is still strongly positive ! Rich households on the other hand gain more from higher interest rates than they loose from lower wages, which adds to the also for them existing insurance effect.



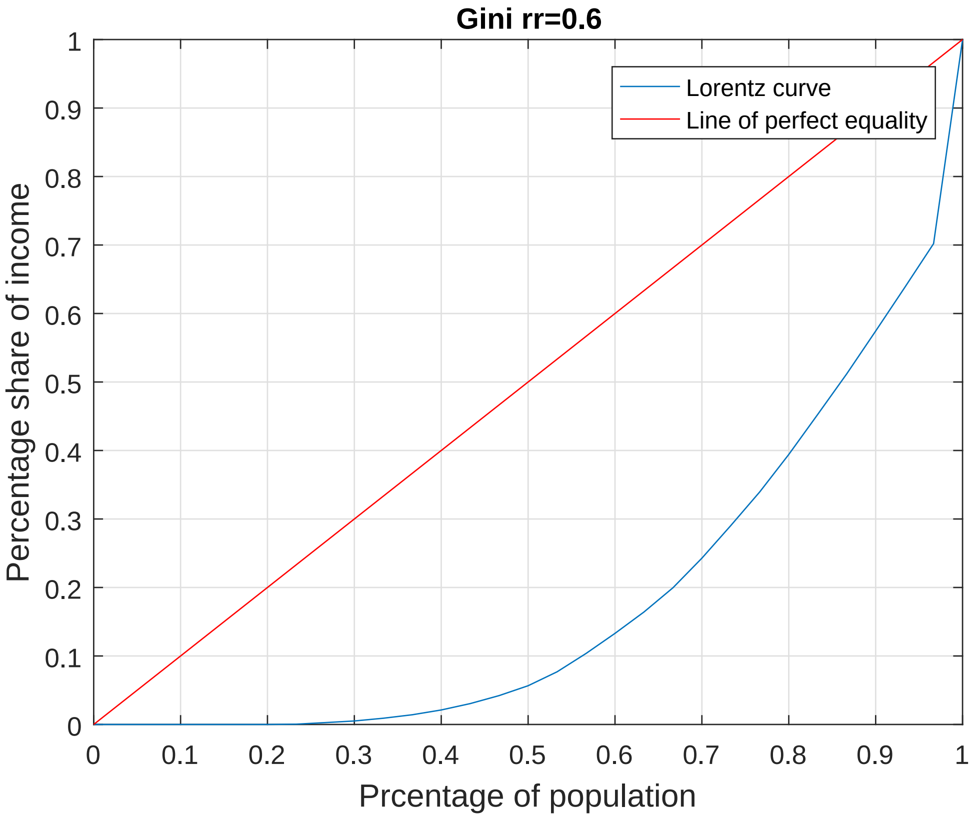


Gini

With no pension system the Gini coefficient of wealth (where i use assets as wealth) is 0.4. Introducing the new pension system increases inequality to a Gini of 0.6. This is in line with the fact that the gains from the new system are higher for households who have initially more assets. The pension system benefits rich households more. This should be again the case since the pension system increases the interest rate, while decreasing wages. We can plot the Lorenz curves as:



Gini $rr=0$
Lorentz curve
Line of perfect equality
Percentage share of income
Prcentage of population
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Compared to real data a gini coefficient of 0.6 is extremely high. I fact by World Bank estimates South Africa has the highest observed Gini coefficient with 0.63. Most central European countries have a gini of below 0.4 as a comparison.

Problem c

Write-Up

1) Start computing the old and the new steady state. We have done that in the exercises before and can use those results.

2) Fix big T. Convergence in OLG models can take many periods, so in

order to solve the model correctly set T high (e.g. 200). In the final solutions kinks the converging variables at the end will indicate that T is too small. (In my attempt of solving the transition I will choose T to be small).

3) Guess a series of interest rates for the whole transition path $(r_t)_{t=1}^{T-1}$. A good first guess is to just guess the interest rate of the new steady state for all $t > 1$.

4) Solve the firm problem given r as before, but now for each period to recover wages in each period.

5) Now solve the household problem backwards in time. The difference to the calculation of the steady state from before is, that r,w and τ are now function of time. That is they are potentially different in age t along the transition. Furthermore we know the set of value functions v_T for each cohort, that is the value function of the new steady state. From there we can solve backwards for each cohort to the old steady state. We also have to be take into account that some cohorts cease to exist as they reach the maximum age along the transition. We receive $(v_t, a_{t+1}, c_t)_{t=1}^{T-1}$

6) Given the policy functions we can aggregate forward in time. That means first get a sequence of $\phi_t(j, a, y)$, where j is age and y is the income shock. We get the sequence by starting with the distribution in the old steady state and then iterating forward using

$$\phi_{t+1} = \int Q(j+1, a, y) d\phi \tag{1}$$

Q is the transition function induced by the markov process and the policy function on assets as before in the steady state calculations.
With this we can sum up the assets forward in time by

$$A' = \int \sum a'(j, a, y) d\phi N_j \tag{2}$$

7) Using the aggregate capital stock update the sequence of r from the firm model

8) Check for convergence in r, if they have not converged use some weighted average of the old and new guess for r to continue with step 4

My Results