## Proof by Contrapositive (2.1)

An implication statement and its contrapositive are equivalent. This means that when it comes to proofs, we can either perform our proof on $p \rightarrow q$ or on $\neg q \rightarrow \neg p$. In some situations, a proof by contrapositive might be easier than a proof of the original statement.

Example: *Informally prove the statement, "For all integers n, if $n^2$ is even, then n is even."*

- Original hypothesis: $n^2$ is even
- Original conclusion: $n$ is even.
- Contrapositive hypothesis: $n$ is odd.
- Contrapositive conclusion: $n^2$ is odd.

Contrapositive: If $n$ is odd, then $n^2$ is odd.

- $n^2=(2k+1)^2$ … $n^2=4k^2+4k+1$ … $n^2=2\cdot(2k^2+2k)+1$ << PROOF

## Proof by Contradiction (2.5)

When we are working with a **direct proof** or with a **proof by contrapositive**, we can think of these as proving that **there cannot possibly be a counterexample to the theorem.**

In order to build a counterexample, it must follow these rules:
- It must make the hypothesis of the implication true.
- It must make the conclusion of the implication false.

For a direct proof, we are showing that if the you select an element that satisfies the first property, that element cannot satisfy the second property.

For a proof-by-contrapositive, we are showing that if you select a property that satisfies the second property, then that element cannot satisfy the first property.

Therefore for both of these – no counterexample is possible.

For **proof by contradiction**, we work by assuming that we *have* found some counterexample (so we assume the hypothesis is true and the conclusion is false). We then show that with this, after crunching the numbers we would end up with a contradiction – so that we show that the statement (the counterexample) is always false and therefore showing that the first and second properties are logically incompatible.

| If $n$ is an odd integer, then $n^2+n$ is even. |
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| • Assume we've found a counterexample:<br>  ○ Hypothesis is *n is an odd integer*, $n=2k+1$<br>  ○ Conclusion is $n^2+n$ *is odd*. $n^2+n=2j+1$<br>• $2j+1=(2k+1)^2+(2k+1)$<br>• $2j+1=4k^2+6k+2$<br>• $2j-4k^2-6k=2-1$<br>• $2(j-2k^2-3k)=1$<br>• $j-2k^2-3k=\frac{1}{2}$<br><br>Since ***j*** and ***k*** are integers, we know that $j-2k^2-3k$ is an integer. In our proof, we end up with ½, which is *not* an integer, and therefore the statement is invalid. |
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