Group Policy: You may collaborate with classmates on this assignment and turn in as a team or solo. For a team, each person should be working out the problems, then discuss to decide what to put on the single turn-in to represent the entire team.

Turn-In: Turn in one copy of this exercise per team.

Team Members:

Goals:

1. Learn the definition of a relation and a function.
2. Be able to recognize relations that are also functions.
3. Be able to build an arrow diagram for a relation.s

Section 1: Terminology of Functions

<u>Definition of a Function:</u> The notation $f : A \to B$ is used for a function, simply called *f*, with a set of inputs *A* (called the *domain*), and a set B (called the *codomain*) that includes all the *outputs*. The function *f* associates with each input in *A* one and only one output in *B*.

$f : A \to B$ is read "*f* is a function from A to B".

<u>Example:</u> Suppose $f : \mathbb{N} \to \mathbb{N}$ is defined by the rule $f(x) = 2x + 1$. We can think of this as meaning, "Given an input $x \in \mathbb{N}$, *f* maps *x* to the output value $2x + 1 \in \mathbb{N}$." Is every element of the codomain an output of one and only one input into the function?

<u>Solution:</u> No, there are codomain elements like $0 \in \mathbb{N}$ that are not the value of *f* at any input value $a \in \mathbb{N}$ – that is, there is no $a \in \mathbb{N}$ for which $2a + 1 = 0$. This does not affect the fact that *f* is a function.

1. Suppose $f : \mathbb{Z} \to \mathbb{Z}$ is defined by the rule $f(x) = x^2$. We think of this as meaning, "Given an input $x \in \mathbb{Z}$, the value of *f* at *x* is $x^2 \in \mathbb{Z}$." Is every element of the codomain an output of one and only one input into the function? **(___/3)**

a. Is $f(x) = x^2$ a function?

b. Are there any two *x* values that end up resulting in the same *f(x)* value?

c. Therefore, *is every element of the codomain an output of <u>one and only one</u> input into the function?*

To completely describe a function, we must do four things:

1. **Give the function a name. *f*, *g*, and *h* are popular names for functions, but it's always okay to be creative and descriptive.**
2. **Describe the domain.**
3. **Describe the codomain.**
4. **Describe the rule.**

2. Name: *f*. Domain: {1, 2, 3, 4, 5}. Codomain: $\mathbb{N}$. Rule: To each number in the domain, associate the square of the number. (___/3)

a. What would the algebraic form of the rule be?

b. List all outputs for each input (from the domain).

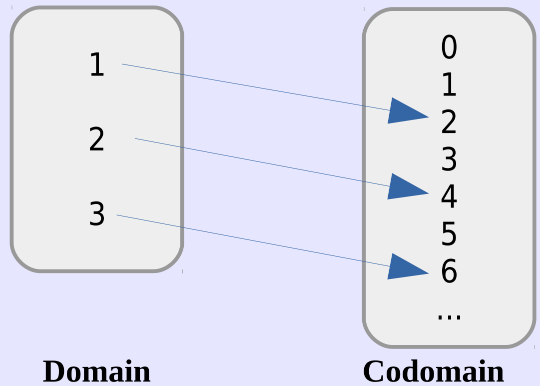
c. Finish the arrow diagram by drawing arrows from the *input* to the *output*:

1 2 3 4 5

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 …

Arrow diagrams list all the elements of the domain and codomain, with arrows pointing from each input, going to each output.

Example: **Function *f*. Domain: {1, 2, 3}.**
Codomain: $\mathbb{N}$, Rule: $2k$.



3. Draw the arrow diagrams for the following functions:

a. Let *f* be the function with the domain {a, b, c} and codomain {1, 2, 3} defined by the set of ordered pairs: { (a, 2), (b, 3), (c, 1) }. **(___/1)**

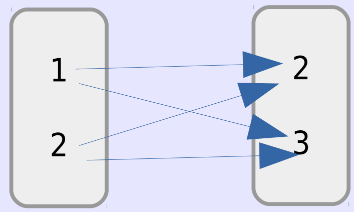
b. Let S = { a, b, c}, and consider the function $n : \wp(S) \rightarrow \{0,1,2,3\}$, where $n(A)$ is the number of elements in the set A. **(___/1)**

*First: What is the **domain?*** *Second: What is the **codomain?***
*Third: What are all the **sizes** of the sets in the power set of S? (This is the relation!)*

Section 2: Binary Relations

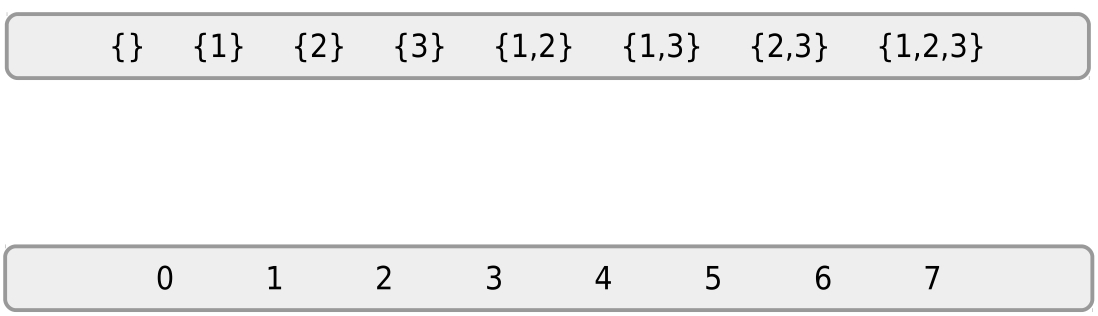
<u>Definition of a Binary Relation:</u> A *binary relation R* consists of three components: a domain *A*, a codomain *B*, and the subset of $A \times B$ called the "rule" for the relation. When we say "a relation between A and B", A is the domain and B is the codomain of R.

<u>Example:</u> Domain A = {1, 2}, Codomain B = {2, 3}, Rule: L = { (1, 2), (1, 3), (2, 2), (2, 3) }.

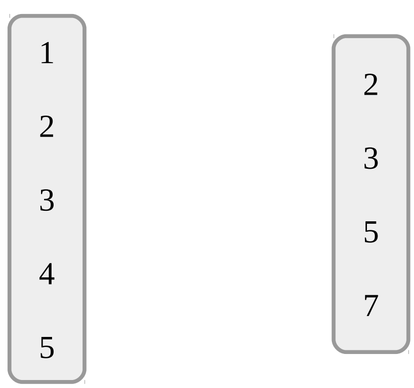


4. Draw the two-set arrow diagram for each relation R described:

a. Domain: The power set, $\wp(\{1,2,3\})$. **(___/2)**
Codomain: The set B = { 0, 1, 2, 3, 4, 5, 6, 7 }.
Rule: $(S,n) \in R$ means that *n* is the **sum** of the elements in S.



b. Domain: A = { 1, 2, 3, 4, 5 } **(___/2)**
Codomain: B = { 2, 3, 5, 7 }
Rule: L = { (1, 2), (1, 3), (1, 5), (1, 7),
(2, 3), (2, 5), (2, 7),
(3, 5), (3, 7),
(4, 5), (4, 7),
(5, 7) }



<u>Definition of a Function that is a Binary Relation:</u> A *function F* from *A to B* is a binary relation with the domain *A* and the codomain *B* with the property that for every $x \in A$, there is exactly one element $y \in B$ for which $(x, y) \in F$.

5. Which of the following relations are actually functions? For each relation that is not a function, give a specific way in which it violates the definition of a function.

a. Let A = { 1, 4, 9, 16, 25 }, and define the relation R from *A* to $\mathbb{N}$ with the rule $(x, n) \in R$ if $n^2 = x$. **(___/1)**

*First: What is the **domain**?*

*Second: What is the **codomain?***

Third: For each input from A, is the output (according to the rule) in $\mathbb{N}$?

Fourth: Does each input in the domain have <u>one and only one</u> output in the codomain?

Therefore, is this relation also a function?

b. Let R be the relation whose domain is { 1, 2, 3, 4, 5 } and whose codomain is $\mathbb{N}$, and whose rule is given by R = { (1, 12), (2, 4), (3, 4), (2, 9), (5, 25) }. **(___/1)**

*First: What is the **domain**?*

*Second: What is the **codomain**?*

Third: For each input from A, is the output (according to the rule) in $\mathbb{N}$?

Fourth: Does each input in the domain have <u>one and only one</u> output in the codomain?

Therefore, is this relation also a function?