Section 1: Element-Wise Proofs (Chapter 3.3)

When we're interested in proving that one set is a subset of another set, we use an element-wise proof. For discrete sets, this is easy:

For sets B = { 2, 4, 6, 8, 10 } and C = { 2, 4 }, we can see that C is a subset of B by checking every element of C: (1) $2 \in B$ **, (2)** $4 \in B$ **.**

1. Prove the following in a systematic way using an Element-Wise Proof

a. Prove: The set A = { q, w, e, r, t, y } is a subset of the set Z = { a, b, c, …, x, y, z }. **(___/1)**
Check off which of the following is true.

1. $q \in Z\,?$
2. $w \in Z\,?$
3. $e \in Z\,?$
4. $r \in Z\,?$
5. $t \in Z\,?$
6. $y \in Z\,?$

b. Prove: The set A = { 2, 4, 6, 8 } is a subset of the set $Z = \{2k : k \in \mathbb{N}\}$ **(___/1)**
Check off which of the following is true.

1. $2 \in Z\,?$
2. $4 \in Z\,?$
3. $6 \in Z\,?$
4. $8 \in Z\,?$

For sets where we cannot possibly check every element of a set against another set, we have to approach things a little differently. Let's approach this a little more generically…

Let A be the set { 0, 10, 20, 30, 40, … } and let B be the set { …, -6, -4, -2, 0, 2, 4, 6, … }. In other words, $A = \{10k : k \in \mathbb{N}\}$ **and** $B = \{k \in \mathbb{Z} : k \text{ is even}\}$ **.**

These sets are infinite, so we have to prove that If $x \in A$ **, then** $x \in B$ **.**

1. **Hypothesis:** $x \in A$ **, so we need to select some *x* such that this is true. The easy way to go here is to say** $x = 10k$ **.**
2. **Conclusion:** $x \in B$ **, or "x is even".**
3. **Another way to phrase this would be that "10k is even".**
4. **Rewrite:** $2(5k)$ **. By the definition of an even integer, we have proven that A is a subset of B.**

2. Prove the following (by rewriting one element's equation as the other element's equation.)

a. **Let there be the sets:** $A = \{4m : m \in \mathbb{Z}\}$ *and* $B = \{2n : n \in \mathbb{Z}\}$ **(___/2)**
Prove that $A \subseteq B$ **.**

1. Hypothesis: If $x \in A$, then $x \in B$.

b. **Let there be the sets:** $A = \{2(m+1) : m \in \mathbb{Z}\}$ *and* $B = \{2n+2 : n \in \mathbb{Z}\}$ **(___/2)**
Prove that $A = B$ **.**

(Hint: We can clearly see that they are the same, but to prove it, we need to do two proofs – $A \subseteq B$ *and* $B \subseteq A$ *!)*