Chapter 4: Functions and Relations (40%)

Calculating function compositions

(5%)

For the given functions, find the following:

f(x)=2x-1 And g(x)=3x, with the domain and codomain both being $\mathbb Z$.

Composition arrow diagrams

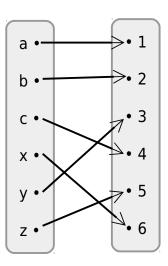
(5%)

Fill in the missing arrow diagram.

f

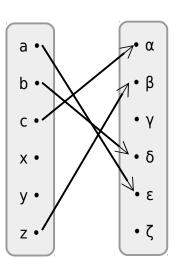
g

 $g \circ f$



1 · 2 · 3 · 4 · 5 · 6 ·

α
β
γ
δ
ε
ζ



Function properties (5%)

Fill in the blank for the following statements.

_____ (a) A function is ______ if every point in the codomain has an arrow ending at that point.

_____ (b) A function is ______ if no point in the codomain has two or more arrows ending at that point.

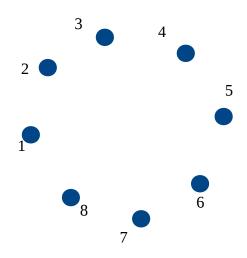
_____ (c) A function is ______ if every point in the codomain has exactly one arrow ending at that point.

Binary relations (10%)

Complete the arrow diagram for each of the relations on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and decide if it has any reflexive, antisymmetric, or transitive properties. For each property that a relation does not have, illustrate this failure with a specific example.

$$R_1 = \{ (1, 1), (1, 2), (1, 4), (1, 8), (2, 2), (2, 4), (2, 8), (3, 3), (3, 6), (4, 4), (4, 8), (5, 5), (6, 6), (7, 7), (8, 8) \}$$

Is this transitive, antisymmetric, and/or reflexive?



Equivalence relations

(15%)

Let $S = \{1, 2, 3\}$. For each of the following relations on $\wp(S)$, draw the arrow diagram and decide if the relation is reflexive, symmetric, or transitive. If it is all three (i.e., an equivalence relation), give the corresponding partition of $\wp(S)$.

$$R_5 = \{(A, B) \in \wp(S) \times \wp(S) : n(A) = n(B)\}$$

