Section 1: Equivalence Relations

Symmetry	A relation R *on set* A is said to be *symmetric* if for all $a, b \in A$, if $(a, b) \in R$ then $(b, a) \in R$. In terms of arrow diagrams, a symmetric relation has the property that every pair of nodes connected by an arrow is actually connected by two arrows, one in each direction.

1. For each of the following relations on $\mathbb{Z}$, decide if the relation is symmetric. If it is not, give a specific example to illustrate this. (Question 2 from the homework.)

a) $R_1 = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a + b \text{ is even}\}$ **(___/1)**

If a + b is even, then is b + a also even?
Therefore, is this relation symmetric?

b) $R_2 = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a + b \text{ is odd}\}$ **(___/1)**

If a + b is odd, then is b + a also odd?
Therefore, is this relation symmetric?

c) $R_3 = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a + 2b \text{ is even}\}$ **(___/1)**

For (a, b), a + 2b is even, is the result of (b, a), b + 2a also even?

a	**b**	**Result: a + 2b. Is it even?**	**Result: b + 2a. Is it even?**
1	1	3, no, not in relation R_3	3, no, not in relation R_3
2	2	2 + 2(2) = 6. Is in R_3 .	2 + 2(2) = 6. Is in R_3 .
4	2		
2	1		

Therefore, is the relation symmetric?

Partitions	For a set A, a *partition of* A is a set S = { $S_{1,} S_{2,} S_{3,} \ldots$ } of subsets of A (each set S_i is called *a part of* S) such that: 1. For all i, $S_i \neq \emptyset$. That is, each part is nonempty. 2. For all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$. That is, different parts have nothing in common. 3. $S_1 \cup S_2 \cup S_3 \cup \ldots = A$. That is, every element in A is in some part.

2. Which of the following are partitions of the set A = {1, 2, 3, 4, 5, 6, 7, 8}? For each that is not, explain why not. (Question 4 from the homework)

a) { 1, 2, {3, 4, 5}, {6, 7, 8} } **(___/1)**

b) { {1, 5}, {6, 7, 2}, {4 3, 5}, {8} } **(___/1)**

c) { {1, 4}, {6, 8, 2}, {3, 5}, {7} } **(___/1)**

d) { {1, 8}, {4, 3, 5}, {7, 2} } **(___/1)**

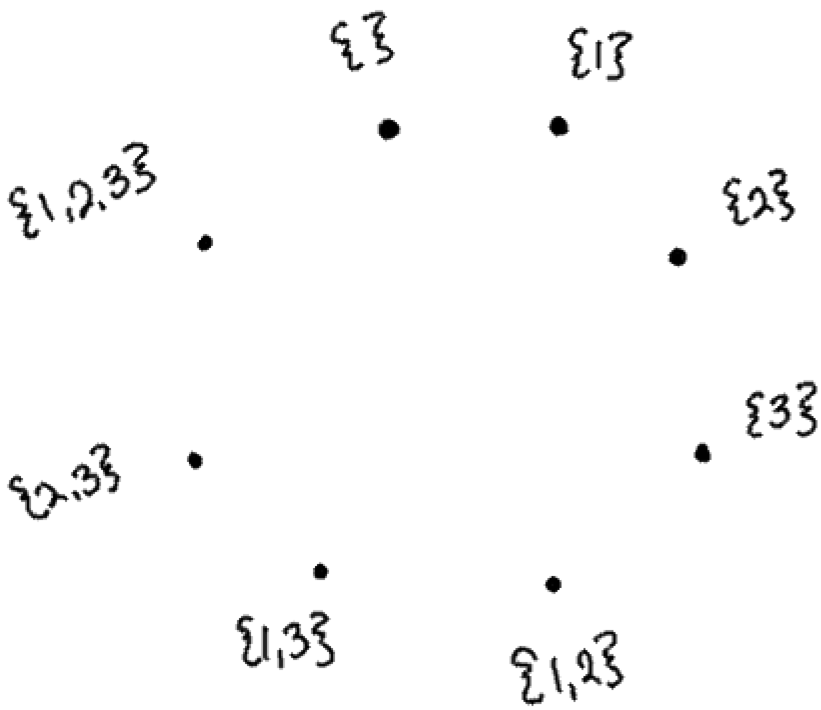
Let *R* be a binary relation on a set *A*.
Remember that a relation can be Reflexive, Irreflexive, or Neither, and it can be Symmetric, Antisymmetric, or Neither.

Reflexive / Irreflexive	*R* is said to be ***reflexive*** if $(a,a)\in R$ for all $a\in A$. On an arrow diagram, this means that every node has a loop. A relation *R* on *A* is ***irreflexive*** if for all $a\in A$, $(a,a)\notin R$. On an arrow diagram, this means no loops.
Symmetric / Antisymmetric	A relation *R on set A* is said to be ***symmetric*** if for all $a,b\in A$, if $(a,b)\in R$ then $(b,a)\in R$ On an arrow diagram, the arrow goes in two directions. A relation *R* is called ***antisymmetric*** if for all $a,b\in A$, if $a\neq b$ and $(a,b)\in R$, then $(b,a)\notin R$ On an arrow diagram, the arrow goes only in one direction.
Transitive	A relation *R* is called ***transitive*** if whenever $(a,b)\in R$ and $(b,c)\in R$, it must also be the case that $(a,c)\in R$ On an arrow diagram, if you can follow two arrows to get from a node *a* to *c*, and you can also get there along a single arrow.

3. Let S = {1, 2, 3}. For each of the following relations on $\wp(S)$, draw the arrow diagram and decide if the relation is reflexive, symmetric, or transitive. If it is all three (i.e., an equivalence relation), give the corresponding partition of $\wp(S)$. (Question 8 from the homework)

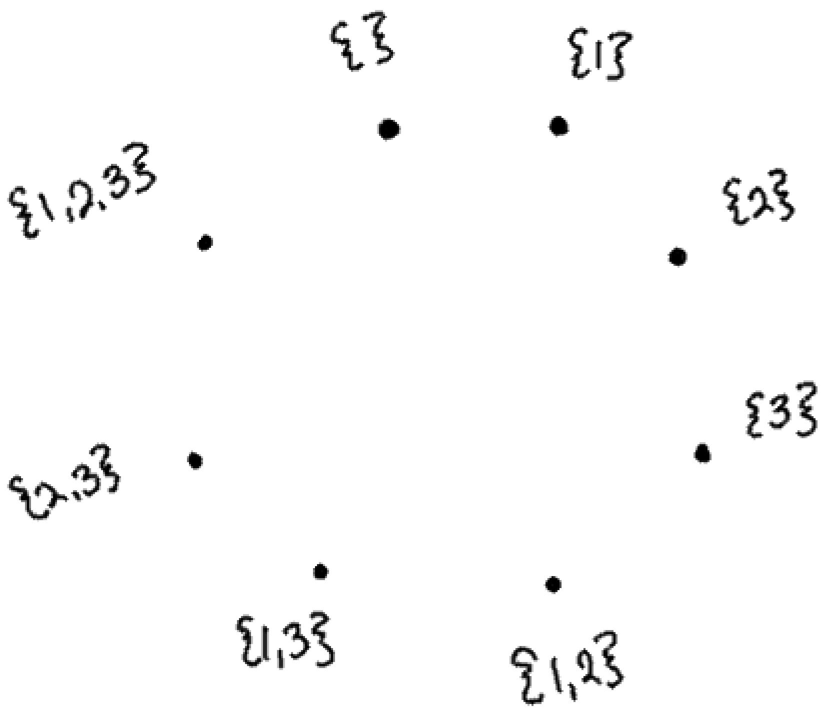
a) $R_1 = \{(A, B) \in \wp(S) \times \wp(S) : A \subseteq B\}$ **(___/2)**

Reflexive/Irreflexive? Symmetric/Antisymmetric? Transitive?



c) $R_3 = \{(A, B) \in \wp(S) \times \wp(S) : A \cap B = \emptyset\}$ **(___/2)**

Reflexive/Irreflexive? Symmetric/Antisymmetric? Transitive?



e) $R_5 = \{(A, B) \in \wp(S) \times \wp(S) : n(A) = n(B)\}$ **(___/2)**

Reflexive/Irreflexive? Symmetric/Antisymmetric? Transitive?