Section 1: Sets

The following are common sets we will see in this chapter:	
$\mathbb{N}$ **: The set of natural numbers**	These are numbers that can answer counting problems. ($\mathbb{N} = 0, 1, 2, 3, \ldots$)
$\mathbb{Z}$ **: The set of integers**	($\mathbb{Z} = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$)
$\mathbb{Q}$ **: The set of rational numbers**	These are characterized as ratios of integers such as $1/2$, $-17/4$, or $3/1$
$\mathbb{R}$ **: The set of real numbers**	These can be thought of as decimal numbers with possibly unending strings of digits after the decimal point.

1. Match numbers to sets **(___/3)**
For the following numbers, which set(s) do they belong to?

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
10				
-5				
12 / 6				
π				
2.40				

2. List three numbers that are in the set of integers $\mathbb{Z}$, but not in the set of natural numbers $\mathbb{N}$. **(___/1)**

3. List three numbers that are in the set of rational numbers $\mathbb{Q}$, but not in the set of integers $\mathbb{Z}$. **(___/1)**

4. List three numbers that are in the set of real numbers $\mathbb{R}$, but not in the set of rational numbers $\mathbb{Q}$. **(___/1)**

Section 2: Subsets

Subsets and existence within sets:	
x exists in A	The notation $x \in A$ means "x is an element of A", which means that x is one of the members of set A.
A is a subset of B	A is a subset of B (written as $A \subseteq B$) if every element in A is also an element in B. Formally, this means that for every *x*, if $x \in A$, then $x \in B$.
A is equal to B	A is equal to B (simply written A = B) means that A and B have exactly the same members. This is expressed formally by saying, " $A \subseteq B$ and $B \subseteq A$ ".
Empty set	A set that contains no elements is called an *empty set*, and is denoted by { } or $\emptyset$.
The universal set	For any given discussion, all the sets will be subsets of a larger set called the *universal set* or *universe*, for short. We commonly use the letter U to denote this set.

5. Given these sets, **(___/9)**

U = { 1, 2, 3, 4, 5, 6 } **A = { 1, 1, 2, 2, 2, 4, 4 }**

B = { 2, 2 } **C = { 1, 2, 4, 5, 6 }**

D = { 1, 4 } **E = { 6, 5, 4, 2, 1 }**

a) Which are the true statements?

1. $B \subseteq C$
2. $D \subseteq A$
3. $U \subseteq \emptyset$
4. $C \subseteq U$
5. $B \subseteq B$
6. $U \subseteq \mathbb{N}$
7. $B \subseteq \mathbb{Z}$
8. $B \subseteq D$
9. $E \subseteq D$

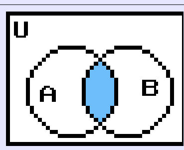
b) Fill in the blanks with either $\subseteq$ (is a subset of), $\not\subseteq$ (not a subset of), or = (equal to) for the following: **(___/6)**

1. D ____ U
2. D ____ C
3. D ____ B
4. C ____ E
5. E ____ U
6. E ____ D

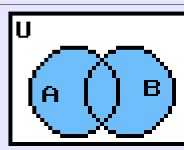
Section 3: Intersections, unions, and differences

The intersection of A and B $A \cap B$	Is the set that contains those elements common to both A and B. In set-builder notation, we write: $A \cap B = \{ x \in U : x \in A \wedge x \in B \}$
The union of A and B $A \cup B$	Is the set that contains those elements in either set A or B. In set-builder notation, we write: $A \cup B = \{ x \in U : x \in A \vee x \in B \}$
The difference of A and B $A - B$	Is the set that contains those elements in A which are not in B. In set-builder notation, we write: $A - B = \{ x \in U : x \in A \wedge x \notin B \}$
Disjoint	Sets A and B are *disjoint* if $A \cap B = \emptyset$
Complement A'	Given a set A with elements from the universe U, the complement of A (written A') is the set that contains those elements of the universal set U which are not in A. That is, $A' = U - A$.

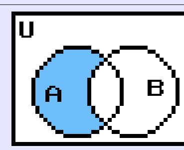
Venn diagrams are used to visually represent relationships between sets. Set A and Set B (or more) are drawn as overlapping, and the shaded-in region is the resulting *set* based on any *intersections, unions, complements* or *differences.*



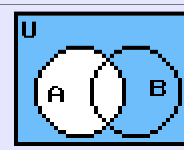
$A \cap B$



$A \cup B$



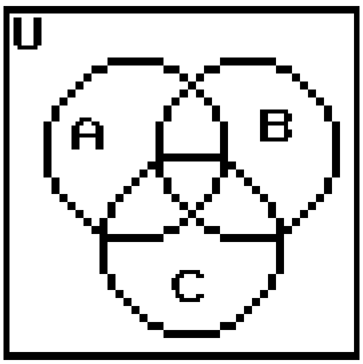
$A - B$



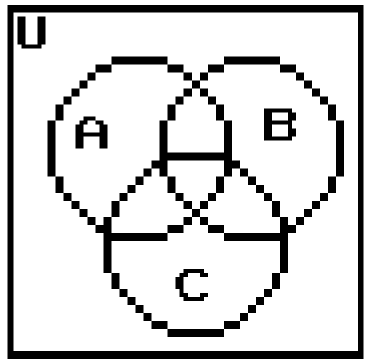
A'

6. Color in the following Venn diagrams to match the statements: **(___/9)**

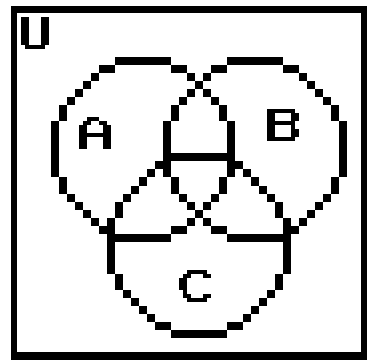
a) $A \cap B$



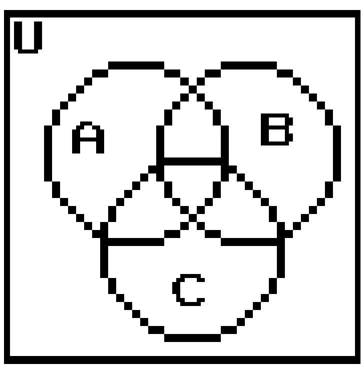
b) $A \cap C$



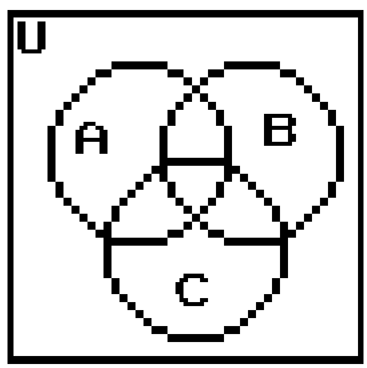
c) $(A \cap B) \cup (A \cap C)$



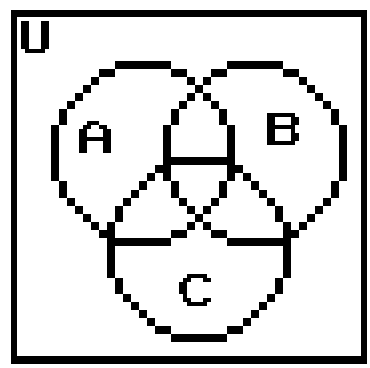
d) $B \cup C$



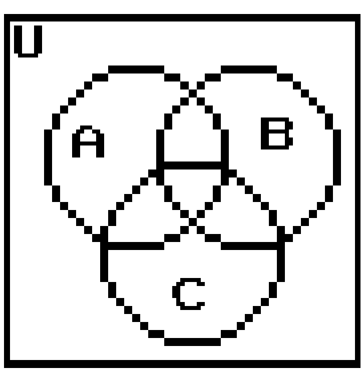
e) $A \cap (B \cup C)$



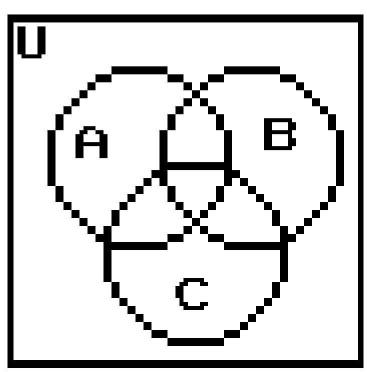
f) $B - C$



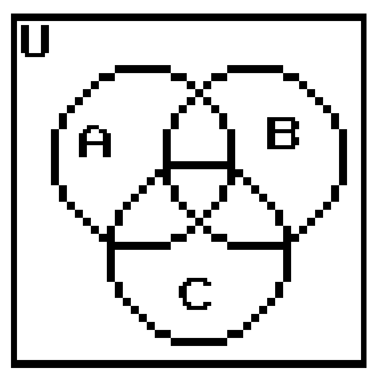
g) B'



h) $(A \cup B) - B$



i) $A \cup (B - C)$



7. Given the following sets, verify each statement.

U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 } **A = { 1, 3, 5 }** **B = { 1, 2, 3, 4 }** **C = { 1, 2, 5, 6, 10 }**

It might help to write out each subset (e.g., $B \cup C$ ***) as you go.***

a) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(___/1)**

b) Verify $(A \cup B)' = A' \cap B'$ **(___/1)**

c) Verify $A \cap (A \cup B) = A$ **(___/1)**

Section 4: Set-builder notation

It is impractical to try to list every element of a set. We use *set-builder notation* to describe most sets. There are two different forms of set-builder notation:

A Property Description is of the form, “The set of all x in U such that x is _____”. The blank is some *property* of x, which determines whether an element of U is or is not in the set.

- The set of even integers: $\{ x \in \mathbb{Z} : x = 2y \text{ for some } y \in \mathbb{Z} \}$
- The set of real numbers bigger than 10: $\{ x \in \mathbb{R} : x > 10 \}$

A Form Description is of the form, “All numbers of the form ____, where x is in set D”, where the first part will be some equation (like “2x” for even).

- The set of integers that are multiples of 3: $\{ 3k : k \in \mathbb{Z} \}$
- The set of perfect square integers: $\{ m^2 : m \in \mathbb{N} \}$, or $\{ m^2 : m \in \mathbb{Z} \}$

8. Write the following statement in form description and property description set-notation:

a) The set of all odd integers. **(___/2)**

1. Let’s use x as our variable.

2. What set does it belong in? $x \in$ _____________
(This is LEFT OF THE : for prop description and RIGHT OF THE : for form description.)

3. In English, what is x? x is _______________
(This is RIGHT OF THE : for prop description)

4. How do you write an odd integer, using x as the variable:
(This is LEFT OF THE : for form description.)

Property Description:
({ set : property })

Form Description:
({ form : set })

b) The set of all even integers. **(___/2)**

Property Description:

Form Description:

9. Convert the following from property description to form description:

a) { $x \in \mathbb{N}$: *x* is twice a perfect square } **(___/2)**

We will be writing the form description with relation to *y*, rather than *x*.

1. If *y* is our variable to-be-squared, how do you write x in terms of y? (x is twice a perfect square)
2. If the variable *y* is a square-root of a perfect square, which set must it be part of? ($\mathbb{N}, \mathbb{Q}, \mathbb{R}, \mathbb{Z}$)
3. Write the equation (without x, in terms of y) as the left-hand side, and the set that y belongs in as the right-hand side:

b) { $x \in \mathbb{Q}$: $x = 2^m$ for some $m \in \mathbb{Z}$ } **(___/2)**

We will be writing the form description with relation to *m*, rather than *x*.

1. What is the equation for *x*?
2. What is the set that *m* exists in?
3. Write the equation (without x, in terms of m) as the left-hand side, and the set that m belongs in as the right-hand side:

c) { $x \in \mathbb{Z}$: x is the product of two consecutive integers } **(___/2)**

We will be writing the form description with relation to *z*, rather than *x*.

1. Write “x is the product of two consecutive integers”, using *z* as the variable for the integer (and z+1) for its next item.
2. What set does *z* belong in?
3. Write the equation (without x, in terms of z) as the left-hand side, and the set that z belongs in as the right-hand side: