

Group Policy: You may collaborate with classmates on this assignment and turn in as a team or solo. For a team, each person should be working out the problems, then discuss to decide what to put on the single turn-in to represent the entire team.

Turn-In: Turn in one copy of this exercise per team.

Team Members:

Goals:

1. Be able to find compositions of functions.
2. Be able to find a function from another function and the compositions
3. Use arrow diagrams to diagram two functions and their composition.

Section 1: Composition of Functions

If $f: A \rightarrow B$ and $g: B \rightarrow C$, then we can build a new function called $(g \circ f)$ that has domain A and codomain C , and that follows the rule $(g \circ f)(x) = g(f(x))$. We call $(g \circ f)$, read “g of f”, the composition of g with f.

Example: $f(x) = 2x + 1$ and $g(x) = x^2 - 1$. What is $(f \circ g)(x)$?

1. Plug $f(x)$ into $g(y)$: $g \circ f = f(g(x)) = 2(x^2 - 1) + 1$
2. Simplify: $= 2x^2 - 2 + 1$
 $= 2x^2 - 1$

We may also need to find the function $g(x)$ based on $f(x)$ and $(f \circ g)(x)$:

Example: $f(x) = 2x + 1$ and $(f \circ g)(x) = 2x^2 - 1$. What is $g(x)$?

1. Let's use the alias “a” to symbolize $g(x)$. $f(g(x))$ therefore will be $f(a)$:
 $2a + 1 = 2x^2 - 1$ Now we can find the value of $g(y)$ by solving for a.
2. $2a = 2x^2 - 1 - 1 \Rightarrow 2a = 2x^2 - 2 \Rightarrow a = x^2 - 1$
3. Therefore, $g(x) = x^2 - 1$

And finding $f(x)$ from $g(x)$ and $(f \circ g)(x)$:

Example: $g(x) = x^2 - 1$ and $(f \circ g)(x) = 2x^2 - 1$. What is $f(x)$?

Method 1:

1. Start with $f(g(x))$. Let's say that the alias “a” will symbolize $g(x)$. So $a = x^2 - 1$.
2. Solve the alias function for the other variable: $x^2 = a + 1 \dots x = \sqrt{a + 1}$.
3. Then, for $f(g(x))$, we use $f(x)$: $f(g(x)) = 2x^2 - 1$
4. Plug in our found value for x : $2(\sqrt{a + 1})^2 - 1$
5. Simplify: $2(a + 1) - 1 \dots = 2a + 2 - 1 \dots = 2a + 1$
6. So $f(x) = 2x + 1$

1. Find the value of $(f \circ g)(x)$ for the following $f(x)$ and $g(x)$ functions:

a. $f(x)=2x-1$ and $g(x)=3x$, what is $(f \circ g)(x)$? (___/1)

b. $f(x)=2x-1$ and $g(x)=3x$, what is $(g \circ f)(x)$? (___/1)

c. $f(x)=x^2$ and $g(x)=x+1$, what is $(f \circ g)(x)$? (___/1)

d. $f(x)=x^2$ and $g(x)=x+1$, what is $(g \circ f)(x)$? (___/1)

2. Find the value of $g(x)$ from the following $f(x)$ and $(f \circ g)(x)$ given:

a. $f(x) = 2x - 1$ and $(f \circ g)(x) = 6x - 1$. What is $g(x)$? (___/1)

b. $f(x) = x^2$ and $(f \circ g)(x) = x^2 + 2x + 1$. What is $g(x)$? (___/1)

c. $f(x) = 3x - 2$ and $(f \circ g)(y) = 12y + 7$. What is $g(x)$? (___/1)

3. Find the value of $f(x)$ from the following $g(x)$ and $(f \circ g)(x)$ given:

a. $g(x)=3x$ and $(f \circ g)(x)=6x-1$. What is $f(x)$? (___/1)

b. $g(x)=x+1$ and $(f \circ g)(x)=x^2+2x+1$. What is $f(x)$? (___/1)

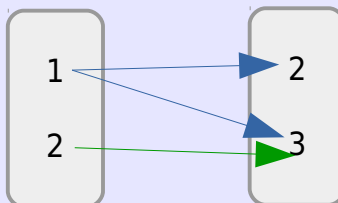
c. $g(x)=2x-1$ and $(f \circ g)(x)=6x-1$. What is $f(x)$? (___/1)

Section 2: More arrow diagrams

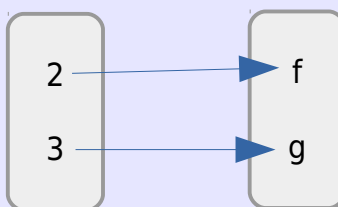
We can also use arrow diagrams to represent compositions of relational functions.

Example:

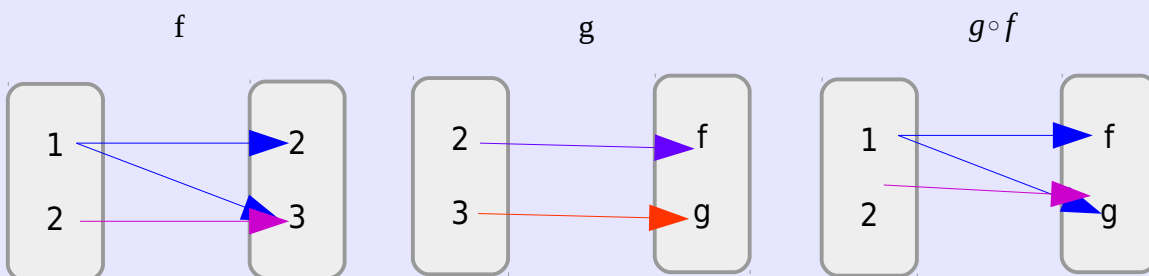
$f(x)$: Domain $A = \{1, 2\}$, Codomain $B = \{2, 3\}$, Rule: $L = \{(1, 2), (1, 3), (2, 3)\}$.



$g(x)$: Domain: $C = \{a, b\}$, Codomain $D = \{f, g\}$, Rule: $M = \{(a, f), (b, g)\}$



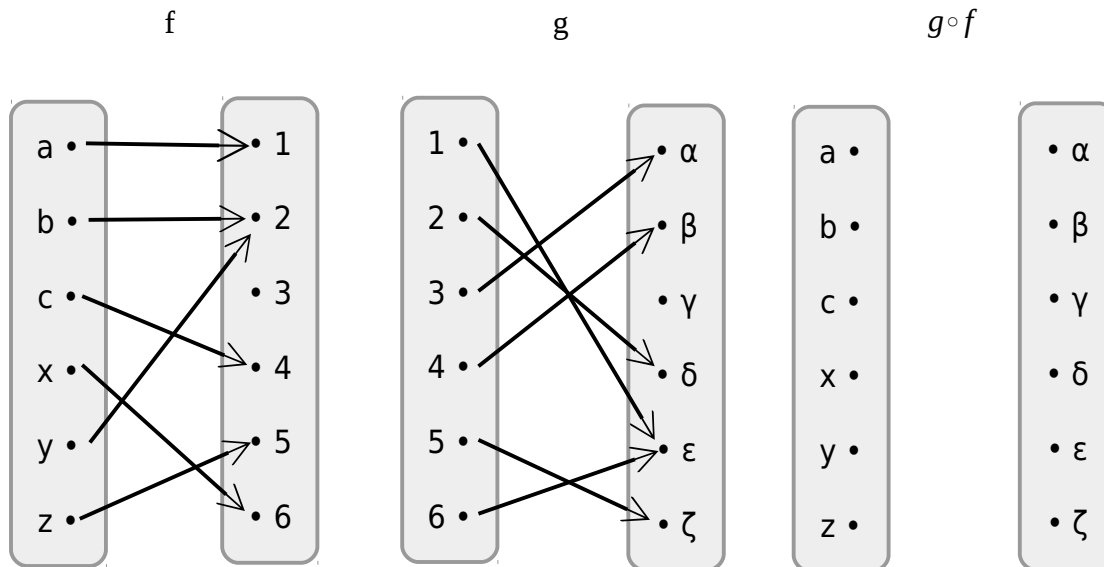
So in order to draw the diagram of $(g \circ f)(x)$, we take the domain of f and the codomain of g . In this example, in f , 2 goes to 3. In g , 3 goes to g , so in $g \circ f$, 2 goes to g .



4. Complete the arrow diagrams

a.

(___/2)



b.

(___/2)

