Team Info (only fill out for the sheet to be turned in)

Team Name:

Group members (up to four): 1. 2.

3. 4.

Instructions
Only one sheet per team will be turned in. Each team member can work on their own sheet for practice, but then the group as a whole should discuss the answers and collaborate on the turn-in sheet. Everyone can take home their own sheets.

Goals
1. Be able to prove that the result of a summation is equivalent to a given formula for each index given.
2. Be able to prove that the result of some formula is *even, odd*, or *divisible by some number* through induction.

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1. Introductory Practice

Practice 1. For the following equations, fill out the table for the resulting values for *n* = some number. For the recursive formula, , where , write out the values for:

	$s_n = s_{(n-1)} + (2n-1)$; $s_1 = 1$	n^2	$\sum_{i=1}^{n} (2i-1)$
a) $n=2$			
b) $n=3$			
c) $n=4$			

2. Sums as Recursive Sequences

Example 1 from the textbook

Consider the sum $\sum_{i=1}^{n}(2i-1)$, which is the same as 1 + 3 + 5 + … + (2n-1). Use the notation s_n to denote this sum. Find a recursive description of s_n .

Step 1: Find the first term, s_1 :

Plug into the summation: $\sum_{i=1}^{1}(2i-1)=(2\cdot 1-1)=1$, so $s_1=1$

Step 2: Restate the result of s_n as $s_{(n-1)}$ plus the final term

$$s_n=s_{(n-1)}+(2n-1)$$

So, for $\sum_{i=1}^{n}(2i-1)$, the recursive formula is: $s_1=1$, $s_n=s_{(n-1)}+(2n-1)$.

Practice 1

Consider the sum $\sum_{i=1}^{n}(3n^2)$. Use the notation s_n to denote this sum. Find a recursive description of s_n .

Step 1: Find the first term, s_1 :

Step 2: Restate the result of s_n as $s_{(n-1)}$ plus the final term

Step 3: Check your answer! Plug in various values into *n* for both the summation and the recursive formula and make sure the result comes out to the same values.

Practice 2

Consider the sum $\sum_{i=1}^{n} \left(2^{(i-1)}+1\right)$. Use the notation s_n to denote this sum. Find a recursive description of s_n .

Practice 3

Consider the sum $\sum_{i=1}^{n} \left(i^3-i\right)$. Use the notation s_n to denote this sum. Find a recursive description of s_n .

3. More proofs by induction

Example 6 from the book
Show that n^3+2n is divisible by 3 for all positive integers *n*. ($D(n)=n^3+2n$)

Step 1: Check for D(1):

$$D(1)=1^3+2\cdot 1=3 \quad \checkmark$$

Step 2: Acknowledge that "Show that n^3+2n is divisible by 3 for all positive integers n." has been proven for D(1) through D(m-1).

Step 3: Write out D(m-1) and simplify:

$$D(m-1)=(m-1)^3+2(m-1)$$
$$D(m-1)=m^3-3m^2+3m-1+2m-2$$

Step 4: Rewrite simplified version so that D(m) is part of the equation:

$$D(m-1)=(m^3+2m)-3m^2+3m-3$$

Step 5: Rewrite with D(m):

$$D(m-1)=D(m)-3m^2+3m-3$$

Step 6: Solve for D(m):

$$D(m)=D(m-1)+3m^2-3m+3$$

Step 7: Remember that *divisibility by 3* has been proven true for D(1) through D(m-1) (from Step 2). Replace D(m-1) with "3K".

$$D(m)=3K+3m^2-3m+3$$

Step 8: Factor out common terms to get final proof that n^3+2n is divisible by 3:

$$D(m)=3(K+m^2-m+1)$$

Practice 4
Use induction to prove that for each integer $n\geq 1$, $2n$ is even.

Practice 5
Use induction to prove that for each integer $n \geq 1$, $4n+1$ is odd.

Practice 6
Use induction to prove that for each integer $n \geq 1$, $n^2 - n$ is even.