Exercise 2.1: Mathematical Writing CS 210 September 19th, 2016

Team Name:

Instructions

<u>Only one sheet per team will be turned in.</u> Each team member can work on their own sheet for practice, but then the group as a whole should discuss the answers and collaborate on the turn-in sheet. Everyone can take home their own sheets.

Team Info (only fill out for the sheet to be turned in)

Team Name:

Group members (up to four): 1. 2.

3. 4.

Introduction 1

This time we're exploring mathematical writing and getting introduced to proofs. This means that we are going to be working more with <u>contrapositives</u> and <u>implications</u> in order to prove statements.

In Example 1 from the book, it asks to rewrite the statement

For every prime *n*, $n^2 - n + 41$ is also prime.

in "if, then" implication form, which comes out to:

If a positive integer *n* is prime, then the number $n^2 - n + 41$ is also prime.

Question 1

Rewrite the following statements as "if, then" statements:

a) Whenever n is an even integer, $n+1$ is an odd integer.

b) All squares have four equal sides.

(Think of representing the square as a variable, and the length of a side as a variable)

Introduction 2

A <u>counterexample</u> is a way to disprove a proposition. For implications, if we can come up with *some hypothesis* that results in the *conclusion* being false, then we can disprove a statement.

In Practice Problem 4 from book, the statement is made…:

For every integer $n \geq 1$, if *n* is odd, then n^2+4 is a prime number.

Several examples given don't disprove this statement:

$$3^2+4=13 \quad , \quad 5^2+4=29 \quad , \quad 7^2+4=53$$

But as long as at least one can be found, then we can use this as a counterexample to disprove it:

$9^2+4=85$, 85 is divisible by 5 and 17.

Question 2

Disprove the following statements with a counterexample:

a) For every even integer *n*, *n+1 is also even*

b) For every integer integer *n*, $n/2$ is also an integer.

Introduction 3
How do we prove that an implication is true?

Given this proposition:
"The result of summing any odd integer with any even integer is an odd integer."

We can write this in mathematical language. First, let's define our numbers:
x is an odd integer, and y is an even integer, so $x + y$ should result in an odd integer.

But how do we symbolize "even" and "odd" numerically? We can come up with more symbols:

$$x=2A+1 \quad , \quad y=2B$$

where *2A+1* will always be odd for an integer A, and *2B* will always be even for an integer.

Then we can form the expression:

$$x+y=(2A+1)+(2B)$$

and simplify:

$$x+y=2A+2B+1$$
$$x+y=2(A+B)+1$$

Notice the similarity between $x=2A+1$ and $2(A+B)+1$ - since $A+B$ is an integer, and 2*x(integer)+1* results in an odd integer, we can conclude that the result will **always** be an odd integer.

Question 3
Prove the following statement: For all integers $n > 0$, if n is odd, then n^2+n is even.

Introduction 4
Several definitions from the book:

Divisible by 4
"An integer *n* is divisible by 4 if it can be written in the form $n = 4M$ for some integer M"

Even
"An integer *n* is even if it can be written in the form $n = 2K$ for some integer K"

Odd
"...an integer *m* is odd if it can be written in the form $m = 2L+1$ for some integer L"

Example: Show that 10 is even: $10=5\cdot 2$

Question 4
Using the definitions of even, odd, and divisible by 4, show that the following statements are true:

a) 12 is even

b) -13 is odd

c) 64 is divisible by 4

d) $8n^2+8n+4$ is divisible by 4
(hint: You don't have to factor it to a FOIL-able state! Look for the common term to factor out)