Sequences

Element: One specific item of a list (or array) at some position. (e.g., $a_1 = 5$, element is 5.)

Index: The position of an item of a sequence. (e.g., $a_1 = 5$, index is 1.)

Closed formula: $a_n = 3n$, the value of the element is based on the index.

Recursive formula: $a_1 = 3$, $a_n = a_{(n-1)} + 3$, the first element (at least) must be specified. All subsequent elements are based on the element at the previous index.

Propositions

A **propositional variable** is a variable that is either **true** or **false**. Propositional variables can be combined with AND ∧, OR ∨, and NOT ¬ to build a statement. This statement also results in **true** or **false**.

Predicates

Predicates are generally denoted as P(x), Q(x), R(x), etc. where *x* is some input value. With this input value plugged in (e.g, P(5)), the result of the predicate is either true or false. Given a predicate with a domain, it is possible for: (a) the predicate to *always* be true for any element of the domain, (b) the predicate to *sometimes* be true given the possible elements of the domain, or even (c) *never* be true given the elements of the domain.

Quantifiers:	For all ∀	There exists (at least one) ∃
	$x \in D$ X is an element of D	$x \notin D$ X is NOT an element of D
	$\mathbb{R}$ The set of all real numbers	$\mathbb{Z}$ The set of all integers

Negations

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x)$

$\neg(\exists x \in D, P(x)) \equiv \forall x \in D, \neg P(x)$

Implication laws

Contrapositive	Converse	Inverse
$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$

This exam review contains the same topics that will be on the actual exam. The exact questions will not be the same, but similar questions will be asked.

Chapter 1.2 topics

1. For the given sequence, find the closed formula. 3, 5, 7, 9, 11

2. For the given sequence, find the closed formula. 2, 5, 10, 17, 26

3. For the given sequence, find the recursive formula. 1, 3, 9, 27, 81

4. For the given sequence, find the recursive formula. 1, 6, 31, 156, 781

5. Evaluate the following summation. $\sum_{k=1}^{k=5}(2k)$

6. Evaluate the following summation. $\sum_{k=1}^{k=5}(3k+1)$

Chapter 1.3 topics

7. Create a truth table for the following expression. $(p \vee q) \wedge \neg (p \wedge q)$

8. Create a truth table for the following expression. $p \wedge (q \vee \neg r)$

9. Translate the following statements into propositional logic with the given variables.

o. Patron has overdue books **m. Patron has the maximum amount of books checked out** **a. book is available at this library**

a. Patron has overdue books and has the maximum amount of books checked out.

b. Patron does not have overdue books, but the book is not available at this library.

c. Patron does not have overdue books and the book is available at the library.

d. Patron does not have overdue books and the book is available at the library, but the patron has the maximum amount of books checked out.

e. Patron either has overdue books, or has the maximum amount of books checked out.

f. Patron either has overdue books, or has the maximum amount of books checked out, but not both.

g. The patron doesn't have overdue books, and doesn't have the maximum checked out, and the book is not available at this library.

10. Given three statements **p**, **q**, and **r**, write a compound statement that will meet the following criteria: The result of the statement will be true if:

a. p and q are true, but not r.

b. r is true, but the other two are false.

c. either p or q is false, and r is true.

11. Given three statements **p**, **q**, and **r**, write a compound statement that will meet the following criteria. Use parenthesis to explicitly define the order of statements.

a. Exactly two statements are true, but not only one or all three.

b. Exactly one statement is true, and not more than one.

c. p and one other statement are true, but not all three.

Chapter 1.4 topics

12. Given the following predicate, define a domain that makes the quantified statement true or false. (Multiple solutions)

a. P(n) is the predicate "n is divisible by 2". Quantified statement: $\forall n \in D, P(n)$ is true

b. P(n) is the predicate "n is divisible by 2". Quantified statement: $\forall n \in D, P(n)$ is false

c. Q(n) is the predicate "n is positive". Quantified statement: $\forall n \in D, P(n)$ is true

d. Q(n) is the predicate "n is positive". Quantified statement: $\forall n \in D, P(n)$ is false

13. Solve the following.

a. Translate the following statement into a quantified statement using predicate logic.
"For every element *x* that is a member of the domain *D*, *x* is greater than 10."
D = { 20, 40, 60, 80, 100 }.

b. Write the negation of the statement from (a). Simplify so that the negation sign ¬ is not present.

c. Which statement (a or b) is true?

14. Solve the following.

a. Translate the following statement into a quantified statement using predicate logic.
"For every element *x* that is a member of the domain *D*, x > 0."
D = { -10, -5, 0, 5, 10 }.

b. Write the negation of the statement from (a). Simplify so that the negation sign ¬ is not present.

c. Which statement (a or b) is true?

Chapter 1.5 topics

15. Solve the following.

a. Translate the following into quantified predicate logic.
"For all integers *x*, if 2 times x is 0, then x is zero." Note that the set of integers is $\mathbb{Z}$

b. Write the negation of the statement from (a). Simplify so that the negation sign ¬ is not present.

c. Which statement (a or b) is true?

16. Given the following statement, find the contrapositive, converse, and inverse.

" $\forall n \in \mathbb{Z}$, if n is even, then $3n$ is even "

Use the predicates *P(n) is "n is even"*, *Q(n) is "3n is even"*

a. Write with quantified predicate logic.

b. Write the contrapositive.

c. Write the converse.

d. Write the inverse.

17. Let *D* be the domain of all people. Given the predicates:

P(x) is "nobody likes x" Q(x) is "everybody hates x" R(x) is "x will go eat worms"

Use the logical operators $\neg, \wedge, \vee, \rightarrow, \exists, \forall$ and write each of the following English statements as Logical statements. Make sure to use $\forall$ or $\exists$ for each.

a. Anybody who is liked by nobody will go eat worms.
(If nobody likes *x*, then *x* will go eat worms.)

b. There exists somebody *x* whom nobody likes, but *x* will not go eat worms.

c. Anybody who is hated by everybody (all possible *x*) then nobody likes *x*.

d. If *x* is not eating worms, then *x* is liked by somebody.
(For all people, if they're not eating worms, then they're liked…)

Answers

1. $a_n = 2n + 1$
2. $a_n = n^2 + 1$
3. $a_1 = 1$ with $a_n = 3a_{n-1}$
3. $a_1 = 1$ with $a_n = 5a_{n-1} + 1$
4. 4 + 7 + 10 + 13 + 16 = 50
5. 2 + 4 + 6 + 8 + 10 = 30

7.

p	q	$(p \vee q)$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

8.

p	q	r	$\neg r$	$q \vee \neg r$	$p \wedge (q \vee \neg r)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

For the propositions, there could be multiple solutions.

9. a. $o \wedge m$ b. $\neg o \wedge \neg a$ c. $\neg o \wedge a$ d. $\neg o \wedge a \wedge m$

e. $o \vee m$ f. $(o \wedge \neg m) \vee (\neg o \wedge m)$ or $(o \vee m) \wedge \neg (o \wedge m)$

g. $\neg o \wedge \neg m \wedge \neg a$ or $\neg (o \vee m \vee a)$

10. a. $p \wedge q \wedge \neg r$ b. $r \wedge \neg p \wedge \neg q$ c. $(\neg p \vee \neg q) \wedge r$

11. a. $(p \wedge q \wedge \neg r) \vee (q \wedge r \wedge \neg p) \vee (p \wedge r \wedge \neg q)$

b. $(p \wedge \neg q \wedge \neg r) \vee (q \wedge \neg p \wedge \neg r) \vee (r \wedge \neg p \wedge \neg q)$

c. $p \wedge (q \vee r) \wedge \neg (p \wedge q \wedge r)$

12. Many solutions

13. a. $\forall x \in D, x > 10$ b. $\exists x \in D, x \leq 10$ c. Statement (a) is true.

14. a. $\forall x \in D, x > 0$ b. $\exists x \in D, x \leq 0$ c. statement (b) is true.

15. a. $\forall x \in \mathbb{Z}, \quad 2x = 0 \rightarrow x = 0$

b. $\neg (\forall x \in \mathbb{Z}, \quad 2x = 0 \rightarrow x = 0)$, simplified: $\exists x \in \mathbb{Z}, (2x = 0) \wedge (x \neq 0)$

c. Statement (a) is true.

16. a. $\forall n \in \mathbb{Z}, P(n) \rightarrow Q(n)$
b. $\forall n \in \mathbb{Z}, \neg Q(n) \rightarrow \neg P(n)$
c. $\forall n \in \mathbb{Z}, Q(n) \rightarrow P(n)$
d. $\forall n \in \mathbb{Z}, \neg P(n) \rightarrow \neg Q(n)$

17. a. $\forall x \in D, P(x) \rightarrow R(x)$
b. $\exists x \in D, P(x) \rightarrow \neg R(x)$
c. $\forall x \in D, Q(x) \rightarrow P(x)$
d. $\forall x \in D, \neg R(x) \rightarrow \neg P(x)$