Sets

$\mathbb{N}$: The set of natural numbers

$\mathbb{Z}$: The set of integers

$\mathbb{Q}$: The set of rational numbers

$\mathbb{R}$: The set of real numbers

Syntax of Boolean Algebra

	Boolean Algebra
Variables	a, b, c
Operations	$\cdot$, $+$, $'$
Special elements	$0, 1$

A Property Description is of the form, "The set of all x in U such that x is _____". The blank is some *property* of x, which determines whether an element of U is or is not in the set.

A Form Description is of the form, "All numbers of the form ____, where x is in set D", where the first part will be some equation (like "2x" for even).

Cartesian product of A and B

Given sets A and B, we define $A \times B = \{(a,b) : a \in A, b \in B\}$.(We read that as "A cross B", and call (a,b) "the ordered pair *a, b*".)

In the common special case that both coordinates are taken from the same set, we often write A^2 instead of $A \times A$.

Partition $\mathcal{S}$ of a set

For a set A, a *partition* of A is a set $\mathcal{S}$ = { $S_1, S_2, S_3, \ldots$ } of subsets of A (each set S_i is called *a part of* $\mathcal{S}$), such that:

1. For all i , $S_i \neq \emptyset$. That is, each part is nonempty.
2. For all i and j , if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$. That is, different parts have nothing in common.
3. $S_1 \cup S_2 \cup S_3 \cup \ldots = A$. That is, every element in A is in some part.

The power of set A $\wp(A)$

$\wp(A) = \{S : S \subseteq A\}$ (We read $\wp(A)$ as "the power set of A")

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Please read and sign

I understand that this exam is a solo effort and the following is not allowed: Copying from a classmate or outside source (including storing extra information on hidden items), using a graphing calculator on this exam, discussing the exam with classmates during the exam period, and other behavior that is deemed academic dishonesty. I understand that, if I am suspected of cheating, I will be asked to leave the classroom and receive a 0 on the exam. I understand that if I need clarification on a question or otherwise need assistance, I can ask the instructor during the exam time.

Your printed name	*Your signed name*

Score:	Total Possible Points: 81

(12 pts) Question 1: Set Operations, Chapter 3.1

Find the following results given the following sets.

एक	$U=\{2,4,6,8,10,12,14,16,18\}$	$A=\{2,4,6,16,18\}$ $C=\{8,10,12\}$	$B=\{2,16\}$ $D=\{16,18\}$
दो	$U=\{1,3,5,7,9,11,13,15,17\}$	$A=\{1,3,5,15,17\}$ $C=\{7,9,11\}$	$B=\{3,15\}$ $D=\{15,17\}$

____ / 2 (a) $A\cap B=$

____ / 2 (b) $U-D=$

____ / 2 (c) $D'=$

____ / 2 (d) $(A\cup C)-B=$

____ / 2 (e) $(A\cap C)'=$

____ / 2 (f) $A'\cap B'=$

(12 pts) Question 2: Venn Diagrams, Chapter 3.1

Draw Venn diagrams for each of the following statements. Remember to include the universe.

____ / 4 (a) एक A'

____ / 4 (b) एक $A - B$

____ / 4 (c) एक $(A - B) \cap C$

(6 pts) Question 3: Set-builder Notation, Chapter 3.1

Write the following as a *form description* of a set builder notation:

____ / 2 (a) एक The set consisting of all integers divisible by 5 and greater than zero.

____ / 2 (b) एक Property description: $\{\ y \in \mathbb{N} :$ y is one more than $2k$, and $k \in \mathbb{N}\ \}$

(9 pts) Question 4: Cartesian Products, Chapter 3.2

For the following, find the Cartesian product ($A \times B$). List all elements, and write the elements in an orderly way.

____ / 3 (a) एक $A = \{1, 2\}$ $B = \{b\}$

$A \times B =$

____ / 3 (b) एक $A = \{a, s, d, f\}$ $B = \{2, 4\}$

$A \times B =$

____ / 3 (b) एक $A = \{a, s, d, f\}$ $B = \{2, 4\}$

$B \times A =$

(8 pts) Question 5: Power Sets, Chapter 3.2

For the following, find the power set and list all elements.

____ / 2 (a) एक $A = \{a, b\}$ $\wp(A) =$

____ / 3 (b) एक $A = \{a, b, c\}$ $\wp(A) =$

____ / 3 (b) एक $A = \{a, b, c, d\}$ $\wp(A) =$

(10 pts) Question 6: Cartesian Products & Power Sets, Chapter 3.2

Given the following sets:

एक $A = \{3\}$ $B = \{1,2\}$

Find the following:

____ / 2 (b) $\wp(A) =$

____ / 2 (c) $\wp(B) =$

____ / 2 (d) $A \times B =$

____ / 2 (d) $\wp(A) \times \wp(B) =$

____ / 2 (d) $\wp(A \times B) =$

(12 pts) Question 7: Partitions, Chapter 3.2

Given the following set, find partitions that satisfies the requirements.

एक $U=\{q,w,e,r,t,y\}$

____ / 3 (a) There are as few parts as possible

____ / 3 (b) No two parts have the same size

____ / 3 (c) Every part has the same size

____ / 3 (d) There are as many parts as possible

(12 pts) Question 8: Boolean Algebra, Chapter 3.4

Rewrite each of these statements in Boolean Algebra notation.

____ / 1 (a) $A \cup B$

____ / 1 (b) $A \cap B$

____ / 1 (c) $A - B$

____ / 2 (c) $A \cap (B \cap C)$

____ / 2 (d) $A \cup (B \cup C)$

____ / 3 (d) $(A \cap C') \cup B$

____ / 3 (d) $B \cap (A \cup C)'$

____ / 3 (d) $((B \cap A') \cup C)'$

(5 pts) Extra Credit

____ / 1 (a)	The result of $A \times B$ and $B \times A$ are the same	True / False
____ / 1 (b)	$a \cdot b = b \cdot a$	True / False
____ / 1 (c)	$a \cdot (b + c) = (a + b) \cdot (a + c)$	True / False
____ / 1 (d)	$a \cdot a' = 1$	True / False
____ / 1 (e)	$0' = 1$	True / False