Team Info (only fill out for the sheet to be turned in)

Team Name:

Group members (up to four): 1. 2.

3. 4.

Instructions

Only one sheet per team will be turned in. Each team member can work on their own sheet for practice, but then the group as a whole should discuss the answers and collaborate on the turn-in sheet. Everyone can take home their own sheets.

Goals

1. Be able to prove that the sequences generated from a given recursive formula and and a given closed formula are equal.
2. Be able to prove that the result of a summation is equivalent to a given formula for each index given.

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1. Introductory Practice

Practice 1. If P(n) is " n^2+1 is prime", write P(1), P(2), P(12), and P(m-1)

P(1) = P(2) = P(12) = P(m-1) =

Which of these are true, if any?

Practice 2. For the sequence defined by recursive formula $a_k = a_{(k-1)} + 4$; $a_1 = 1$, write out the values for each value of *k*:

P(1) = P(2) = P(3) = P(4) =

Practice 3. For the sequence defined by closed formula $a_n = 4n - 3$, write out the values for each value of *n*:

P(1) = P(2) = P(3) = P(4) =

2. Recursive ↔ Closed formula equivalence

Question 3a from the textbook
Show that the sequence defined by $a_k = a_{(k-1)} + 4 \,;\, a_1 = 1$ for $k \geq 2$ is equivalently described by the closed formula $a_n = 4n - 3$.

Step 1: Check values results of both formulas for a_1 :

Recursive: $a_1 = 1$ (provided) Closed: $a_1 = 4(1) - 3 = 1$ ✓

Step 2: Rewrite the recursive formula in terms of *m*:

$$a_m = a_{(m-1)} + 4$$

Step 3: Find the equation for $a_{(m-1)}$ through the closed formula:

$$a_n = 4n - 3 \quad \rightarrow \quad a_{(m-1)} = 4(m-1) - 3 \quad \rightarrow \quad a_{(m-1)} = 4m - 7$$

Step 4: Plug $a_{(m-1)}$ into the recursive formula from step 2 and simplify.

$$a_m = a_{(m-1)} + 4 \quad \rightarrow \quad a_m = (4m - 7) + 4 \quad \rightarrow \quad a_m = 4m - 3$$

PROOF: $a_m = 4m - 3$ and the closed formula $a_n = 4n - 3$ match so the closed formula and recursive formula are equivalent.

Practice 4
Show that the sequence defined by $b_k = 4b_{(k-1)} + 3$ for $k \geq 2$, where $b_1 = 3$, is equivalently described by the closed formula $b_n = 2^{(2n)} - 1$.

Step 1: Check values results of both formulas for b_1 :

Recursive: Closed:

Step 2: Rewrite the recursive formula in terms of *m*:

Step 3: Find the equation for $b_{(m-1)}$ through the recursive formula:

Step 4: Plug $b_{(m-1)}$ into the recursive formula from step 2 and simplify.

Result:

Practice 5

Show that the sequence defined by $a_n = a_{(n-1)} + 2$ for $k \geq 2$, where $a_1 = 5$, is equivalently described by the closed formula $a_n = 2n + 3$.

Practice 6

Show that the sequence defined by $a_k = 2a_{(k-1)} + 1$ for $k \geq 2$, where $a_1 = 1$, is equivalently described by the closed formula $a_n = 2^n - 1$.

3. Summation ↔ formula equivalence

Question 8a from the textbook
Use induction to prove each of the following. As part of your proof, write and verify each statement for at least *n = 1, n = 2, n = 3, and n = 4.*

$$\textbf{Proposition:} \quad \sum_{i=1}^{n}(2i-1)=n^2 \quad \text{for each} \quad n\geq 1$$

Step 1: "Trace" the proof for a few initial values…

✓ $i=1$	$\sum_{i=1}^{1}(2i-1)=(2\cdot 1-1)=1$	$1^2=1$
✓ $i=2$	$\sum_{i=1}^{2}(2i-1) \quad =(2\cdot 1-1)+(2\cdot 2-1) \quad =1+3=4$	$2^2=4$
✓ $i=2$	$\sum_{i=1}^{3}(2i-1) \quad =(2\cdot 1-1)+(2\cdot 2-1)+(2\cdot 3-1)$ $=1+3+5=9$	$3^2=9$

Step 2: Redefine the proposition in terms of *m-1*

$$\sum_{i=1}^{m-1}(2i-1)=(m-1)^2$$

Step 3: Given the value of the summation from $i=1$ to $m-1$, write the proposition in terms of the summation from $i=1$ to $m-1$ plus the result of $(2i-1)$ for the last run of the summation, at i = m.

$$\sum_{i=1}^{n}(2i-1)=[(m-1)^2]+(2m-1)$$

Step 4: Work out and simplify.

$$[(m-1)^2]+(2m-1)$$

$$=(m-1)(m-1)+(2m-1)$$

$$=m^2-2m+1+2m-1$$

$$=m^2$$

Proof: The result for $\sum_{i=1}^{n}(2i-1)$ results in n^2 .

Practice 7

Use induction to prove $\sum_{i=1}^{n}(2i+4)=n^2+5n$ for each $n\geq 1$. Make sure to also build the table for *n = 1, n = 2, and n = 3.*

Step 1: "Trace" the proof for n = 1, n = 2, and n = 3

Step 2: Redefine the proposition in terms of *m-1*

Step 3: Given the value of the summation from $i=1$ to $m-1$, write the proposition in terms of the summation from $i=1$ to $m-1$ plus the result of $(2m+4)$ for the last run of the summation, at i = m.

Step 4: Work out and simplify.

Result:

Practice 8

Use induction to prove $\sum_{i=1}^{n}(2^i - 1) = 2^{(n+1)} - n - 2$ for each $n \geq 1$. Make sure to also build the table for
n = 1, n = 2, and n = 3.