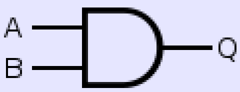
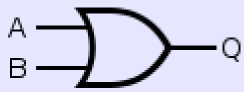
Section 1: Logic Circuits

We are going to be using logic gates as one way to represent our Boolean algebra expressions graphically. The gates we'll be using are:

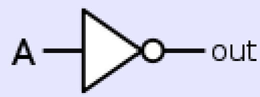
AND



OR



NOT



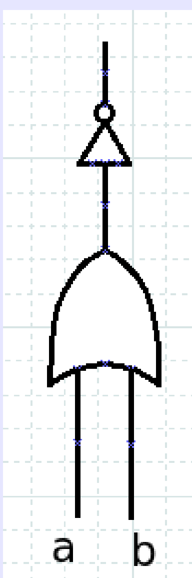
A	**B**	**A · B**
0	0	0
0	1	0
1	0	0
1	1	1

A	**B**	**A + B**
0	0	0
0	1	1
1	0	1
1	1	1

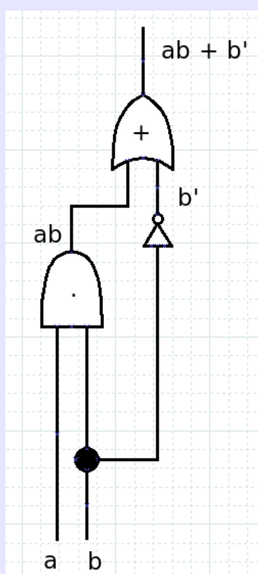
A	**A'**
0	1
1	0

We can connect gates together in order to build an expression. For example:

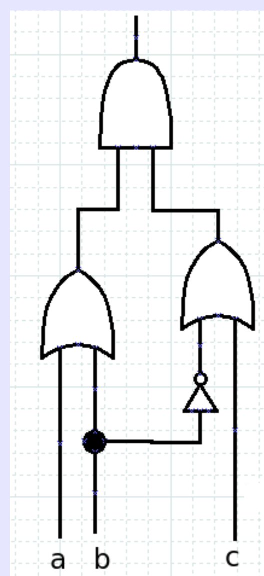
$(a+b)'$



$ab+a'b$



$(a+b)\cdot(b'+c)$

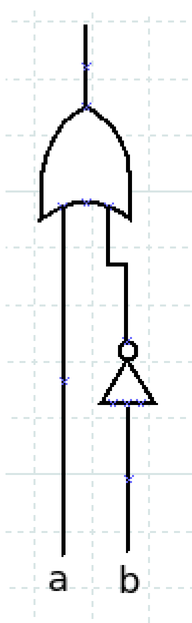


1. Draw a circuit diagram for the following Boolean expressions:

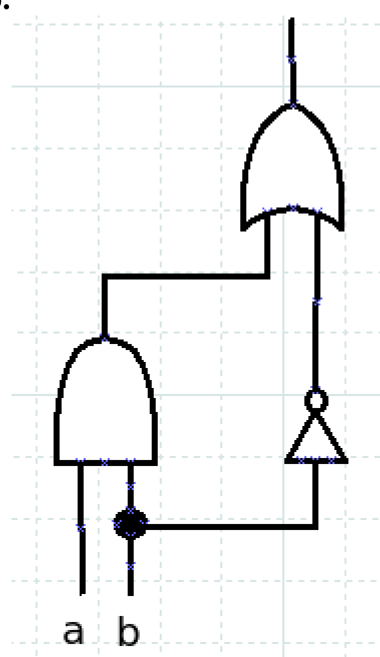
a. $a+b'$ **b.** $a'\cdot b'$ **c.** $a+(b\cdot c)$

2. Write out the Boolean expression for the following diagrams:

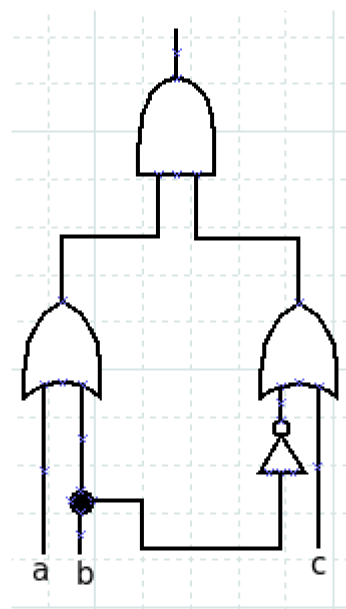
a.



b.

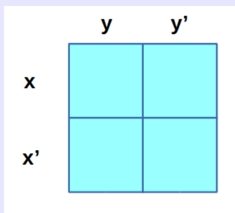


c.



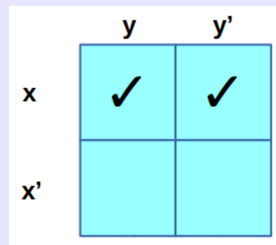
Section 2: 2-variable Karnaugh Maps

We can use Karnaugh maps to visually represent Boolean expressions, in order to simplify them. For an expression with two variables, our map looks like this.

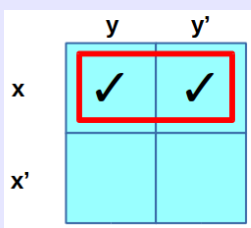


You check off any <u>products</u> in an expression. The separate products (ANDs) are summed together (ORs).

$xy+xy'$



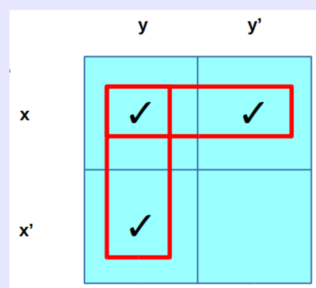
After writing out all of the products, any <u>adjacent</u> cells can be grouped off within a rectangle region.
Here, all the cells in the "x" row are filled, so this represents that we can replace $xy+xy'$ **with** x



$xy+xy' \rightarrow x$

If we have multiple rectangles, our result will be the sum of these regions.

If there are no adjacent regions, the expression cannot be simplified further.



$xy+xy'+x'y \rightarrow x+y$

3. Figure out the (not-simplified) expression the following Karnaugh maps represent.

a.

	y	y’
x		✓
x’		✓

b.

	y	y’
x		
x’	✓	✓

c.

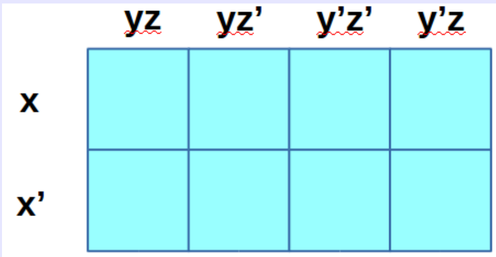
	y	y’
x		✓
x’	✓	✓

4. From the Karnaugh maps above, simplify and write out the simplest form expression.

5. Map out the following Boolean expressions in a 2-variable Karnaugh map, and write out the simplest form, if there is one.

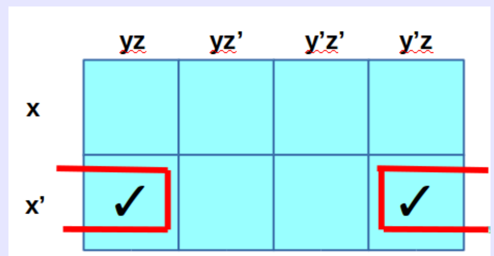
a. $xy+x'y$ **b.** $xy+x'y'$ **c.** $x+y'$

Section 2: 3-variable Karnugh Maps



Once we have three variables, we will write the map such that each column represents two variables together.

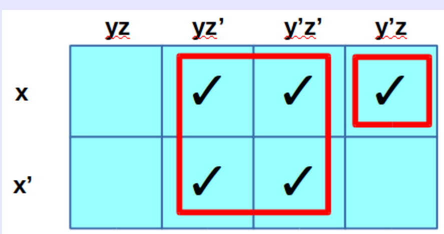
Note that for each column, the difference in the variables is only a difference of 1. You cannot change two variables in one step (e.g., you cannot go from yz → y'z'.)



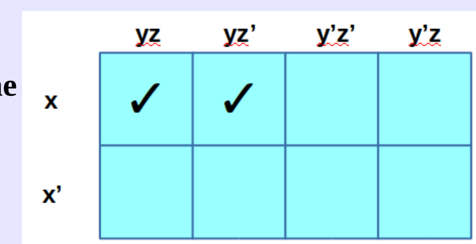
When we're drawing our regions to simplify, we will be able to "wrap-around" horizontally.

In this example, $x'yz + x'y'z \rightarrow x'z$.
Since *y* changes between *y* and *y'*, but x' and z' are the same for both cells, this is what the simplified form is. (The value of y doesn't actually affect it.)

You can also only have <u>rectangles of length 0, 1, 2, or 4.</u> You cannot have a rectangle whose region is 3, as it will not fully cover any one variable.



In order to get the simplest expression, you should also <u>choose the smallest amount of rectangles, and so that each rectangle is as large as possible.</u>



Karnaugh map for *xy*

And finally, if we have an expression where one of the terms is missing a variable (e.g., *xy*), in order to map it we recognize that, in this case, *z* has no bearing on the result… Therefore, we can expand xy to $xyz + xyz'$.

6. Write out the Boolean algebra expressions (don't simplify) for the given Karnaugh maps

a.

	yz	yz'	y'z'	y'z
x	✓	✓		
x'		✓		

b.

	yz	yz'	y'z'	y'z
x	✓	✓	✓	✓
x'			✓	

7. Fill out the Karnaugh maps given the following Boolean algebra expressions. Simplify (if possible) and also write out the simplest Boolean algebra expression.

a. $x'yz + x'yz' + x'y'z' + xy'z$

b. $yz + yz' + y'z$

8. Map out the following Boolean expressions in a 3-variable Karnaugh map, and write out the simplest form, if there is one.

a. $xyz + x'yz$

b. $xyz + xy'z + x'yz + x'y'z$

c. $x'yz' + x'y'z'$

d. $xyz + xy'z$