Section 1: Functions that are one-to-one, onto, and invertible

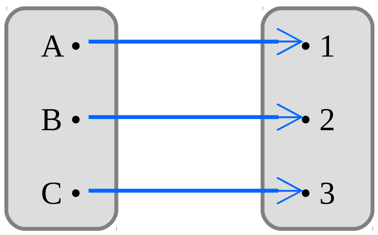
Functions that are onto	A function *f* is *onto* if everything in the codomain really is an output of *f*. That is, for every element *y* in the codomain, there must be (at least one) x in the domain where f(x) = y.
Functions that are one-to-one	A function *f* is *one-to-one* if nothing in the codomain is an output via two different inputs. That is, for every choice of different elements x_1 and x_2 in the domain, $f(x_1)$ and $f(x_2)$ must be different.
Functions that have a one-to-one correspondence (invertible)	A function *f* is a *one-to-one correspondence* if it is both *one-to-one* and *onto*. This is equivalent to saying that *f is invertible*. Another way of stating this is: The function $f: A \rightarrow B$ is *invertible* if there is a function $f^{-1}: B \rightarrow A$ such that $f(x)=y$ if and only if $f^{-1}(y)=x$. The notation f^{-1} is read as "*f* inverse" and the symmetry of the definition means that $(f^{-1})^{-1}=f$.

In diagramming terms:

- **A function is *onto* if every point in the codomain has an arrow ending at that point.**
- **A function is *one-to-one* if no point in the codomain has two or more arrows ending at a point.**
- **A function is a *one-to-one correspondence (invertible)* if every point in the codomain has *exactly one arrow* ending at that point.**

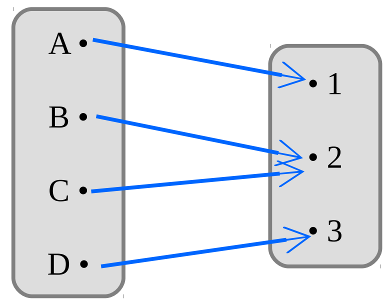
**1. Determine whether these functions are one-to-one, onto, and/or invertible.
If not, state why not.**

(___/1) a.



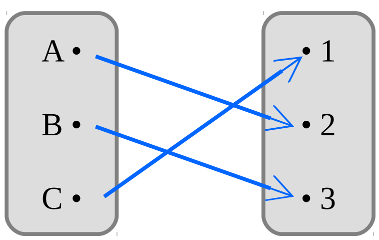
Onto?
One-to-one?
Invertible?

(___/1) b.



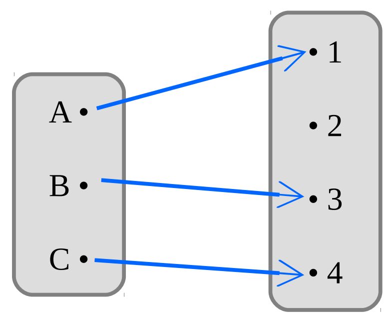
Onto?
One-to-one?
Invertible?

(___/1) c.



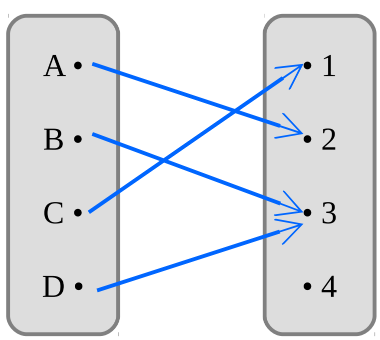
Onto?
One-to-one?
Invertible?

(___/1) d.



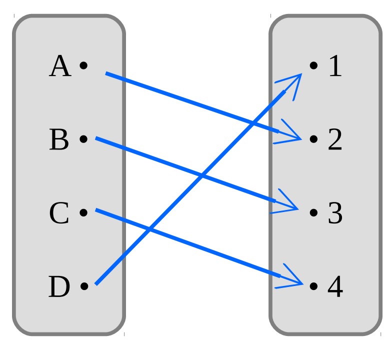
Onto?
One-to-one?
Invertible?

(___/1) e.



Onto?
One-to-one?
Invertible?

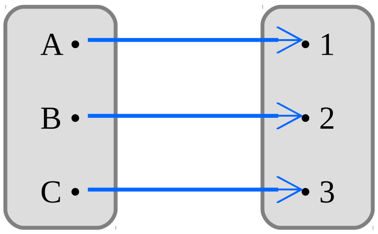
(___/1) e.



Onto?
One-to-one?
Invertible?

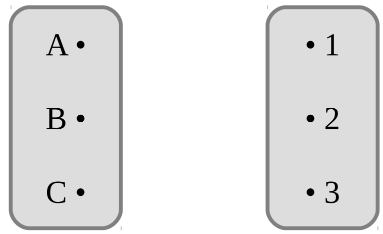
2. Draw the inversions of the following functions:

(___/1) a. $f: A \rightarrow B$

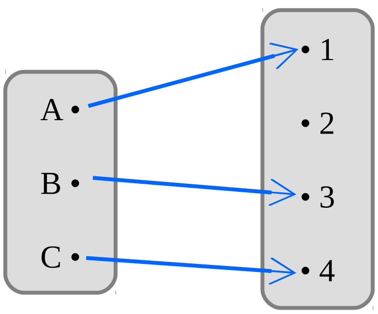


Draw $f^{-1}: B \rightarrow A$

Is this a valid function?

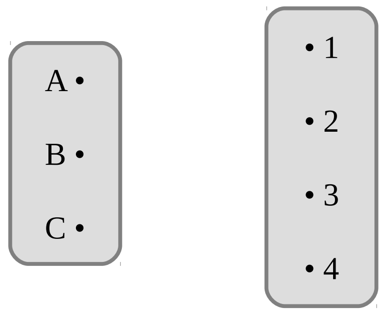


(___/1) b. $g: A \rightarrow B$

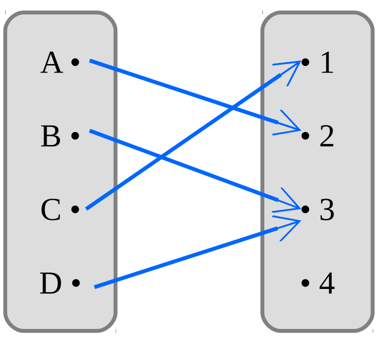


Draw $g^{-1}: B \rightarrow A$

Is this a valid function?

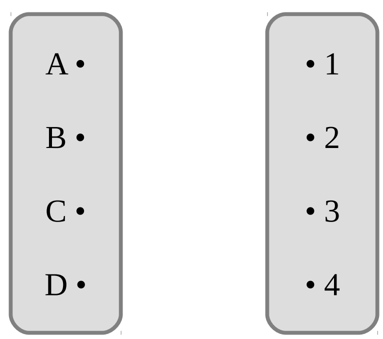


(___/1) c. $h: A \rightarrow B$



Draw $h^{-1}: B \rightarrow A$

Is this a valid function?



3. This function is not onto. Give an example of an element in the codomain and explain why no element in the domain is associated with it. **(___/1)**

$f: \mathbb{R} \rightarrow \mathbb{R}$, with $f(x) = x^2 + 4x + 1$

4. This function is not one-to-one. To demonstrate this, provide an example of two elements of the domain that are associated with the same element of the codomain. **(___/1)**

$f: \mathbb{R} \rightarrow \mathbb{R}$, with $f(x) = x^2 + 4x + 1$

3. Which of the following functions are invertible? For each noninvertible function, explain why it is not one-to-one or not onto. It might help to list out several mappings from domain → codomain to see the results.

a. $c:\mathbb{Z}\rightarrow\mathbb{Z}$, with $c(x)=x^3$ for all $x\in\mathbb{Z}$ **(___/1)**

What result of c(x) can you find where x is not in the set of all integers?

b. $s:\mathbb{N}\rightarrow\mathbb{N}$, defined so that $s(x)$ is the closest whole number to $\sqrt{(x)}$ for all $x\in\mathbb{N}$ **(___/1)**

Are there any elements in the codomain that have multiple inputs?

c. $h:\{0,1,2,3,4\}\rightarrow\{1,2,4,6,8\}$, given so that $h(n)$ is the ones' digit of 2^n for all $n\in\{0,1,2,3,4\}$ **(___/1)**

d. $g:\{0,1,2,3,4,5,6,7,8,9\}\rightarrow\{0,1,2,3,4,5,6,7,8,9\}$, given so that $g(n)$ is the ones digit of 2^n for all $n\in\{0,1,2,3,4,5,6,7,8,9\}$ **(___/1)**