Team Info (only fill out for the sheet to be turned in)

Team Name:

Group members (up to four): 1. 2.

3. 4.

Instructions
Only one sheet per team will be turned in. Each team member can work on their own sheet for practice, but then the group as a whole should discuss the answers and collaborate on the turn-in sheet. Everyone can take home their own sheets.

Goals
1. Be able to prove a proposition using “proof by contradiction”.

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1. Introductory Practice

For the statement, “If n mod 3 = 1, then n mod 9 $\neq$ 5.”

a) What is the **hypothesis *p*?** (__/1)

b) What is the **conclusion *q*?** (__/1)

c) Remember that the negation of an implication is: $\neg(p \rightarrow q) \equiv p \wedge \neg q$ (__/1)
Write out the original statement (in English terms) as a contradiction.

2. Proof by contradiction

Example: "Prove by contradiction: If n^2 is even, then n is even."

Hypothesis: n^2 is even Conclusion: n is even

A contradiction would be $\neg(p \rightarrow q) \equiv p \wedge \neg q$, or in English:
Hypothesis: n^2 is even Conclusion: n is odd

Write in math terms: $n^2=2K$ $n=2L+1$

Make equation: $(2L+1)^2=2K$
Simplify: $4L^2+4L+1=2K$
$1=2K-4L^2-4L$
$\frac{1}{2}=K-2L^2-2L$

Result: As K and L are both integers, and through the closure property of integers (addition, subtraction, and multiplication of integers result in an integer), as $K-2L^2-2L$ results in something that is *not an integer*, it shows that our counterexample is false and no counterexample can exist.

Practice 1 (__/1)

"Prove by contradiction: If n^2 is odd, then n is odd."

Practice 2 (__/1)

"Use proof by contradiction to explain why it is impossible for a number *n* to be of form 5K+3 and of the form 5L+1 for integers *K* and *L*."

(Hint: $n=5K+3$ and $n=5L+1$, so $5K+3=5L+1$ is your starting point, and keep in mind the *closure principle of integers*!)