Theorem 1:
Fundamental Theorem of Arithmetic

Every integer greater than 1 can be expressed as the product of a list of prime numbers.

This time, we will be finding a recursive formula (aka "recurrence relation") for summations.

WRITE A SUM AS A RECURSIVE FORMULA
Example 1 from the book:

Consider the sum $\sum_{i=1}^{n}(2i-1)$

Which is the same as
1 + 3 + 5 + … + (2n-1).

Use the notation s_n to denote this sum. For example, s_5 means 1 + 3 + 5 + 7 + 9.

Find a recursive description of s_n
…

So remember that we need to get all the values from 1 to n-1 for the summation. In this case, that would be $s_{(n-1)}$

So we express the summation as the sum of the first n-1 terms, plus the final term:

$s_n=\sum_{i=1}^{n}(2i-1)$ … $=[1+3+5+...+(2n-3)]+(2n-1)$

(Sum from 1 to n-1) (Final term)

$=s_{(n-1)}+(2n-1)$ (Written in terms of *s*)

Result: $s_n=s_{(n-1)}+(2n-1)$ Also need a starting value

Look at first term: $\sum_{i=1}^{1}(2i-1)=2\cdot 1-1=1$, so $s_1=1$

Resulting recursive formula: $s_n=s_{(n-1)}+(2n-1)$, $s_1=1$

OTHER USES OF INDUCTION

Show that n^3+2n is divisible by 3 for all positive integers n.

First, check D(1):

$D(n)=n^3+2n \quad \rightarrow \quad D(1)=1^3+2\cdot 1 \quad \rightarrow \quad D(1)=1+2=3$

Next, let's say that some positive integer m is given, such that we've been able to check D(1) through D(m-1) are true.

$D(n)=n^3+2n \quad \rightarrow \quad D(m-1)=(m-1)^3+2(m-1)$

$\rightarrow \quad D(m-1)=m^3-3m^2+3m-1+2m-2$ pull out terms to form D(m)…

$\rightarrow \quad D(m-1)=m^3-3m^2+3m+2m-3 \quad \rightarrow \quad D(m-1)=(-3m^2+3m-3)+m^3+2m$

We know that $D(n)=n^3+2n$, so we can swap it out…

$D(m-1)=(-3m^2+3m-3)+D(m)$

Rewrite in terms of D(m)…

$D(m)=D(m-1)-(-3m^2+3m-3) \quad \rightarrow \quad D(m)=D(m-1)+3m^2-3m+3$

We know previously that D(1) through D(m-1) are true for the statement "divisible by 3 for all positive integers",
so we will rewrite D(m-1) as 3K and sub it out.

$D(m)=3K+3m^2-3m+3 \quad \rightarrow \quad D(m)=3(K+m^2-m+1)$

So our proof is that D(m) is divisible by 3.