Note: Proofs dealing with even/odd/divisibility <u>must</u> be simplified to one of these forms in order to get full credit!

Definition of an even integer

$n = 2K$

Definition of an odd integer

$m = 2L + 1$

Definition of a divisible integer

$n = 4M$

Is an integer divisible by M

Divisibility

An integer *n* is divisible by nonzero integer *k* if:

$n = k \cdot q$

The division theorem

For all integers *a* and *b*, there is an integer *q* (quotient) and an integer *r* (remainder):

$a = b \cdot q + r$

Modulus

73 mod 6 = 1

Proof by induction:

Recursive ↔ Closed formula equivalence

1. Check a_1 for both the recursive and closed formulas match.
2. Rewrite recursive formula as a_m
3. Find $a_{(m-1)}$ through the closed formula.
4. Plug $a_{(m-1)}$ into the recursive formula from step 2.
5. Simplify to show that resulting a_m and the closed formula a_n match.

Summation ↔ other formula equivalence

1. Redefine the whole proposition in terms of $m-1$.
2. Rewrite the summation in terms of: The sum from $i=1$ to $m-1$, plus the final term of the summation at $i=m$.
3. Replace the summation from $i=1$ to $m-1$ with the *other formula*, written in terms of $m-1$.
4. Work out and simplify, so that the result matches the original *other formula*.

Summations → Recursive sequence

1. Find the value of s_1 through the summation given.
2. Restate the summation as s_n , including $s_{(n-1)}$ plus the final term of the summation for $i=n$.
3. Simplify to get the recursive formula.
   <u>Final answer is both</u> s_1 **<u>and</u>** s_n **<u>!</u>**

Other proofs by induction

1. Check to make sure D(1) is true.
2. Assume that the proposition has been proven for D(1) through D(m-1).
3. Write out D(m-1) and simplify so that D(m) is part of the equation.
4. Replace part of the equation with D(m).
5. Solve for D(m).
6. Remember step 2, replace D(m-1) with the proper (“already proven”) definition.
7. Simplify to prove the original proposition.

For proof by contradiction, your counterexample must make the **hypothesis true and the conclusion false.**

Decimal representation:

$$X = \sum_{i=0}^{n} d_i \cdot 10^i$$
$$= d_n \cdot 10^n + d_{(n-1)} \cdot 10^{(n-1)} + \ldots + d_2 \cdot 10^2 + d_1 \cdot 10^1 + d_0 \cdot 10^0$$

Algorithm to write a number in base *b*:

1. Input a natural number *n*
2. While n > 0, do the following:
   1. Divide *n* by *b* and get a quotient *q* and remainder *r*.
   2. Write *r* as the next (right-to-left) digit.
   3. Replace the value of *n* with *q*, and repeat.

Table of Contents

Please read and sign

I understand that this exam is a solo effort and the following is not allowed: Copying from a classmate or outside source (including storing extra information on hidden items), using a graphing calculator on this exam, discussing the exam with classmates during the exam period, and other behavior that is deemed academic dishonesty. I understand that, if I am suspected of cheating, I will be asked to leave the classroom and receive a 0 on the exam. I understand that if I need clarification on a question or otherwise need assistance, I can ask the instructor during the exam time.

Your printed name *Your signed name*

Score: Total Possible Points: 67

(10 pts) Question 1: Direct proofs, Chapter 2.1

Prove the following statements using a **direct proof**:

____ / 5 — (a) For any two integers ***m*** and ***n***, where ***m*** is even and ***n*** is odd, $m+n$ is always odd.

____ / 5 — (b) For any two integers ***m*** and ***n***, where ***m*** is odd and ***n*** is divisible by 4, $m \cdot n$ is always divisible by 4.

(5 pts) Question 2: Proof by contrapositives , Chapter 2.1

Prove the following statements using a **proof by contrapositive**

(Reminder, for $p \to q$, the contrapositive is $\neg q \to \neg p$)

____ / 5 — For all integers *n*, if n^2 is even, then *n* is even.

(8 pts) Question 3: Division theorem, Chapter 2.2

Fill in the blanks in the style of the **division theorem: (Note: remainder must be positive)**

		Quotient		Remainder
____ / 2 — (a)	9 =	________	$\times 4+$	________
____ / 2 — (b)	23 =	________	$\times 5+$	________
____ / 2 — (c)	-31 =	________	$\times 4+$	________
____ / 2 — (d)	-5 =	________	$\times 2+$	________

(20 pts) Question 4: Induction, Chapter 2.3

Use **induction** to prove the following statements:

____ / 10 — (a) The sequence defined by $a_k = a_{(k-1)} + 4$ for $k \geq 2$ where $a_1 = 1$ is equivalently described by $a_n = 4n - 3$

Use **induction** to prove the following statements:

____ / 10 — (b) $\sum_{i=1}^{n}(2i-1)=n^2$ for each $n\geq 1$.

(10 pts) Question 5: Proof by contradiction, Chapter 2.5

Use **proof by contradiction** to prove the following statements:

____ / 10 — If $n^2 - 1$ is divisible by 5, then n is not divisible by 5.

(6 pts) Question 6: Numeric representations, Chapter 2.6

Write the following numbers as the sum of multiples of powers of the base. Do **not** simplify!

____ / 2 — (a) $(246)_{10} =$

____ / 2 — (b) $(0100\,1001)_2 =$

____ / 2 — (c) $(F00D)_{16} =$

(4 pts) Question 7: Numeric representations, Chapter 2.6

Hex	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hex	8	9	A	B	C	D	E	F
Binary	1000	1001	1010	1011	1100	1101	1110	1111

____ / 2 — (a) Convert $(1111\ \ 0000\ \ 0000\ \ 1101)_2$ to base-16

____ / 2 — (b) Convert $(CAF\,3)_{16}$ to base-2

(4 pts) Question 8: Numeric representations, Chapter 2.6

____ / 2 — (a) Convert $(75)_{10}$ to base-2

____ / 2 — (b) Convert $(0101\,1010)_2$ to base-10

Extra Credit

(1 pt) 1. Fill in the blanks in the style of the division theorem:

$$(9k^2+5) = \_\_\_\_\_\_\_\_\_\_ \cdot 3 + \_\_\_\_\_\_\_\_\_\_$$

(2 pts) 2. Prove existence:
There exists a positive integer n such that $n \bmod 7 = 1$ and $n \bmod 3 = 2$.

(0.25 pts) 3. Draw a cat:

Survey (If desired, tear off and turn in separately without name)

1. Please tell me whether the following content was useful:

    a. Online video lectures

    b. In-class exercises

    c. In-class quizzes

    d. Online quizzes and webpage projects

    e. Exam review material (Chapter guide, review questions)

2. Did you feel like you had enough resources for you as you studied for the exam?

3. Any suggestions for how to improve the course?

4. Any comments on what is going well with the course?