Section 1: Cartesian Products

The result of a Cartesian Product is an ordered pair, such as (a, b) – like what you would see in Algebra.

For some sets of numbers A and B, the result of $A \times B$ **will be a combination of all of each set's elements together.** $A \times B = \{(a,b) : a \in A, b \in B\}$

For example, for A = { 1, 2 } and B = { 4, 5, 6 },
The elements of $A \times B$ **are:**

B → ***A*** ↓	**4**	**5**	**6**
1	(1,4)	(1,5)	(1,6)
2	(2,4)	(2,5)	(2,6)

So $A \times B = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6)\}$

1. Given the following sets, calculate each Cartesian Product. Write it out as a table, like above.
A = { 1, 2 } B = { 3, 4 }

a. $A \times B$ **(___/2)**

B → ***A*** ↓	**3**	**4**
1		
2		

b. $B \times A$ **(___/2)**

A → ***B*** ↓	**1**	**2**
3		
4		

2. Calculate the Cartesian Product. Write it out as a table.

A = { 2, 4, 6 } B = { 1, 3 }

a. $A \times B$ **(___/2)**

B → ***A*** ↓	**1**	**3**
2		
4		
6		

b. $B \times A$ **(___/2)**

A → ***B*** ↓	**2**	**4**	**6**
1			
3			

3. Calculate the Cartesian Products. Write it out as a set of coordinate pairs.

A = { 2, 4 } B = { 1, 3 } C = { 3, 4, 5, 6 }

a. $A \times B$ **(___/2)**

b. $A \times C$ **(___/2)**

c. $B \times C$ **(___/2)**

d. $C \times B$ **(___/2)**

e. A^2 **(aka** $A \times A$ **)** **(___/2)**

4. For the given sets, find the intersections, unions, and differences.

$$A = \{1\} \qquad B = \{3,5,7\} \qquad C = \{3,5,9,11\}$$
$$A \times B = \{(1,3),(1,5),(1,7)\} \qquad A \times C = \{(1,3),(1,5),(1,9),(1,11)\}$$

a. $(A \times B) - (A \times C)$ *remember, the result is the first set's elements, but none of the elements that are shared by the second set.* **(___/2)**

b. $(A \times C) - (A \times B)$ **(___/2)**

c. $A \times (B \cup C)$ **(___/2)**

d. $A \times (B \cup C) \cap (A \times B)$ **(___/2)**

e. $(A \times B) \cup (A \times C)$ **(___/2)**

Section 2: Partitions

The Partition of a set, usually denoted by S, is a set of subsets that, when combined together, form the original set.

Definition:
For a set A, a *partition* of A is a set S = { $S_1, S_2, S_3, ...$ } of subsets of A (each set S_i is called *a part of* S), such that:

1. For all i, $S_i \neq \emptyset$. That is, each part is nonempty.
2. For all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$. That is, different parts have nothing in common.
3. $S_1 \cup S_2 \cup S_3 \cup ... = A$. That is, every element in A is in some part.

So if we have the set A = { 1, 2, 3, 4 }, we could form various partitions, such as:
Partition version 1: { {1}, {2}, {3}, {4} }
Partition version 2: { {1, 2}, {3, 4} }
Partition version 3: { {1, 2, 3}, {4} }
Partition version 4: { {1, 2, 3, 4 } }

Essentially, it can be any combination of subsets of whatever size, so long as all elements of A are represented in the partition.

5. For the given set, write out all possible partitions. A = {1, 2} **(___/1)**

Partition 1:

Partition 2:

6. For the given set, write out all possible partitions. B = { 1, 2, 3 }. *Remember that order of elements in a set do not matter. ({{2, 3}, {1}} and {{1}, {2, 3}} is the same)* **(___/2)**

Partition 1:

Partition 2:

Partition 3:

Partition 4:

Partition 5:

7. For the given set, find partitions that meet the requirements. (There could be more than one answer). **A = { 1, 2, 3, 4, 5, 6 }**

a. Find a partition where each part has the same size. **(___/1)**

b. Find a partition where no two parts have the same size. **(___/1)**

c. Write out the partition that has as many parts as possible. **(___/1)**

d. Write out the partition that has as few parts as possible. **(___/1)**

8. Which of the following are partitions of the set A = { 1, 2, 4, 8, 16, 32, 64, 128 }? For those that are not, explain why.

a. { 1, 2, { 4, 8, 16 }, { 32, 64, 128 } } **(___/2)**

b. { { 1, 16 }, { 32, 64, 2 }, { 8, 4, 16 }, { 128 } } **(___/2)**

c. { {1, 128}, { 8, 4, 16 }, { 64, 2 } } **(___/2)**

d. { {8, 2, 4}, {16, 1, 128}, {32, 64} } **(___/2)**

Section 3: Power Sets

The Power Set of A is defined as $\wp(A) = \{S : S \subseteq A\}$. For example,
A = {1, 2, 3, 4}, $\wp(A)$

= { $\emptyset$, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4} }

Essentially, all the possible subsets of A.

As a simpler example, the Power Set $\wp(\{1\})$ = { $\emptyset$, {1} }

9. Find the Power Set for each set.

a. $\wp(\{1,2\})$ **(___/1)**

b. $\wp(\{3,4\})$ **(___/1)**

c. $\wp(\{1,2,3\})$ **(___/2)**