Benchmarking Hybrid Quantum Gates and Algorithms Under Noise

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ABSTRACT

- **Hybrid quantum computers**, which use discrete variables (qubits) and continuous variables (oscillators/qumodes) may be able to simulate molecular dynamics beyond the capacity of classical or purely qubit-based computers.
- The Hamiltonian of a system represents its total energy and is used to study how a system evolves over time.
- Hamiltonian simulation on hybrid quantum architectures is made difficult by approximation errors and **noise**. General methods for characterizing hybrid gate behavior under noisy conditions remains an open research challenge.
- A specific model (the Morse Hamiltonian) is synthesized to study the effect of approximation and noise on the behavior of simulations.

BACKGROUND

Qubits and qumodes are basic units of quantum information. Qubits can be in the 0 or 1 state (physically realized by the bottom 2 levels of an anharmonic oscillator, seen in figure 2), or a superposition of those states. Qumodes can be in any of an infinite number of different states or any superposition of those states (physically realized by the energy levels of harmonic oscillators).

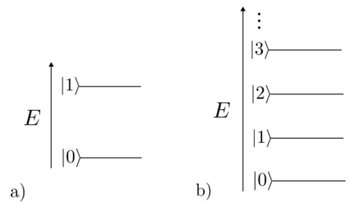

Figure 1: Distinct states of a qubit (a) compared with the infinitely distinguishable states of an oscillator (b)[1].

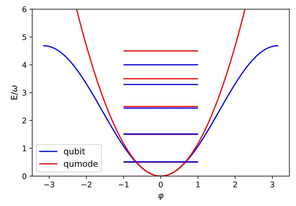

Figure 2: First five energy levels of a transmon qubit (blue) and an oscillator (red)[1].

The Morse Potential is a specific interatomic interaction model that describes the potential energy of a diatomic molecule. This model has a limited number of defined energy states, seen in figure 3.

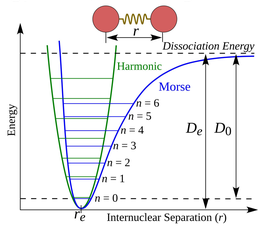

Figure 3: Limited energy levels supported by the Morse Potential Compared to the infinite levels of an Oscillator[2].

$$\hat{H} = \frac{1}{2\mu}\hat{p}^2 + D(1 - e^{-\beta(\hat{q}-q_0)})^2$$

Equation 1: Full equation for the Morse Hamiltonian. Where μ is the diatomic mass, $\hat{p}$ is the momentum operator, D is the well depth (seen in figure 3), β controls the width of the potential (seen in figure 3), $\hat{q}$ is the distance between atoms, and q_0 is the equilibrium length of the bond.

There are many sources of error in quantum systems, such as amplitude damping (a loss of energy that causes decay towards the ground state), dephasing (fluctuations in phase which destroy superposition), and leakage in qubits (transition to a higher energy state than 1, which is an undefined state). Physically, these may be caused by thermal fluctuations, external fields, or fluctuations in the system itself.

OBJECTIVE

Build an optimization function which allows for the study of error from approximation in Hamiltonian simulation and the study of the behavior of hybrid quantum gates under noise.

METHODS

Custom python code was developed using QuTip[3] to optimize the parameters of a gate sequence to match the time evolution of the Morse Hamiltonian. The optimization was bounded to the defined energy levels of the Morse Potential.

$$U = \prod_{j=1}^{d} D(\alpha_j)R_{\phi_j}(\theta_j)V(t_j) = [D(\alpha_d)R_{\phi_1}(\theta_d)V(t_d)]\cdots[D(\alpha_1)R_{\phi_1}(\theta_j)V(t_1)]\,.$$

Equation 2: Native gate sequence used to match the time evolution of the Morse Hamiltonian. U is the exponential of the Morse Hamiltonian, similar to the exponentiation of the always-on Hamiltonian in equation 4. The right hand side the gate sequence used to synthesize the time evolution.

$$H' = (\chi\sigma_z + \omega I)\hat{n} = \begin{bmatrix}(\omega+\chi)\hat{n} & 0\\ 0 & (\omega-\chi)\hat{n}\end{bmatrix} \qquad V(t) = e^{-iH't} = \begin{bmatrix}e^{-i(\omega+\chi)t\hat{n}} & 0\\ 0 & e^{-i(\omega-\chi)t\hat{n}}\end{bmatrix}$$

Equation 3: The always-on Hamiltonian.

Equation 4: Time evolution of the always-on Hamiltonian.

$$R_\phi(\theta) = e^{-i\frac{\theta}{2}(\sigma_x\cos\phi + \sigma_y\sin\phi)}$$

Equation 5: Qubit xy-rotation gate.

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

Equation 6: Displacement gate.

RESULTS

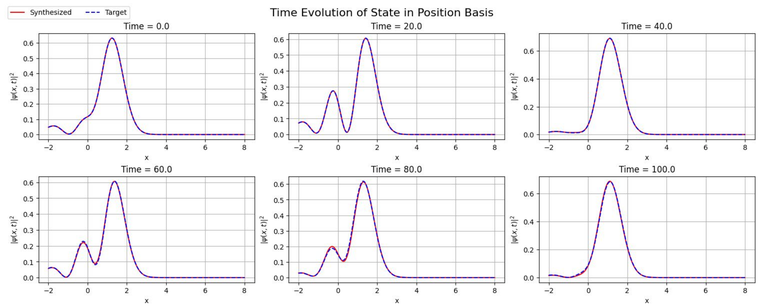

Figure 4: Time evolution of the wavefunction (position probabilities) of a coherent (classical-like) state in the bounded space from a synthesis of depth 12 optimized over time t=1. The red line is the synthesized time evolution, the blue line is the true time evolution.

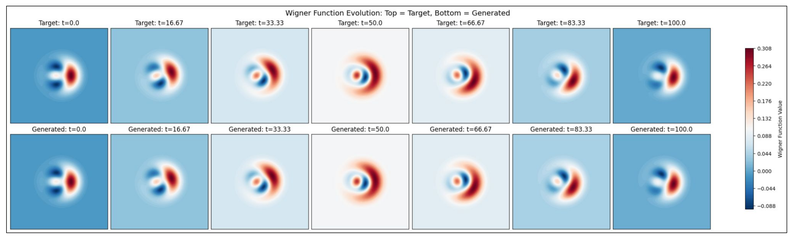

Figure 5: Time evolution of the wigner function (visualization of the distribution of the position and momentum information) of a coherent state from the same synthesized time evolution in figure 4. The top is the true time evolution, the bottom is the synthesized time evolution.

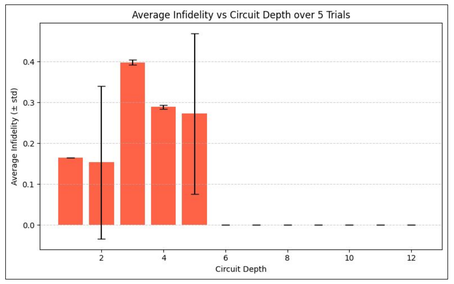

Figure 6: The effect of circuit depth on the infidelity of the synthesized time evolution (less is better).

DISCUSSION

Accurate simulation of the Morse Hamiltonian is achievable when the system is strictly bounded to the defined energy levels of the Morse Potential. Under these conditions, synthesized time evolutions closely match the true dynamics for durations up to roughly 100 times the optimization window. Beyond that, the deviation from the true dynamics has significant error. These results highlight the importance of a focus on the defined behavior of a system when optimizing a gate sequence to match its dynamics. Notably, with such an approach, accurate synthesis can be consistently achieved with circuit depths as low as six.

LIMITATIONS & FUTURE DEVELOPMENT

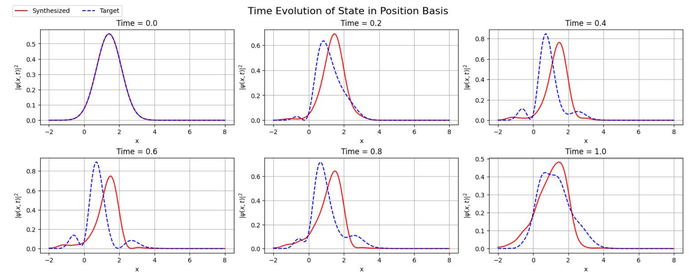

Figure 7: Time evolution of the wavefunction of a coherent state in a large space (includes energy levels outside the defined range of the Morse Potential) from a synthesis of depth 8 optimized over time t=1.

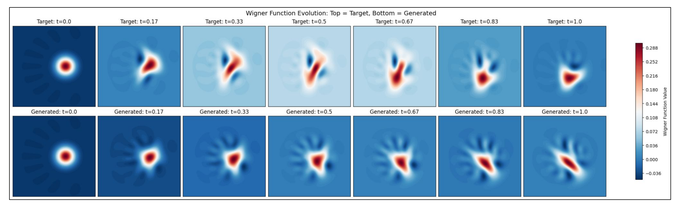

Figure 8: Time evolution of the wigner function of a coherent state from the same synthesized time evolution in figure 7.

When optimizing over a space that contains energy levels higher than the bounded levels of the Morse Potential, the optimization fails to generate accurate behavior, even within the optimization window. Without extra energy levels included, noise cannot be faithfully represented by the simulation, thus changes need to be made to the optimization to successfully synthesize time evolutions in larger systems.

Once such an optimization is created. Engineering noise can be introduced in the simulation to study the effect of noise on the behavior of hybrid quantum gates. Data from such simulations will be used to develop a protocol for benchmarking the behavior of any hybrid quantum gate under noise.

REFERENCES & ACKNOWLEDGEMENTS

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