Laplace Transform

Why?

(1) Reduces calculus to algebra for Linear Time Invariant D.Eqs.

(2) The complex realm of this transform provides great insight into the time behavior of the solutions of the D.Eqs.

(3) It is the only real place that we can <u>design</u>.

It is easy to convert a D.Eq. to its Laplace equivalent. The only heartburn comes from any initial conditions. Oh happy day, we are looking for the Laplace Transform to find a transfer function and by great good fortune, transfer functions are defined to be initial condition free. (initial conditions all zero!)

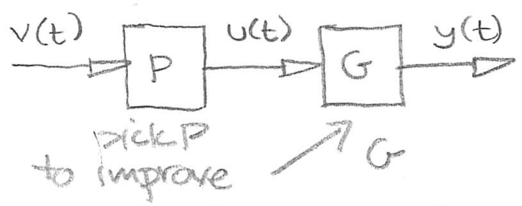
Questions?

Do you understand how to convert D.E. to the Laplace domain and find transfer functions?

Do you understand that we can use circuit analysis to go directly to the Laplace domain?

The Laplace Transform

Why bother? Why not just use the differential equation describing the system?



with $G: \dddot{y} + A_1 \ddot{y} + A_2 \dot{y} + A_3 y = B_1 \dot{u} + B_2 u$

$P: \ddot{u} + C_1 \dot{u} + C_2 u = E_1 \dot{v} + E_2 v$

The combined equation is

convolution →

$$y^{(5)} + (A_1 + C_1) y^{(4)} + (C_2 + A_1 C_1 + A_2) y^{(3)} + (A_1 C_2 + A_2 C_1 + A_3) y^{(2)}$$
$$+ (A_2 C_2 + A_3 C_1) y^{(1)} + A_3 C_2 y = B_1 E_1 v^{(2)} + (B_1 E_2 + B_2 E_1) v^{(1)} + B_2 E_2 v$$

Now pick C_1 and C_2 to improve σ.

It is very difficult to know what to do. Transfer functions are more suited to control synthesis because of simplified operations between blocks. Convolution is replaced by multiplication. We can change inputs without re-convolving. Laplace Transforms, despite their reputation, are our friends.

Formally defined the Laplace Transform ($F(s)$) of a function ($f(t)$) is

$$\mathcal{L}(f(t)) = F(s) = \int_{0-}^{\infty} f(t)\, e^{-st}\, dt$$

where s is a complex variable

$s = \sigma + j\omega$

important later on

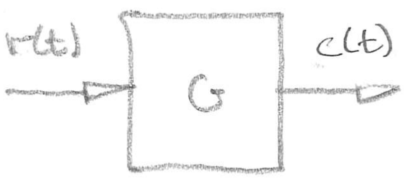
Not to worry though, we will prefer tables to the integral in this class.

We go to tables for Laplace Transforms of inputs (signals) and to invert system responses.

$f(t)$	$F(s)$	
$u(t)$	$1/s$	(step)
$tu(t)$	$1/s^2$	(ramp)

(see book for more)

We convert differential equations to the Laplace domain through the simplest of procedures. And since we are after transfer functions we do not worry about initial conditions (the very definition of transfer functions eliminates initial conditions)



generally

$$G: \quad a_n \frac{d^n}{dt^n} c(t) + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} c(t) + \dots + a_0 c(t)$$

$$= b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1}}{dt^{m-1}} r(t) + \dots + b_0 r(t)$$

Laplace Transforming G is simple n^{th} order derivatives are replaced by s^n, $(n-1)^{th}$ with s^{n-1} etc., signals ($c(t)$ + $r(t)$) are replaced w/ their Laplace transforms $C(s)$ + $R(s)$ respectively.

so

$$G: \quad a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \dots + a_0 C(s)$$

$$= b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \dots + b_0 R(s)$$

$C(s)$ and $R(s)$ can be easily factored out

$$C(s)\left[a_n s^n + a_{n-1} s^{n-1} + \dots + a_0\right] = R(s)\left[b_m s^m + b_{m-1} s^{m-1} + \dots + b_0\right]$$

the transfer function is output/input $\left[C(s)/R(s)\right]$

so $$\frac{C(s)}{R(s)} = G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0}$$

note that if $R(s) = 1$ $r(t)$ is an impulse and therefore $G(s)$ is also called the impulse response