Let's make some transfer functions!

#1 The Dynamic Balance (dual spring/mass)

$$m_1\ddot{x}_1 + D_1\dot{x}_1 + K_1 x_1 + K_{12}(x_1 - x_2) = f$$

$$m_2\ddot{x}_1 + K_{12}(x_2 - x_1) = 0$$

This is a coupled (through spring K_{12}) system of differential equations

Laplace transform $(x_1(0), \dot{x}_1(0), x_2(0), \dot{x}_2(0)) = 0$

$$(m_1 s^2 + D_1 s + K_1 + K_{12})X_1(s) - K_{12}X_2(s) = F(s)$$

$$(m_2 s^2 + K_{12})X_2(s) - K_{12}X_1(s) = 0$$

Oh no! How can I obtain the transfer function $\dfrac{X_1(s)}{F(s)}$ or $\dfrac{X_2(s)}{F(s)}$?

Simultaneously Solve for $X_1(s)$ and $X_2(s)$
(algebra with awful looking coefficients)

$$A_1 = m_1 s^2 + D_1 s + K_1 + K_{12}$$
$$B_1 = -K_{12}$$
$$C_1 = -K_{12}$$
$$D_1 = m_2 s^2 + K_{12}$$

$$A_1(s)X_1(s) + B_1(s)X_2(s) = F(s)$$

$$C_1(s)X_1(s) + D_1(s)X_2(s) = 0$$

$$X_1(s) = \frac{\begin{vmatrix} F(s) & B_1(s) \\ 0 & D_1(s) \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ C_1 & D_1 \end{vmatrix}} \qquad X_2(s) = \frac{\begin{vmatrix} A_1 & F \\ C_1 & 0 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ C_1 & D_1 \end{vmatrix}}$$

common (called the characteristic equation)

$$= A_1 D_1 - B_1 C_1$$

$$(m_1 s^2 + D_1 s + K_1 + K_{12})(m_2 s^2 + K_{12}) - K_{12}^2$$

$$X_1(s) = \frac{F(s)\,(m_2 s^2 + K_{12})}{(m_1 s^2 + D_1 s + K_1 + K_{12})(m_2 s^2 + K_{12}) - K_{12}^2}$$

zeros

poles

I claimed that motion would stop for a certain frequency, when would this occur?

$$m_2 s^2 + K_{12} \Big|_{s=j\omega} = 0$$

$$\omega^2 = \frac{K_{12}}{m_2}$$

$$\omega = \sqrt{\frac{K_{12}}{m_2}}$$

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What about systems that aren't linear? (most aren't)

We can often linearize about an operating point, call this new point zero, and proceed as if nothing had happened.

Functions

Linear	Affine	Linear, Affine, or Non-Linear
$y = mx$	$y = mx + b$	$y = f(x)$

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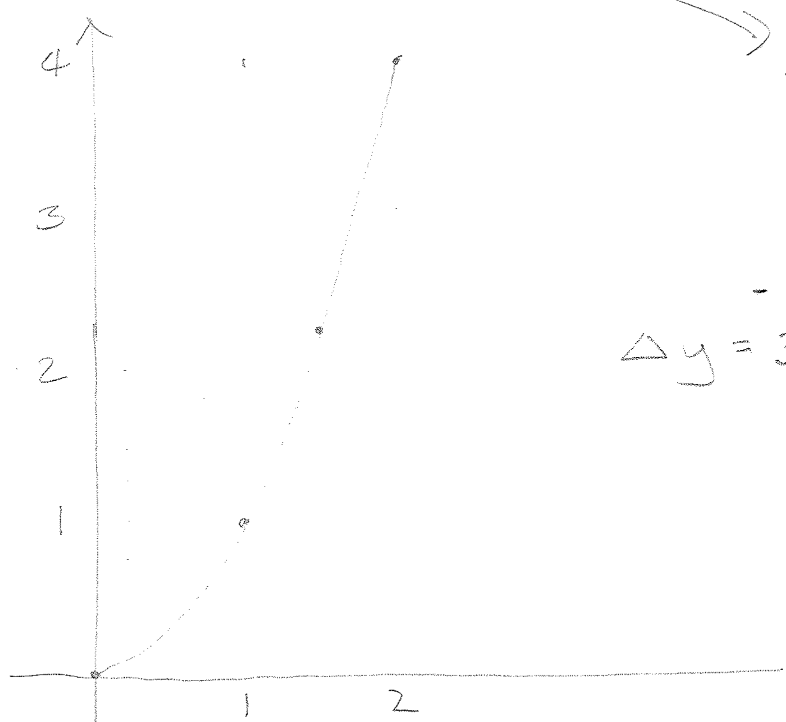
We must work with linear functions

$\Delta y = m \Delta x$, linearized about some operating point

Let $y = x^2$ $x_0 = 1.5$ → $y = 2.25$



$y' = 2x$, $y'|_{x=x_0} = 3$

$\Delta y = 3 \Delta x$ at $x_0 = 1.5$

new 'zero'

Procedure

(1) select (or be given) your operating pt. x_0

(2) take the derivative $\left.\frac{dy}{dx}\right|_{x=x_0}$

(3) $\Delta y = \left.\frac{dy}{dx}\right|_{x=x_0} \Delta x$

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Review of Electric Circuit Theory

3-Laws

Ohm's Law

$\frac{V}{I} = R$

generalizes to

$\frac{V}{I} = Z$ ← impedance

Kirchhoff's Voltage Law

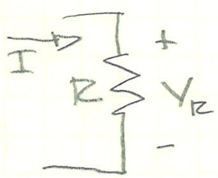
The sum of voltages around any circuit loop is zero.

(We cannot accumulate voltage on a perfect conductor.)

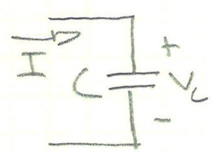
Kirchhoff's Current Law

The net current into or out of a circuit node is zero.

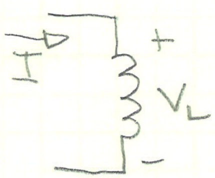
(Charge can't bunch at a node.)



$\frac{V_R}{I} = R$



$\frac{V_C}{I} = Z_C = \frac{1}{sC}$



$\frac{V_L}{I} = Z_L = sL$

2 - Primary Analysis Techniques

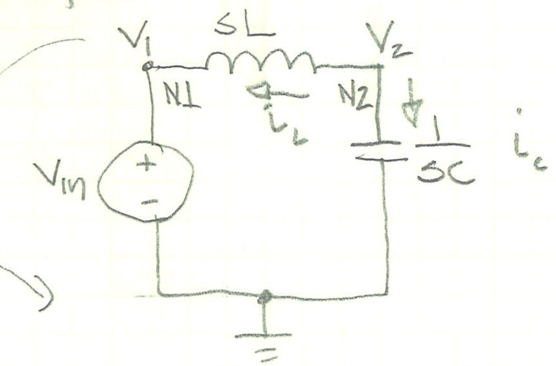
Node Voltage Method

Assign a voltage to every node in the circuit including a zero voltage point called ground.

Using the assigned voltages calculate the current into (or out of the node) use the same direction for each current don't try to guess. Solve for the voltages.

Example

referenced to ground



N1 $V_1 = V_{in}$ (no equation necessary)

N2 currents out of the node

$$\frac{V_2 - V_{in}}{sL} = i_L \qquad \frac{V_2 - V_{in}}{sL} + V_2 sC = 0$$

$$\frac{V_2 - 0}{\frac{1}{sC}} = i_c$$

solve for the transfer function $\frac{V_2}{V_{in}} = ?$

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Steps to Node Voltage Analysis

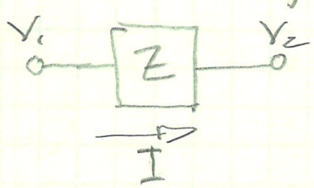
(1) Choose one node to be the common or ground node in the circuit.

(2) Assign a named voltage to every other circuit node ($V_1, V_2, \ldots$); ($V_A, V_B, \ldots$), etc.

(3) Choose a node current direction. Choose all into the node or all out of the node (never both!).

(4) Assign current names from (or into) each non-ground circuit node. ($i_1, i_2, \ldots$), ($i_A, i_B, \ldots$), etc. (or choose something more descriptive.

(5) Using Ohm's Law, write an equation for each node current



$$I = \frac{V_1 - V_2}{Z}$$

(note that you will know some voltages (the sources))

(6) Sum the currents for each individual node to zero.

(7) Solve for the required ratio.