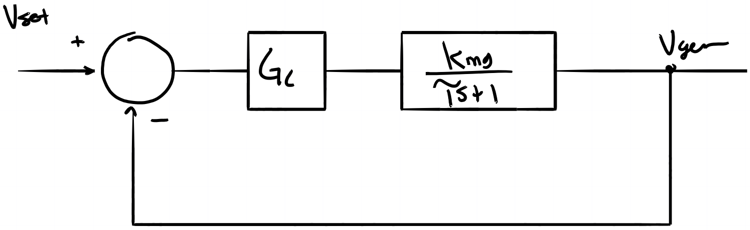
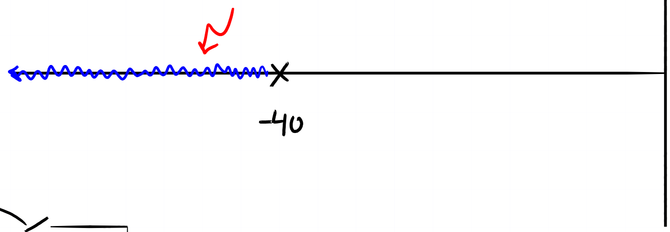


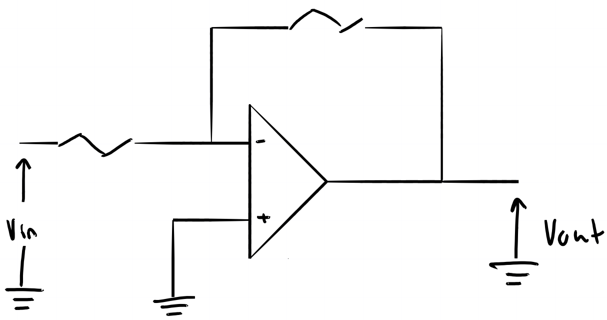
$$G_{mg} = \frac{36.4}{s+40}$$

Method 1: Gain Factor compensation ($G = k =$ constant)

Because this is on the axis, it will never overshoot

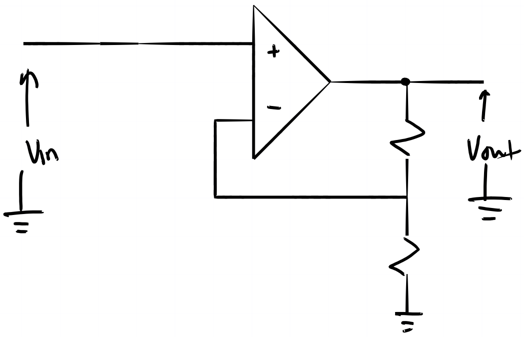
s-plane



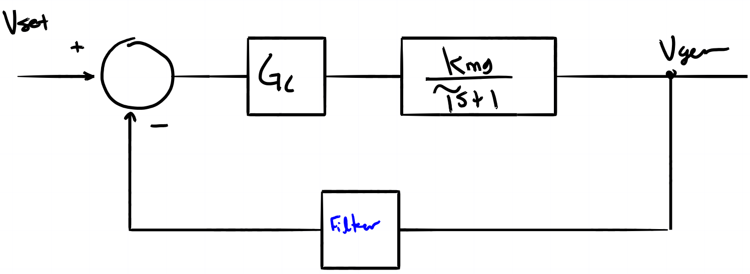


Closed loop transfer function

$$\frac{36.4k_c}{s+40+k_c}$$

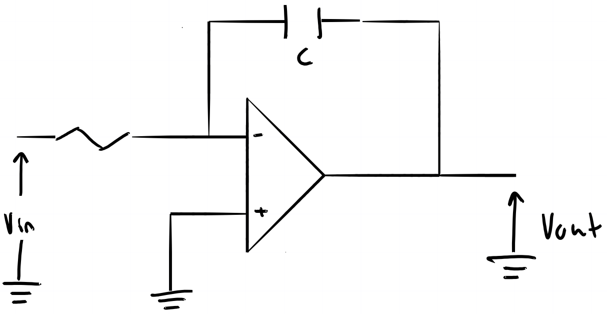


2: Filter

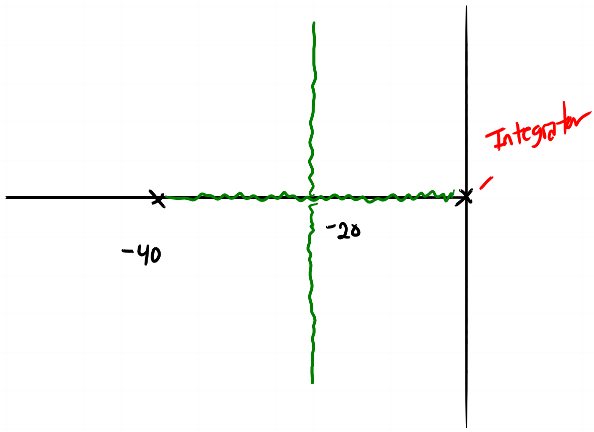


The filter would cause oscilatory effects.

Method 2: Integrator/Gain Compensation



Integrators tend to drift, but placing it in a closed loop circuit will compensate for the drift by correcting it.



Advantages

- Step input has zero steady state error

Disadvantages

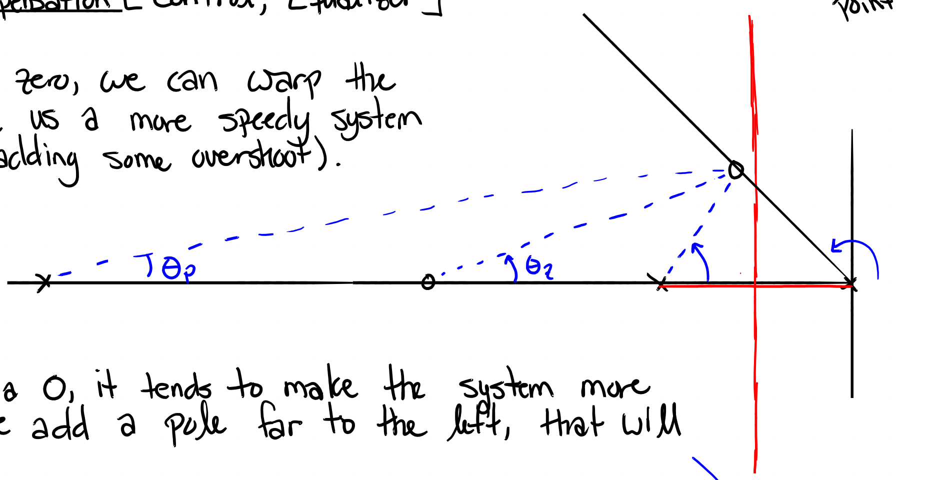
- Slower
- Potential for Overshoot

Gain = 36.4 · K → so if we want to operate at -20:

$$400 = 36.4 \cdot K$$
$$\Rightarrow K = \frac{400}{36.4}$$

Lead Compensation [Control, Equalizer]

By adding a zero, we can warp the locus to give us a more speedy system (also while adding some overshoot).



When we add a 0, it tends to make the system more noisy. If we add a pole far to the left, that will help.

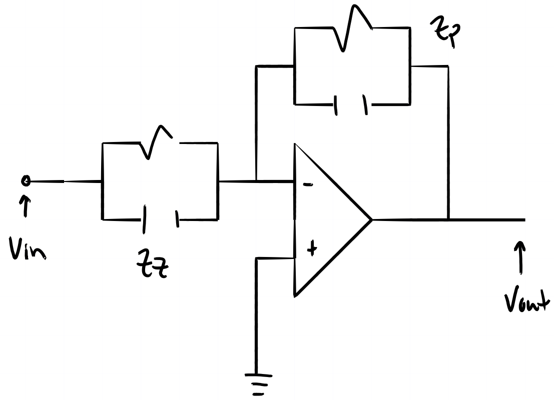
↳ Takes care of high frequency

↳ ~ 10 times the distance of the zero

zeros tend to amplify noise

How do you build it?

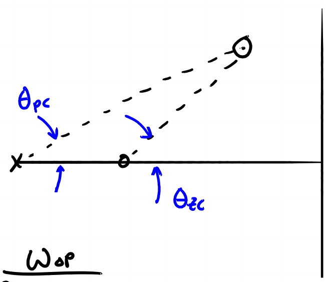
$$\frac{V_{out}}{V_{in}} = -\frac{Z_P}{Z_Z}$$

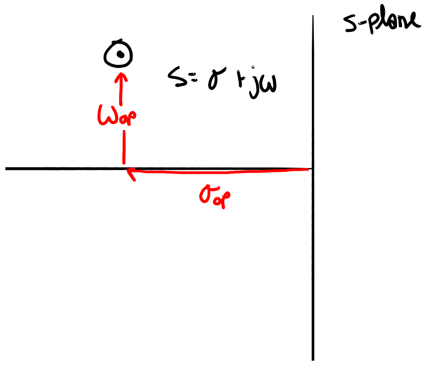


Now lets approach this design analytically

Net angular contribution:

$\theta_{zc} - \theta_{pc}$





$$\Rightarrow \tan^{-1}\frac{\omega_{op}}{z_c - \sigma_{op}} - \tan^{-1}\frac{\omega_{op}}{p_c - \sigma_{op}}$$

$$\underbrace{\tan\left(\tan^{-1}\frac{\omega_{op}}{z_c - \sigma_{op}} - \tan^{-1}\frac{\omega_{op}}{p_c - \sigma_{op}}\right)} = \tan(\theta_{net})$$

Math tables to simplify $\longrightarrow$ $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow \frac{\omega_{op}(p_c - \sigma_{op}) - \omega_{op}(z_c - \sigma_{op})}{(z_c - \sigma_{op})(p_c - \sigma_{op}) + \omega_{op}^2} = \tan(\theta_{net})$$

$$\Rightarrow \tan(\theta_{net}) = \frac{\omega_{op} p_c - \omega_{op}\sigma_{op} - \omega_{op} z_c + \omega_o \sigma_{op}}{z_c p_c - \sigma_{op}(z_c + p_c) + \sigma_{op}^2 + \omega_{op}^2} = \frac{\omega_{op}(p_c - z_c)}{z_c p_c - \sigma_{op}(z_c + p_c) + \sigma_{op}^2 + \omega_{op}^2}$$

Let $p_c = \alpha z_c$ where α is typically ≤ 10

Further simplifying

$$\Rightarrow \frac{\omega_{op}(\alpha z_c - z_c)}{\alpha z_c^2 - \sigma_{op}(\alpha+1)z_c + \sigma_{op}^2 + \omega_{op}^2} = \frac{\omega_{op}(\alpha-1)z_c}{\alpha z_c^2 - \sigma_{op}(\alpha+1)z_c + \sigma_{op}^2 + \omega_{op}^2}$$

$$\Rightarrow \alpha z_c^2 \tan\theta_{net} - [\sigma_{op}(\alpha+1)\tan\theta_{net} + \omega_{op}(\alpha-1)]z_c + (\sigma_{op}^2 + \omega_{op}^2)\tan\theta_{net} = 0$$

$$z_c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = \alpha \tan\theta_{net}$
$b = -[\sigma_{op}(\alpha+1)\tan\theta_{net} + \omega_{op}(\alpha-1)]$
$c = (\sigma_{op}^2 + \omega_{op}^2)\tan\theta_{net}$

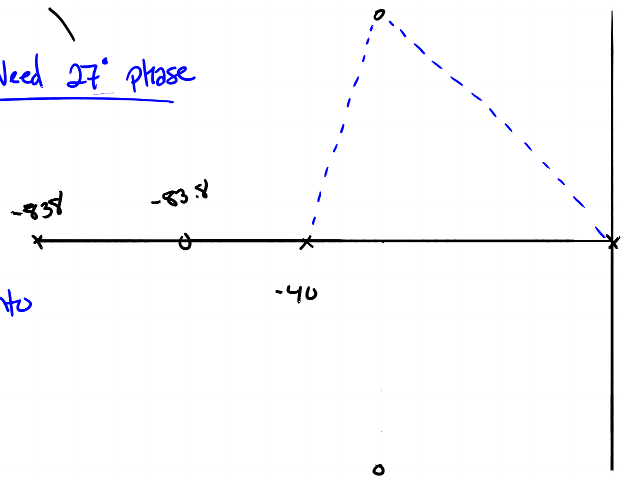
$207 - 180 = 27^\circ$

Need 27° phase

$\sum \angle zeros - \sum \angle poles$

$\Rightarrow 0 - (135^\circ - 72^\circ) = -207$

We know θ_{net}, lets plug it into the EQ we just found



$$z_c = \frac{30\cdot 11\cdot \tan 27^\circ + 30\cdot 9 \pm \sqrt{(30\cdot 11 \tan 27^\circ + 30\cdot 9)^2 - 40[2\cdot 30^2]\tan^2 27^\circ}}{20\tan 27^\circ}$$

$\Rightarrow z_c \simeq 83.8$ ← Zero location & rule of thumb that next pole is $10 \times z_c$ further away

↳ z_c also was -21.6 which does not really make sense.

Finding Gain (k)

$$\frac{G}{1+GH} \rightarrow 1 + k\frac{n(s)}{d(s)} = 0$$

$$\Rightarrow k\frac{n(s)}{d(s)} = -1$$

Now lets say we have : $\frac{k(53.12)}{s+62.5}$

$$\Rightarrow k = \frac{s}{53.13}$$

Where s is the point at which we would like to operate.