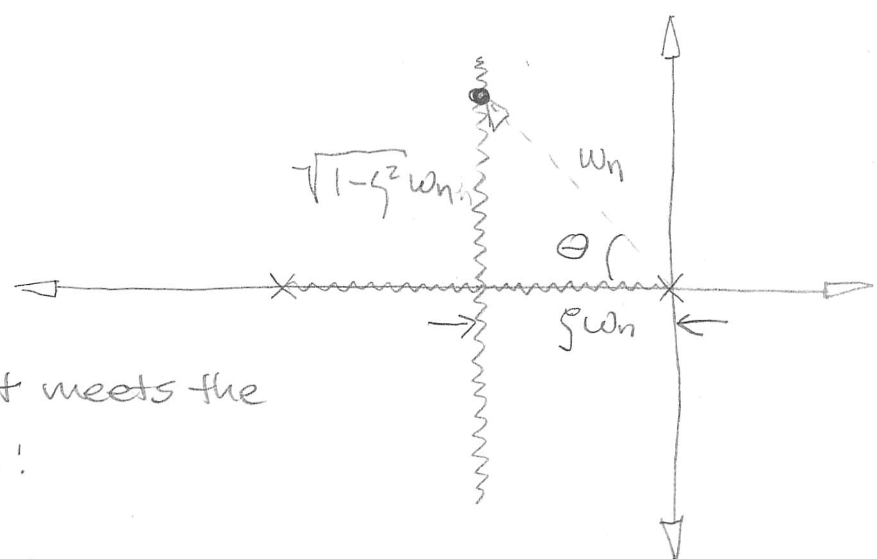
ECE/MAE 5310

Designing w/ Root Locus

Obtaining a map of how the closed-loop poles of a system $T(s)$ vary based on the open loop system $KG(s)$ is interesting, but is it useful?

Simple Example

$$KG(s) = \frac{K}{s(s+3)}$$



We want a system that meets the following specifications:

<u>Steady State Error to a Step Input</u>

0

<u>Steady State Error to a Unit Ramp Input</u>

< 1

<u>P.O.</u> $< 5\%$

$T_p < 2.5$ s

Can meet the Specifications w only a gain (K):

$$P.O. = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100 < 5$$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 5/100$$

$$-\zeta/\sqrt{1-\zeta^2} = \frac{\ln 5/100}{\pi}$$

$$\frac{\zeta^2}{1-\zeta^2} = \left(\frac{\ln 5/100}{\pi}\right)^2$$

$$\zeta^2\left(1+\left(\frac{\ln 5/100}{\pi}\right)^2\right) = \left(\frac{\ln 5/100}{\pi}\right)^2$$

$$\zeta^2 = \frac{\frac{\ln 5/100}{\pi}}{1+\left(\frac{\ln 5/100}{\pi}\right)^2} =$$

$$\zeta = \sqrt{\frac{\ln 5/100/\pi}{1+\left(\frac{\ln 5/100}{\pi}\right)^2}} = .69$$

using $\Theta = 45^\circ$ ensures P.O. < 5%

$\cos\Theta = \zeta$

$\Theta = \cos^{-1} .69 = 46.4^\circ$

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Peak Time

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \text{but} \quad \zeta\omega_n = 1.5$$

$$\omega_n = \frac{1.5}{.707} = 2.12$$

45°

$$= \frac{\pi}{2.12\sqrt{1-.5}} = 2.1 \text{ sec.}$$

Steady-State Error to a Step is zero (why?)

Steady-State Error to a Ramp is

$$\lim_{s\to 0} \frac{s}{s^2}\frac{1}{1+KG(s)} = \lim_{s\to 0}\frac{1}{s}\cdot\frac{1}{1+KG(s)} = \lim_{s\to 0}\frac{1}{s}\frac{1}{1+\frac{K}{s(s+3)}}$$

$$= \lim_{s\to 0}\frac{1}{\cancel{s}}\frac{\cancel{s}(s+3)}{s^2+3s+K}$$

$$= \frac{3}{K}$$

What is K at the operating point?

$$K = (2.12)^2 = 4.5$$

How did we do? (show simulink simulation)

The system (admittedly canned) meets the specifications using only an adjustable gain. Note that we didn't have to iterate or guess at all!

What happens when we can't meet specification with a gain?

Example Again

S.S. Ramp error too high! What would I have to do to improve this spec. to .1?

$3/K = .1 \quad K = \underline{30} = \omega_n^2 \quad \omega_n = \sqrt{30} \quad ; \; 2\zeta\omega_n = 1.5$

$\zeta = \frac{1.5}{\sqrt{30}} = .2739$

40 to 50% overshoot range!!

see graph p. 49 (Pushing on a balloon)

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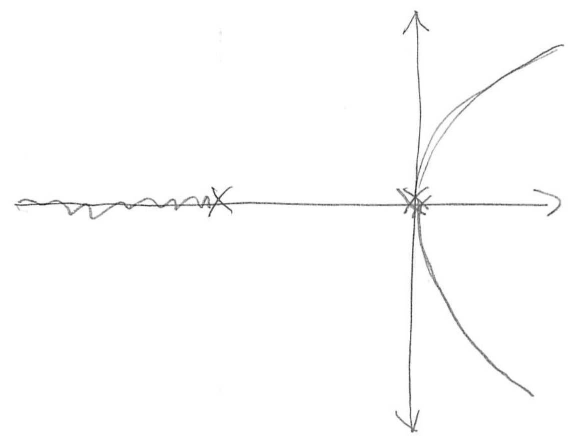
What else could I do?

You become the designers.

(1) Add an integrator



(2) ?