# Systems of Linear Equations and Matrices

## Rules of Matrix Arithmetic

1) $A \pm B = B \pm A$
2) $(A \pm B) \pm C = A \pm (B \pm C)$
3) $A(BC) = (AB)C$
4) $A(B \pm C) = (AB \pm AC)$
5) $(B \pm C)A = BA \pm CA$
6) $a(A \pm B) = aA \pm aB$
7) $ab(C) = a(bC)$
8) $a(BC) = (aB)C = B(aC)$

   A product of invertible matrices is always invertible, and the inverse of the product is the product of the inverse in reverse order.

## Elementary Matrices and Method for Finding $A^-1$

Definition > An $nxn$ matrix is called an *elementary matrix* if it can be obtained from the $nxn$ identity matrix by performing a single elementary row operation.

Theorem > If the elementary matrix $E$ results from performing a certain row operation on $I_m$ and if A is an $mxn$ matrix, the product $EA$ is the matrix that results when this same row operation is performed on A.

### Example

Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix}$$

and consider the elementary matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

which results in adding 3 times the first row of $I_3$ to the third row. This product $EA$ is then

$$EA = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}$$

***Remark***: Theorem 8 is primarily of theoretical interest and will be used for developing some results about matrices of linear equations.

Theorem > Every elementary matrix is invertible, and the inverse is also an elementary matrix.

Matrices that can be obtained from one another by a finite sequence of elementary row operations are said to be *row equivalent*.

The inverse of a matrix can be found by multiplying it by a sequence of elementary matrices until the identity matrix is created.

$$A^{-1} = E_1 \cdot E_n I_n$$

### Further Results on Systems of Equations and Invertibility

If we have a square, invertible matrix and an equation

$$AX = B$$

then we can say

$$X = A^-1B$$

## References


- Elementary Linear Algebra - Howard Anton