Linear-Time-Invariant System

$$a\frac{dx}{dt} + b = 0$$ Has the form of

An LTI system is stable if all poles are in the left hand plane.

$$G(s) = \frac{n(s)}{d(s)}$$

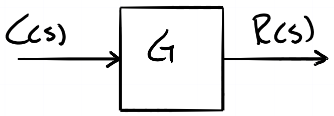
$d(s)=0$ if all s's that solve $d(s)=0$ are negative, the system is stable.

You only need one pole to be positive to make the entire system unstable.

Given a transfer function

$$G(s) = \frac{1}{s^5 + as^4 + 3s^4 + 4s^2 + s + 10}$$

There is no real way of solving this. Especially if we want to find the range for which a is stable.



Bounded Input / Bounded Output

BIBO

Example

$$G(s) = \frac{10}{(s+1)(s+10)} \qquad R(s) = \frac{1}{s}$$

$$C(s) = \frac{10}{s(s+1)(s+10)}$$

Use tables to find $\mathcal{L}^{-1}\{C(s)\}$

$$c(t) = \underline{u(t)}\left[1 - \frac{10}{9}e^{-t} + \frac{1}{9}e^{-10t}\right]$$

The unit step function is implied in the book so we have to add it.

Example

$$G(s) = \frac{1}{(s+1)(s-1)} \qquad R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s(s+1)(s-1)}$$

This function is not stable because there is a positive root.

$$c(t) = u(t)\left[-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}\right]$$

Routh Stability Criterion

Used to find stability of equations without having to factor the characteristic equation.

1) Forming Routh Array

$$a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0 = 0$$

If there are sign changes in the characteristic equation, we know by looking at it that it is unstable. If they are all positive, it <u>may</u> be stable, but it is not guaranteed.

s^n	a_n	a_{n-1}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	$\cdots$
s^{n-1}	b_1	b_2	b_3
$\vdots$			
s^0	c_1	c_2	c_3

$$b_1 = \frac{-\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}}{a_{n-1}} \qquad b_2 = \frac{-\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}}{a_{n-1}}$$

$$c_1 = \frac{-\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}}{b_1} \qquad c_2 = \frac{-\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}}{b_1}$$

The number of sign changes in the first column of the array is the number of positive roots in the system.

Example

$$G(s) = \frac{1}{s^3 + 2s^2 + as + 6}$$

Find range of stability of a.

$$-\frac{6-2a}{2}$$

s^3	1	a	0
s^2	2	6	0
s^1	$-3+a$	0	
s^0	6	0	
	0		

$-3+a > 0$

$\Rightarrow a > 3$

The ensures the system will be stable.

When we be the b row to be 0 (by setting $a = 3$ in this case) we can write the auxillary equation

$$2s^2 + 6 = 0 \Rightarrow s^2 = -3$$

$$s = \pm j\sqrt{3}$$

Example: $T(s) = \dfrac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$

↳ What you do when you divide by 0

- Choose ϵ to be positive. It is still 0, but we can now see the sign of the values →

+	s^5	1	3	5
+	s^4	2	6	3
+	s^3	0	$7/2$	
−	s^2	$\frac{6\epsilon-7}{\epsilon}$		
+	s^1	$\frac{42\epsilon-49-6\epsilon^2}{12\epsilon-14}$		
+	s^0	3		

Replace w/ ϵ

$$s^2 \to \frac{6(0)-7}{0} \to (-)$$

- Now choose ϵ to be negative

$+ \; + \; - \; + \; + \; +$

Reciprocal Root

$3s^5+5s^4+6s^3+3s^2+2s+1$ → Place opposite values together

$s^5 a_0 + s^4 a_1 + \cdots$

Gives the same results as above.

s^5	3	6	2
s^4	5	3	1
s^3	$21/5$	$7/5$	
s^2	$4/3$	1	
s^1	$-7/4$		
s^0	1		