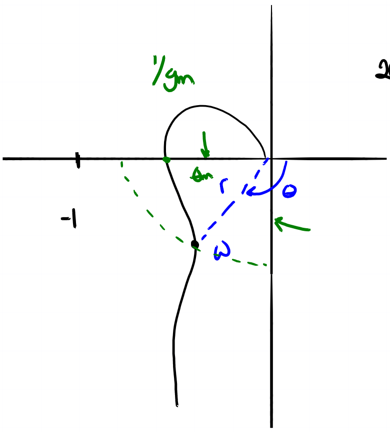
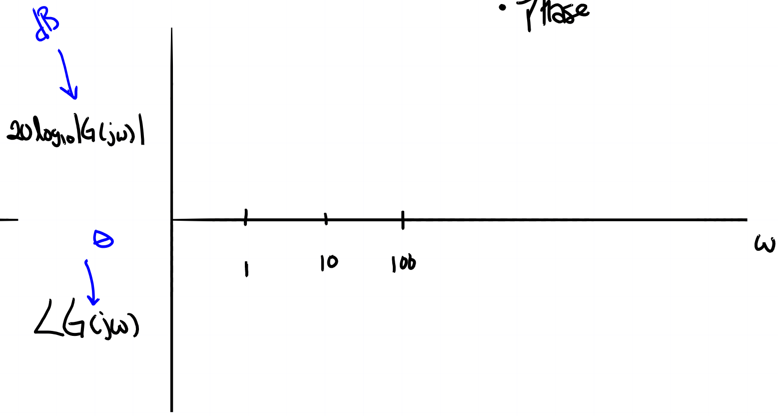
Nyquist Plot



Bode Plot : Need 2 plots

- Magnitude
- Phase



Asymptotic Plots make it easier to see poles easier to see on magnitude plots. It is better to plot the exact Phase angle.

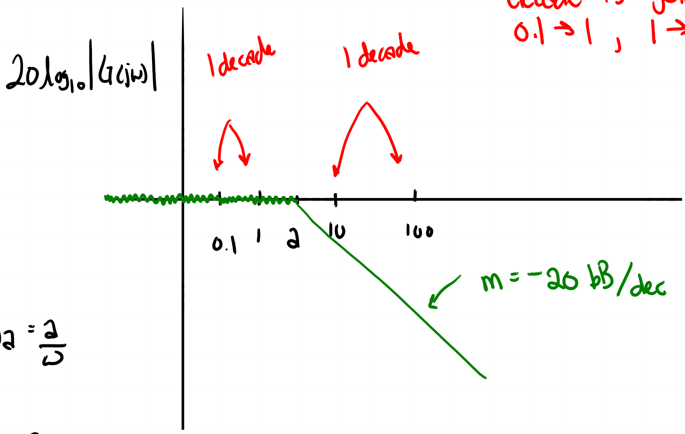
Single Pole

let $G(j\omega) = \dfrac{a}{j\omega + a} = \dfrac{1}{\frac{j\omega}{a} + 1}$

$-a \rightarrow$ Pole location

$\frac{1}{a} = \tau$ (time constant)

$$|G(j\omega)| = \frac{1}{\sqrt{\left(\frac{\omega}{a}\right)^2 + 1}}$$



decade is going from $0.1 \rightarrow 1$, $1 \rightarrow 10$, $10 \rightarrow 100$

$|G(j\omega)|$ when $\omega/a << 1 = 0$

$|G(j\omega)|$ when $\omega/a >> 1 = 1/\omega a = \frac{a}{\omega}$

$$20\log_{10}\frac{a}{\omega} = \underset{m}{-}\underbrace{20\log_{10}\omega}_{x} + \underbrace{20\log_{10}a}_{b}$$

$20\log_{10}|G(j\omega)| = 0 \Rightarrow 0 = -2\log_{10}\omega + 20\log_{10}a \Rightarrow 20\log_{10}\omega = 20\log_{10}a$

To plot phase, lets change the denominator to polar form

$$|G(j\omega)| = \frac{|top| \angle top}{|bottom| \angle bottom} \qquad \angle G(j\omega) = \angle top - \angle bottom$$

$$G(j\omega) = \frac{1 e^{j0^\circ}}{\sqrt{(\omega/a)^2+1}\; e^{j\tan^{-1}\left(\frac{\omega/a}{1}\right)}}$$

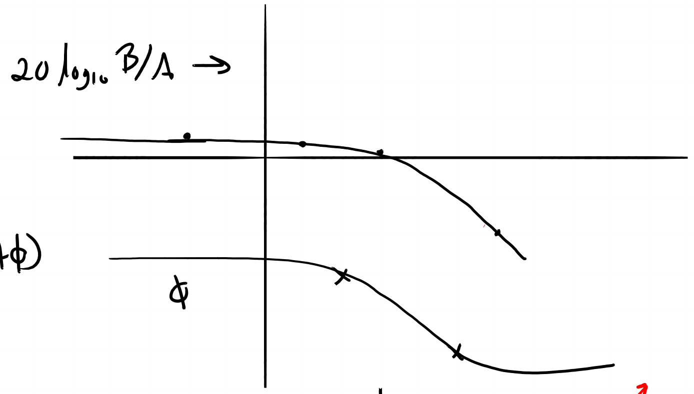
Use trigonometry to find angle

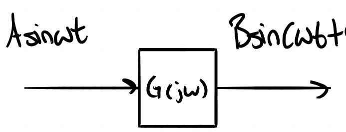
$$\Rightarrow \angle G(j\omega) = 0^\circ - \tan^{-1} \omega/a$$

Bandwidth are the frequencies that a system can respond to.

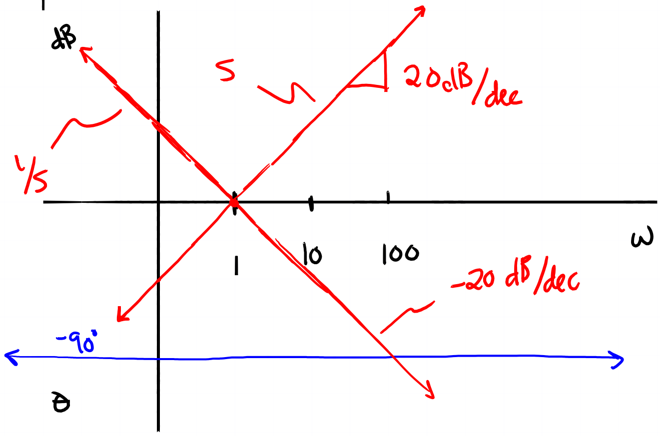
↳ 3dB below steady state (the area where low frequencies are applied) is the start of the bandwidth.

↳ for example: when you apply an ossilatory input to a motor the point where you apply a frequency and the system no longer responds.





		$\lvert W \rvert$	$\angle$
s	$j\omega$	ω	90°
$1/s$	$1/j\omega$	$1/\omega$	-90°



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{s^2/\omega_n^2 + \frac{2\zeta s}{\omega_n} + 1}$$

$$\Rightarrow G(j\omega) = \frac{1}{\frac{-\omega^2}{\omega_n^2} + j\frac{2\zeta\omega}{\omega_n} + 1}$$

for $1 - \frac{\omega^2}{\omega_n^2} >> \frac{2\zeta\omega}{\omega_n}$

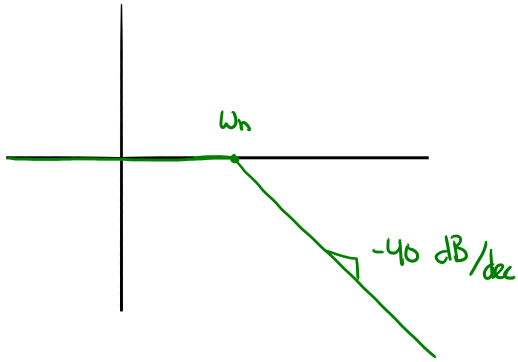
true only if

$\frac{\omega^2}{\omega_n^2} << 1$

for $\frac{\omega^2}{\omega_n^2} >> 1$

and $\frac{\omega^2}{\omega_n^2} >> \frac{2\zeta\omega}{\omega_n}$

$|G(j\omega)| \simeq \frac{1}{\omega^2/\omega_n}$



$$20\log_{10}|G(j\omega)| \simeq 20\log_{10}\frac{\omega_n^2}{\omega^2}$$

$$= -40\,[\log_{10}\omega] + 40\log_{10}\omega_n$$

Given an open loop transfer function $G(s)$

1) Manipulate $G(s)$ into 'time constant' form by

   a) Factoring $G(s)$ into pole/zero form

      Note: Complex conjugate pole or zero are left unfactored

   b) Put $G(s)$ into time constant form

<u>Time constant form</u>

$$\frac{(s/z_1 + 1)(s/z_2 + 1)}{(s/p_1 + 1)\left(\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n^2} + 1\right)}$$

c) Substitute $j\omega$ for s

Example:

$$G(s) = \frac{(s+5)(s+10)}{s(s+1)(s^2+20s+225)}$$

$$\Rightarrow \frac{5(s/5+1)\,10(s/10+1)}{s(s+1)\,225\left(\frac{s^2}{225}+\frac{20s}{225}+1\right)} = \frac{50}{225}\,\frac{(s/5+1)(s/10+1)}{s(s+1)\left(\frac{s^2}{225}+\frac{20s}{225}+1\right)}\Bigg|_{s=j\omega}$$

2) Remove any free integrators or differentiators
Isolate and plot the DC. (zero freq. gain) of $G(j\omega)$ and call K_{DC} or whatever.

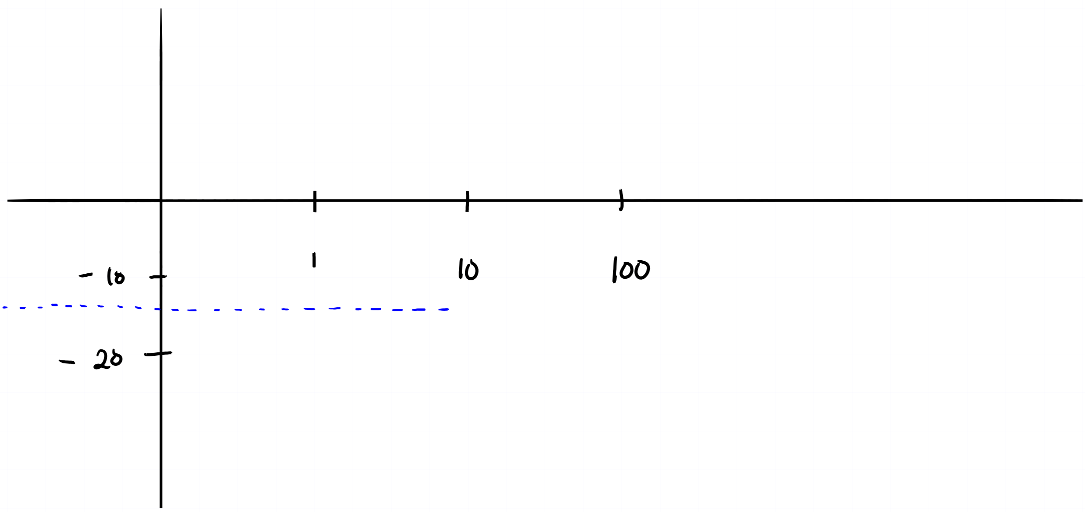
↳ We don't use these much, they amplify noise especially at high frequency.

Example Cont.

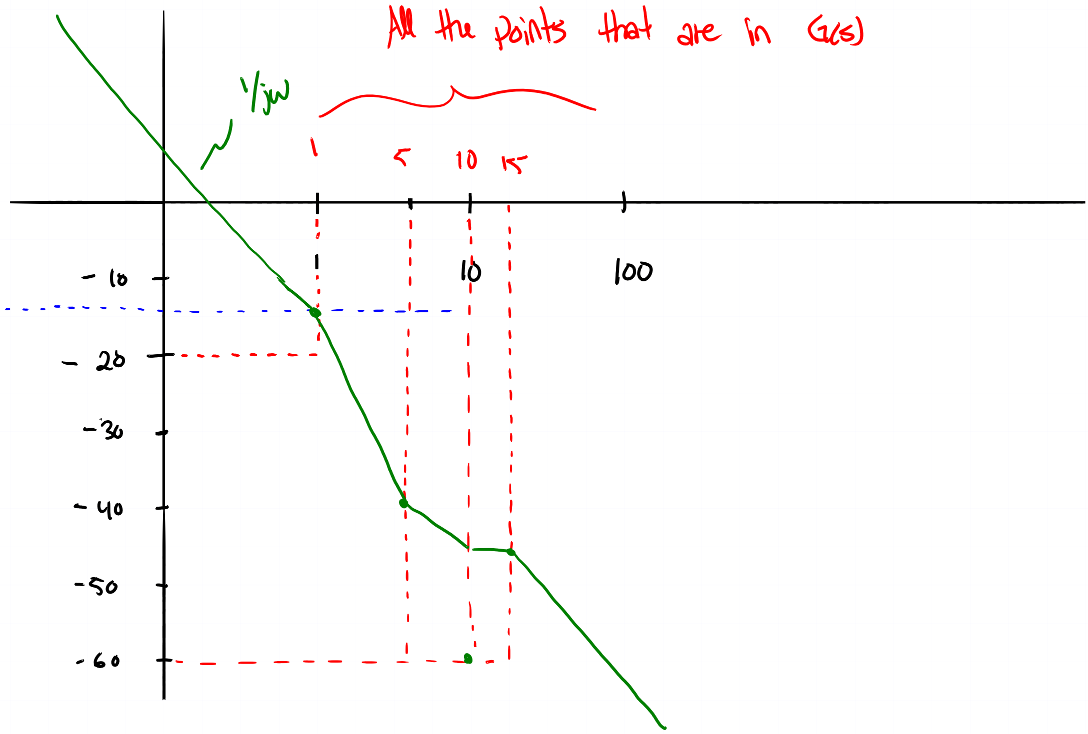
$$G(j\omega) = \frac{(j\omega/5+1)(j\omega/10+1)}{j\omega\,(j\omega+1)\left(-\omega^2/225 + 20/225\, j\omega + 1\right)}$$

Ignoring & plugging in $\omega=0$, we get a gain of 1.

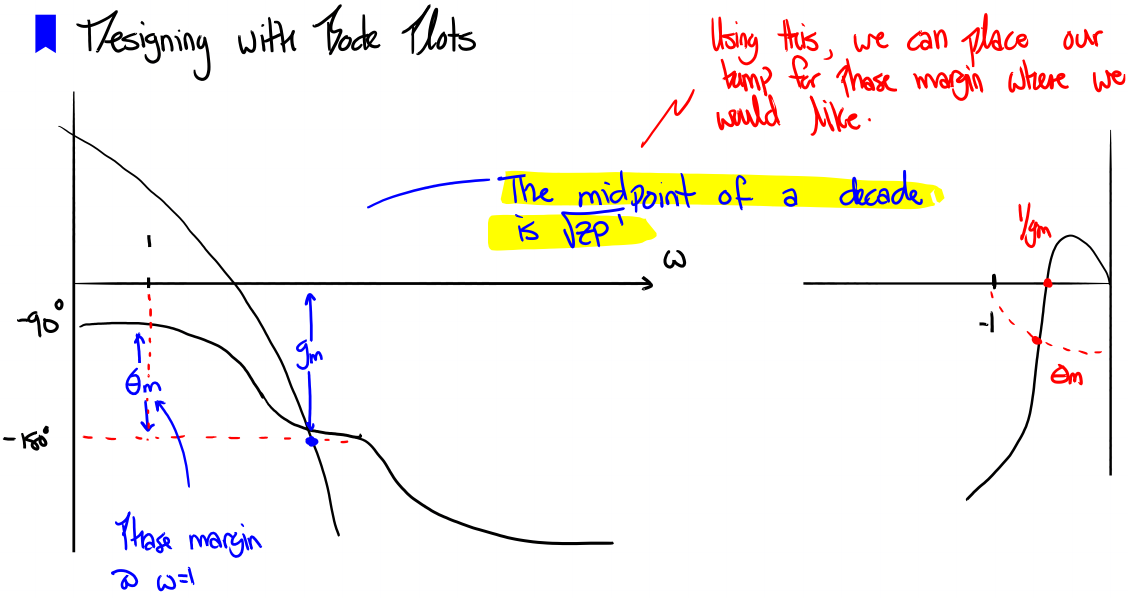
$$\Rightarrow K_{DC_{dB}} = 20\log_{10} K_{DC} = -13\text{ dB}\Big|_{\omega=0}$$



3) If there is a free integrator or differentiator draw in the integrator/differentiator at $\omega = 1$.



Designing with Bode Plots



- With this plot you can determine stability & the range of stability.

Typical Reasonable Phase margin

$45^\circ < \theta_m < 60^\circ$

Typical Reasonable Gain margin

$g_m > 10\,dB$ (linear gain $\simeq 3$)

Lead Compensation Design

$$G_c = k\,\frac{(s/z + 1)}{(s/p + 1)}$$

Time Constant Form. Tends to be easier to design with when in the frequency domain.

Adds negative phase (lag)

Adds positive phase (lead)

Adjusts the zero dB crossing point.

Requirements:

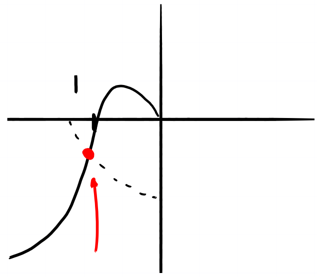
1) Adds no gain at the zero dB crossover frequency
2) Adds phase lead (positive phase) at the crossover frequency
3) Can be analytically designed

$$\angle G_c = \angle\, k\,\frac{(j\omega/z + 1)}{(j\omega/p + 1)} = \angle\,\frac{|\,|e^{j\theta_z}}{|\,|e^{j\theta_p}} = \theta_z - \theta_p$$

$$\Rightarrow G_c = \tan^{-1}\frac{\omega/z}{1} - \tan^{-1}\frac{\omega/p}{1}$$

It is important to divide by one to remember that this is quadrant sensitive.

We want to add a phase 'hump' at the point where the curve on a nyquist plot crosses r=1.



$$\frac{d\angle G_c}{d\omega} = \frac{1/z}{1+\omega^2/z^2} - \frac{1/p}{1+\omega^2/p^2} = 0$$

$$\Rightarrow \frac{d\angle G_c}{d\omega} = \frac{1/z\,(1+\omega^2/p^2) - 1/p\,(1+\omega^2/z^2)}{(1+\omega^2/z^2)(1+\omega^2/p^2)} = 0$$

$$\Rightarrow \frac{d\angle G_c}{d\omega} = \frac{1}{z}(1+\omega^2/p^2) - \frac{1}{p}(1+\omega^2/p^2) = 0$$

$$\Rightarrow \omega^2\left[\frac{1}{zp^2} - \frac{1}{pz^2}\right] = \frac{1}{p} - \frac{1}{z}$$

$$\Rightarrow \omega^2\left[\frac{z-p}{z^2p^2}\right] = \frac{z-p}{zp}$$

$$\Rightarrow \omega^2 = \frac{z-p}{zp}\;\frac{z^2p^2}{z-p} = zp$$

$$\Rightarrow \omega = \sqrt{zp}$$

<u>let this $\omega = \omega_c$</u> $\rightarrow$ $\omega_c = \sqrt{zp}$

$$\angle G_c(j\omega_c) = \tan^{-1}\frac{\omega_c}{z} - \tan^{-1}\frac{\omega_c}{p} = \tan^{-1}\frac{\sqrt{zp}}{z} - \tan^{-1}\frac{\sqrt{zp}}{p}$$

$$= \tan^{-1}\sqrt{\frac{p}{z}} - \tan^{-1}\sqrt{\frac{z}{p}}$$

Using tables: $\tan^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) + 90^\circ$

$$\Rightarrow \angle G_c(j\omega_c) = 90^\circ - 2\tan^{-1}\sqrt{\frac{z}{p}}$$ (in degrees)