Predictive Control

Main Concepts

Optimization Problems

An optimization problem is generally formulated as

$$inf_{z \in S \subseteq Z} f(z)$$

Where $inf(\cdot)$ resembles finding the optimal value within the subset.

Solving this problem means to compute the least possible cost $f*$.

$$f* = inf_{z \in S} f(z)$$

The number $f*$ is the **optimal value** of $inf_{z \in S \subseteq Z} f(z)$, i.e.:

$$f(z) \geq f(z*) = f* \; \forall z \in S, \; with \; z* \in S$$

Continuous Problems

Nonlinear mathematical program

$$\begin{array}{lll} inf_z & f(z) & \\ subj. \; to & g_i(z) \leq 0 & for \; i = 1, \cdots, m \\ & h_j(z) = 0 & for \; j = 1, \cdots, p \\ & z \in Z & \end{array}$$

A point $\bar{z} \in \mathbb{R}^s$ is **feasible** for the continuous optimization problems if:

1. it belongs to Z
2. It satisfies the inequality and equality constraints

Integer and Mixed-Integer Problems

If the optimization problem

$$inf_{z \in S \subseteq Z} f(z)$$

is finite, then the optimization problem is called *combinatorial* or *finite*. If $Z \subseteq 0, 1^s$, then the problem is said to be *integer*. If Z is a subset of the Cartesian product of an integer set and real Euclidian space, then the problem is said to be *mixed-integer*. The standard form of a mixed-integer nonlinear program is:

$$\begin{array}{lll} inf_{[z_c, z_b]} & f(z_c, z_b) & \\ subj. \; to & g_i(z_c, z_b) \leq 0 & for \; i = 1, \cdots, m \\ & h_j(z_c, z_b) = 0 & for \; j = 1, \cdots, p \\ & z_c \in \mathbb{R}^{s_c}, \; z_b \in 0, 1^{s_b} & \end{array}$$

Convexity

Theorem 1.1

Consider a convex optimization problem and let $\bar{z}$ be a local optimizer. Then $\bar{z}$ is a global optimizer.

Optimality Conditions

References

- Predictive Control - Borrelli, Bemporad, Marari