Frequency Domain Response

Example

Given $G(s)$ then $G(j\omega) = G(s)\big|_{s=j\omega}$

Let $G(s) = \dfrac{1}{s+1}$ $\qquad G(j\omega) = \dfrac{1}{j\omega+1}$

$$G(j\omega) = \frac{1}{j\omega+1} \cdot \frac{-j\omega+1}{-j\omega+1} = \frac{-j\omega+1}{\omega^2+1}$$

Complex Conjugate to move imaginary part to numerator.

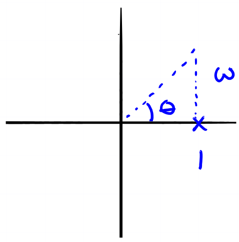
$$\Rightarrow G(j\omega) = \mathrm{Re}\,G(j\omega) + j\,\mathrm{Im}\,G(j\omega) = \frac{1}{\omega^2+1} + j\,\frac{-\omega}{\omega^2+1}$$

Polar Form

$$G(j\omega) = \frac{1}{j\omega+1} = |G(j\omega)|\, e^{j\angle G(j\omega)}$$

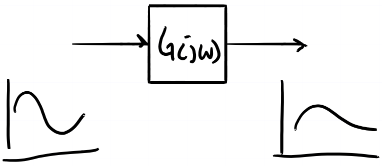
$$\Rightarrow G(j\omega) = \frac{1}{\sqrt{\omega^2+1}}\, e^{j(\angle num - \angle den)}$$

$$\Rightarrow G(j\omega) = \frac{1}{\sqrt{\omega^2+1}}\, e^{j(0 - \tan^{-1}\omega/1)}$$



$\theta_1 = 0$

$\theta_\omega = \tan^{-1}\dfrac{\omega}{1}$



Asinωt   Bsin(ωt+φ)

'Gain' $= \frac{B}{A}$

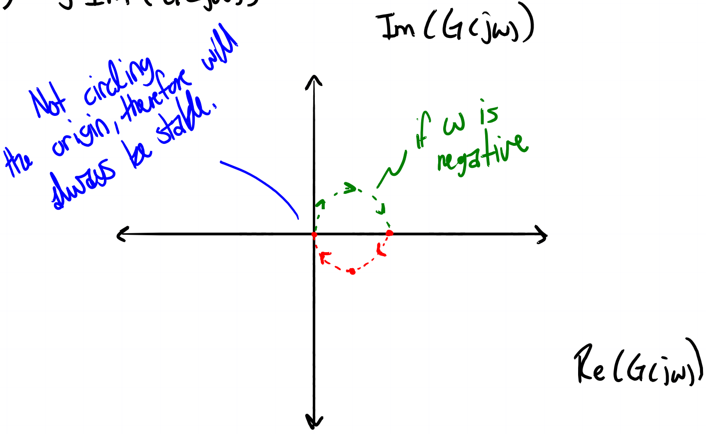
'Phase' $= \phi$

Nyquist Plot: Plot $Re(G(j\omega)) + j\,Im(G(j\omega))$

$Re(G(j\omega)) = \frac{1}{1+\omega^2}$

$Im(G(j\omega)) = \frac{-\omega}{1+\omega^2}$

	Re	Im
$\omega = 0$	1	0
$\omega = 1$	1/2	−1/2
$\omega = 2$	1/5	−2/5
$\omega = \infty$	0	0



Why use this instead of Root Locus?

- Can gain information that is hard to get out of a root locus plot.
- Shows all responses of any given ω.
- It is a mapping of s plane to frequency domain

* For every unstable pole, the Nyquist plot will circle the origin.

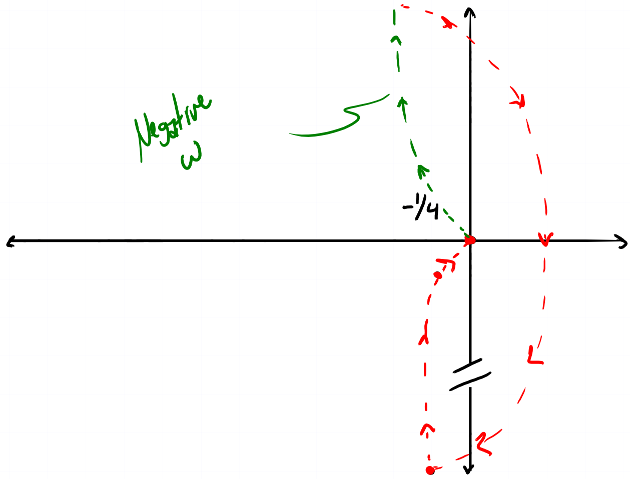
Example

$$G(s) = \frac{1}{s(s+2)} \qquad G(j\omega) = \frac{1}{j\omega(j\omega+2)}$$

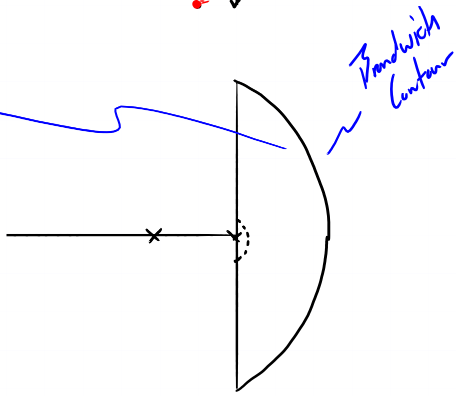
$$= \frac{-1}{\omega^2+4} + j\,\frac{-2}{\omega^3+4\omega}$$ Cartesian

$$= \frac{1}{\omega\sqrt{\omega^2+4}}\, e^{-j(90^\circ + \tan^{-1}\omega/2)}$$ Polar

ω	Re	Im	$\lvert\ \rvert$	$\angle$
0	$-1/4$	$-\infty$	∞	$-90°$
1	$-1/5$	$-2/5$	$1/\sqrt{5}$	$-117°$
∞	0	0	0	$-180°$



Circle the right half plane to find any poles that will lead to instability.

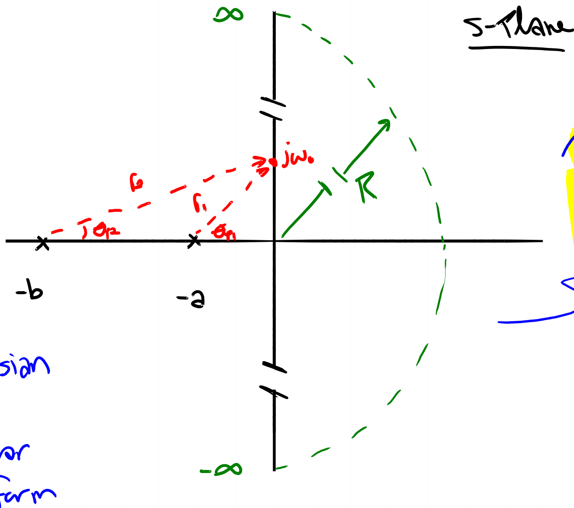


The Bromwich Contour

$$G(s) = \frac{1}{(s+b)(s+a)}$$

$$G(j\omega) = \frac{1}{(j\omega+b)(j\omega+a)}$$ Cartesian

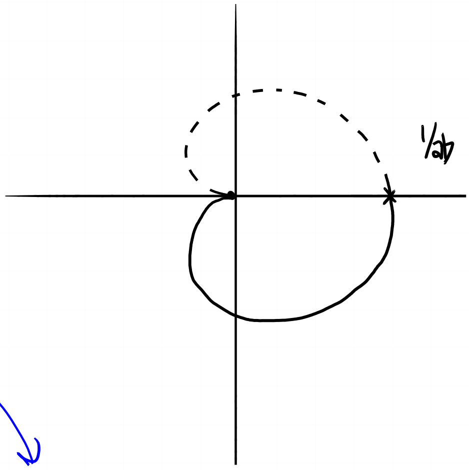
$$= \frac{1}{r_1 e^{j\theta_{p1}}\, r_2 e^{j\theta_{p2}}}$$ Polar form



Remember the plot is symmetric about x axis

- Each pole can add a maximum angle contribution of $-90°$ because as $j\omega \to \infty$ the angle becomes $90°$.

Plotting the equation above:



We have only been dealing w/ open loop transfer function. Now lets look at the transfer function

$$1 + GH = 0$$

$$\Rightarrow GH = -1$$

For closed loop

This is an interesting result, because instead of checking to see if we circle the origin, now we will check to see if we circle -1.

↳ If we circle -1, then the system has instability.

* 2^{nd} order systems will never go unstable

* 3^{rd} order systems can become unstable

Think of Root Locus

These plots can show how stable our system is. The value we multiply by to go unstable is gain margin.

A similar concept can be applied. If we rotate the system about the axis, can we cause instability? This is called Phase margin.

→ These two give us margins of stability like safety factors.

- Produces calculable stable systems

Closed Loop Stability Information

- Plot $G(j\omega)H(j\omega)$
- Look for encirclement of the -1 point.