ECE/MAE 5310 Sensitivity — Chap. 9 Material

$$S(s) = \frac{1}{1+L}$$

Define the complementary sensitivity function as

$$C(s) = \frac{L}{1+L}$$

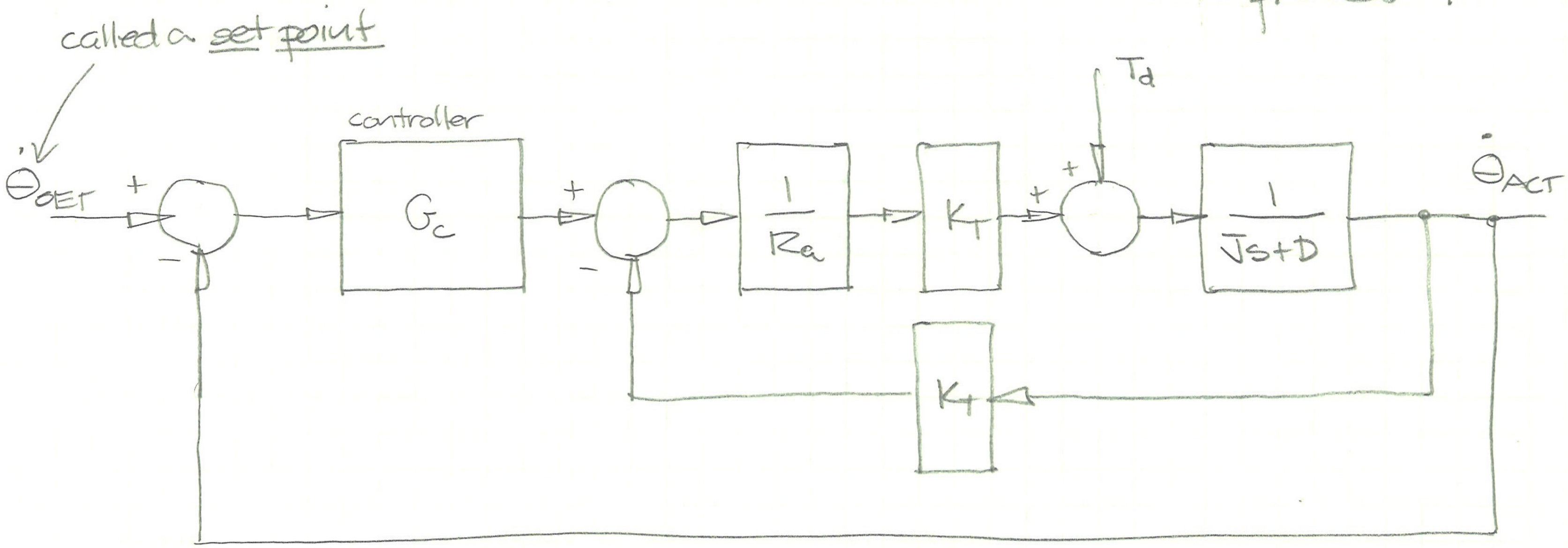
then

$$E(s) = S(s)R(s) - G_p S(s) T_d(s) + C(s) N(s)$$

Note that

$S(s) + C(s) = 1 \Rightarrow$ $S(s)$ small

$C(s)$ close to 1

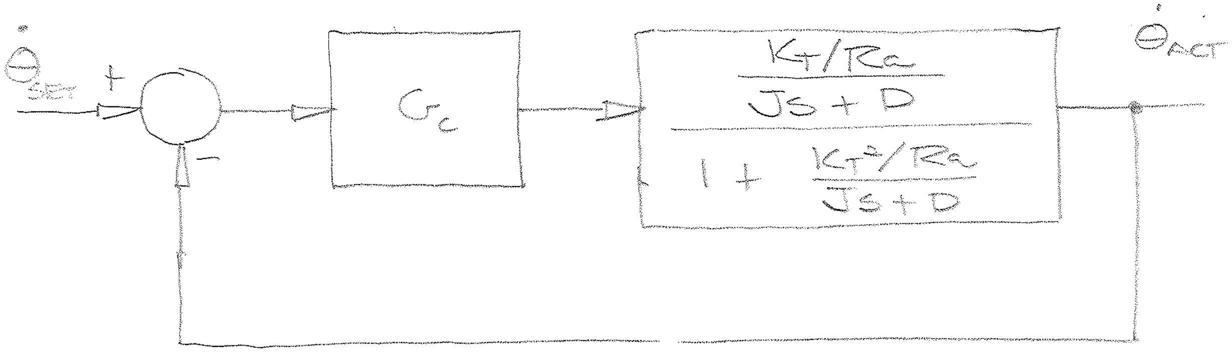


What can we do to make performance better?

Let's look at the effects of adding the control feedback loop.

Let's look at the responses of the controlled system
(Using superposition to deal w/ one input at a time! (critical!!))

Reducing the system when the disturbance torque is 0.



$$\frac{\dot{\Theta}_{ACT}}{\dot{\Theta}_{SET}} = \frac{G_c \dfrac{K_T/R_a}{Js+D+K_T^2/R_a}}{1 + G_c \dfrac{K_T/R_a}{Js+D+K_T^2/R_a}} = \frac{G_c K_T/R_a}{Js+D+K_T^2/R_a + G_c K_T/R_a}$$

In steady state we want $\dfrac{\dot{\Theta}_{ACT}}{\dot{\Theta}_{SET}} = ?$ (1, why?) (we could settle for almost 1) in steady state ($s \to 0$)

$$\frac{\dot{\Theta}_{ACT}}{\dot{\Theta}_{SET}} = \frac{G_c K_T/R_a}{D + K_T^2/R_a + G_c K_T/R_a}$$

this should be close to 1 for the controller to be effective. How can we achieve this?

$$G_c K_T/R_a \gg D + K_T^2/R_a$$ (why does this work?)

or

$$\frac{G_c K_T/R_a}{D + K_T^2/R_a} \gg 1$$

We will settle (for now) for $\left| \frac{\Theta_{ACT}}{T_D} \right|_{t \to \infty}$ small. When is this small?

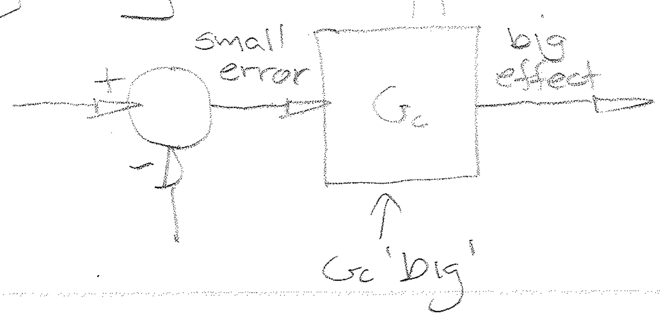
$$G_c K_T / R_a \gg D + K_T^2 / R_a$$

$$\frac{G_c K_T / R_a}{D + K_T^2 / R_a} \gg 1 \quad \text{(look familar)}$$

We can only set G_c so what do we need to do to G_c?

make it big!

Physically what happens when we set G_c to be 'big'?



Sensitivity - The Math

We can determine a relative measure of how sensitive a system is to a change in parameter or transfer function.

Fractional change in T(s)

$$S = \frac{\Delta T(s) / T(s)}{\Delta G(s)/G(s)}$$

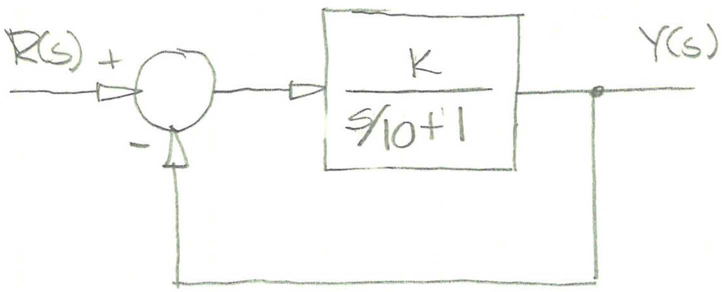
Fractional change in G(s)

$$S = \lim_{\Delta T(s), \Delta G(s) \to 0} \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)} = \frac{\partial T(s)}{\partial G(s)} \frac{G(s)}{T(s)} = \frac{\partial \ln T}{\partial \ln G} \quad \left(d(\ln A) = \frac{\partial A}{A} \right)$$

Alternatively, if we are interested in the change in a parameter only

$$S_\alpha^T = S_G^T S_\alpha^G = \frac{\partial T}{\partial G}\frac{G}{T} \cdot \frac{\partial G}{\partial \alpha}\frac{\alpha}{G} \quad \text{or directly} \quad S_\alpha^T = \frac{\partial T}{\partial \alpha}\frac{\alpha}{T}$$

Example #1



$$\frac{Y(s)}{R(s)} = T(s) \qquad G(s) = \frac{K}{s/10+1}$$

$$= \frac{G(s)}{1+G(s)}$$

$$\frac{\partial}{\partial G(s)}\left(\frac{G(s)}{1+G(s)}\right) = \frac{(1+G(s))1 - (G(s))1}{(1+G(s))^2}$$

$$S^T_G = \frac{\partial T(s)}{\partial G(s)} \frac{G(s)}{T(s)} \qquad = \frac{1}{(1+G(s))^2}$$

$$= \frac{1}{(1+G(s))^2} \frac{G(s)}{G(s)} (1+G(s)) = \frac{1}{1+G(s)}$$

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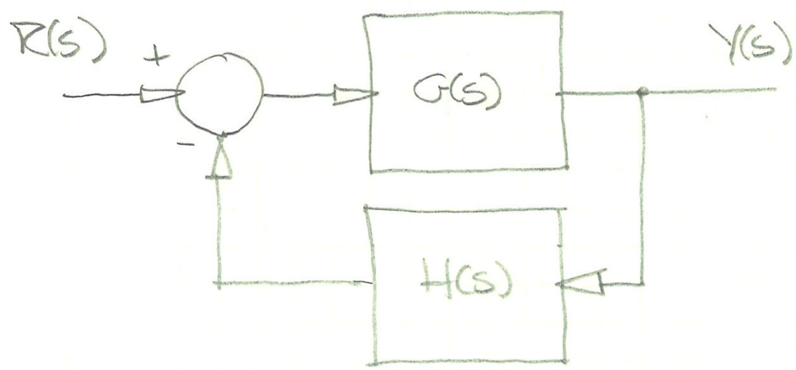
to find S^T_K

$$\frac{\partial G(s)}{\partial K} = \frac{1}{s/10+1}$$

$$S^T_K = S^T_G S^G_K = \frac{1}{1+G(s)} \frac{\partial G(s)}{\partial K} \frac{K}{G(s)} = \frac{1}{1+G(s)} \frac{1}{s/10+1} \frac{s/10+1}{K} K = \frac{1}{1+G(s)}$$

Example #2



$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$S_H^T = \frac{\partial T}{\partial H}\frac{H}{T} = \frac{-GH}{1+GH}$$