Predictive Control

Main Concepts

Model predictive control is the only advanced control technique that has see widespread impact on industrial process control. ***It's the only control technology that can deal with constraints***.

Principles of Predictive Control

- Prediction
  - Why is prediction important: We care about the goal to be achieved more than what is currently happening within the system.
- Receding Horizon
  - What is receding horizon: Some fixed interval (period of time) over which we consider the future (Car headlight analogy).

Optimization Problems

An optimization problem is generally formulated as

$$inf_{z \in S \subseteq Z} f(z)$$

Where $inf(\cdot)$ resembles finding the optimal value within the subset.

Solving this problem means to compute the least possible cost $f*$.

$$f* = inf_{z \in S} f(z)$$

The number $f*$ is the **optimal value** of $inf_{z \in S \subseteq Z} f(z)$, i.e.:

$$f(z) \geq f(z*) = f * \forall z \in S, \; with \; z* \in S$$

Continuous Problems

Nonlinear mathematical program

$$\begin{array}{lll} inf_z & f(z) & \\ subj.\ to & g_i(z) \leq 0 & for\ i = 1, \cdots, m \\ & h_j(z) = 0 & for\ j = 1, \cdots, p \\ & z \in Z & \end{array}$$

A point $\bar{z} \in \mathbb{R}^s$ is **feasible** for the continuous optimization problems if:

1. it belongs to Z
2. It satisfies the inequality and equality constraints

Integer and Mixed-Integer Problems

If the optimization problem

$$inf_{z \in S \subseteq Z} f(z)$$

is finite, then the optimization problem is called *combinatorial* or *finite*. If $Z \subseteq 0, 1^s$, then the problem is said to be *integer*. If Z is a subset of the Cartesian product of an integer set and real Euclidian space, then the problem is said to be *mixed-integer*. The standard form of a mixed-integer nonlinear program is:

$$
\begin{array}{lll}
inf_{[z_c, z_b]} & f(z_c, z_b) & \\
subj.\ to & g_i(z_c, z_b) \leq 0 & for\ i = 1, \cdots, m \\
 & h_j(z_c, z_b) = 0 & for\ j = 1, \cdots, p \\
 & z_c \in \mathbb{R}^{s_c},\ z_b \in 0, 1^{s_b} &
\end{array}
$$

Convexity

Theorem 1.1

Consider a convex optimization problem and let $\bar{z}$ be a local optimizer. Then $\bar{z}$ is a global optimizer.

Optimality Conditions

Optimality Conditions For Unconstrained Problems

Theorem 1.2 (Necessary Condition):

Suppose that $f : \mathbb{R}^s \to \mathbb{R}$ is differential at $\bar{z}$. If there exists a vector d such that $\nabla f(\bar{z})' d < 0$, then there exists a $\delta > 0$ such that $f(\bar{z} + \lambda d) < f(\bar{z})$ for a $\lambda \in (0, \delta)$.

Theorem 1.3 (Sufficient Condition):

Suppose that $f : \mathbb{R}^s \to \mathbb{R}$ is twice differentiable at $\bar{z}$. If $\nabla f(\bar{z}) = 0$ and the Hessian $(\nabla^2 f(\bar{z}))$ of $f(z)$ at $\bar{z}$ is positive definite, then $\bar{z}$ is a local minimizer.

Theorem 1.4: (Necessary and Sufficient Condition):

Supposed that $f : \mathbb{R}^s \to \mathbb{R}$ is differentiable at $\bar{z}$. If f is convex, then $\bar{z}$ is a global minimizer iff $\nabla f(\bar{z}) = 0$

Lagrange Duality Theory

Consider the optimality problem. Any feasible point $\bar{z}$ provides an upper bound to the optimal value $f(\bar{z}) \geq f*$ ($f*$ being the optimal value). The Lagrange Duality Theory generates a lower boundary for $f*$.

Starting from the same problem, we construct another problem with different variables and constrains. In other words, from the primal problem, we will develop the dual problem.

$$L(z, u, v) = f(z) + u_1 g_1(z) + \cdots + u_m g_m(z) + v_1 h_1(z) + \cdots + v_p h_p(z)$$

References

- Predictive Control - Borrelli, Bemporad, Marari