Example 1

Given $G(s)$ then $G(j\omega) = G(s)\big|_{s=j\omega}$ (called the steady-state sinusoidal response)

Let $G(s) = \dfrac{1}{s+1}$, $G(j\omega) = \dfrac{1}{j\omega+1}$

Form 1 $Re + j\,Im$ (Useful for adding transfer functions)

$$G(j\omega) = \frac{1}{j\omega+1}\,\frac{-j\omega+1}{-j\omega+1} = \frac{1-j\omega}{\omega^2+1}$$

from

$$\begin{array}{rrr} & j\omega & +\ 1 \\ & -j\omega & +\ 1 \\ \hline & +j\omega & +\ 1 \\ \omega^2 & -j\omega & \\ \hline \omega^2 & & +\ 1 \end{array}$$

$$= \frac{1}{\omega^2+1} + j\,\frac{-\omega}{\omega^2+1}$$

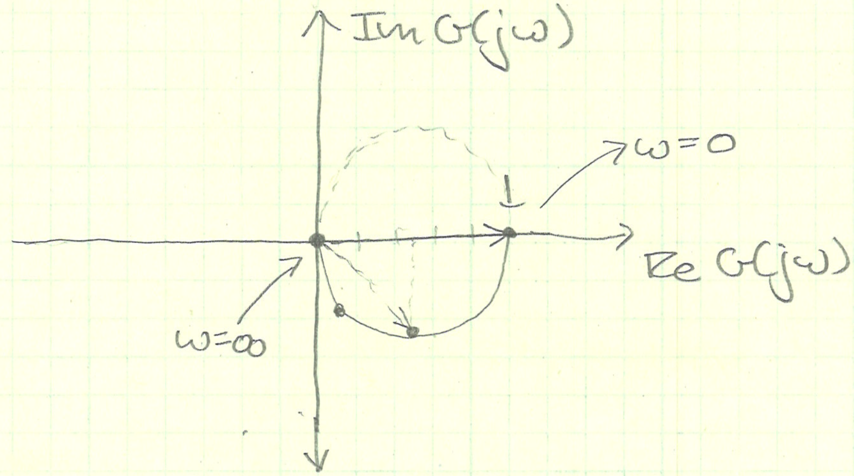
Form 2

$$G(j\omega) = \frac{1}{j\omega+1} = |G(j\omega)|\,e^{j\angle G(j\omega)}$$

$$= \frac{1}{\sqrt{\omega^2+1}}\,e^{j(\angle num - \angle den)} = \frac{1}{\sqrt{\omega^2+1}}\,e^{j(0^\circ - \tan^{-1}\frac{\omega}{1})}$$

Plotting $\text{Re}\, G(j\omega) + j\,\text{Im}\, G(j\omega)$

	Re	Im
$\omega = 0$	1	0
$\omega = 1$	$\frac{1}{2}$	$-\frac{1}{2}$
$\omega = 2$	$\frac{1}{5}$	$-\frac{2}{5}$
$\omega = \infty$	0	



Equivalently $|G(j\omega)|\ \angle G(j\omega)$ from the same plot

$\omega = 0 \quad |G(j\omega)| = 1 \qquad \angle G(j\omega) = 0^\circ$

$\omega = 1 \quad |G(j\omega)| = 1/\sqrt{2} \qquad \angle G(j\omega) = -\tan^{-1}(1) = -45^\circ$

$\omega = 2 \quad |G(j\omega)| = 1/\sqrt{5} \qquad \angle G(j\omega) = -\tan^{-1}\left(\frac{2}{1}\right) =$

$\vdots$

$\omega = \infty \quad |G(j\omega)| = 0 \qquad \angle G(j\omega) = -\tan^{-1}(\infty) = ?\quad (-90^\circ)$

Given $G(s)$ then $G(j\omega) = G(s)\big|_{s=j\omega}$

Two representations

real + imaginary, magnitude + phase

$G(j\omega) = \mathrm{Re}\,G(j\omega) + j\,\mathrm{Im}\,G(j\omega)$

$= |G(j\omega)|e^{j\arg(G(j\omega))}$

all complex numbers are vectors $x+jy$ or $re^{j\tan^{-1}y/x}$

Example 2

$$G(s) = \frac{1}{s(s+2)} \qquad G(j\omega) = \frac{1}{j\omega(j\omega+2)}$$

The two forms of $G(j\omega)$ are:

$$G(j\omega) = \frac{1}{-\omega^2+j2\omega} = \frac{-\omega^2-j2\omega}{\omega^4+4\omega^2} = \underbrace{\frac{-1}{\omega^2+4}}_{\mathrm{Re}} + j\underbrace{\frac{-2}{\omega^3+4\omega}}_{\mathrm{Im}}$$

1 is a complex number $1+j0$

$$= \frac{1e^{j0}}{\omega e^{j90^\circ}\left(\sqrt{\omega^2+4}\,e^{j\tan^{-1}(\omega/2)}\right)}$$

a word about atan2

$$= \frac{1}{\omega}e^{-j90^\circ}\frac{1}{\sqrt{\omega^2+4}}e^{-j\tan^{-1}(\omega/2)} = \frac{1}{\omega\sqrt{\omega^2+4}}e^{-j(90^\circ+\tan^{-1}(\omega/2))}$$

but what does it mean?

ω	$G(j\omega)$ Re	$G(j\omega)$ Im	$\lvert\ \rvert$	$\angle$
0	$-1/4$	$-\infty$	∞	$-90°$
1	$-1/5$	$-2/5$	$1/\sqrt{5}$	$-117°$
2				
3				
4				
5				
6				
7				
8				
9				
10				
⋮				
∞	0	0	0	$-180°$

$\underbrace{\text{Re} \quad \text{Im}}$ top goes to ∞ 'slower' than bottom

$-90° - \tan^{-1}(2/-1) =$