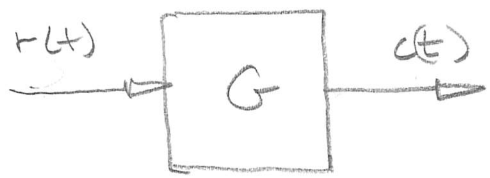
Example with numbers



$$G: \quad \frac{dc(t)}{dt} + 2c(t) = r(t)$$

Laplace Transform of $\quad (s+2)C(s) = R(s)$

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

Note that we can easily make the input what we want it to be. Let's choose $R(s)$ as a unit step $R(s) = \frac{1}{s}$

then $\quad C(s) = \dfrac{R(s)}{s+2} = \dfrac{1}{s(s+2)}$

That's all well and good but what is $c(t)$?

We have spent last two or so weeks reviewing and learning about finding models of physical systems.

We typically want to arrive at a transfer function of general form

$$\frac{C(s)}{R(s)} = \underbrace{K}_{\text{a constant}} \underbrace{\frac{(s+z_1)(s+z_2)(s+z_3)\cdots(s+z_n)}{(s+p_1)(s+p_2)(s+p_3)\cdots(s+p_m)}}_{= G(s)}$$

where $-z_n$ are the zeroes
where $-p_m$ are the poles

If we choose an input (or if the problem forces one on us) we can find the system time response

$$C(s) = G(s)R(s),$$

taking the inverse Laplace transform (via partial fraction expansion or other method).

Example

$$C(s) = \frac{10}{s(s+10)(s+1)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+1}$$

$$= \frac{1}{s} + \frac{1/9}{s+10} + \frac{-10/9}{s+1}$$

$$c(t) = \left[1 + \tfrac{1}{9}e^{-10t} - \tfrac{10}{9}e^{-t}\right]u(t)$$

Let's take a qualitative look at this equation. We may get to stay in the complex plane longer if we know what is going on here.

The exponential forms e^{-10t} and e^{-t} can be written in 'time constant' form, $e^{-t/\tau}$.

$e^{-10t} \rightarrow e^{-t/.1} \rightarrow \tau_1 = .1\,s$

$e^{-t} \rightarrow e^{-t/1} \rightarrow \tau_2 = 1\,s$

It instructive to examine the sizes of these exponential terms as integral numbers of time constants pass

Since the time constants are integer multiples of each other let's choose $t = .1$ (one τ_1 constant) (

$.2$ (two τ_2 constants)

$.3$

$\vdots$

1.0

Then let's look at the relative sizes of the exponential terms

t	0	.1	.2	.3	.4	.5	.6	.7	.8
$e^{-t/.1}$	1	.37	.14	.05	.02	.007	.003	.001	.0003
$e^{t/1}$	1	.9	.82	.74	.67	.61	.56	.5	.45

τ_1 effect $\simeq$ 3% of τ_2 effect

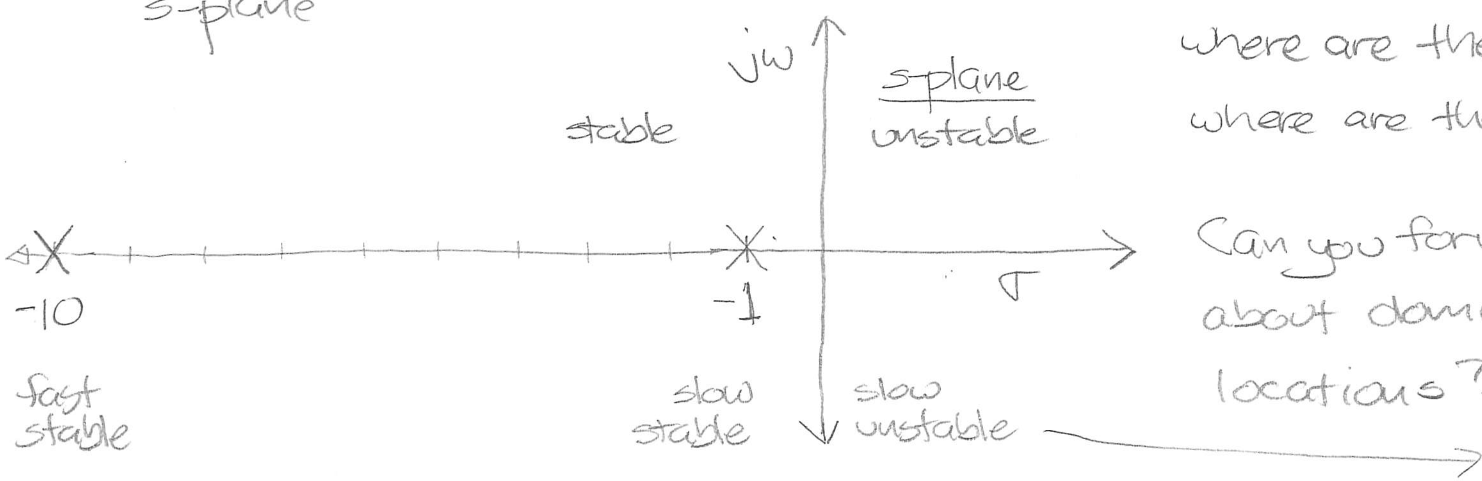
As you can see after 4τ, (.4 s) $e^{-.4/.1}$ is very small, and $e^{-.4/1}$ is ~ 30 times as large.

What conclusions can you draw from this fact?

(1) The effects of the $\frac{B}{(s+10)}$ term diminish rapidly.

(2) The transient response is dominated by the $\frac{C}{s+1}$ term.

Let's look at a pole-zero map of our transfer function $G(s)$ in the s-plane



where are the poles of $G(s)$?

where are the zeroes of $G(s)$?

Can you form a conclusion about dominant pole locations?