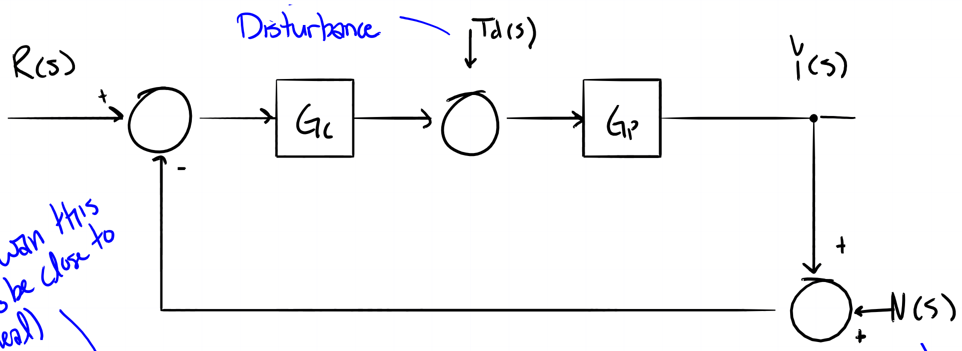


Noise

We want this to be close to 1 (ideal)

$$\frac{Y(s)}{R(s)} = \frac{G_c G_p}{1+G_c G_p} \qquad \frac{Y(s)}{T_d(s)} = \frac{G_p}{1+G_c G_p}$$

Want this to be small

$$\frac{Y(s)}{N_o(s)} = \frac{-G_c G_p}{1+G_c G_p}$$

- The only way to limit noise is to have good sensors.

Define Sensitivity:

$$S(s) = \frac{1}{1+G_c G_p}$$

Loop Transmission : $L(s) = G_c G_p$

$$\Rightarrow S(s) = \frac{1}{1+L}$$

Complimentary Sensitivity Function

$$C(s) = \frac{L}{1+L}$$

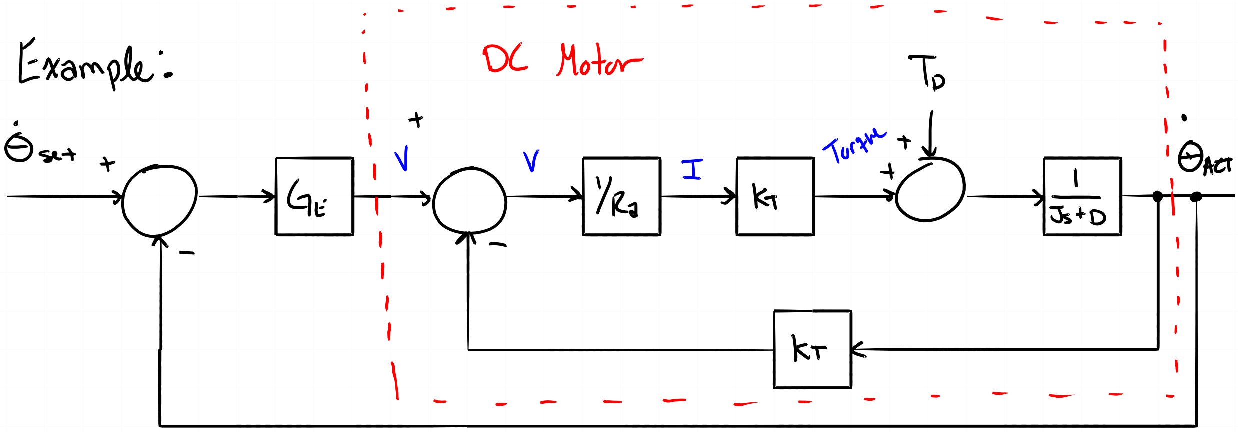
$$\boxed{\Rightarrow C(s) + S(s) = 1}$$

$C(s) + R(s) = 1$

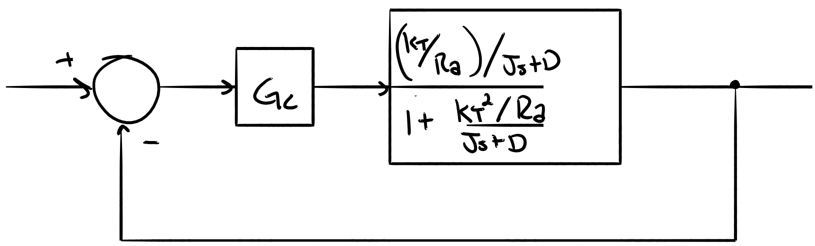
If $C(s) \simeq 1$ $\therefore$ Sensitivity $\simeq 0$

As $S(s) \rightarrow 1$ the system becomes more sensitive.

Example:



Set $T_D = 0$

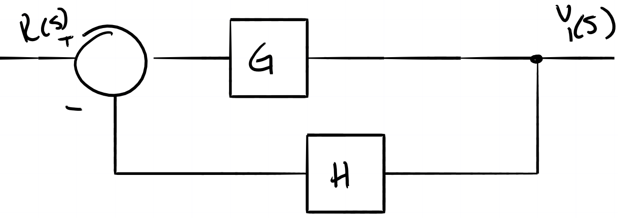


$$\Rightarrow \frac{\dot{\theta}_{ACT}}{\dot{\theta}_{set}} = \frac{G_c \; K_T/R_a}{Js + D + \frac{K_T^2}{R_a} + \frac{G_c K_T}{R_a}}$$

$$\left.\frac{\dot{\theta}_{ACT}}{\dot{\theta}_{set}}\right|_{\text{Stead State}} =$$

↳ Replace s w/ 0. Because s's are derivatives we are setting the equation @ steady state.

Sensitivity Math



$$S = \frac{\left(\dfrac{\Delta T(s)}{T(s)}\right)}{\left(\dfrac{\Delta G(s)}{G(s)}\right)}$$

Fractional change T(s)

Fractional change of G(s)

Insensitive is close to 0 for this function.

$$S = \lim_{\Delta T(s),\, \Delta G(s) \to 0} \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)} = \frac{\delta T(s)}{\delta G(s)} \frac{G(s)}{T(s)} = \frac{\delta \ln T}{\delta \ln G}$$

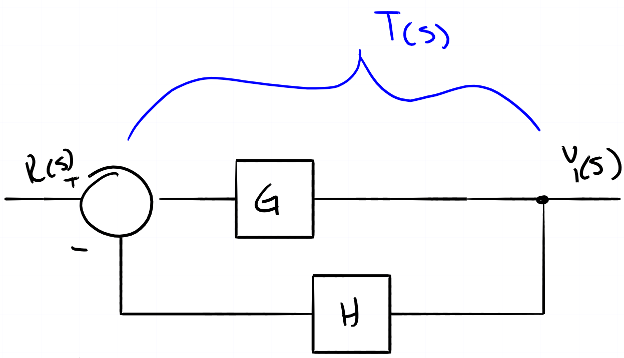
$$\partial(\ln A) = \frac{dA}{A}$$

Manipulation used.

$$S_\alpha^T = S_G^T S_\alpha^G = \frac{\partial T}{\partial G} \frac{G}{T} \frac{\partial G}{\alpha} \frac{\alpha}{G}$$

↳ Sensitivity of parameter α in a system with a transfer function T.

Example



$$G(s) = \frac{K}{\frac{s}{10} + 1}, \quad H = 1$$

$$\frac{Y(s)}{R(s)} = T(s) = \frac{G(s)}{1 + G(s)}$$

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T}$$

Using Quotient Rule

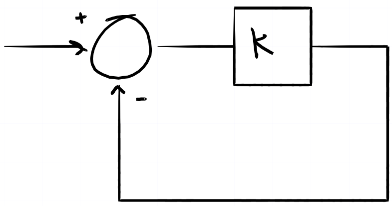
$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G(s)}{1+G(s)}\right) = \frac{(1+G(s))(1) - G(s)\,1}{(1+G(s))^2}$$

$$\Rightarrow \frac{\partial T}{\partial G} = \frac{1}{[1+G(s)]^2}$$

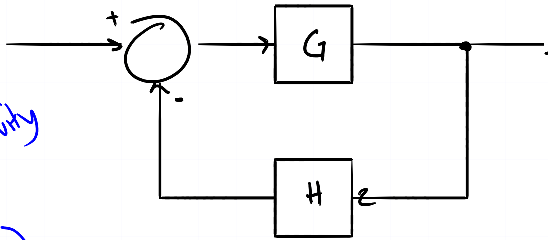
$$\Rightarrow S_G^T = \frac{1}{[1+G(s)]^2}\,G(s)\cdot\frac{1+G(s)}{G(s)} = \frac{1}{1+G(s)}$$

Example

$$T(s) = \frac{K}{1+K}$$



Example



$$T(s) = \frac{G}{1+GH}$$

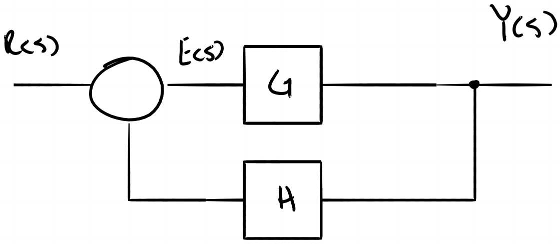
1 to 1 sensitivity

$$S_H^T = \frac{\partial T}{\partial H}\frac{H}{T} = \frac{-GH}{1+GH}$$

← This says if you have variation then the sensor will reflect that back to the input.

* This is why it is important to have a good sensor because if you don't (as we can see in the sensitivity) the sensor will make your system do whatever it wants.

Steady State Error



$$\frac{Y(s)}{R(s)} = \frac{G}{1+GH}$$

$$Y(s) = G\,E(s)$$

$$\Rightarrow \frac{G\,E(s)}{R(s)} = \frac{G}{1+GH}$$

$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

System error due to any input

Example

$$G = \frac{1}{s(s+1)} \qquad H(s) = 1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+\frac{1}{s(s+1)}} = \frac{s(s+1)}{s^2+s+1}$$

If $R(s)$ is a unit step function $R(s) = \frac{1}{s}$

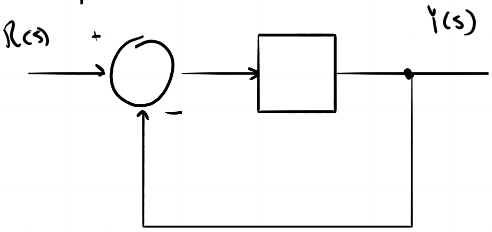
$$E(s) = \frac{1}{s}\,\frac{s(s+1)}{s^2+s+1} = \frac{s+1}{s^2+s+1}$$

Final value Theorem

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s)$$

$$\lim_{s \to 0} s\left[\frac{s+1}{s^2+s+1}\right] = 0$$

Example



Let $G(s) = \frac{1}{s+1}$

$$\downarrow$$

$$G(s) = \frac{k}{\tau s+1}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+\frac{1}{s+1}} = \frac{s+1}{s+2}$$

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

lets assume $R(s) = 1/s$

$$\Rightarrow \lim_{s \to 0} s\left[\frac{1}{s}\,\frac{s+1}{s+2}\right] = \underline{\underline{\frac{1}{2}}}$$

This means that this system has a finite steady state error.

* Integrators accumulate their inputs.

The number of free integrators define the system type.

↳ Free integrators are those s values that can be factored out by themselves.

Number of free integrators set the steady error output based on the input type.

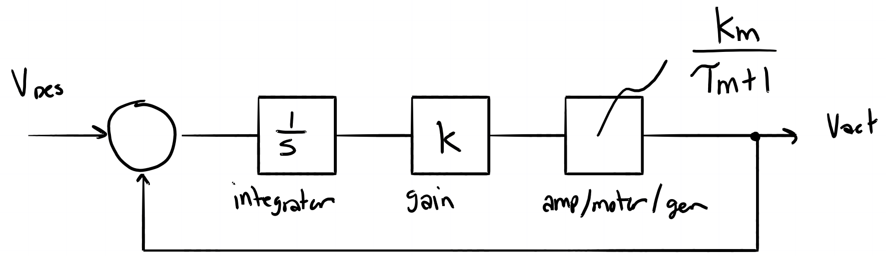
Suppose we have a system

$$G = \frac{1}{s(s+1)}$$

open loop

Free integrator

There is 0 steady state error for a step input in feedback.



finite steady state error for ramp input

∞ steady state error for parabolic input.

2 Free integrators:

0 ss. error for step

0 ss error for ramp

finite ss error for parabola

* Triple free integrators are difficult to control.

Position Error constant

$$R(s) = \frac{1}{s} \qquad K_p \equiv \lim_{s \to 0} G = \lim \frac{K\, n(s)}{s^l\, d(s)} = \begin{cases} \dfrac{K\, n(0)}{d(0)} & l = 0 \\ \infty & \text{for } l > 0 \end{cases}$$

defined as

$$e(\infty) = \frac{1}{1 + K_p}$$