Bus Charging Schedule Simulated Annealing with MILP Constraints

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July 18, 2023

Contents

1 Preliminaries

1.1 Mixed Integer Linear Program

A mixed integer linear programming (MILP) problem is a class of constrained optimization in which one seeks to find a set of continuous or integer values that maximizes or minimizes an objective function while satisfying a set of constraints (Chen, Der-San and Batson, Robert G and Dang, Yu, 2010). Given an objective function J, decision variables (i.e. variables of optimization) $x_j \in \mathbb{R}$ and $y_k \in \mathbb{Z}^+$, and input parameters $c_j, d_k, a_{ij}, g_{ik}, b_i \in \mathbb{R}$, a MILP has the mathematical structure represented in autoref:eq:milp-structure (Chen, Der-San and Batson, Robert G and Dang, Yu, 2010).

$$\begin{array}{lll} \text{Maximize} & J = \sum_j c_j x_j + \sum_k d_k y_k & \\ \text{subject to} & \sum_j a_{ij} x_j + \sum_k g_{ik} y_k \leq b_i & (i = 1, 2, ..., m) \\ & x_j \geq 0 & (j = 1, 2, ..., n) \\ & y_k \in \mathbb{Z}^+ 0 & (k = 1, 2, ..., n) \end{array} \tag{1}$$

This formulation of the MILP is also referred to as “crisp”. By this it is meant that each variable in the formulation acts as an injective mapping to its number representation. In other words, no values on the formulation are fuzzy (Jagdeep Kaur and Amit Kumar, 2016).

1.2 Fuzzy Sets Theory

This section introduces the notion of fuzzy numbers and some basic definitions. Concepts from this section are pulled from (H.-J. Zimmermann, 2001, Sapan Kumar Das and T. Mandal and S. A. Edalatpanah, 2016, M. Yaghobi and M. Rabbani and M. Adabitabar Firozja and J. Vahidi, 2014, Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019).

1.2.1 Fuzzy Sets

Let's begin with what a fuzzy number is not. A classical (crisp) set is defined as a collection of elements $x \in X$. Crisp sets are binary, either an element either belongs in the set, or it does not (H.-J. Zimmermann, 2001). For a fuzzy set, what is known as the characteristic functions applies various degrees of membership for elements of a given set (H.-J. Zimmermann, 2001).

Definition 1.1 *Let X be a collection of objects (often called the universe of discourse (Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019)). If X is denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs as shown in autoref:eq:membership-function.*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\} \tag{2}$$

$\mu_{\tilde{A}}$ is called the membership function where $\mu_{\tilde{A}}$ is the mapping $\mu_{\tilde{A}} : X \to [0, 1]$; which assigns a real number to the interval $[0, 1]$. The value of $\mu_{\tilde{A}}$ represents the degree of membership of x in $\tilde{A}$.

This paper will use fuzzy sets defined on the real numbers $\mathbb{R}$. The membership function describes the shape of the fuzzy number. As an example, consider the following definition.

Definition 1.2 *A fuzzy number that is represented by $\tilde{A} = (a, b, c)$ is said to be triangular if its membership function is defined as autoref:eq:triangular-fuzzy-number.*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x \leq b \\ \frac{(d-x)}{(d-b)} & c \leq x \leq d \\ 0 & otherwise \end{cases} \tag{3}$$

Definition 1.3 *The fuzzy set tildeA in $\mathbb{R}$ is normal if $max_x \mu_{\tilde{A}}(x) = 1$.*

Definition 1.4 *A fuzzy set $\tilde{A}$ in $\mathbb{R}$ is convex if and only if the membership function of $\tilde{A}$ satisfies the inequality*

$$\mu_{\tilde{A}}[\beta x_1 + (1 - \beta)x_2] \geq min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)] \; \forall x_1, x_2 \in \mathbb{R} \; \beta \in [0, 1]$$

Definition 1.5 *A fuzzy number is a normal convex fuzzy set in $\mathbb{R}$.*

Definition 1.6 *The triangular fuzzy number $\tilde{A}$ is nonnegative $\iff$ $a \geq 0$.*

A more general definition of fuzzy numbers is known as LR fuzzy numbers (Jagdeep Kaur and Amit Kumar, 2016, H.-J. Zimmermann, 2001).

Definition 1.7 *A function $L : [0, \infty] \to [0, 1]$ (or $R : [0, \infty] \to [0, 1]$) is said to be reference a function of the fuzzy number if and only if*

1. *$L(0) = 1$ (or $R(0) = 1$)*
2. *L (or R) is non-increasing on $[0, \infty)$*

Definition 1.8 *A fuzzy number $\tilde{A}$ defined on the set of real numbers, $\mathbb{R}$, denoted as $(m, n, \alpha, \beta)_{LR}$, is said to be an LR flat fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is defined as*

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}) & x \leq m, \alpha > 0 \\ R(\frac{m-n}{\beta}) & x \geq m, \beta > 0 \\ 1 & m \leq x \leq n \end{cases} \tag{4}$$

Definition 1.9 *An LR flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be a non-negative LR flat fuzzy number if and only if $m - \alpha \geq 0$ and is said to be non-positive LR flat fuzzy number if and only if $m - \alpha \leq 0$ is a real number.*

Definition 1.10 *An LR flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be an unrestricted LR flat fuzzy number if and only if $m - \alpha$ is a real number.*

1.2.2 Fuzzy Arithmetic

If two triangular fuzzy numbers $\tilde{a}_1 = \{a_1, a_2, a_3\}$ and $\tilde{b}_1 = \{b_1, b_2, b_3\}$ are nonnegative then the following operations are defined in autoref:eq:fuzzy-arithmetic.

$$\begin{array}{rcl} \tilde{a} \oplus \tilde{b} & = & (a_1 + b_1, a_2 + b_2, a_3 + b_3) \\ \tilde{a} \ominus \tilde{b} & = & (a_1 + b_3, a_2 + b_2, a_3 + b_1) \\ \tilde{a} \otimes \tilde{b} & = & (a_1 b_1, a_2 b_2, a_3 b_3) \end{array} \tag{5}$$

1.2.3 Comparing Fuzzy Numbers

Fuzzy numbers do not directly provide a method of ordering nor do they always provide an ordered set like real numbers (Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019). There are multiple methods for ordering fuzzy numbers, each coming with advantages and disadvantages. Different properties have been applied to justify comparison of fuzzy numbers, such as: preference, rationality, and robustness (Mariano Jiménez and Mar Arenas and Amelia Bilbao and M. Victoria Rodrı´guez, 2007, Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019, Jagdeep Kaur and Amit Kumar, 2016). These methods are commonly known as ranking functions or ordering functions (Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019, Sapan Kumar Das and T. Mandal and S. A. Edalatpanah, 2016, Jagdeep Kaur and Amit Kumar, 2016). Commonly, including in this work, the First index of Yager (Ronald R. Yager, 1981) is used (autoref:eq:first-index-yager).

$$\mathfrak{R}(\tilde{A}) = \frac{\sum_i a_i}{|\tilde{A}|} \tag{6}$$

where $|\cdot|$ represents the cardinality of the fuzzy number. In words, autoref:eq:first-index-yager is merely the average of the values in the fuzzy number.

1.3 Fully Fuzzy Linear Programming

Much like the MILP, fully fuzzy linear programs (FFLP) it is a class of constrained optimization in which one seeks to find a set of continuous variables that either maximizes or minimizes an objective function, J, while satisfying a set of constraints. The key difference in FFLP is that it is designed to accommodate imprecise information (Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019, Jagdeep Kaur and Amit Kumar, 2016). In FFLP, the parameters and decision variables are fuzzy and linear. A general FFLP is represented as shown in autoref:eq:general-fflp. The subscripts $\cdot_e$, $\cdot_l$, and $\cdot_g$ indicate to equality, less than, and greater than constraints, respectively.

$$\begin{array}{ll} \text{Maximize} & J = \sum_j \tilde{C}_j \otimes \tilde{X}_j \\ \text{subject to} & \sum_j \tilde{a}_{ej} \otimes \tilde{x}_j = \tilde{b}_e \quad \forall e = 1, 2, 3, ... \\ & \sum_j \tilde{a}_{lj} \otimes \tilde{x}_j \leq \tilde{b}_l \quad \forall l = 1, 2, 3, ... \\ & \sum_j \tilde{a}_{gj} \otimes \tilde{x}_j \geq \tilde{b}_l \quad \forall g = 1, 2, 3, ... \end{array} \tag{7}$$

There are many methods of solving FFLP (Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019, Jagdeep Kaur and Amit Kumar, 2016, Ali Ebrahimnejad, 2016, Nasseri, SH and Behmanesh, E and Taleshian, F and Abdolalipoor, M and TAGHI, NEZHAD NA, 2013); however, most solution methods convert the fuzzy model into a crisp model that can be solved using traditional methods (Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019). In (Nasseri, SH and Behmanesh, E and Taleshian, F and Abdolalipoor, M and TAGHI, NEZHAD NA, 2013, Marilyn Bello and Gonzalo Nápoles and Ivett Fuentes and Isel Grau and Rafael Falcon and Rafael Bello and Koen Vanhoof, 2019), the method of converting the FFLP into a crisp MILP is simply by applying the ranking function to the objective function and breaking the constraints down into a set of crisp constraints as shown in autoref:eq:nasseri-solution. The constraints are separated according to the definition of fuzzy set multiplication defined in autoref:eq:fuzzy-arithmetic. The fuzzy number index is represented is the exponent rather than the subscript to clearly distinguish between the indexed value in the fuzzy number and the constraint index (i.e. $\tilde{A} = (a^1, a^2, a^3)$). Furthermore, it is assumed that the fuzzy numbers are nonnegative.

$$\begin{array}{lll}
\text{Maximize} & J = \Re\Big(\sum_j (c_j^1, c_j^2, c_j^3)(x_j^1, x_j^2, x_j^3) \Big) & \\
\text{subject to} & \sum_j a_{ej}^1 x_j^1 = b_e^1 & \forall e = 1, 2, 3, ... \\
& \sum_j a_{lj}^1 x_j^1 \leq b_l^1 & \forall l = 1, 2, 3, ... \\
& \sum_j a_{gj}^1 x_j^1 \geq b_l^1 & \forall g = 1, 2, 3, ... \\
& \sum_j a_{ej}^2 x_j^2 = b_e^2 & \forall e = 1, 2, 3, ... \\
& \sum_j a_{lj}^2 x_j^2 \leq b_l^2 & \forall l = 1, 2, 3, ... \\
& \sum_j a_{gj}^2 x_j^2 \geq b_l^2 & \forall g = 1, 2, 3, ... \\
& \sum_j a_{ej}^3 x_j^3 = b_e^3 & \forall e = 1, 2, 3, ... \\
& \sum_j a_{lj}^3 x_j^3 \leq b_l^3 & \forall l = 1, 2, 3, ... \\
& \sum_j a_{gj}^3 x_j^3 \geq b_l^3 & \forall g = 1, 2, 3, ... \\
& x_j^2 - x_j^1 \geq 0 & x_j^3 - x_j^2 \geq 0
\end{array} \tag{8}$$

To be more succinct, the FFLP can also equivalently be written as autoref:eq:nasseri-solution-condensed.

$$\begin{array}{lll}
\text{Maximize} & J = \Re\Big(\sum_j (c_j^1, c_j^2, c_j^3) \otimes (x_j^1, x_j^2, x_j^3) \Big) & \\
\text{subject to} & \sum_j a_{ej}^k x_j^k = b_e^k & \forall e = 1, 2, 3, ... \\
& \sum_j a_{lj}^k x_j^k \leq b_l^k & \forall l = 1, 2, 3, ... \\
& \sum_j a_{gj}^k x_j^k \geq b_l^k & \forall g = 1, 2, 3, ... \\
& x_j^2 - x_j^1 \geq 0 & x_j^3 - x_j^2 \geq 0 \\
& \forall k \in \{1, 2, ...\} &
\end{array} \tag{9}$$

Where k has a max value equal to the cardinality to the type of fuzzy number being utilized. This can be further be elaborated on by rewriting the inequality constraints as equality constraints by introducing slack as equality constraints by introducing slack as equality constraints by introducing slack variables. This is useful as it represents the formulation in a standard form (Chen, Der-San and Batson, Robert G and Dang, Yu, 2010, Robert J. Vanderbei, 2020). It also has the slightly less useful benefit of (mostly) providing the solver a set of equations called a hyperplane (Chen, Der-San and Batson, Robert G and Dang, Yu, 2010).

The given method is called the Kumar and Kaurs method. Generally speaking, it is designed to solve FFLP problems with inequality constraints having LR flat fuzzy numbers. Given the FFLP autoref:eq:general-fflp and assuming that $\tilde{x}_j$ is an LR flat fuzzy number, the problem can be reformulated as autoref:eq:kumar-kaurs-fuzzy (Jagdeep Kaur and Amit Kumar, 2016).

$$\begin{array}{lll}
\text{Maximize} & J = \sum_j \tilde{C}_j \otimes \tilde{X}_j & \\
\text{subject to} & \sum_j \tilde{a}_{ej} \otimes \tilde{x}_j = \tilde{b}_e & \forall e = 1, 2, 3, ... \\
& \sum_j \tilde{a}_{lj} \otimes \tilde{x}_j \oplus \tilde{S}_l = \tilde{b}_l \oplus \tilde{S'}_l & \forall l = 1, 2, 3, ... \\
& \sum_j \tilde{a}_{gj} \otimes \tilde{x}_j \oplus \tilde{S}_e = \tilde{b}_l \oplus \tilde{S'}_g & \forall g = 1, 2, 3, ... \\
& \Re(\tilde{S}_l) - \Re(\tilde{S}'_l) \geq 0 & \forall l = 1, 2, 3, ... \\
& \Re(\tilde{S}_g) - \Re(\tilde{S}'_g) \leq 0 & \forall g = 1, 2, 3, ...
\end{array} \tag{10}$$

Expanding the set of equation and using the condensed notation in autoref:eq:nasseri-solution-condensed we find autoref:eq:kumar-kaurs-crisp (Jagdeep Kaur and Amit Kumar, 2016).

$$
\begin{array}{lll}
\text{Maximize} & J = \Re\Big(\sum_j (c_j^1, c_j^2, c_j^3) \otimes (x_j^1, x_j^2, x_j^3) \Big) & \\
\text{subject to} & \sum_j a_{ej}^k x_j^k = b_e^k & \forall e = 1,2,3,... \\
 & \sum_j a_{lj}^k x_j^k s_l^k \leq s_l^{'k} b_l^k & \forall l = 1,2,3,... \\
 & \sum_j a_{gj}^k x_j^k s_g^k \geq s_l^{'k} b_l^k & \forall g = 1,2,3,... \\
 & \Re(\tilde{S}_l) - \Re(\tilde{S}_l') = 0 & \forall l = 1,2,3,... \\
 & \Re(\tilde{S}_g) - \Re(\tilde{S}_g') = 0 & \forall g = 1,2,3,... \\
 & x_j^2 - x_j^1 \geq 0 & x_j^3 - x_j^2 \geq 0 \\
 & s_j^2 - s_j^1 \geq 0 & s_j^3 - s_j^2 \geq 0 \\
 & s_j^{'2} - s_j^{'1} \geq 0 & s_j^{'3} - s_j^{'2} \geq 0 \\
 & \forall k \in \{1,2,...\} &
\end{array}
\tag{11}
$$

2 The Crisp BAP and PAP

2.1 The Berth Allocation Problem

The BAP models the optimal distribution of container ships to terminals in order to be serviced. The allocation of the ships depends primarily on the size of the ship and its service time (Pablo Frojan and Juan Francisco Correcher and Ramon Alvarez-Valdes and Gerasimos Koulouris and Jose Manuel Tamarit, 2015, Akio Imai and Etsuko Nishimura and Stratos Papadimitriou, 2001, Katja Buhrkal and Sara Zuglian and Stefan Ropke and Jesper Larsen and Richard Lusby, 2011). Most BAP models assume the service time, size, and preferred terminals to be the input parameters and have delay, deviation from ideal position to be the decision variables (Pablo Frojan and Juan Francisco Correcher and Ramon Alvarez-Valdes and Gerasimos Koulouris and Jose Manuel Tamarit, 2015, Akio Imai and Etsuko Nishimura and Stratos Papadimitriou, 2001, Katja Buhrkal and Sara Zuglian and Stefan Ropke and Jesper Larsen and Richard Lusby, 2011). A general formulation for the BAP of a single quay is described in autoref:eq:generalbap. The variables are as described in autoref:tab:bapvariables.

The equations will now explained. autoref:subeq:bapobj is the objective function for the BAP. In this form, it is attempting to minimize the total time from arrival to service completion. autoref:subeq:baptemporal is a big-M constraint that is used to check if ship i's service time ends before ship i. That is $\sigma_{ij} = 1$ if $a_j \geq a_i - s_i$ and $\sigma_{ij} = 0$ otherwise. Similarly, autoref:subeq:bapspatial checks if ship i is asbelow ship j. That is $\psi_{ij} = 1$ if $v_j \geq v_i - s_i$ and $\psi_{ij} = 0$ otherwise. The equations autoref:subeq:bapvalidpos - autoref:subeq:bappsi ensure that ship j is either assigned after ship i has finished its service and/or j is assigned below ship i; however, $\sigma_{ij} = \sigma_{ji} \neq 1$ or $\psi_{ij} = \psi_{ji} \neq 1$. That is to say a ship cannot be queued before and after another or be queued above and below another simultaneously.

$$\text{Minimize} \sum_{i=1}^{I} (e_i - a_i) \tag{12a}$$

$$\text{subject to } a_j - a_i - s_i - (\sigma_{ij} - 1)T \geq 0 \tag{12b}$$

$$v_j - v_i - s_i - (\psi_{ij} - 1)S \geq 0 \tag{12c}$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1 \tag{12d}$$

$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{12e}$$

$$\psi_{ij} + \psi_{ji} \leq 1 \tag{12f}$$

$$s_i + a_i = e_i \tag{12g}$$

$$a_i \leq a_i \leq (T - s_i) \tag{12h}$$

$$\sigma_{ij} \in \{0,1\},\ \psi_{ij} \in \{0,1\}\ v_i \in [0\ S] \tag{12i}$$

Table 1: Table of variables used for the BAP

Variable	**Description**
Input constants	
I	Number of total ships
Input variables	
a_i	Arrival time of ship i
e_i	Time ship i must departs the berth
Decision Variables	
ψ_{ij}	Tracks spatial overlap for ships (i, j)
σ_{ij}	Tracks temporal overlap for ships (i, j)
s_i	Service time for ship i
u_i	Service start time for ship i
v_i	Assigned quay for ship i

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