definition

Contents

1 Preliminaries

1.1 Mixed Interger Linear Program

A mixed integer linear programming (MILP) problem is a class of constrained optimization in which one seeks to find a set of continuous or integer values that maximizes or minimizes an objective function while satisfying a set of constraints [1]. Given an objective function J, decision variables (i.e. variables of optimization) $x_j \in \mathbb{R}$ and $y_k =\in \mathbb{Z}^+$, and input parameters $c_j, d_k, a_{ij}, g_{ik}, b_i \in \mathbb{R}$, a MILP has the mathematical structure represented in Equation 1 [1].

$$\begin{array}{lll} \text{Maximize} & J = \sum_j c_j x_j + \sum_k d_k y_k & \\ \text{subject to} & \sum_j a_{ij} x_j + \sum_k g_{ik} y_k \leq b_i & (i = 1, 2, ..., m) \\ & x_j \geq 0 & (j = 1, 2, ..., n) \\ & y_k \in \mathbb{Z}^+ 0 & (k = 1, 2, ..., n) \end{array} \tag{1}$$

This formulation of the MILP is also referred to as crisp. By this it is meant that each variable in the formulation acts as an injective mapping to its number representation. In other words, no values on the formulation are fuzzy [2].

1.2 Fuzzy Sets and LR Fuzzy Numbers

This section introduces the notion of fuzzy numbers and some basic definitions. Let's begin with what a fuzzy number is not. A classical (crisp) set is defined as a collection of elements $x \in X$. Each element either belongs in the set, or it does not [3]. For a fuzzy set, what is known as the characteristic functions applies various degrees of membership for elements of a given set [3].

Definition 1.1 *Let X be a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\} \tag{2}$$

$\mu_{\tilde{A}}$ is called the membership function where $\mu_{\tilde{A}}$ is the mapping $\mu_{\tilde{A}} : X \to [0, 1]$; which assigns a real number to the interval $[0, 1]$. The value of $\mu_{\tilde{A}}$ represents the degree of membership of x in $\tilde{A}$.

References

[1] Chen, D.-S., Batson, R. G., and Dang, Y. *Applied integer programming.* Wiley, 2010.

[2] Kaur, J., and Kumar, A. *An Introduction to Fuzzy Linear Programming Problems.* Studies in Fuzziness and Soft Computing. Springer International Publishing, 2016.

[3] Zimmermann, H.-J. *Fuzzy Set Theory-and Its Applications.* []. Springer Netherlands, 2001.