A SIMULATED ANNEALING APPROACH TO THE BATTERY ELECTRIC BUS CHARGING PROBLEM

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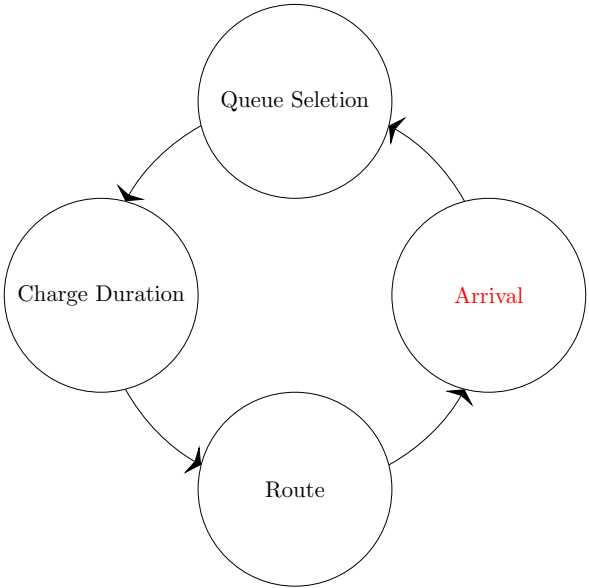
Outline

Topic

Brief State Of The Art

Problem Description



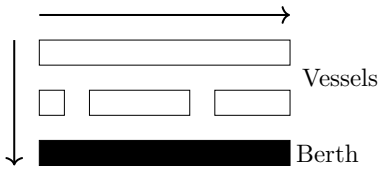
The Berth Allocation Problem[1]

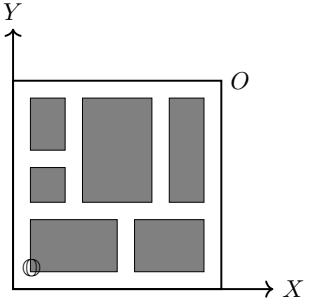
[1] https://www.mdpi.com/2077-1312/11/7/1280

The Berth Allocation Problem

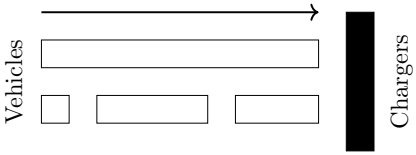
- Vessels move down toward the quay
- Receive service
- Exit to the right

- A variant of the rectangle packing problem
- Solves the problem of optimally assigning incoming vessels to be serviced

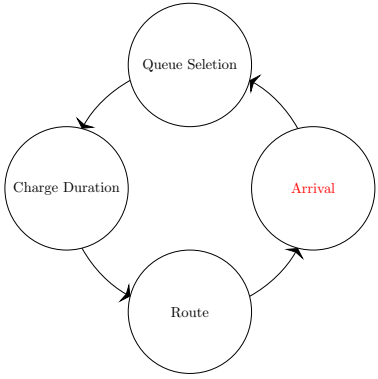




The Position Allocation Problem



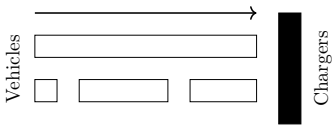
- Service flow is left to right
- Single charger type
- All arrivals are considered unique
- Service times are assumed to be known

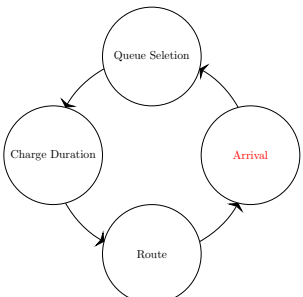


Topic

Requirements For BEB Implementation

- Charges must be able to be tracked
- Service time is unknown
- Accommodate chargers of different speeds
- Minimize charger count
- Minimize consumption cost
- Encourage slow charger use for battery health





Constraints

$$\min \sum_{i=1}^{n_V} \sum_{q=1}^{n_Q} \left(w_{iq} m_q + g_{iq} \epsilon_q \right)$$

$$u_j - u_i - s_i - (\sigma_{ij} - 1) T \geq 0$$

$$v_j - v_i - (\psi_{ij} - 1) n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

$$\sum_{q=1}^{n_Q} w_{iq} = 1$$

$$v_i = \sum_{q=1}^{n_Q} q w_{iq}$$

$$s_i + u_i = d_i$$

$$a_i \leq u_i \leq (T - s_i)$$

$$d_i \leq \tau_i$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i = \eta_{\gamma_i}$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i \geq \nu_{\Gamma_i} \kappa_{\Gamma_i}$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q \leq \kappa_{\Gamma_i}$$

Constraints

$$u_j - u_i - s_i - (\sigma_{ij} - 1)T \geq 0$$

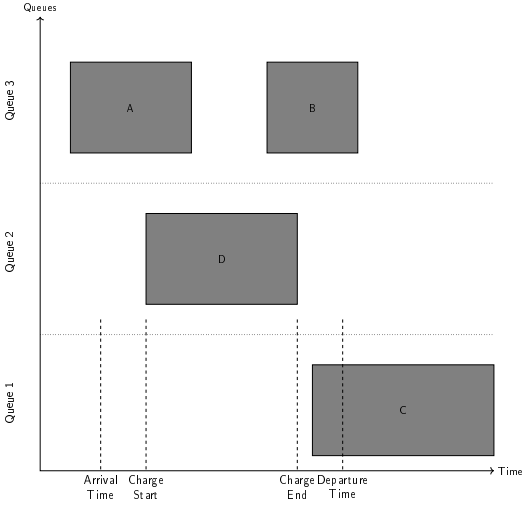
$$v_j - v_i - (\psi_{ij} - 1)n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

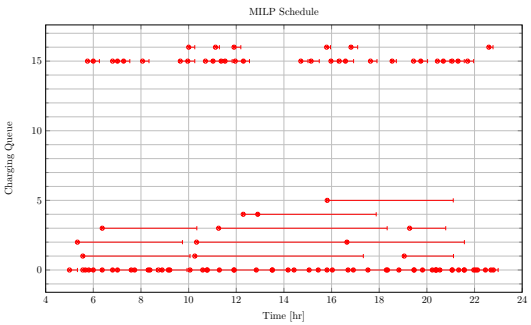
- Used to ensure that gray rectangles do not overlap
- σ_{ij} establishes temporal ordering when active
- ψ_{ij} establishes spacial ordering when active



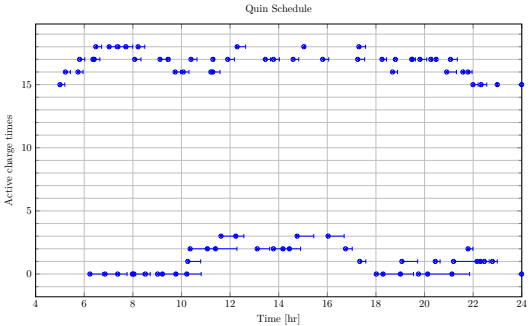
Parameters

- Executed for 7200 seconds (2 hours)
- $T = 24$
- $n_V = 338$
- $n_A = 35$
- $\alpha_i = 90\%;\ \nu_i = 20\%;\ \beta_i = 70\%$
- $\forall q \in \{n_B+1, n_B+2, ..., n_B+n_C\};\ m_q = 1000q$

Schedules

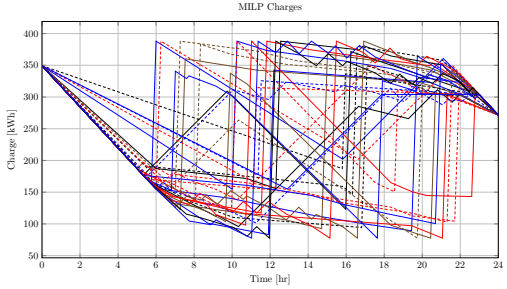


- Utilized more slow chargers for longer durations
- Fast chargers utilized more sparingly

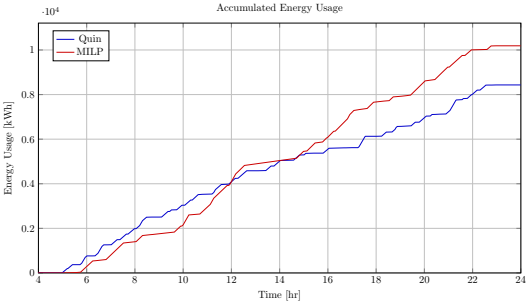


- Extensive use of fast chargers
- Buggy version of Qin, keep because that is in the publication?

SOC And Energy Use



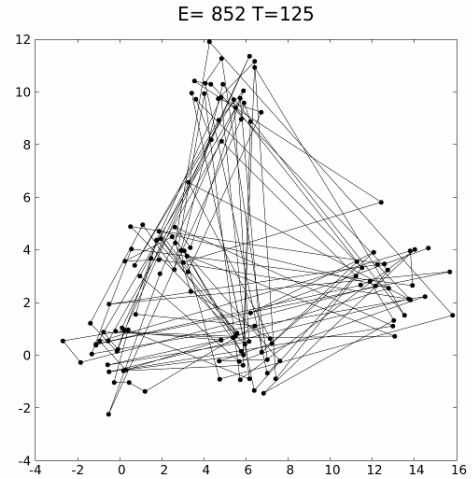
- SOC of Qin allowed to drop to 0
- PAP maintained minimum SOC and final SOC
- PAP accumulated energy is larger due to minimum SOC constraints



Topic

Simulated Annealing[2]

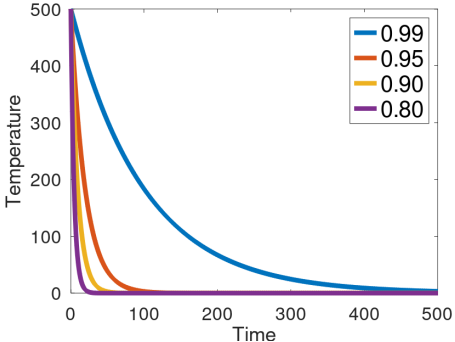
- A probabilistic technique for approximating the global optimum of a given function.
- Often applied to problems that contain many local solutions
- Three key components:
  - Cooling Schedule
  - Acceptance Criteria
  - Generation Mechanisms



[2] https://en.wikipedia.org/wiki/Simulated_annealing

Cooling Schedule

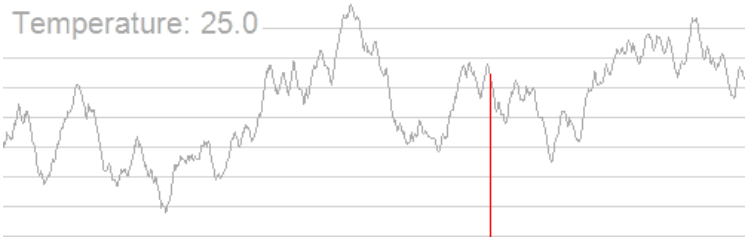
- The cooling equation models the rate at which the temperature decreases over time in the SA process.
- The temperature is high, SA encourages exploration. As the temperature decreases, exploitation is encouraged.

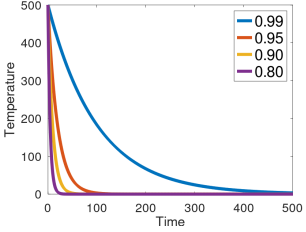


$$t_m = \beta t_{m-1}$$

Acceptance Criteria[3]

$$f(\mathbb{I}, \bar{\mathbb{I}}, t_m) = \begin{cases} 1 & \Delta E > 0 \\ e^{-\frac{\Delta E}{t_m}} & \text{otherwise} \end{cases}$$

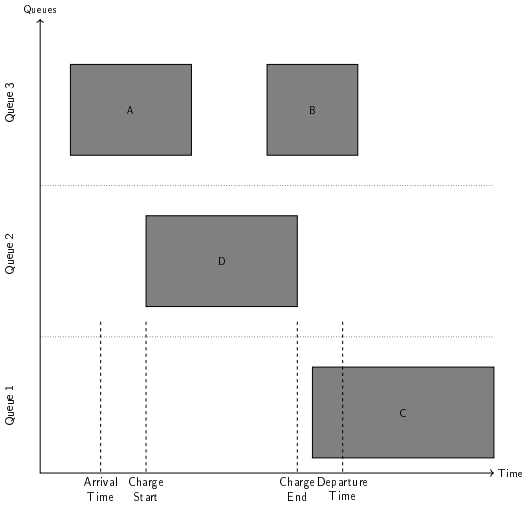




[3] https://en.wikipedia.org/wiki/Simulated_annealing

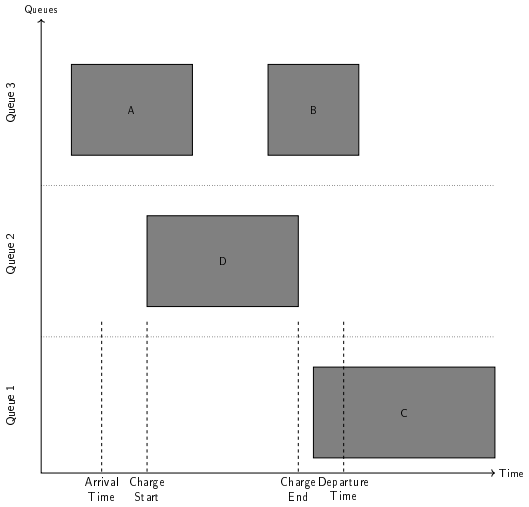
Generation Mechanisms - Primitive Functions

- New Visit: Move a bus from a wait queue to charge queue
- Slide Visit: Change the charge duration of a visit
- New Charger: Move a visit to a new charger
- Wait: Move a visit to its idle queue
- New Window: Execute Wait then New Visit primitives



Generation Mechanisms - Wrapper Functions

- Charge Schedule Generation: Iterate through each visit and execute New Visit
- Perturb Schedule: Randomly execute one of the primitives with a weighted distribution



Objective Function

$$J(\mathbb{I}) = z_d p_d + \sum_{i=1}^{n_V} \left[\epsilon_{q_i} r_{q_i} + z_p \phi_i(\eta_i - \nu_{b_i} \kappa_{b_i}) + z_c r_{q_i} s_i \right]$$

- Demand cost
  - $p_{T_p}[h] = \frac{1}{T_p} \sum_{h - \frac{T_p}{dt}+1}^{h} p_h$
  - $p_d = \max(p_{fix}, p_{max})$
  - $p_{max} = \max_{h \in H} p_{T_p}[h]$

- $\epsilon_{q_i} r_{q_i}$: Assignment Cost
- $z_p \phi_i(\eta_i - \nu_{b_i} \kappa_{b_i})$: Penalty Function
- $z_c r_{q_i} s_i$: Consumption Cost

Constraints

$$u_j - d_i - (\sigma_{ij} - 1)T \geq 0$$
$$q_j - q_i - 1 - (\psi_{ij} - 1)Q \geq 0$$
$$\sigma_{ij} + \sigma_{ji} \leq 1$$
$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$
$$s_i = d_i - u_i$$
$$\eta_{\xi_i} = \eta_i + r_{q_i} s_i - \Delta_i$$
$$\kappa_{\Xi_i} \geq \eta_i + r_{q_i} s_i$$
$$a_i \leq u_i \leq d_i \leq e_i \leq \mathcal{T}$$

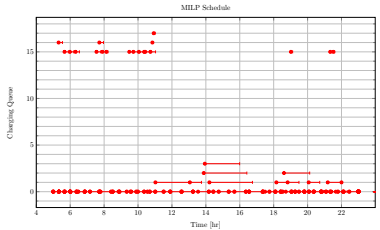
Parameters

Model	Execution Time [s]	Iteration [s]
MILP	1900	N/A
Quick	1532.8	0.4
Heuristic	1916	0.5

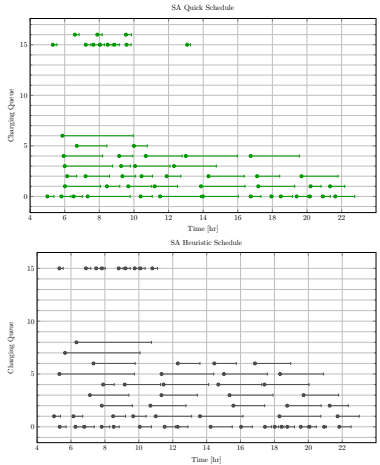
- $T_0 = 90000$
- $\beta = 0.997$
- $|t| = 3797$
- $n_K = 500$

Schedule

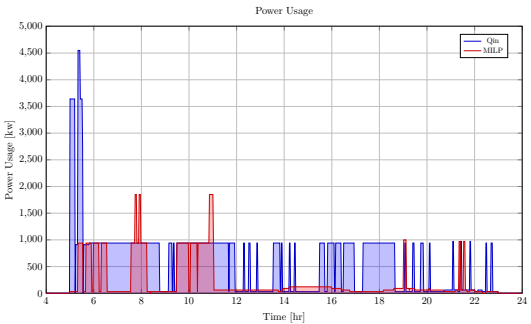
- Qin heavily utilizes fast charger
- PAP emphasizes slow queues, but utilizes fast chargers readily



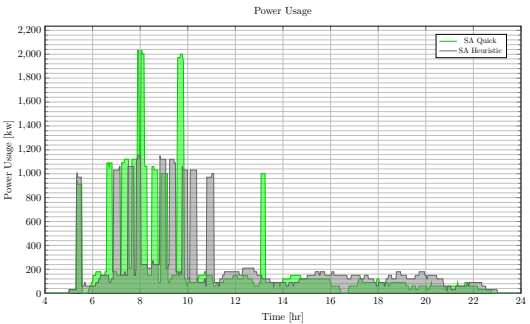
- SA techniques heavily utilize slow charging
- SA techniques utilize fast chargers more sparingly



Power



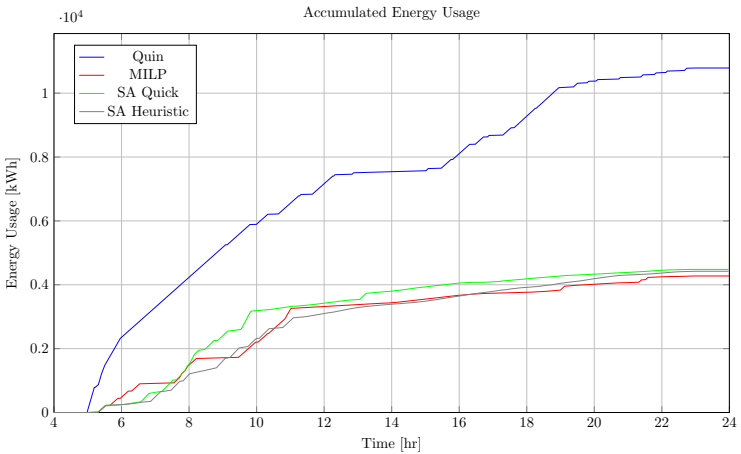
- Heuristic SA maintained the lowest demand peak



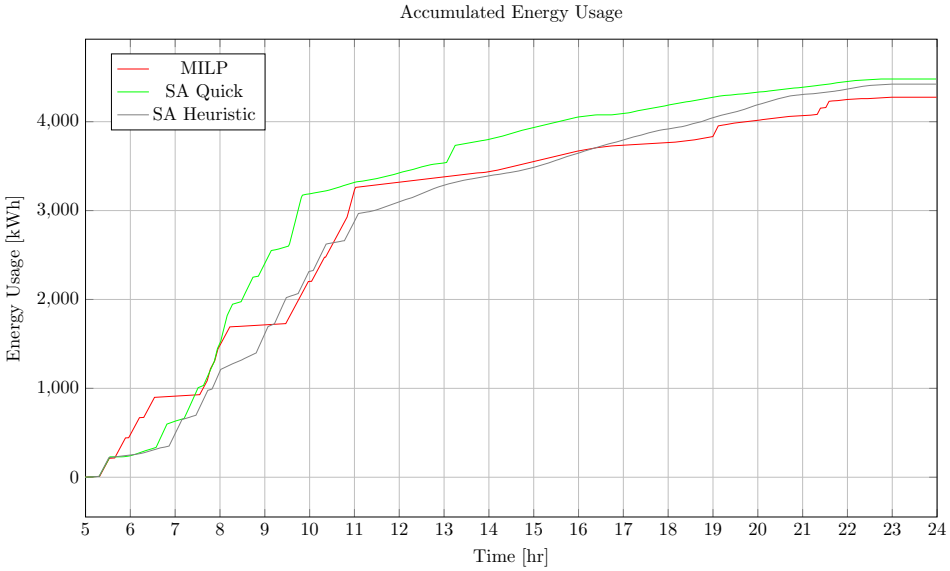
- Quick SA peak demand comparable to the PAP peak

Energy

- PAP maintained lowest energy consumption
- Second lowest consumed energy by the heuristic SA (185 kW difference)
- Trade off of the lower peak demand



Energy


Accumulated Energy Usage
MILP
SA Quick
SA Heuristic
Energy Usage [kWh]
0
1,000
2,000
3,000
4,000
5
6
7
8
9
10
11
12
13
14
15
16
17
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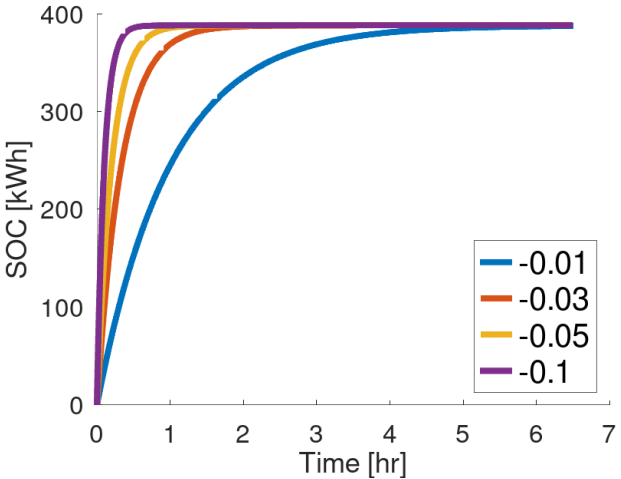
19
20
21
22
23
24
Time [hr]

Topic

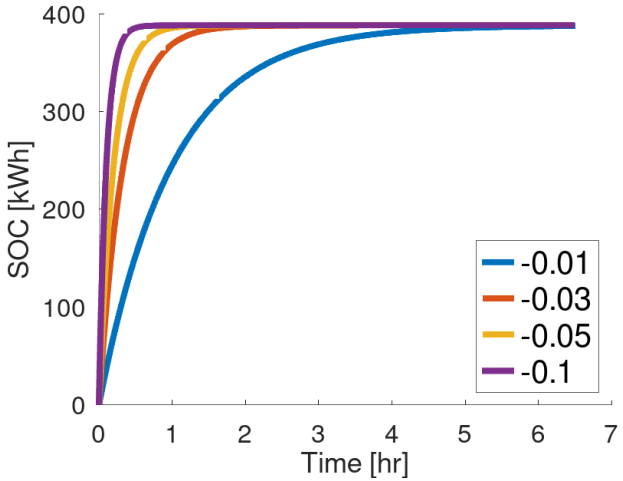
- Higher fidelity in approximating charge at high SOC
- Implemented in SA for simplicity



Non-Linear Battery Dynamics Model

$$\eta_{\xi_i} = \bar{a}_q \eta_i - \bar{b}_q \kappa_{\Xi_i}$$

$$\bar{a}_q = e^{a_q dt} \quad \bar{b}_q = e^{a_q dt} - 1$$



Results

- Figures!

Appendix

Supplemental Material

Mixed Integer Linear Programming

$$\max J = \sum_j c_j x_j + \sum_k d_k y_k$$

$$\text{subject to } \sum_j a_{ij} x_j + \sum_k g_{ik} y_k \leq b_i \qquad (i = 1, 2, ..., m)$$

$$x_j \geq 0 \qquad (j = 1, 2, ..., n)$$

$$y_k \in \mathbb{Z}^+ \qquad (k = 1, 2, ..., n)$$

- J: Objective function
- $x_j \in \mathbb{R}$ and $y_k \in \mathbb{Z}^+$: Decision Variables
- $c_j, d_k, a_{ij}, g_{ik}, b_i \in \mathbb{R}$: Parameters

SA Primitive Generators

New Visit

Algorithm 1: New visit algorithm

Algorithm: New Visit

Input: $\mathbb{S}$

Output: $\bar{\mathbb{S}}$

begin

  $i \leftarrow \mathbb{S}_i$; /* Extract visit index */

  $a \leftarrow \mathbb{I}_{i.a}$; /* Extract the arrivial time for visit i */

  $e \leftarrow \mathbb{I}_{i.e}$; /* Extract the departure time for visit i */

  $q \leftarrow \mathbb{I}_{i.q}$; /* Extract the current charge queue for visit i */

  $\bar{q} \leftarrow \mathcal{U}_Q$; /* Select a random charging queue with a uniform distribution */

  $C \leftarrow \mathcal{U}_{\mathbb{C}_q}$; /* Select a random time slice from $\mathbb{C}_q$ */

  if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime`$(C, i, q, a, e) \notin \varnothing$ **then** /* If there is time available in C_q^j */

    return $(i, (\bar{q}, \bar{u}, \bar{d}), \bar{C})$ /* Return visit */

  end

  return $(\varnothing)$; /* Return nothing */

end

Algorithm 2: Slide Visit Algorithm

Algorithm: Slide Visit

Input: $\mathbb{S}$

Output: $\bar{\mathbb{S}}$

begin
  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$ `Purge`$(\mathbb{S})$; `/* Purge visit` i `from charger availibility matrix */`
  $C \leftarrow \bar{C}_{i.q_i}$; `/* Get the time availability of the purged visit */`
  `/* If there is time available in` C `*/`
  if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime`$(C, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \varnothing$ **then**
    return $(i, \mathbb{I}, (\mathbb{I}_{i.q_i}, \bar{u}, \bar{d}), \bar{C})$ `/* Return updated visit */`
  end
  return $(\varnothing)$; `/* Return nothing */`
end

New Charger

Algorithm 3: New Charger Algorithm

Algorithm: New Charger

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$ `Purge`$(\mathbb{S})$; `/* Purge visit` i `from charger availibility matrix */`
  $q \leftarrow \mathcal{U}_Q$; `/* Select a random charging queue with a uniform distribution */`
  if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime`$(\bar{\mathbb{C}}_{i.q}, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \varnothing$ **then** `/* If there is time available in` C_q `*/`
    `/* Return visit, note` u `and` d `are the original inital/final charge times. */`
    return $(i, \mathbb{I}, (q, \mathbb{I}_{i.u}, \mathbb{I}_{i.d}), \bar{\mathbb{C}})$
  end
  return $(\varnothing)$; `/* Return nothing */`
end

Wait

Algorithm 4: Wait algorithm

Algorithm: Wait

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$ `Purge`$(\mathbb{S})$; `/* Purge visit` i `from charger availibility matrix */`
  $\bar{\mathbb{C}}'_{\mathbb{I}_{i.\Gamma_i}} \leftarrow \mathbb{C}' \cup \{[\mathbb{I}_{i.a}, \mathbb{I}_{i.e}]\}$; `/* Update the charger availability matrix for wait queue` $\bar{\mathbb{C}}_{i.q_i}$ `*/`
  return $(i, \mathbb{I}, (\mathbb{I}_{i.b}, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}), \bar{\mathbb{C}})$ `/* Return visit */`
end

New Window

Algorithm 5: New window algorithm

Algorithm: New Window

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

```
1 begin
2     S̄ ← Wait(S);                          /* Assign visit to its respective idle queue */
3     if S̄_i ← NewVisit(S̄) ∉ ∅ then          /* Add visit i back in randomly */
4         return S̄_i                          /* Return visit */
5     end
6     return (∅);                            /* Return nothing */
7 end
```

SA Wrapper Functions

Charge Schedule Generation

Algorithm 6: Charge schedule generation algorithm

Algorithm: Candidate Solution Generator

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

1 **begin**
  /* Select an unscheduled BEB visit from a randomly indexed set of visits */
2 **foreach** $\mathbb{I}_i \in \mathbb{I}$ **do**
3 $(i, \bar{\mathbb{I}}, \bar{\mathbb{C}}) \leftarrow$ `NewVisit`$(\mathbb{I}_i, \mathbb{I}, \mathbb{C})$; /* Assign the bus to a charger */
4 **end**
5 **return** $(0, \bar{\mathbb{I}}, \bar{\mathbb{C}})$
6 **end**

Charge Schedule Perturbation

Algorithm 7: Perturb schedule algorithm

Algorithm: Perturb Schedule

Input: $\mathbb{S}$

Output: $\bar{\mathbb{S}}$

begin

  $p \leftarrow [false; n_A]$; /* Create vector to track priority routes */

  $y^i \leftarrow [1.0; n_V]$; /* Create weight vector for index selection */

  /* Loop through the visits in reverse order */

  foreach $\mathbb{I}_i \leftarrow \mathbb{I}_{|\mathbb{I}|}$ *TO* $\mathbb{I}_1$ **do**

    /* If the current visit is part of a priority route */

    if $p_{\mathbb{I}_{i,b}} = true$ **then**

      $y^i_{\mathbb{I}_i} = y^i_{\mathbb{I}_{i,\xi}}$;

    end

    /* Else if the current visit's SOC does below the allowed threshold */

    else if $\mathbb{I}_{i,\eta} \leq \nu_{\mathbb{I}_{i,b}} \kappa_{\mathbb{I}_{i,b}}$ **then**

      $p_{\mathbb{I}_{i,b}} = true$; /* Indicate the current BEB's routes are to be prioritized */

      $y^i_{\mathbb{I}_i} = \kappa_{\mathbb{I}_{i,b}} (\nu_{\mathbb{I}_{i,b}} \kappa_{\mathbb{I}_{i,b}} - \mathbb{I}_{i,\eta})$; /* Calculate the weight of the current visit */

    end

  end

  $\mathbb{I}_i \leftarrow \mathcal{W}^{y^i}_{\mathbb{I}}$; /* Select an index with a weighted distribution */

  $i \leftarrow \mathbb{I}_i$; /* Extract visit index */

  $y^p \leftarrow [y^p_1, y^p_2, ...]$; /* Define the weight of each primitive generator */

  $PGF \leftarrow \mathcal{W}^{y^p}_{[1,n_G]}$; /* Select a generator function with weighted distribution */

  $\bar{\mathbb{S}} \leftarrow \mathtt{PGF}((i, \mathbb{I}, \mathbb{C}))$; /* Excecute the generator function */

  return $(0, \mathbb{I}, \bar{\mathbb{C}})$

end

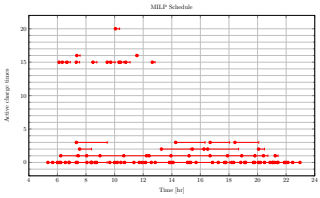
SA Thesis Results

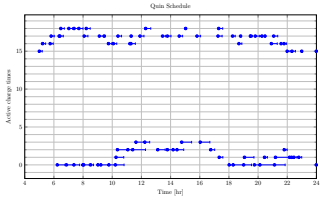
Parameters

Model	Execution Time [s]	Iteration [s]
MILP	3600	N/A
Quick	2275.25	0.25
Heuristic	3640.4	0.4

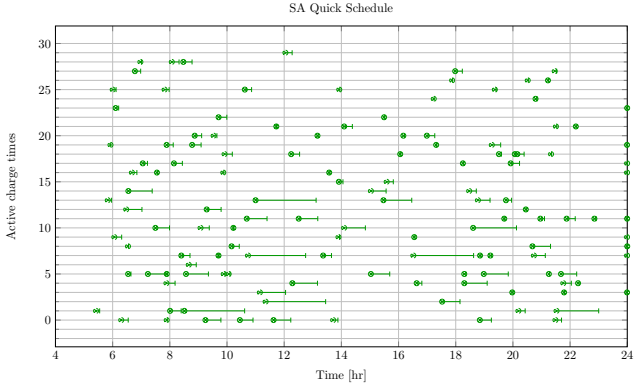
- $T_0 = 99999$
- $\beta = 0.999$
- $|t| = 3797$
- $n_K = 500$

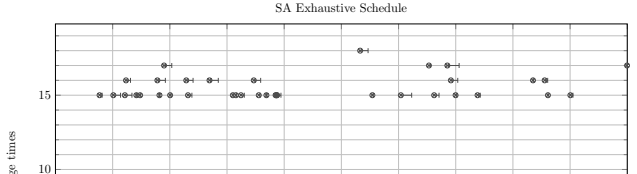
Schedule


MILP Schedule
Active charge times
Time [hr]
0
5
10
15
20
4
6
8
10
12
14
16
18
20
22
24

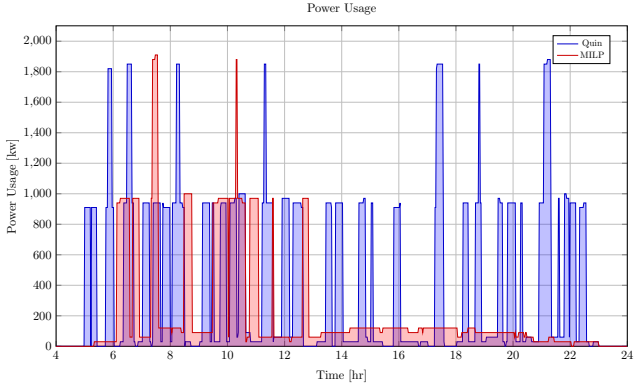

Quin Schedule
Active charge times
Time [hr]
0
5
10
15
4
6
8
10
12
14
16
18
20
22
24

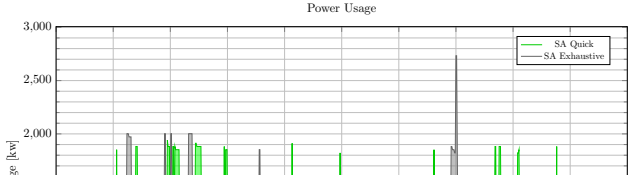
Schedule


SA Quick Schedule
Active charge times
Time [hr]
0
5
10
15
20
25
30
4
6
8
10
12
14
16
18
20
22
24

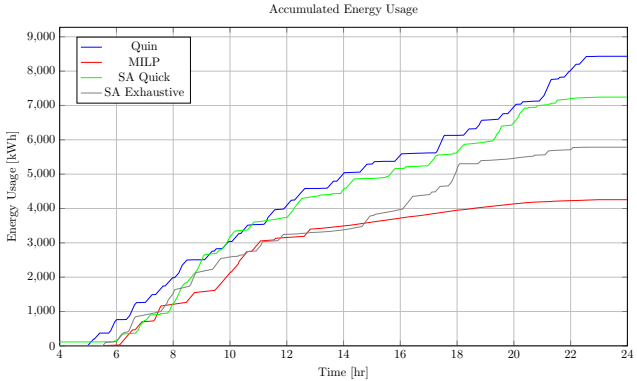

SA Exhaustive Schedule
15
10

Power


Power Usage
Quin
MILP
Power Usage [kw]
Time [hr]
0
200
400
600
800
1,000
1,200
1,400
1,600
1,800
2,000
4
6
8
10
12
14
16
18
20
22
24

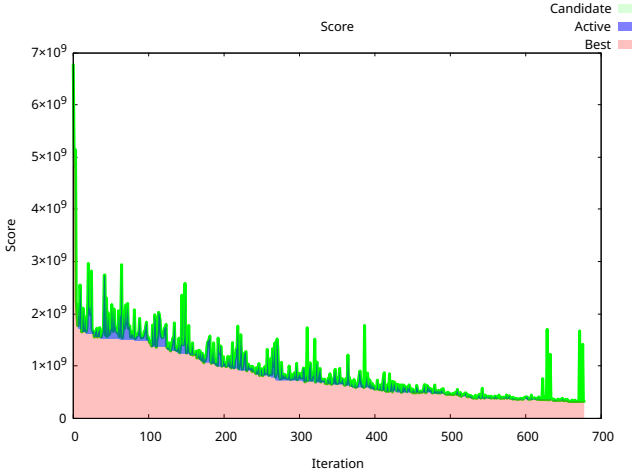

Power Usage
SA Quick
SA Exhaustive
2,000
2,500
3,000

Energy


Accumulated Energy Usage
Quin
MILP
SA Quick
SA Exhaustive
Energy Usage [kWh]
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
4
6
8
10
12
14
16
18
20
22
24
Time [hr]

What Happened?

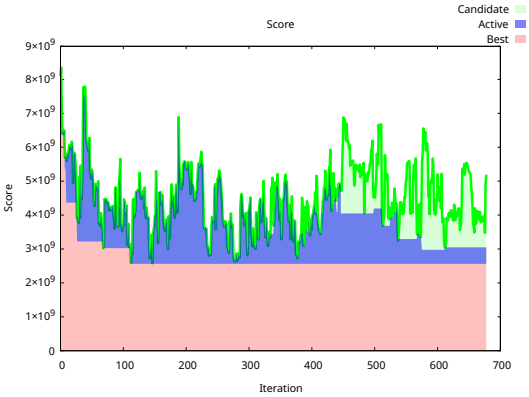
How To Resolve This Problem?



- Reverse search and weight the visit indices
- Be more aggressive in exploiting the best solution

Score Convergence Comparison

Before Fix



After Fix