A POSITION ALLOCATION PROBLEM APPROACH TO THE BATTERY ELECTRIC BUS CHARGING PROBLEM

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May 18, 2024

Outline

Topic

Problem Description

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-	-	6:11 AM	6:16 AM	6:25 AM
6:30 AM	6:34 AM	6:41 AM	6:46 AM	6:55 AM
7:00 AM	7:04 AM	7:11 AM	7:16 AM	7:25 AM
7:30 AM	7:34 AM	7:41 AM	7:46 AM	7:55 AM
8:00 AM	8:04 AM	8:11 AM	8:16 AM	8:25 AM
8:30 AM	8:34 AM	8:41 AM	8:46 AM	8:55 AM
9:00 AM	9:04 AM	9:11 AM	9:16 AM	9:25 AM
9:30 AM	9:34 AM	9:41 AM	9:46 AM	9:55 AM
10:00 AM	10:04 AM	10:11 AM	10:16 AM	10:25 AM
10:30 AM	10:34 AM	10:41 AM	10:46 AM	10:55 AM
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12:00 PM	12:04 PM	12:11 PM	12:16 PM	12:25 PM
12:30 PM	12:34 PM	12:41 PM	12:46 PM	12:55 PM
1:00 PM	1:04 PM	1:11 PM	1:16 PM	1:25 PM
1:30 PM	1:34 PM	1:41 PM	1:46 PM	1:55 PM
2:00 PM	2:04 PM	2:11 PM	2:16 PM	2:25 PM
2:30 PM	2:34 PM	2:41 PM	2:46 PM	2:55 PM
3:00 PM	3:04 PM	3:11 PM	3:16 PM	3:25 PM
3:30 PM	3:34 PM	3:41 PM	3:46 PM	3:55 PM
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7:30 PM	7:34 PM	7:41 PM	7:46 PM	7:55 PM
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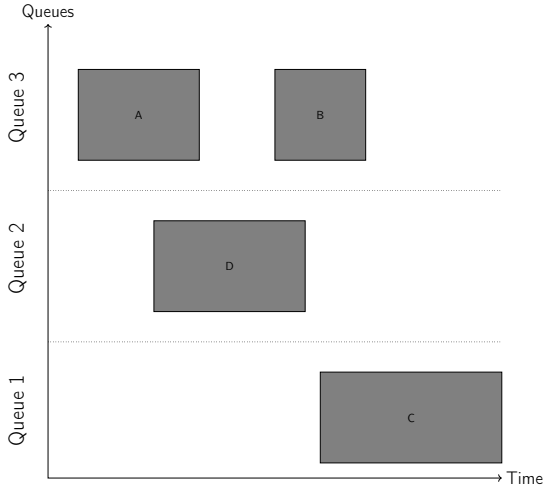


Overall Objective

Model Objectives

Brief State Of The Art

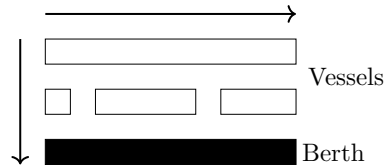
Ref	Consumption	Demand	Linear	Non-linear	Charger Min.	Battery Health
[1]	✓		✓			
[2]	✓	✓	✓			
[3]	✓	✓	✓	✓		
[4]	✓	✓	✓			
[5]	✓	✓	✓			
[6]	✓		✓			
[7]	✓			✓	✓	✓
[8]	✓		✓			
[9]	✓		✓		✓	
BPAP	✓		✓		✓	✓
SA BPAP	✓	✓	✓	✓	*	✓

The Berth Allocation Problem[1]

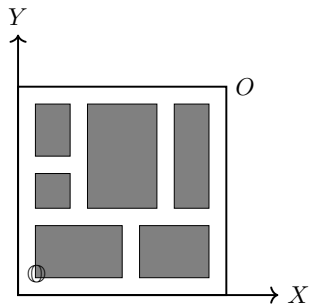
[1] https://www.mdpi.com/2077-1312/11/7/1280

The Berth Allocation Problem

- Vessels move down toward the quay
- Receive service
- Exit to the right



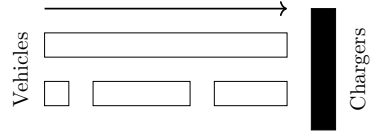
- A variant of the rectangle packing problem
- Solves the problem of optimally assigning incoming vessels to be serviced



The Position Allocation Problem

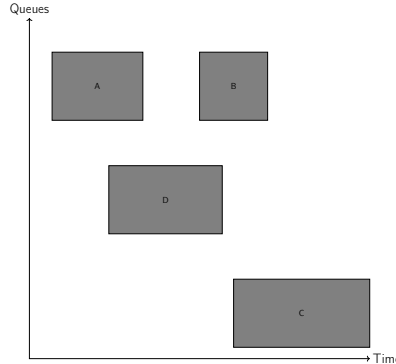
PAP Behavior

- Service flow is left to right
- Single charger type
- All arrivals are considered unique
- Service times are assumed to be known



Desired Behavior

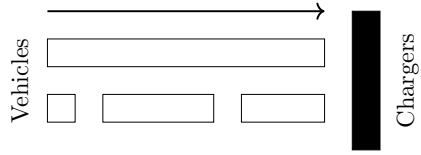
- Discrete charger queues
- Multiple charger types
- Bus can have multiple visits
- Propagate battery SOC across visits

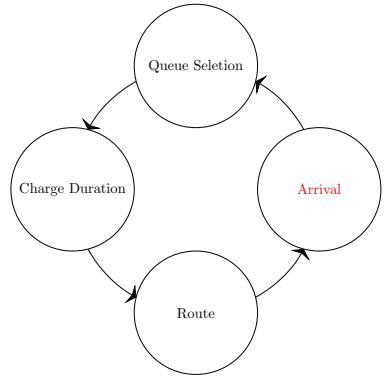


Topic

Requirements For BEB Implementation

- Charges must be able to be tracked
- Service time is unknown
- Accommodate different charger rates
- Minimize charger count
- Minimize consumption cost
- Encourage slow charger use for battery health





Mixed Integer Linear Program

$$\min \sum_{i=1}^{n_V} \sum_{q=1}^{n_Q} \left(w_{iq} m_q + g_{iq} \epsilon_q \right)$$

$$u_j - u_i - s_i - (\sigma_{ij} - 1) T \geq 0$$
$$v_j - v_i - (\psi_{ij} - 1) n_Q \geq 1$$
$$\sigma_{ij} + \sigma_{ji} \leq 1$$
$$\psi_{ij} + \psi_{ji} \leq 1$$
$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

$$s_i + u_i = d_i$$
$$a_i \leq u_i \leq (T - s_i)$$
$$d_i \leq \tau_i$$

$$\sum_{q=1}^{n_Q} w_{iq} = 1$$
$$v_i = \sum_{q=1}^{n_Q} q w_{iq}$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i = \eta_{\gamma_i}$$
$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i \geq \nu_{\Gamma_i} \kappa_{\Gamma_i}$$
$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q \leq \kappa_{\Gamma_i}$$
$$\eta_{\Gamma_b^0} = \alpha_{\Gamma_i} \kappa_{\Gamma_i}$$
$$\eta_{\Gamma_b^f} \geq \beta_{\Gamma_b} \kappa_{\Gamma_b}$$

Queuing Constraints

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$$u_j - u_i - s_i - (\sigma_{ij} - 1)T \geq 0$$

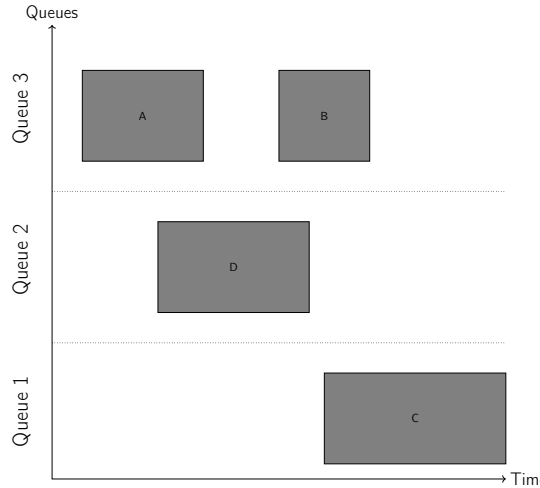
$$v_j - v_i - (\psi_{ij} - 1)n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

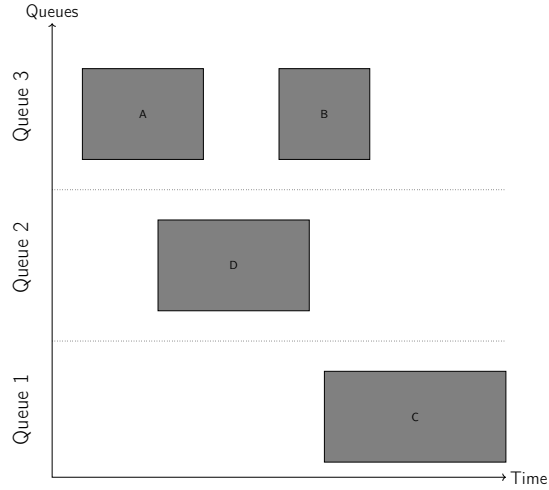
$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

- Used to ensure that gray rectangles do not overlap
- σ_{ij} establishes temporal ordering when active
- ψ_{ij} establishes spacial ordering when active

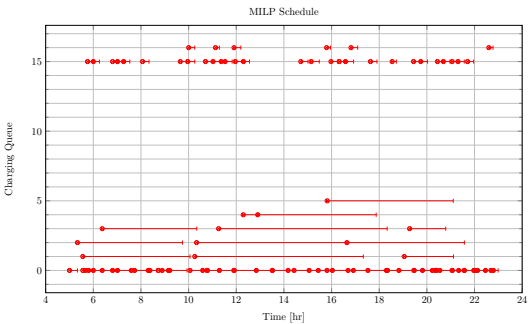


Parameters

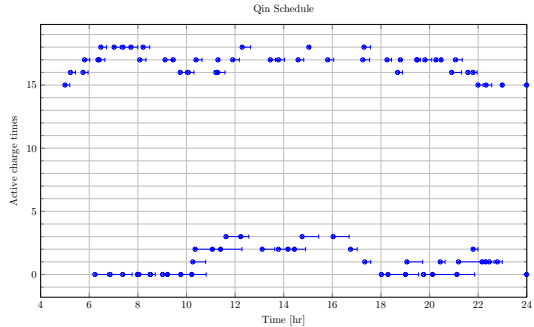
Parameter	Value
Execution	2 hr
T	24 hr
n_V	338
n_A	35
α_i	90%
ν_i	25%
β_i	70%
m_q	1000q



Schedules



- Slow chargers utilized more frequently
- Fast chargers utilized sparingly

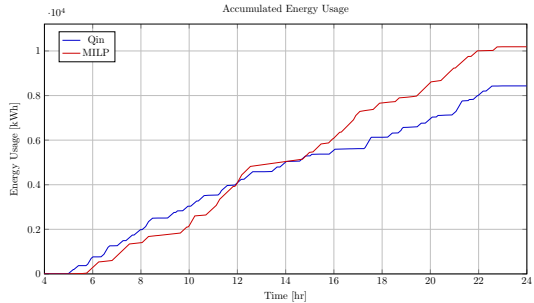


- Slow chargers used for short durations
- Extensive use of fast chargers

SOC And Energy Use

	MILP [kWh]	Qin [kWh]
Mean	265.873	355.93
Min	97.04	0.000
Max	388	368.354

- SOC of Qin allowed to drop to 0
- PAP maintained minimum SOC and final SOC



- PAP accumulated energy is larger due to minimum SOC constraints

Topic

Objective Function And Constraints

$$J(\mathbb{I}) = z_d p_d + \sum_{i=1}^{n_V} \left[\epsilon_{q_i} r_{q_i} + z_p \phi_i(\eta_i - \nu_{b_i} \kappa_{b_i}) + z_c r_{q_i} s_i \right]$$

- p_d: Demand cost
  - $p_{T_p,h} = \frac{1}{T_p} \sum_{k=h-\frac{T_p}{dt}+1}^{h} p_k \; dt$
  - $p_{max} = \max_{k \in [h-\frac{T_p}{dt}+1, h]} p_{T_p,h}$
  - $p_d = \max(p_{fix}, p_{max})$

$$s_i = d_i - u_i$$
$$a_i \leq u_i \leq d_i \leq e_i \leq \mathcal{T}$$

$$\eta_{\xi_i} = \eta_i + r_{q_i} s_i - \Delta_i$$
$$\kappa_{b_i} \geq \eta_i + r_{q_i} s_i$$

- $\epsilon_{q_i} r_{q_i}$: Assignment Cost
- $z_p \phi_i(\eta_i - \nu_{b_i} \kappa_{b_i})$: Penalty Function
- $z_c r_{q_i} s_i$: Consumption Cost

$$u_j - d_i - (\sigma_{ij} - 1)T \geq 0$$
$$q_j - q_i - 1 - (\psi_{ij} - 1)Q \geq 0$$
$$\sigma_{ij} + \sigma_{ji} \leq 1$$
$$\psi_{ij} + \psi_{ji} \leq 1$$
$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

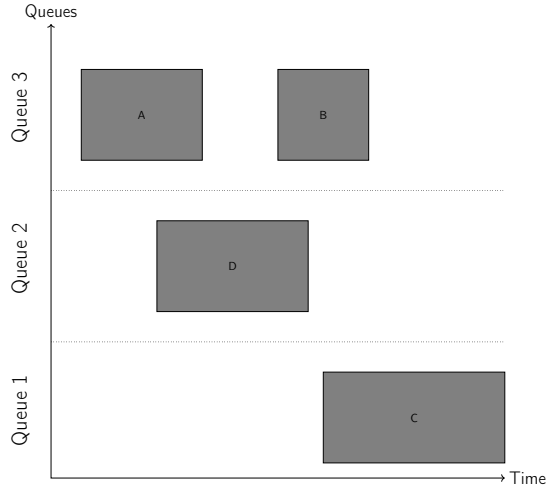
Parameters

NOTES:

- Results are different from what is presented in the thesis
- BPAP model does not take demand cost into consideration

Model	Execution Time [s]
BPAP	1900
Quick	1533
Heuristic	1916

- $T_0 = 90000$
- $\beta = 0.997$
- $|t| = 3797$
- $n_K = 500$



Schedule

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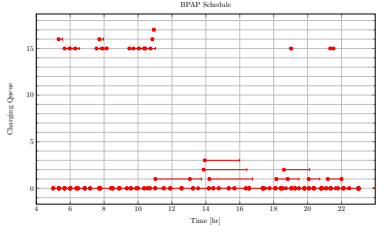
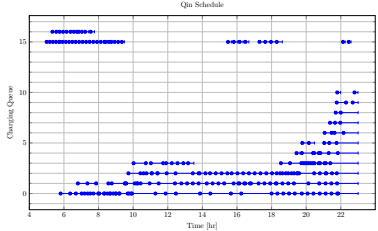
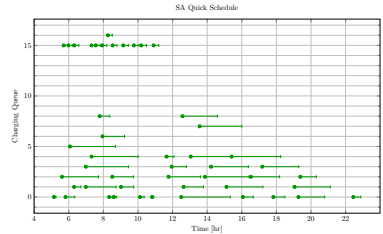
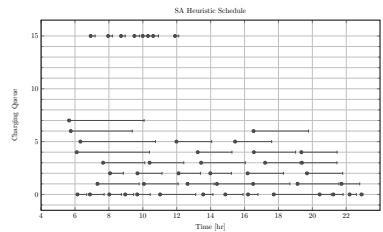
- Qin heavily utilizes fast charger
- BPAP emphasizes slow queues, but utilizes fast chargers readily



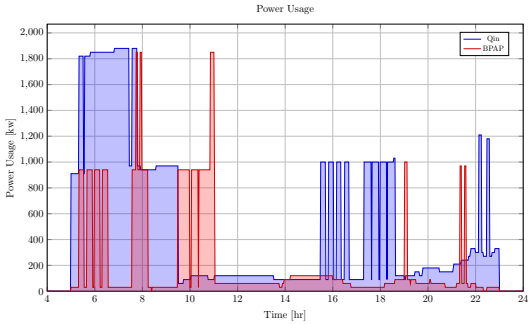


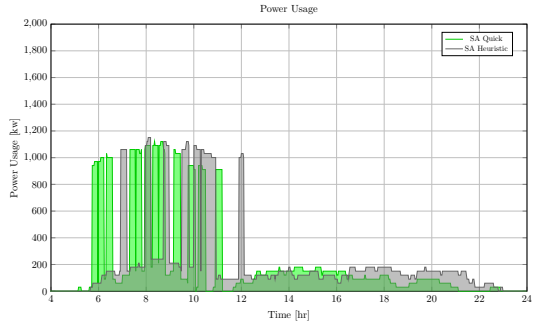
- SA techniques heavily utilize slow charging
- SA techniques utilize fast chargers more sparingly





Power

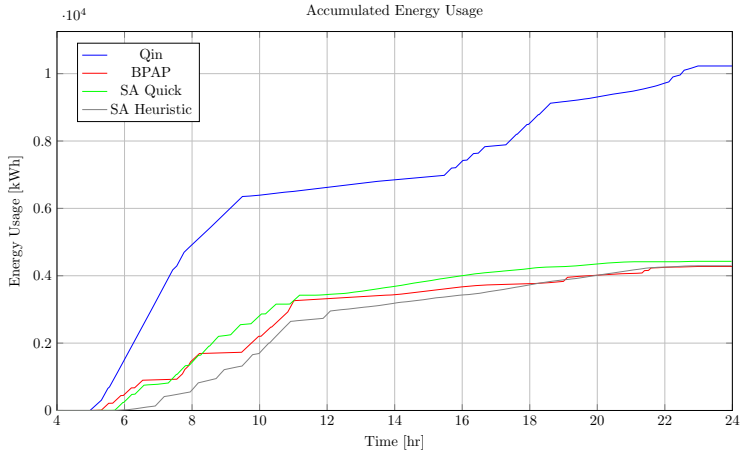




- Heuristic SA maintained the lowest demand peak
- Quick SA peak demand comparable to the PAP peak

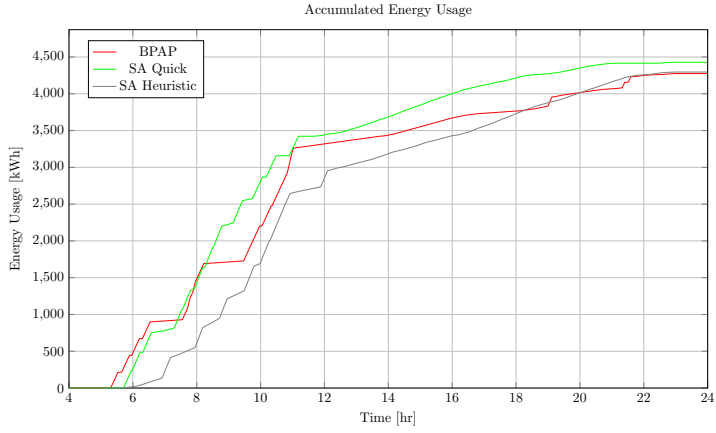
SOC and Energy

- SOC threshold deficits:
  - quick: 2.24 kWh
  - heuristic: 5.74 kWh
- Energy delta:
  - quick: 191.47 kWh
  - heuristic: 58.46 kWh
- Trade off of the lower peak demand



Scores and Energy

Schedule	Score
BPAP	18,500,000
Qin-Modified	34,578,526
Heuristic	11,673,937
Quick	11,234,577



Topic

Summary

Publications

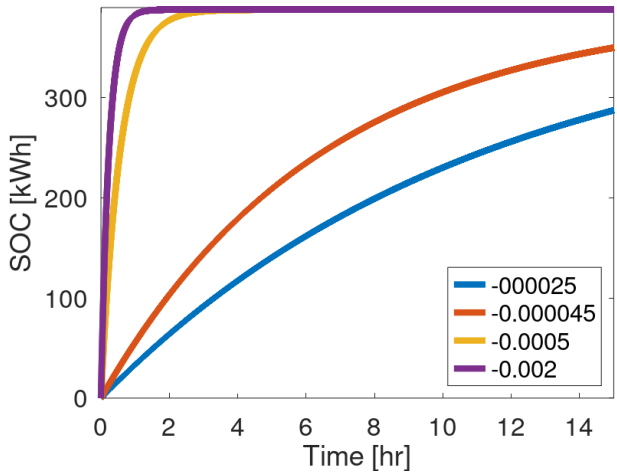
Questions?

Appendix

SA with Non-Linear Battery Dynamics

Introduction

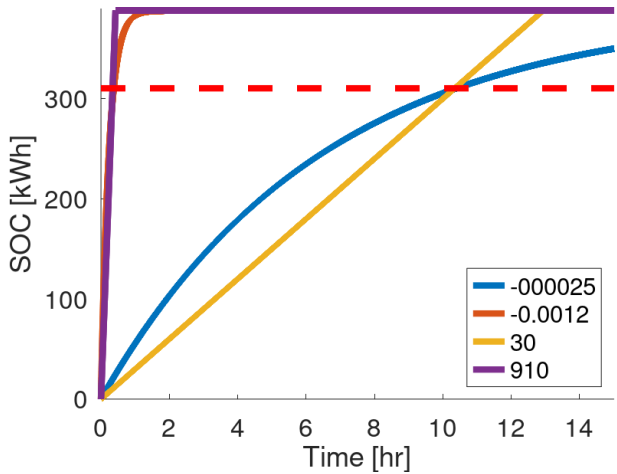
- Higher fidelity in approximating charge at high SOC
- Implemented in SA for simplicity



Non-Linear Battery Dynamics Model

$$\eta_{\xi_i} = \bar{a}_q \eta_i - \bar{b}_q \kappa_{b_i}$$

$$\bar{a}_q = e^{a_q dt} \quad \bar{b}_q = e^{a_q dt} - 1$$



Results

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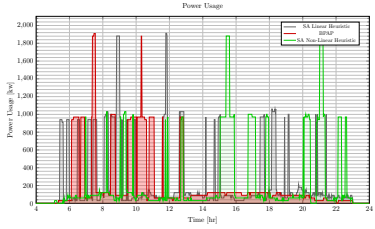
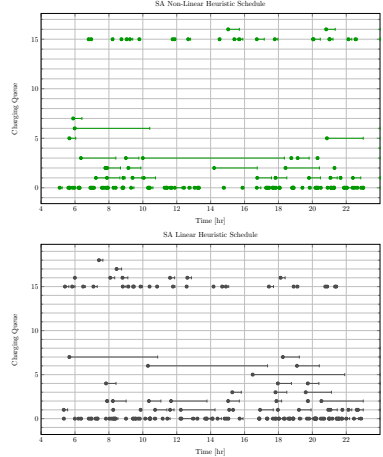
- Minimized demand power
- Larger energy consumption due to fast charger duration



- Similar schedule output, longer fast charger durations due to nonlinear model
- Minimum SOC of 85 kWh



Mixed Integer Linear Programming

$$\max J = \sum_j c_j x_j + \sum_k d_k y_k$$

$$\text{subject to } \sum_j a_{ij} x_j + \sum_k g_{ik} y_k \leq b_i \qquad (i = 1, 2, ..., m)$$

$$x_j \geq 0 \qquad (j = 1, 2, ..., n)$$

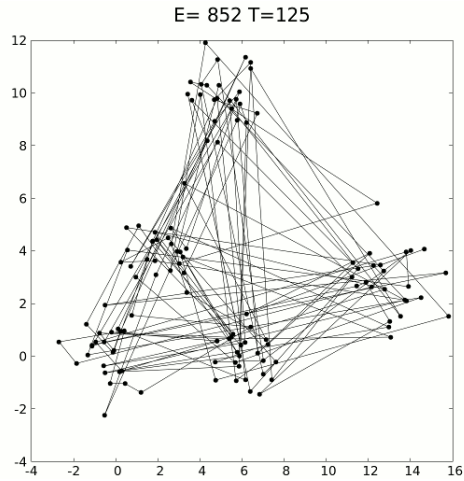
$$y_k \in \mathbb{Z}^+ \qquad (k = 1, 2, ..., n)$$

- J: Objective function
- $x_j \in \mathbb{R}$ and $y_k \in \mathbb{Z}^+$: Decision Variables
- $c_j, d_k, a_{ij}, g_{ik}, b_i \in \mathbb{R}$: Parameters

Simulated Annealing

Simulated Annealing[2]

- A probabilistic technique for approximating the global optimum of a given function.
- Often applied to problems that contain many local solutions
- Three key components:
  - Cooling Schedule
  - Acceptance Criteria
  - Generation Mechanisms



[2] https://en.wikipedia.org/wiki/Simulated_annealing

Cooling Schedule

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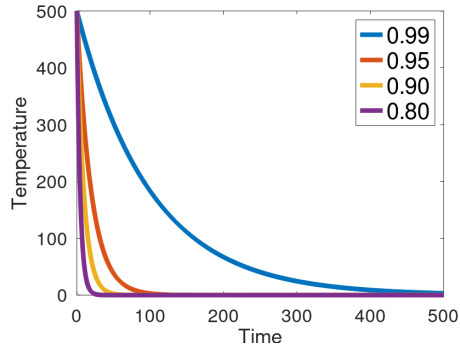
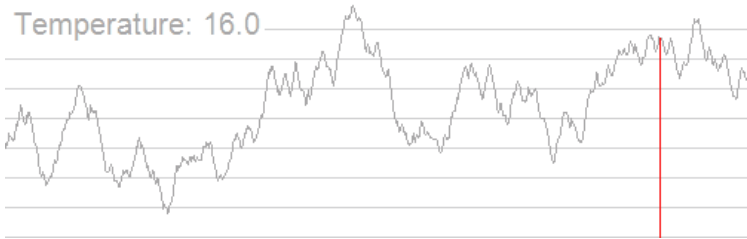
- The cooling equation models the rate at which the temperature decreases over time in the SA process.
- The temperature is high, SA encourages exploration. As the temperature decreases, exploitation is encouraged.

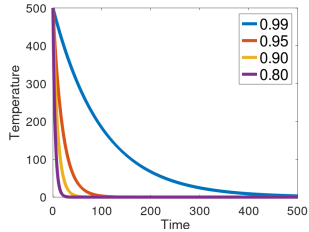


$$t_m = \beta t_{m-1}$$

Acceptance Criteria[3]

$$f(\mathbb{I}, \bar{\mathbb{I}}, t_m) = \begin{cases} 1 & \Delta E > 0 \\ e^{-\frac{\Delta E}{t_m}} & \text{otherwise} \end{cases}$$

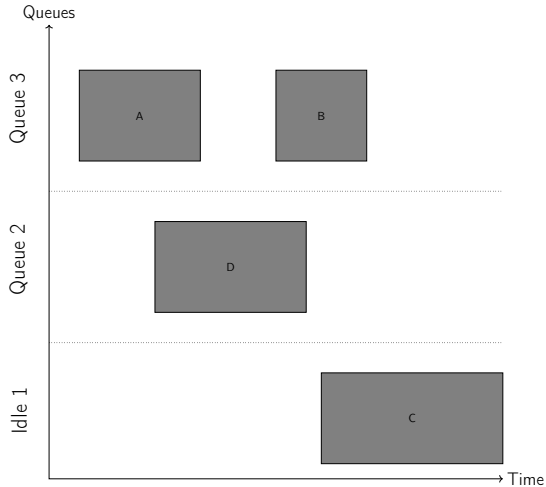




[3] https://en.wikipedia.org/wiki/Simulated_annealing

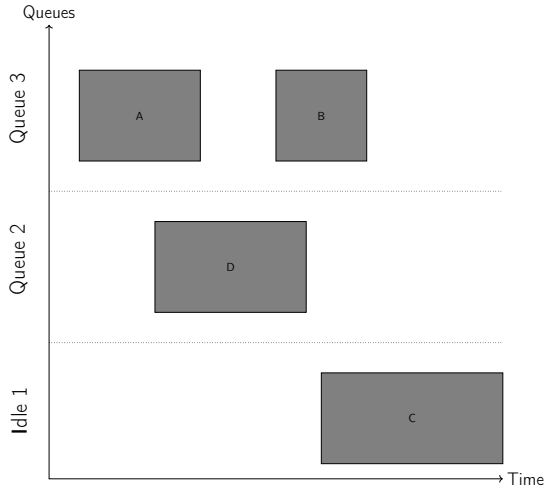
Generation Mechanisms - Primitive Functions

- New Visit: Move a bus from a wait queue to charge queue
- Slide Visit: Change the charge duration of a visit
- New Charger: Move a visit to a new charger
- Wait: Move a visit to its idle queue
- New Window: Execute Wait then New Visit primitives



Generation Mechanisms - Wrapper Functions

- Charge Schedule Generation: Iterate through each visit and execute New Visit
- Perturb Schedule: Randomly execute one of the primitives with a weighted distribution



Algorithm 1: New visit algorithm

Algorithm: New Visit

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  $i \leftarrow \mathbb{S}_i$; /* Extract visit index */
  $a \leftarrow \mathbb{I}_{i.a}$; /* Extract the arrivial time for visit i */
  $e \leftarrow \mathbb{I}_{i.e}$; /* Extract the departure time for visit i */
  $q \leftarrow \mathbb{I}_{i.q}$; /* Extract the current charge queue for visit i */
  $\bar{q} \leftarrow \mathcal{U}_Q$; /* Select a random charging queue with a uniform distribution */
  $C \leftarrow \mathcal{U}_{\mathbb{C}_q}$; /* Select a random time slice from $\mathbb{C}_q$ */
  if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime`$(C, i, q, a, e) \notin \varnothing$ **then** /* If there is time available in C_q^j */
    return $(i, (\bar{q}, \bar{u}, \bar{d}), \bar{C})$ /* Return visit */
  end
  return $(\varnothing)$; /* Return nothing */
end

Algorithm 2: Slide Visit Algorithm

Algorithm: Slide Visit

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$Purge$(\mathbb{S})$; /* Purge visit i from charger availibility matrix */
  $C \leftarrow \bar{C}_{i.q_i}$; /* Get the time availability of the purged visit */
  /* If there is time available in C */
  if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ findFreeTime$(C, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \varnothing$ **then**
    return $(i, \mathbb{I}, (\mathbb{I}_{i.q_i}, \bar{u}, \bar{d}), \bar{C})$ /* Return updated visit */
  end
  return $(\varnothing)$; /* Return nothing */
end

Algorithm 3: New Charger Algorithm

Algorithm: New Charger

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$Purge$(\mathbb{S})$; /* Purge visit i from charger availibility matrix */
  $q \leftarrow \mathcal{U}_Q$; /* Select a random charging queue with a uniform distribution */
  if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ findFreeTime$(\bar{\mathbb{C}}_{i.q}, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \varnothing$ **then** /* If there is time available in C_q */
    /* Return visit, note u and d are the original inital/final charge times. */
    return $(i, \mathbb{I}, (q, \mathbb{I}_{i.u}, \mathbb{I}_{i.d}), \bar{\mathbb{C}})$
  end
  return $(\varnothing)$; /* Return nothing */
end

Wait

Algorithm 4: Wait algorithm

Algorithm: Wait

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

1 **begin**
2 $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$Purge$(\mathbb{S})$; /* Purge visit i from charger availibility matrix */
3 $\bar{\mathbb{C}}'_{\mathbb{I}_{i.\Gamma_i}} \leftarrow \mathbb{C}' \cup \{[\mathbb{I}_{i.a}, \mathbb{I}_{i.e}]\}$; /* Update the charger availability matrix for wait queue $\bar{\mathbb{C}}_{i.q_i}$ */
4 **return** $(i, \mathbb{I}, (\mathbb{I}_{i.b}, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}), \bar{\mathbb{C}})$ /* Return visit */
5 **end**

New Window

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Algorithm 5: New window algorithm

Algorithm: New Window

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

```
1 begin
2     S̄ ← Wait(S);                          /* Assign visit to its respective idle queue */
3     if S̄ ← NewVisit(S̄) ∉ ∅ then           /* Add visit i back in randomly */
4         return S̄                           /* Return visit */
5     end
6     return (∅);                           /* Return nothing */
7 end
```

Charge Schedule Generation

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Algorithm 6: Charge schedule generation algorithm

Algorithm: Candidate Solution Generator

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  /* Select an unscheduled BEB visit from a randomly indexed set of visits */
  foreach $\mathbb{I}_i \in \mathbb{I}$ **do**
    $(i, \bar{\mathbb{I}}, \bar{\mathbb{C}}) \leftarrow$ `NewVisit(`$(\mathbb{I}_i, \mathbb{I}, \mathbb{C})$`)`; /* Assign the bus to a charger */
  end
  return $(0, \bar{\mathbb{I}}, \bar{\mathbb{C}})$
end

Charge Schedule Perturbation

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Algorithm 7: Perturb schedule algorithm

Algorithm: Perturb Schedule
Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$
begin
  $p \leftarrow [false; n_A]$; /* Create vector to track priority routes */
  $y^i \leftarrow [1.0; n_V]$; /* Create weight vector for index selection */
  /* Loop through the visits in reverse order */
  foreach $\mathbb{I}_i \leftarrow \mathbb{I}_{|\mathbb{I}|}$ *TO* $\mathbb{I}_1$ **do**
    /* If the current visit is part of a priority route */
    if $p_{\mathbb{I}_{i,b}} = true$ **then**
      $y^i_{\mathbb{I}_i} = y^i_{\mathbb{I}_{i,\xi}}$;
    end
    /* Else if the current visit's SOC does below the allowed threshold */
    else if $\mathbb{I}_{i,\eta} \leq \nu_{\mathbb{I}_{i,b}} \kappa_{\mathbb{I}_{i,b}}$ **then**
      $p_{\mathbb{I}_{i,b}} = true$; /* Indicate the current BEB's routes are to be prioritized */
      $y^i_{\mathbb{I}_i} = \kappa_{\mathbb{I}_{i,b}}(\nu_{\mathbb{I}_{i,b}} \kappa_{\mathbb{I}_{i,b}} - \mathbb{I}_{i,\eta})$; /* Calculate the weight of the current visit */
    end
  end
  $\mathbb{I}_i \leftarrow \mathcal{W}^{y^i}_{\mathbb{I}}$; /* Select an index with a weighted distribution */
  $i \leftarrow \mathbb{I}_i$; /* Extract visit index */
  $y^p \leftarrow [y^p_1, y^p_2, ...]$; /* Define the weight of each primitive generator */
  $PGF \leftarrow \mathcal{W}^{y^p}_{[\mathbb{1}, n_G]}$; /* Select a generator function with weighted distribution */
  $\bar{\mathbb{S}} \leftarrow \mathrm{PGF}((i, \mathbb{I}, \mathbb{C}))$; /* Excecute the generator function */
  return $(0, \mathbb{I}, \bar{\mathbb{C}})$
end

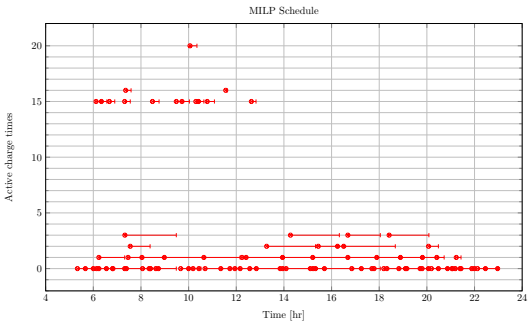
Thesis Results

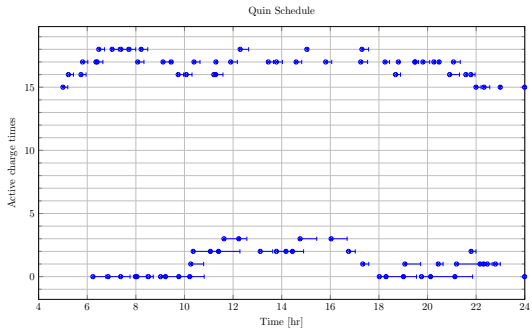
Parameters

Model	Execution Time [s]	Iteration [s]
MILP	3600	N/A
Quick	2275.25	0.25
Heuristic	3640.4	0.4

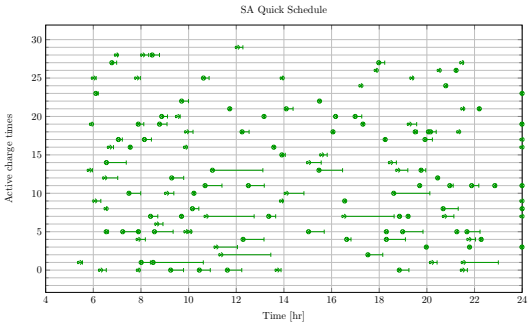
- $T_0 = 99999$
- $\beta = 0.999$
- $|t| = 3797$
- $n_K = 500$

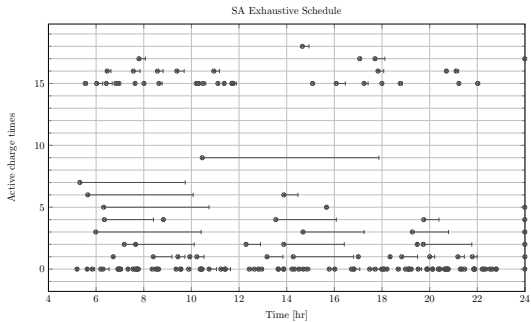
Schedule


MILP Schedule
Active charge times
Time [hr]
0
5
10
15
20
4
6
8
10
12
14
16
18
20
22
24

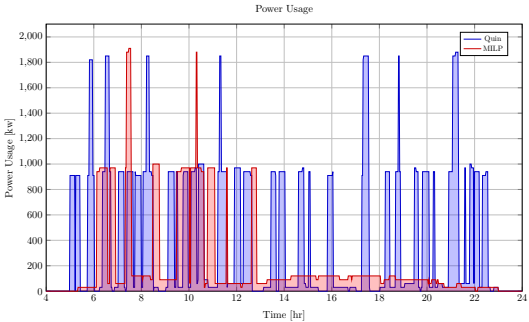

Quin Schedule
Active charge times
Time [hr]
0
5
10
15
4
6
8
10
12
14
16
18
20
22
24

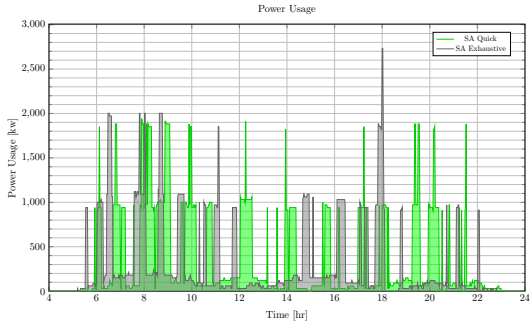
Schedule


SA Quick Schedule
Active charge times
Time [hr]
0
5
10
15
20
25
30
4
6
8
10
12
14
16
18
20
22
24

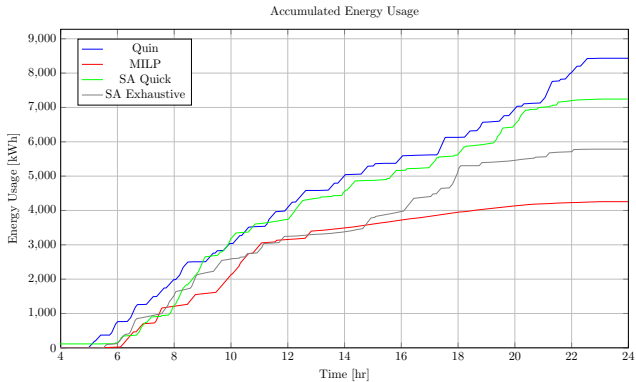

SA Exhaustive Schedule
Active charge times
Time [hr]
0
5
10
15
4
6
8
10
12
14
16
18
20
22
24

Power

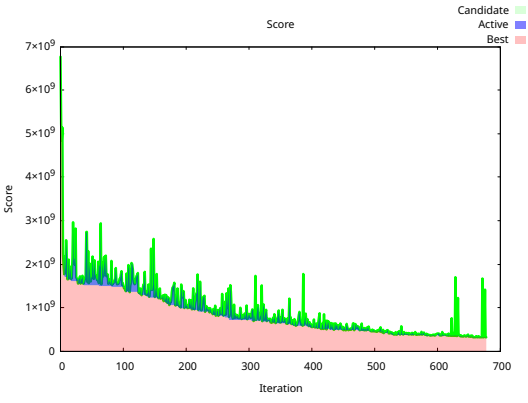

Power Usage
Power Usage [kw]
Time [hr]
Quin
MILP
0
200
400
600
800
1,000
1,200
1,400
1,600
1,800
2,000
4
6
8
10
12
14
16
18
20
22
24


Power Usage
Power Usage [kw]
Time [hr]
SA Quick
SA Exhaustive
0
500
1,000
1,500
2,000
2,500
3,000
4
6
8
10
12
14
16
18
20
22
24

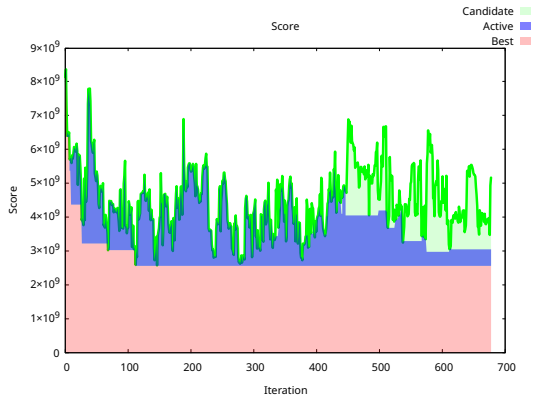
Energy


Accumulated Energy Usage
Quin
MILP
SA Quick
SA Exhaustive
Energy Usage [kWh]
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
4
6
8
10
12
14
16
18
20
22
24
Time [hr]

How To Resolve This Problem?



- Reverse search and weight the visit indices
- Be more aggressive in exploiting the best solution



- Candidate solutions diverge
- Hard time handling “difficult” routes

Score Convergence Comparison

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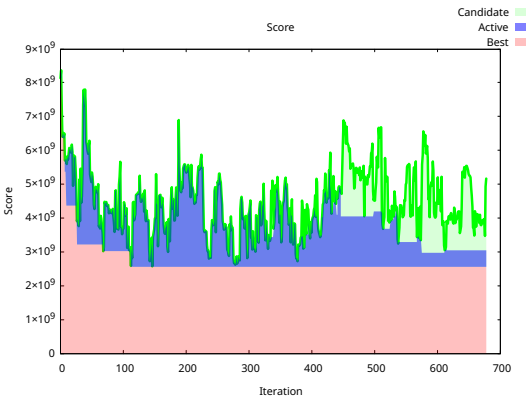
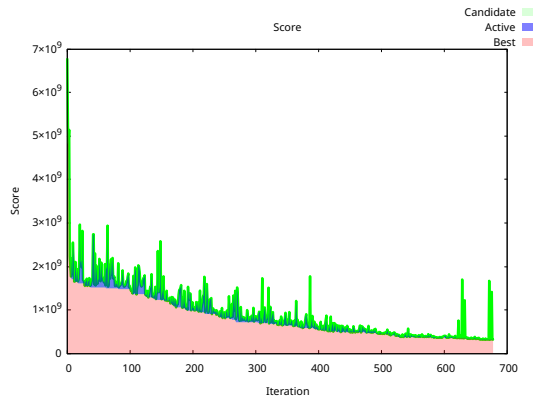
Before Fix



After Fix



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