

A POSITION ALLOCATION PROBLEM APPROACH TO THE BATTERY ELECTRIC BUS CHARGING PROBLEM

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Outline

Introduction

The Position Allocation Problem Approach With Linear Battery Dynamic

The Simulated Annealing Approach With Linear Battery Dynamics

The Simulated Annealing Approach With Non-Linear Battery Dynamics

Introduction

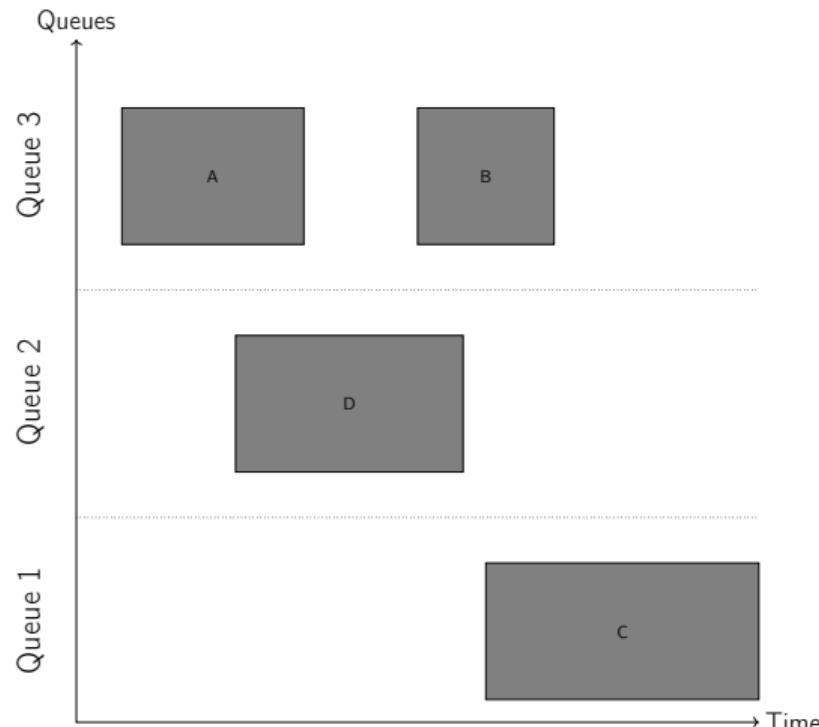
The Position Allocation Problem Approach With Linear Battery Dynamic

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Problem Description

-	-	6:11 AM	6:16 AM	6:25 AM
6:30 AM	6:34 AM	6:41 AM	6:46 AM	6:55 AM
7:00 AM	7:04 AM	7:11 AM	7:16 AM	7:25 AM
7:30 AM	7:34 AM	7:41 AM	7:46 AM	7:55 AM
8:00 AM	8:04 AM	8:11 AM	8:16 AM	8:25 AM
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6:30 PM	6:34 PM	6:41 PM	6:46 PM	6:55 PM
7:00 PM	7:04 PM	7:11 PM	7:16 PM	7:25 PM
7:30 PM	7:34 PM	7:41 PM	7:46 PM	7:55 PM
8:00 PM	8:04 PM	8:11 PM	8:46 PM	8:25 PM



Brief State Of The Art

Ref	Consumption	Demand	Linear	Non-linear	Charger Min.	Battery Health
[1]	✓		1			
[2]	✓	✓	✓			
[3]	✓	✓	✓	✓		
[4]	✓	✓	✓			
[5]	✓	✓	✓			
[6]	✓		✓			
[7]	✓			✓	✓	✓
[8]	✓		2			
[9]	✓		3		✓	
BPAP	✓		✓		✓	✓
SA BPAP	✓	✓	✓	✓	4	✓

The Berth Allocation Problem¹

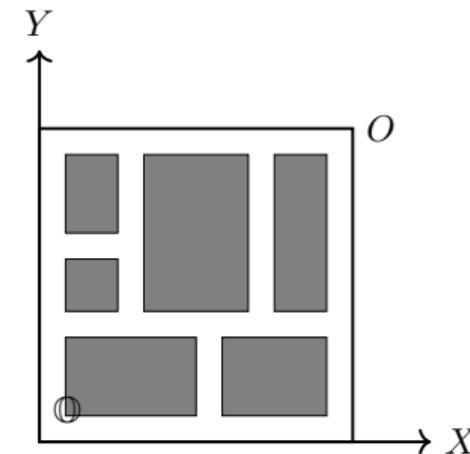
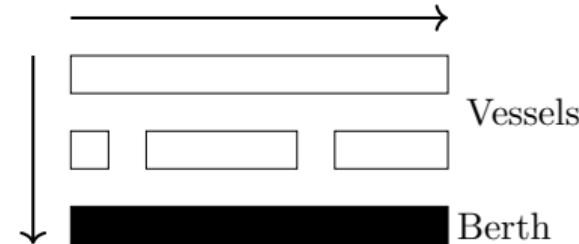


¹<https://www.mdpi.com/2077-1312/11/7/1280>

The Berth Allocation Problem

- ▶ Vessels move down toward the quay
 - ▶ Receive service
 - ▶ Exit to the right
-

- ▶ A variant of the rectangle packing problem
- ▶ Solves the problem of optimally assigning incoming vessels to be serviced

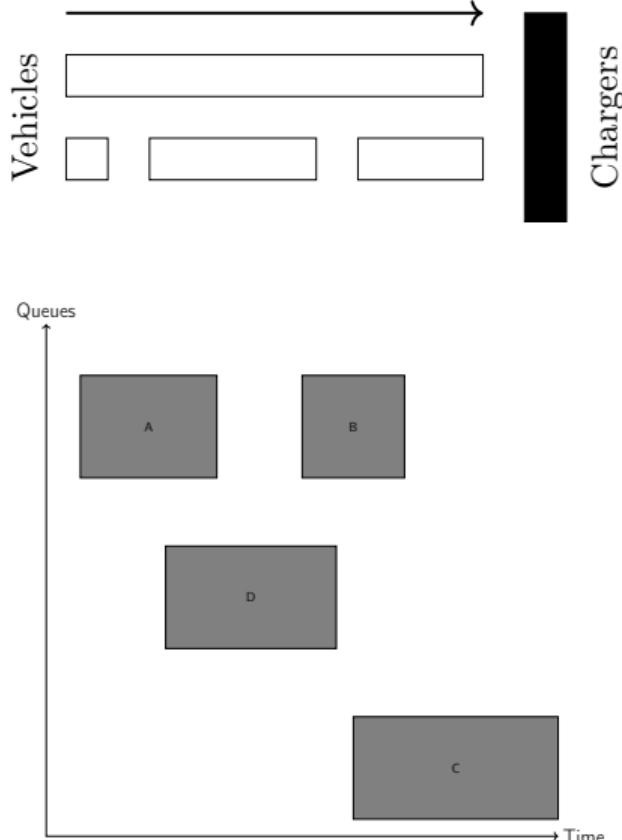


The Position Allocation Problem



- ▶ Service flow is left to right
- ▶ Single charger type
- ▶ All arrivals are considered unique
- ▶ Service times are assumed to be known

-
- ▶ Discrete charger queues
 - ▶ Multiple chargers
 - ▶ Bus can have multiple visits
 - ▶ Propagate battery SOC across visits
 - ▶ Unknown charge times



Introduction

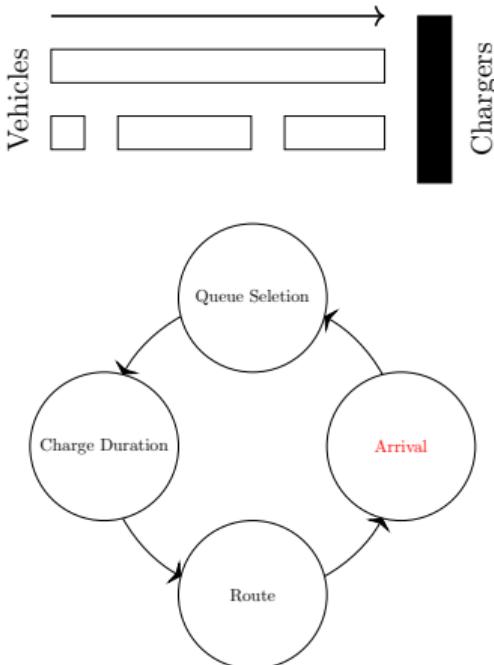
The Position Allocation Problem Approach With Linear Battery Dynamic

The Simulated Annealing Approach With Linear Battery Dynamics

The Simulated Annealing Approach With Non-Linear Battery Dynamics

Requirements For BEB Implementation

- ▶ Charges must be able to be tracked
- ▶ Service time is unknown
- ▶ Accommodate chargers of different speeds
- ▶ Minimize charger count
- ▶ Minimize consumption cost
- ▶ Encourage slow charger use for battery health



Mixed Integer Linear Program



$$\min \sum_{i=1}^{n_V} \sum_{q=1}^{n_Q} (w_{iq} m_q + g_{iq} \epsilon_q)$$

$$u_j - u_i - s_i - (\sigma_{ij} - 1)T \geq 0$$

$$v_j - v_i - (\psi_{ij} - 1)n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

$$\sum_{q=1}^{n_Q} w_{iq} = 1$$

$$v_i = \sum_{q=1}^{n_Q} q w_{iq}$$

$$s_i + u_i = d_i$$

$$a_i \leq u_i \leq (T - s_i)$$

$$d_i \leq \tau_i$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i = \eta_{\gamma_i}$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i \geq \nu_{\Gamma_i} \kappa_{\Gamma_i}$$

$$\eta_i + \sum_{q=1}^{n_Q} g_{iq} r_q \leq \kappa_{\Gamma_i}$$

$$\eta_{\Gamma_b^0} = \alpha_{\Gamma_i} \kappa_{\Gamma_i}$$

$$\eta_{\Gamma_b^f} \geq \beta_{\Gamma_b} \kappa_{\Gamma_b}$$

Constraints

$$u_j - u_i - s_i - (\sigma_{ij} - 1)T \geq 0$$

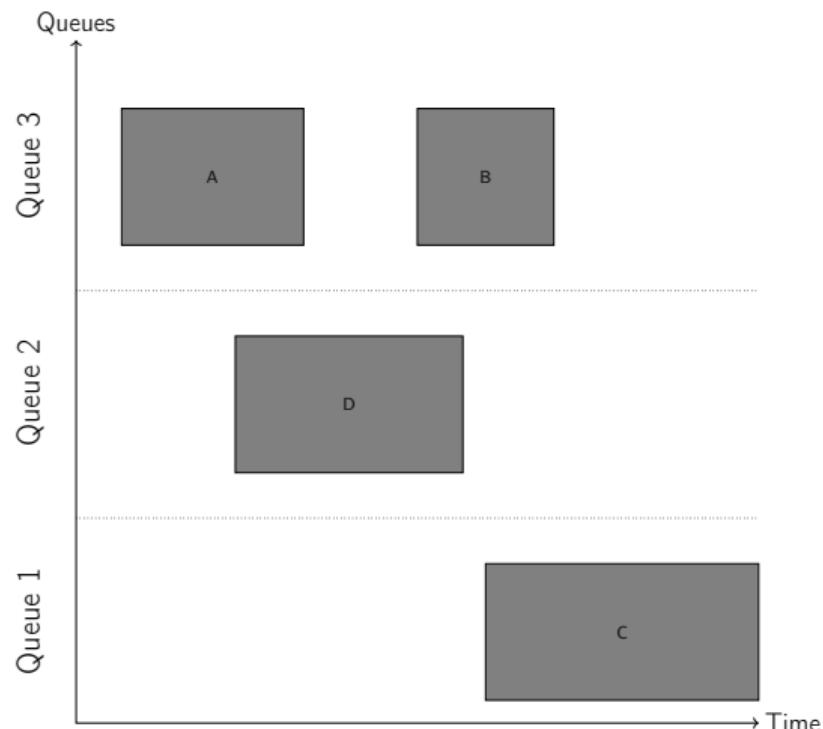
$$v_j - v_i - (\psi_{ij} - 1)n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

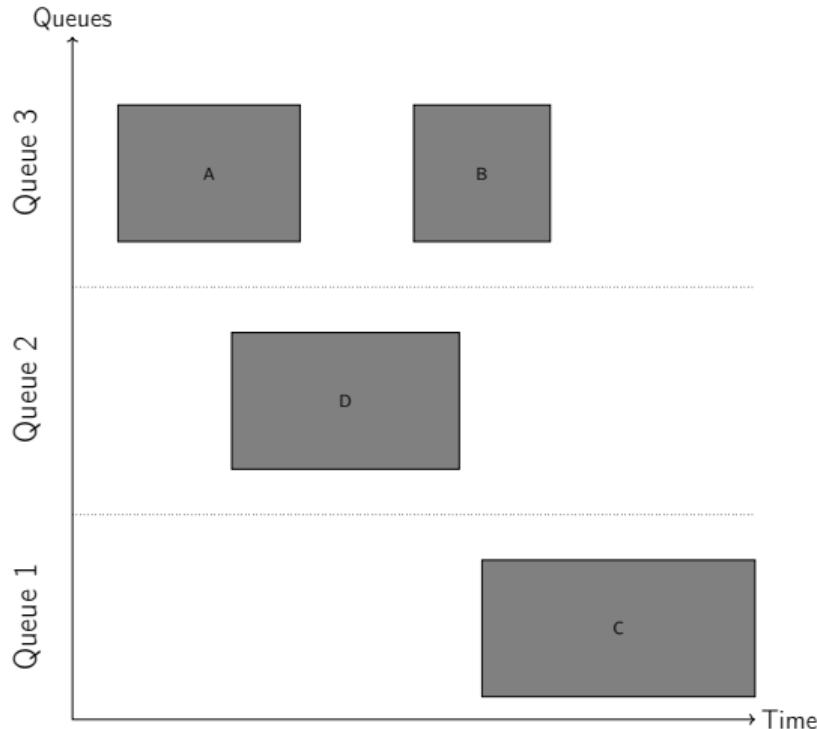
$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

-
- ▶ Used to ensure that gray rectangles do not overlap
 - ▶ σ_{ij} establishes temporal ordering when active
 - ▶ ψ_{ij} establishes spacial ordering when active

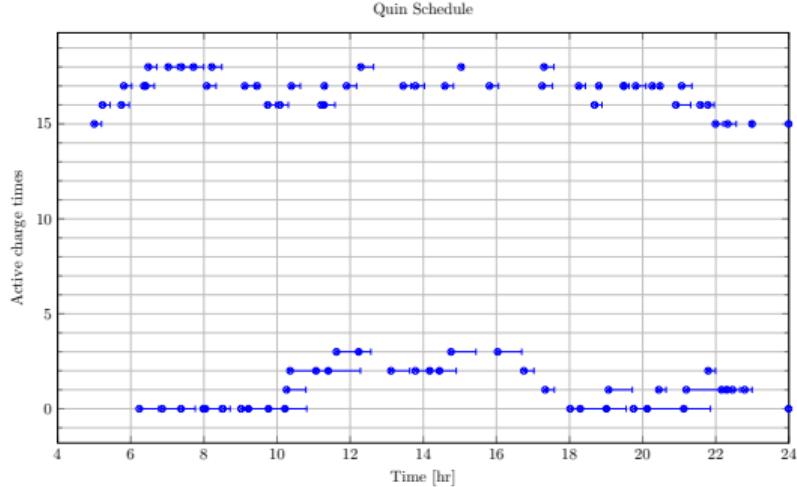
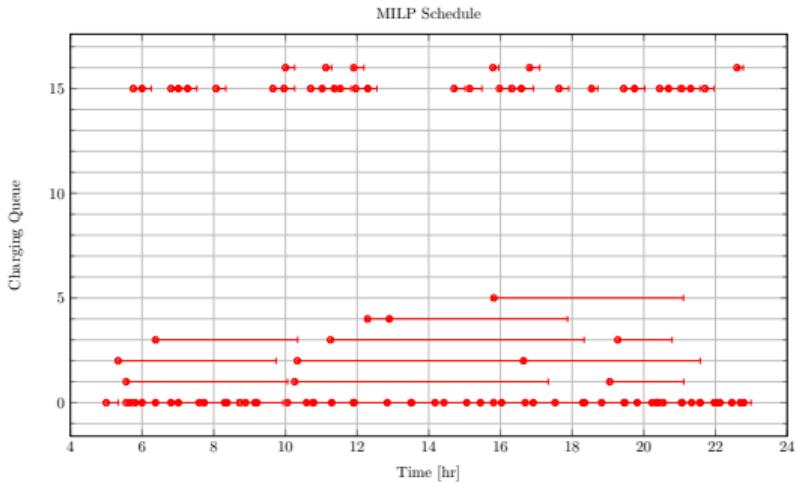


Parameters

Execution	2 hr
T	24 hr
n_V	338
n_A	35
α_i	90%
ν_i	25%
β_i	70%
m_q	1000q



Schedules



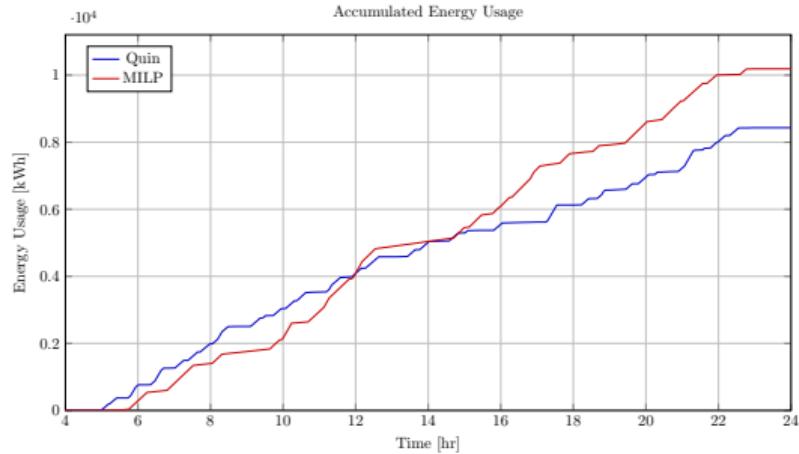
- ▶ Slow chargers utilized for frequently
- ▶ Fast chargers utilized more sparingly

- ▶ Slow chargers used for short durations
- ▶ Extensive use of fast chargers

SOC And Energy Use



	MILP	Qin
Mean	265.873	355.93
Min	97.04	0.000
Max	388	368.354



- ▶ SOC of Qin allowed to drop to 0
- ▶ PAP maintained minimum SOC and final SOC

- ▶ PAP accumulated energy is larger due to minimum SOC constraints

Introduction

The Position Allocation Problem Approach With Linear Battery Dynamic

The Simulated Annealing Approach With Linear Battery Dynamics

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Simulated Annealing²



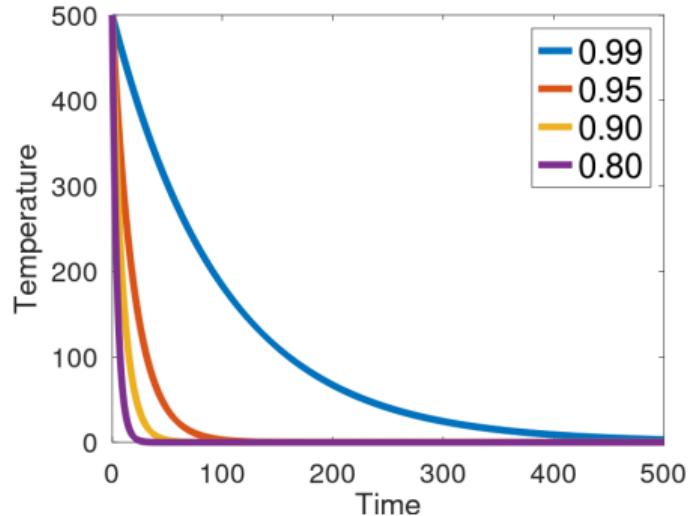
- ▶ A probabilistic technique for approximating the global optimum of a given function.
- ▶ Often applied to problems that contain many local solutions
- ▶ Three key components:
 - ▶ Cooling Schedule
 - ▶ Acceptance Criteria
 - ▶ Generation Mechanisms

²https://en.wikipedia.org/wiki/Simulated_annealing

Cooling Schedule



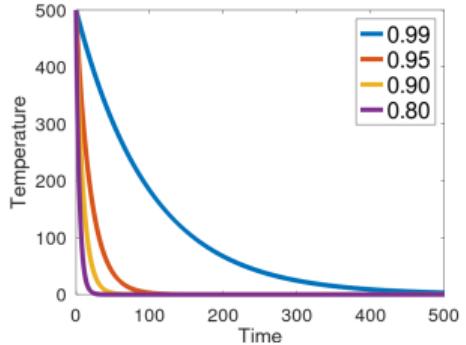
- ▶ The cooling equation models the rate at which the temperature decreases over time in the SA process.
- ▶ The temperature is high, SA encourages exploration. As the temperature decreases, exploitation is encouraged.



$$t_m = \beta t_{m-1}$$

Acceptance Criteria³

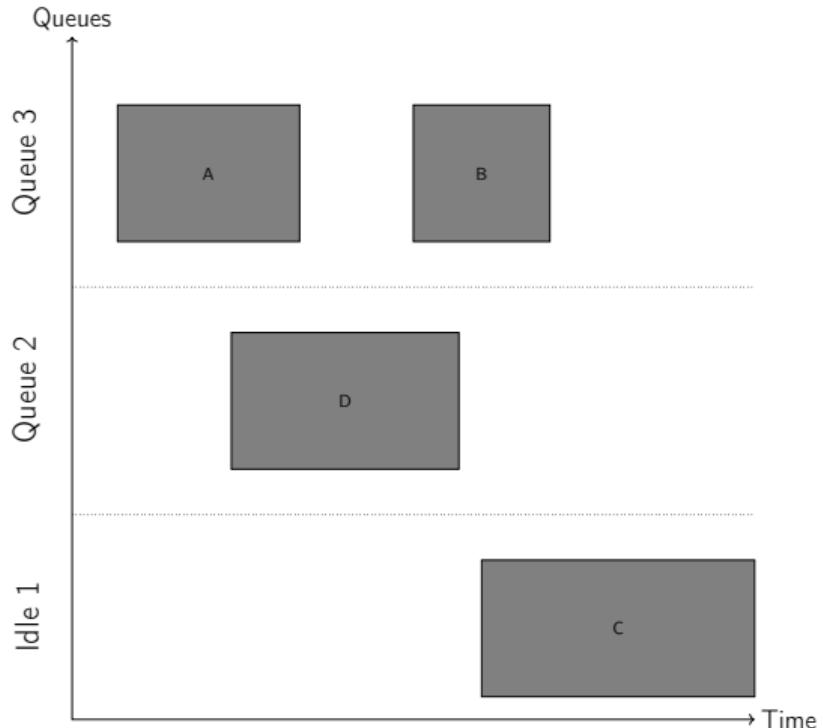
$$f(\mathbb{I}, \bar{\mathbb{I}}, t_m) = \begin{cases} 1 & \Delta E > 0 \\ e^{-\frac{\Delta E}{t_m}} & \text{otherwise} \end{cases}$$



³https://en.wikipedia.org/wiki/Simulated_annealing

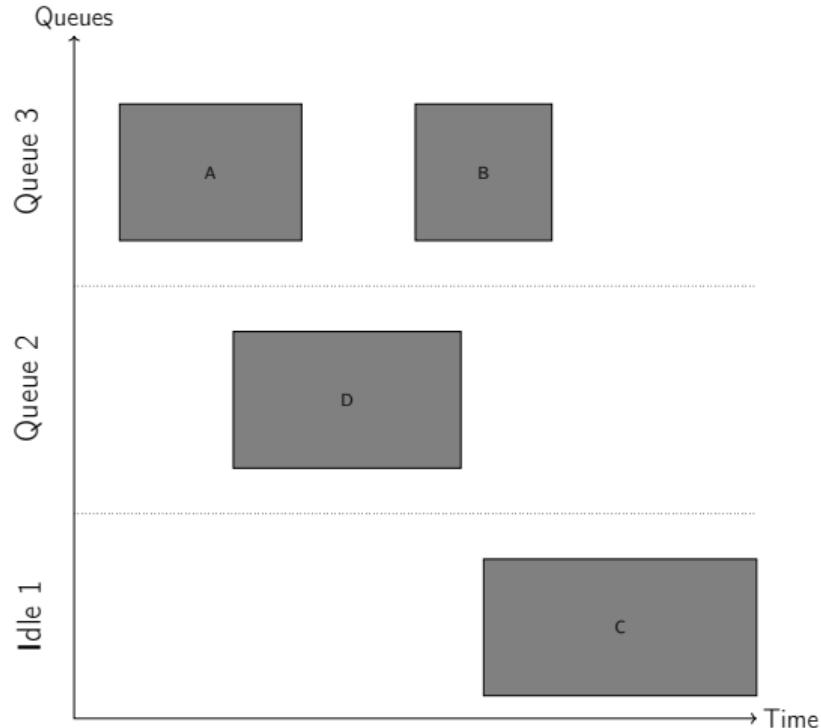
Generation Mechanisms - Primitive Functions

- ▶ New Visit: Move a bus from a wait queue to charge queue
- ▶ Slide Visit: Change the charge duration of a visit
- ▶ New Charger: Move a visit to a new charger
- ▶ Wait: Move a visit to its idle queue
- ▶ New Window: Execute Wait then New Visit primitives



Generation Mechanisms - Wrapper Functions

- ▶ Charge Schedule Generation:
Iterate through each visit and execute New Visit
- ▶ Perturb Schedule: Randomly execute one of the primitives with a weighted distribution



Objective Function



$$J(\mathbb{I}) = z_d p_d + \sum_{i=1}^{n_V} \left[\epsilon_{q_i} r_{q_i} + z_p \phi_i (\eta_i - \nu_{b_i} \kappa_{b_i}) + z_c r_{q_i} s_i \right]$$

► Demand cost

- $p_{T_p, h} = \frac{1}{T_p} \sum_{k=h-\frac{T_p}{dt}+1}^h p_k dt$
- $p_{max} = \max_{k \in [h-\frac{T_p}{dt}+1, h]} p_{T_p, h}$
- $p_d = \max(p_{fix}, p_{max})$

- $\epsilon_{q_i} r_{q_i}$: Assignment Cost
- $z_p \phi_i (\eta_i - \nu_{b_i} \kappa_{b_i})$: Penalty Function
- $z_c r_{q_i} s_i$: Consumption Cost

Constraints

$$u_j - d_i - (\sigma_{ij} - 1)T \geq 0$$

$$q_j - q_i - 1 - (\psi_{ij} - 1)Q \geq 0$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

$$s_i = d_i - u_i$$

$$\eta_{\xi_i} = \eta_i + r_{q_i} s_i - \Delta_i$$

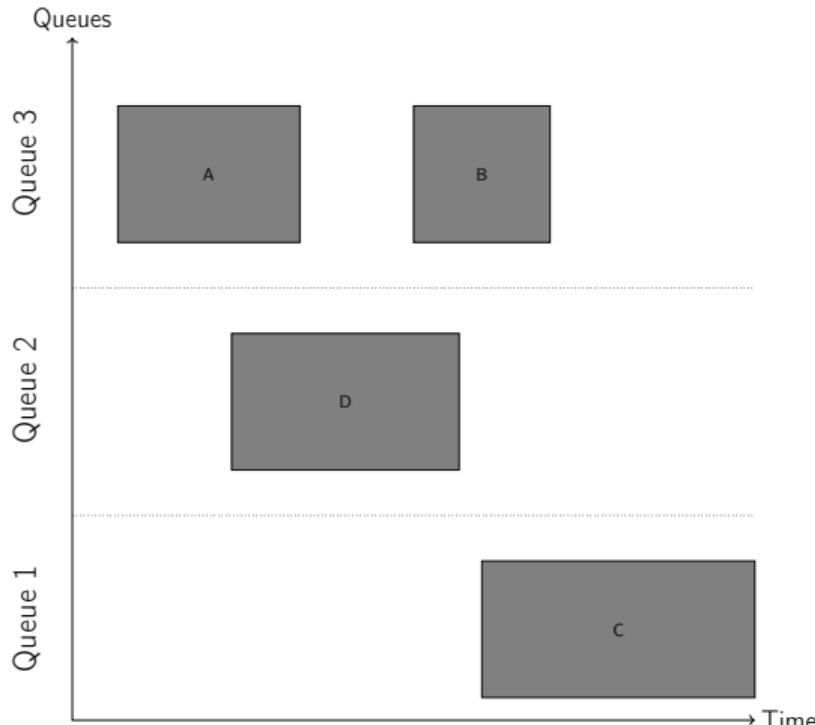
$$\kappa_{b_i} \geq \eta_i + r_{q_i} s_i$$

$$a_i \leq u_i \leq d_i \leq e_i \leq T$$

Parameters

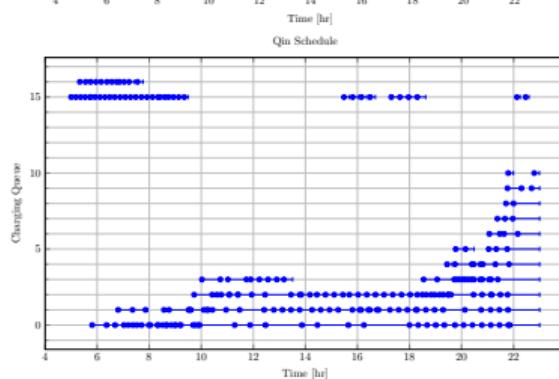
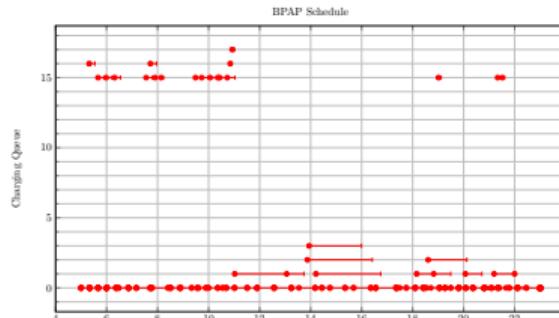
Model	Execution Time [s]
MILP	1900
Quick	1532.8
Heuristic	1916

- ▶ $T_0 = 90000$
- ▶ $|t| = 3797$
- ▶ $\beta = 0.997$
- ▶ $n_K = 500$

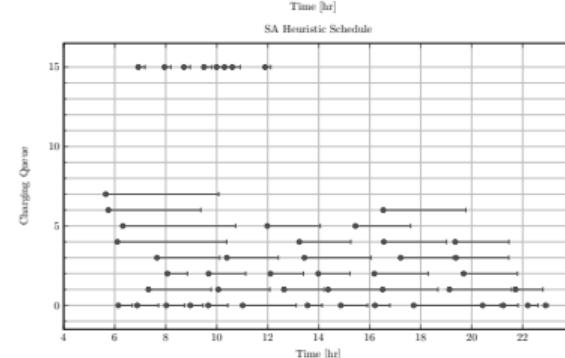
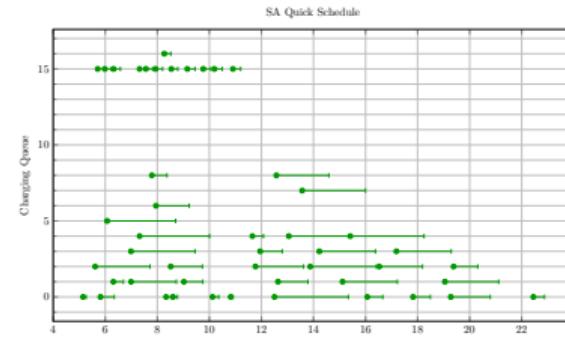


Schedule

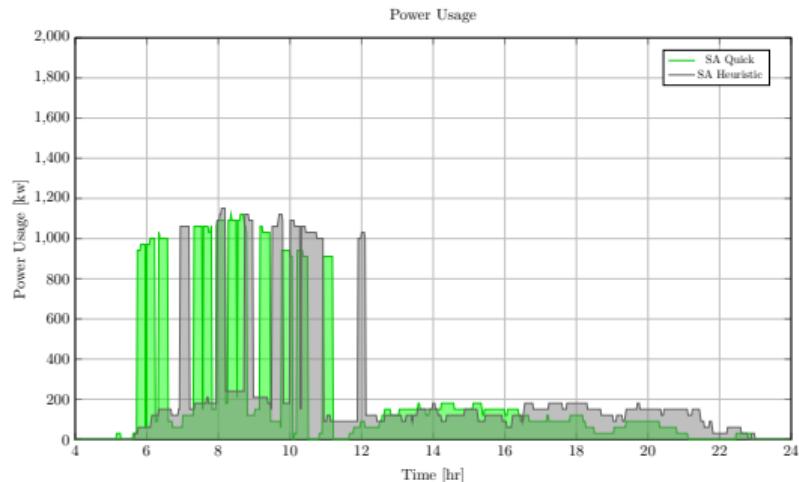
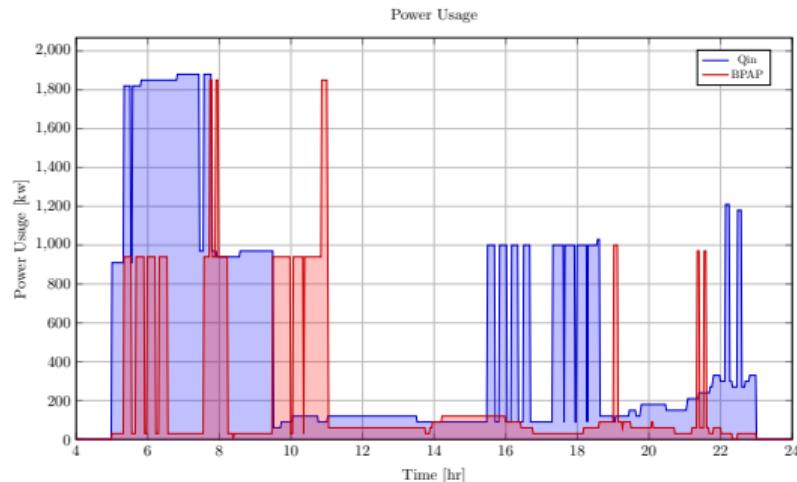
- ▶ Qin heavily utilizes fast charger
- ▶ PAP emphasizes slow queues, but utilizes fast chargers readily



- ▶ SA techniques heavily utilize slow charging
- ▶ SA techniques utilize fast chargers more sparingly



Power

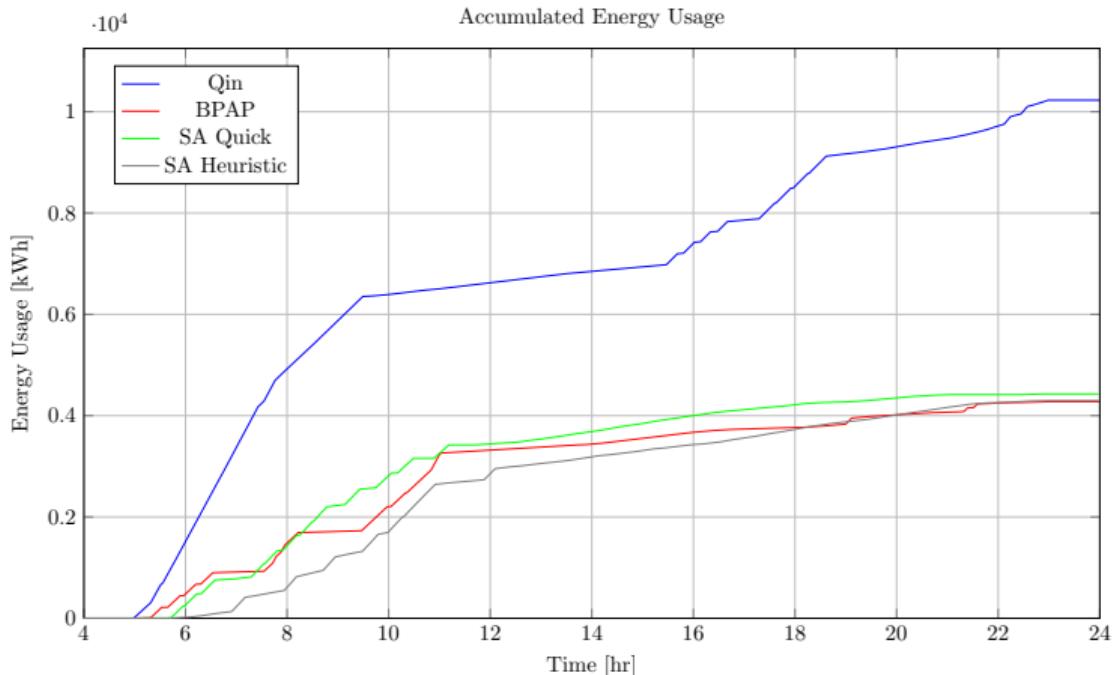


- ▶ Heuristic SA maintained the lowest demand peak
- ▶ Quick SA peak demand comparable to the PAP peak

Energy and SOC

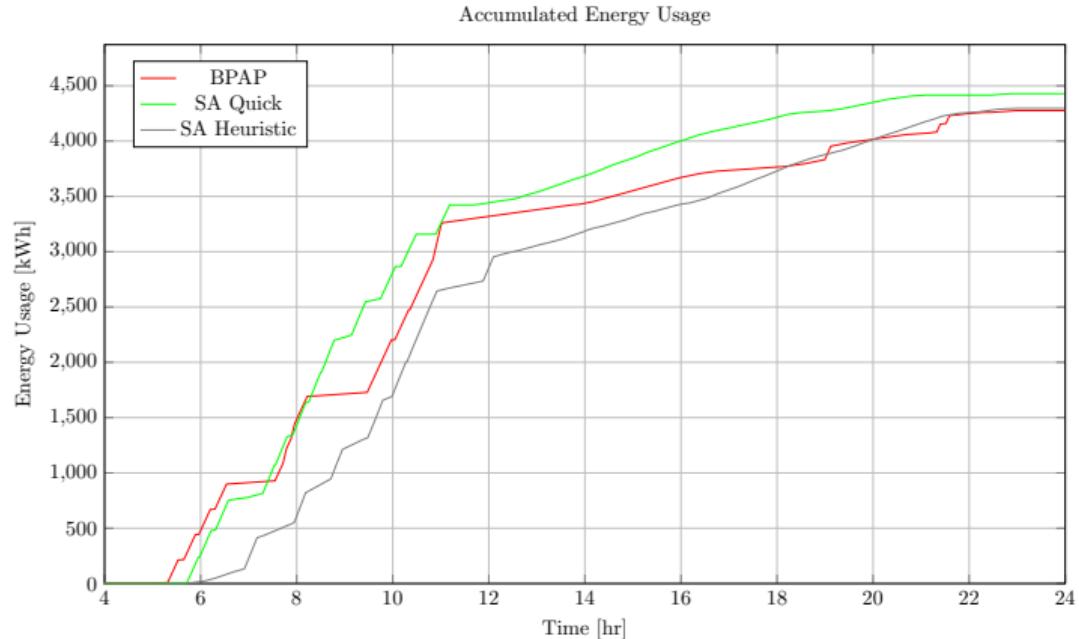


- ▶ SOC threshold deficits:
 - ▶ quick: 2.24 kWh
 - ▶ heuristic: 5.74 kWh
- ▶ Energy delta:
 - ▶ quick: 191.47 kWh
 - ▶ heuristic: 58.46 kWh
- ▶ Trade off of the lower peak demand



Energy and Scores

Schedule	Score
BPAP	18,500,000
Qin-Modified	34,578,526
Heuristic	11,673,937
Quick	11,234,577



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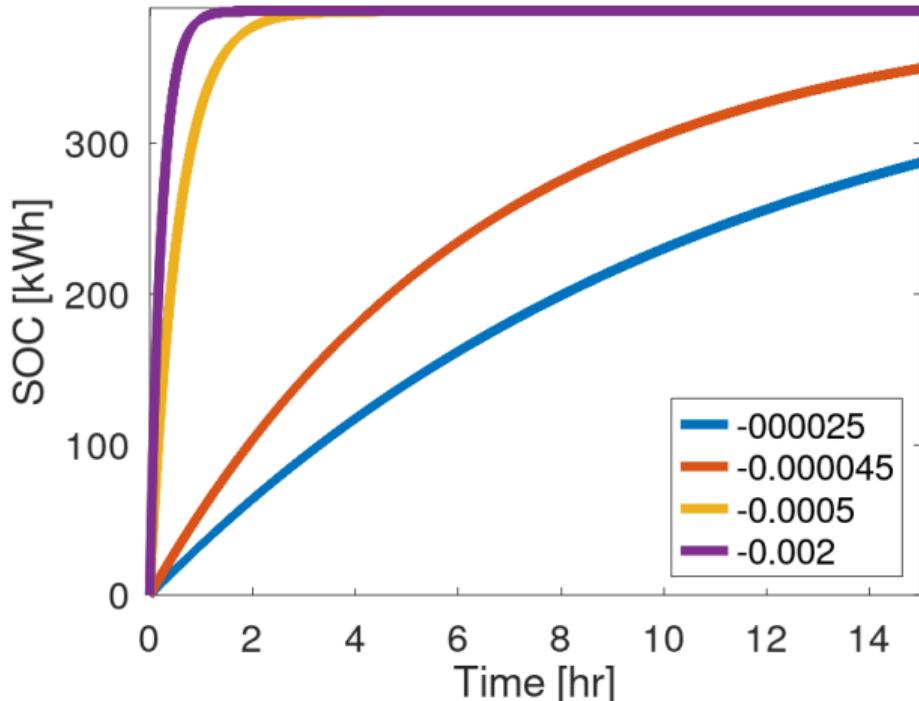
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The Simulated Annealing Approach With Non-Linear Battery Dynamics

Introduction



- ▶ Higher fidelity in approximating charge at high SOC
- ▶ Implemented in SA for simplicity

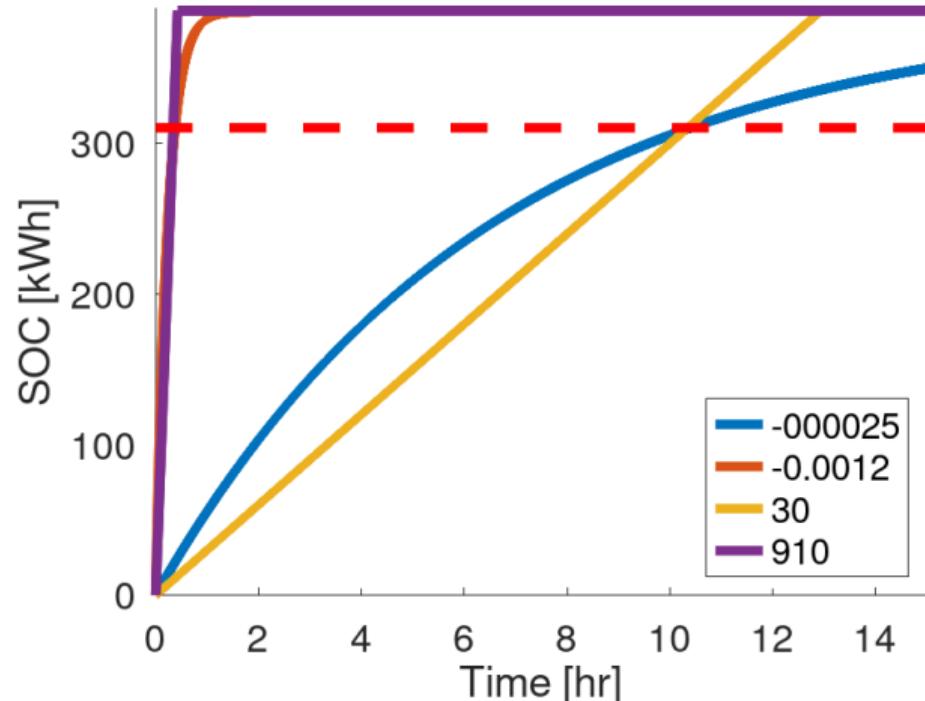


Non-Linear Battery Dynamics Model



$$\eta_{\xi_i} = \bar{a}_q \eta_i - \bar{b}_q \kappa_{\Xi_i}$$

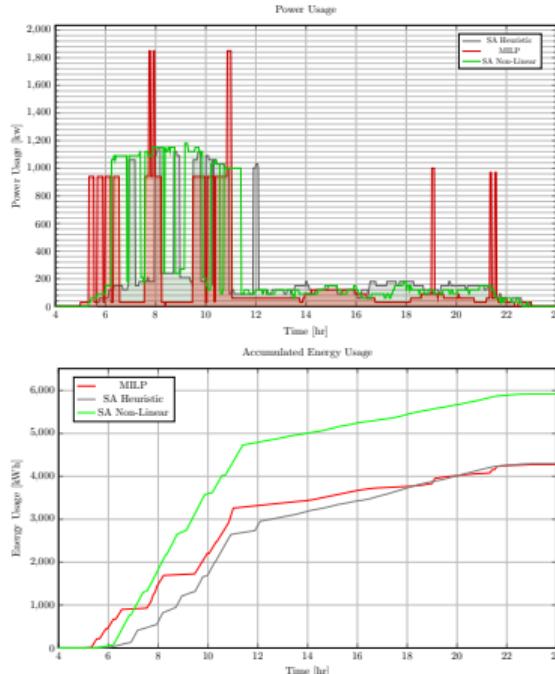
$$\bar{a}_q = e^{a_q dt} \quad \bar{b}_q = e^{a_q dt} - 1$$



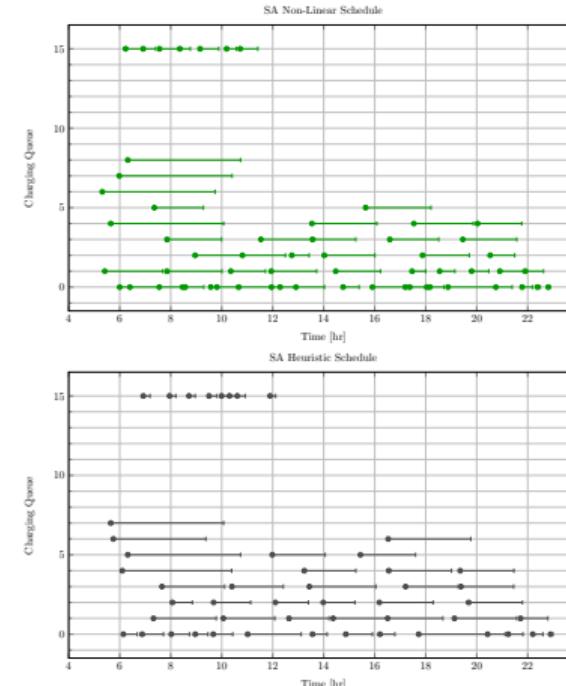
Results



- ▶ Minimized demand power
- ▶ Larger energy consumption due to fast charger duration



- ▶ Similar schedule output, longer fast charger durations due to nonlinear model
- ▶ Minimum SOC of 85 kWh



Appendix



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Mixed Integer Linear Programming



$$\max J = \sum_j c_j x_j + \sum_k d_k y_k$$

$$\text{subject to } \sum_j a_{ij} x_j + \sum_k g_{ik} y_k \leq b_i \quad (i = 1, 2, \dots, m)$$

$$x_j \geq 0 \quad (j = 1, 2, \dots, n)$$

$$y_k \in \mathbb{Z}^+ \quad (k = 1, 2, \dots, n)$$

- ▶ J : Objective function
- ▶ $x_j \in \mathbb{R}$ and $y_k \in \mathbb{Z}^+$: Decision Variables
- ▶ $c_j, d_k, a_{ij}, g_{ik}, b_i \in \mathbb{R}$: Parameters

SA Primitive Generators

Algorithm 0: New visit algorithm

Algorithm: New Visit**Input:** \mathbb{S} **Output:** $\bar{\mathbb{S}}$

```
1 begin
2      $i \leftarrow \mathbb{S}_i$ ;                                /* Extract visit index */
3      $a \leftarrow \mathbb{I}_{i.a}$ ;                            /* Extract the arrival time for visit  $i$  */
4      $e \leftarrow \mathbb{I}_{i.e}$ ;                            /* Extract the departure time for visit  $i$  */
5      $q \leftarrow \mathbb{I}_{i.q}$ ;                            /* Extract the current charge queue for visit  $i$  */
6      $\bar{q} \leftarrow \mathcal{U}_Q$ ;                          /* Select a random charging queue with a uniform distribution */
7      $C \leftarrow \mathcal{U}_{\mathbb{C}_q}$ ;                      /* Select a random time slice from  $\mathbb{C}_q$  */
8     if  $(\bar{C}, \bar{u}, \bar{d}) \leftarrow \text{findFreeTime}(C, i, q, a, e) \notin \emptyset$  then    /* If there is time available in  $C_q^j$  */
9         | return  $(i, (\bar{q}, \bar{u}, \bar{d}), \bar{C})$                                 /* Return visit */
10    end
11    return  $(\emptyset)$ ;                                /* Return nothing */
12 end
```

Algorithm 1: Slide Visit Algorithm

Algorithm: Slide Visit

Input: \mathbb{S}

Output: $\bar{\mathbb{S}}$

```
1 begin
2    $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow \text{Purge}(\mathbb{S});$            /* Purge visit  $i$  from charger availability matrix */
3    $C \leftarrow \bar{\mathbb{C}}_{i,q_i};$                       /* Get the time availability of the purged visit */
4   /* If there is time available in  $C$  */
5   if  $(\bar{C}, \bar{u}, \bar{d}) \leftarrow \text{findFreeTime}(C, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \emptyset$  then
6     | return  $(i, \mathbb{I}, (\mathbb{I}_{i.q_i}, \bar{u}, \bar{d}), \bar{\mathbb{C}})$           /* Return updated visit */
7   end
8   return  $(\emptyset);$                                 /* Return nothing */
9 end
```

Algorithm 2: New Charger Algorithm

Algorithm: New Charger

Input: S

Output: \$

1 begin

```

2   |  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow \text{Purge}(\mathbb{S});$            /* Purge visit  $i$  from charger availability matrix */
3   |  $q \leftarrow \mathcal{U}_Q;$            /* Select a random charging queue with a uniform distribution */
4   | if  $(\bar{C}, \bar{u}, \bar{d}) \leftarrow \text{findFreeTime}(\bar{\mathbb{C}}_{i,q}, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i,a}, \mathbb{I}_{i,e}) \notin \emptyset$  then /* If there is time available in
      |  $C_q$  */
      |   /* Return visit, note  $u$  and  $d$  are the original initial/final charge times. */
      |   | return  $(i, \mathbb{I}, (q, \mathbb{I}_{i,u}, \mathbb{I}_{i,d}), \bar{\mathbb{C}})$ 
5   | end
6   | return  $(\emptyset);$            /* Return nothing */
7
8 end

```

Algorithm 3: Wait algorithm

Algorithm: Wait**Input:** \mathbb{S} **Output:** $\bar{\mathbb{S}}$

```
1 begin
2    $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow \text{Purge}(\mathbb{S});$            /* Purge visit  $i$  from charger availability matrix */
3    $\bar{\mathbb{C}}'_{\mathbb{I}_{i,r_i}} \leftarrow \mathbb{C}' \cup \{[\mathbb{I}_{i.b}, \mathbb{I}_{i.e}]\};$  /* Update the charger availability matrix for wait queue  $\bar{\mathbb{C}}_{i,q_i}$  */
4   return  $(i, \mathbb{I}, (\mathbb{I}_{i.b}, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}), \bar{\mathbb{C}})$            /* Return visit */
5 end
```

Algorithm 4: New window algorithm

Algorithm: New Window

Input: \mathbb{S}

Output: $\bar{\mathbb{S}}$

```
1 begin
2   |    $\bar{\mathbb{S}} \leftarrow \text{Wait}(\mathbb{S});$            /* Assign visit to its respective idle queue */
3   |   if  $\bar{\mathbb{S}} \leftarrow \text{NewVisit}(\bar{\mathbb{S}}) \neq \emptyset$  then
4   |   |   return  $\bar{\mathbb{S}}$                          /* Add visit  $i$  back in randomly */
5   |   end                                         /* Return visit */
6   |   return  $(\emptyset);$                          /* Return nothing */
7 end
```

SA Wrapper Functions



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Algorithm 5: Charge schedule generation algorithm

Algorithm: Candidate Solution Generator

Input: \mathbb{S}

Output: $\bar{\mathbb{S}}$

```
1 begin
2   | /* Select an unscheduled BEB visit from a randomly indexed set of visits */ 
3   | foreach  $\mathbb{I}_i \in \mathbb{I}$  do
4   |   |  $(i, \bar{\mathbb{I}}, \bar{\mathbb{C}}) \leftarrow \text{NewVisit}((\mathbb{I}_i, \mathbb{I}, \mathbb{C}))$ ;           /* Assign the bus to a charger */
5   |   end
6   | return  $(0, \bar{\mathbb{I}}, \bar{\mathbb{C}})$ 
7 end
```

Charge Schedule Perturbation



Algorithm 6: Perturb schedule algorithm

Algorithm: Perturb Schedule

Input: \mathbb{S}

Output: $\tilde{\mathbb{S}}$

```
1 begin
2      $p \leftarrow [\text{false}; n_A]$ ;                                /* Create vector to track priority routes */
3      $y^i \leftarrow [1.0; n_V]$ ;                                /* Create weight vector for index selection */
4     /* Loop through the visits in reverse order */                */
5     foreach  $\mathbb{I}_i \leftarrow \mathbb{I}_{|\mathbb{I}|} \text{ TO } \mathbb{I}_1$  do
6         /* If the current visit is part of a priority route */      */
7         if  $p_{\mathbb{I}_i.b} = \text{true}$  then
8              $y_{\mathbb{I}_i}^i = y_{\mathbb{I}_i,\xi}^i$ ;                            /* */
9         end
10        /* Else if the current visit's SOC does below the allowed threshold */ */
11        else if  $\nu_{\mathbb{I}_i,\eta} \leq \nu_{\mathbb{I}_i,b}\kappa_{\mathbb{I}_i,b}$  then
12             $p_{\mathbb{I}_i.b} = \text{true}$ ;                                /* Indicate the current BEB's routes are to be prioritized */
13             $y_{\mathbb{I}_i}^i = \kappa_{\mathbb{I}_i,b}\kappa_{\mathbb{I}_i.b} - \nu_{\mathbb{I}_i,\eta}$ ; /* Calculate the weight of the current visit */
14        end
15    end
16     $\mathbb{I}_i \leftarrow \mathcal{W}_1^{y^i}$ ;                                /* Select an index with a weighted distribution */
17     $i \leftarrow \mathbb{I}_i$ ;                                    /* Extract visit index */
18     $y^P \leftarrow [y_1^P, y_2^P, \dots]$ ;                  /* Define the weight of each primitive generator */
19     $PGF \leftarrow \mathcal{W}_{[1,n_G]}^{y^P}$ ;                  /* Select a generator function with weighted distribution */
20     $\tilde{\mathbb{S}} \leftarrow PGF((i, \mathbb{I}, \mathbb{C}))$ ;          /* Execute the generator function */
21    return  $(0, \mathbb{I}, \tilde{\mathbb{C}})$ 
22 end
```

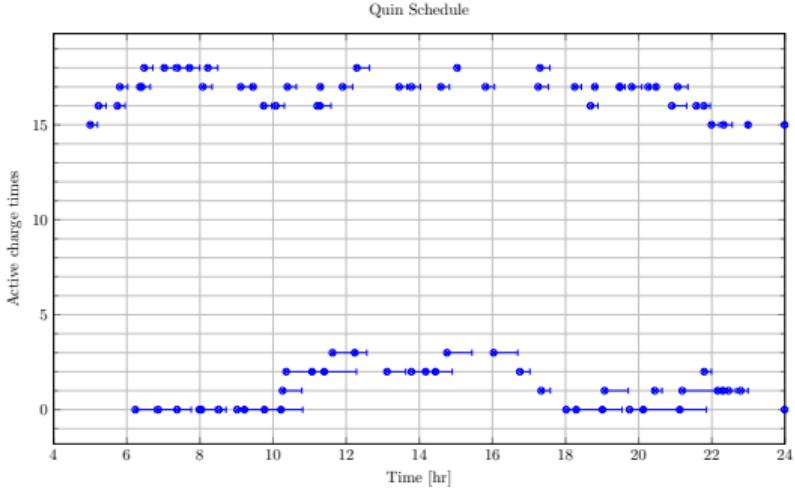
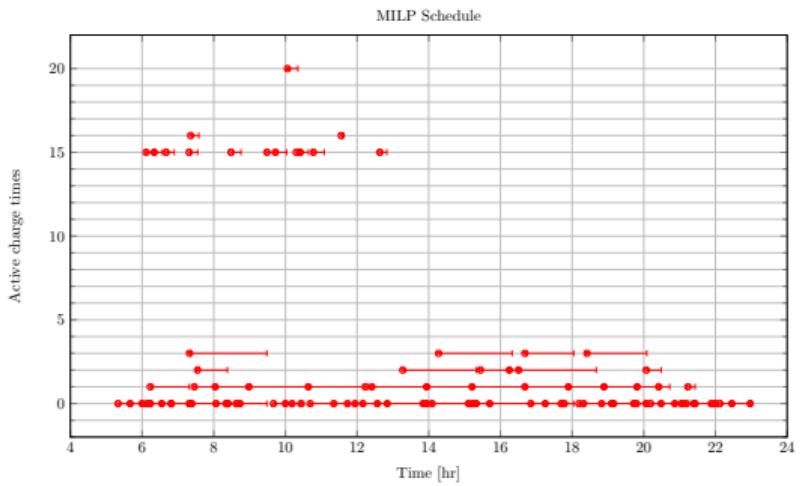
SA Thesis Results

Parameters

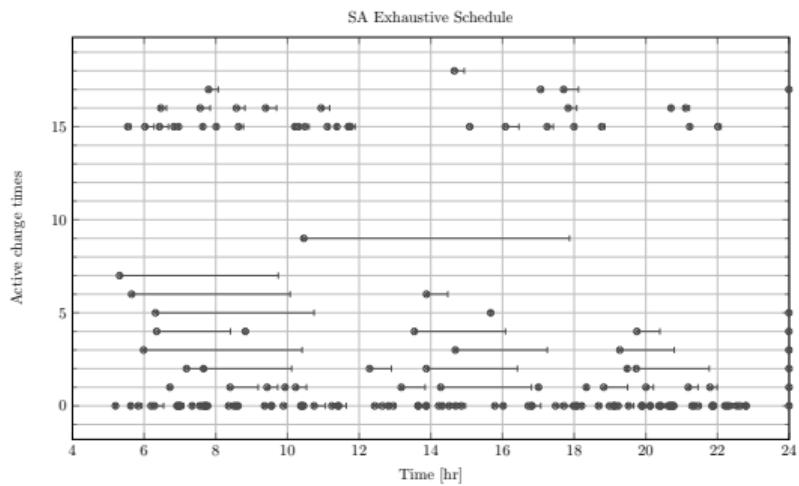
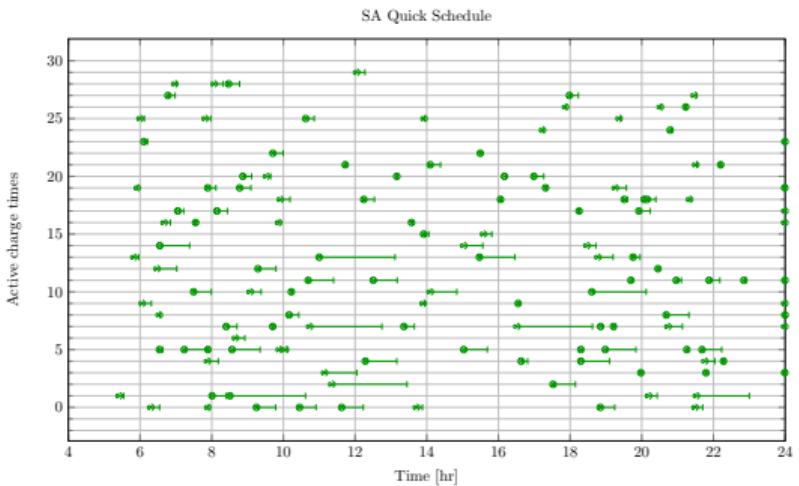
Model	Execution Time [s]	Iteration [s]
MILP	3600	N/A
Quick	2275.25	0.25
Heuristic	3640.4	0.4

- ▶ $T_0 = 99999$
- ▶ $\beta = 0.999$
- ▶ $|t| = 3797$
- ▶ $n_K = 500$

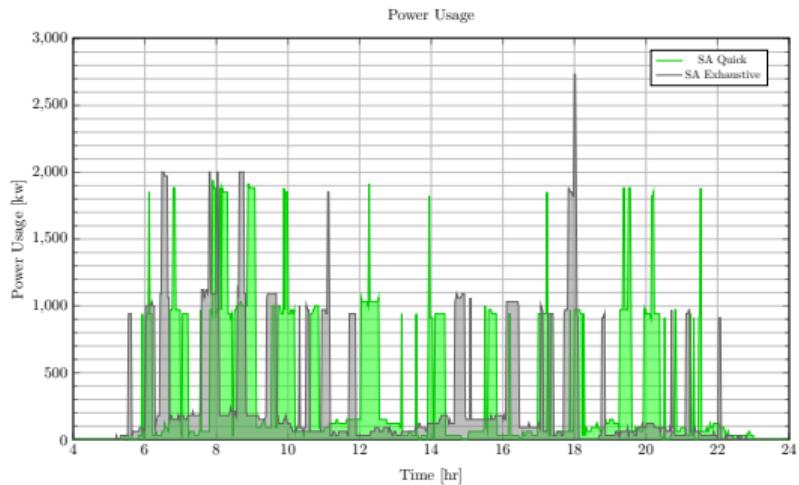
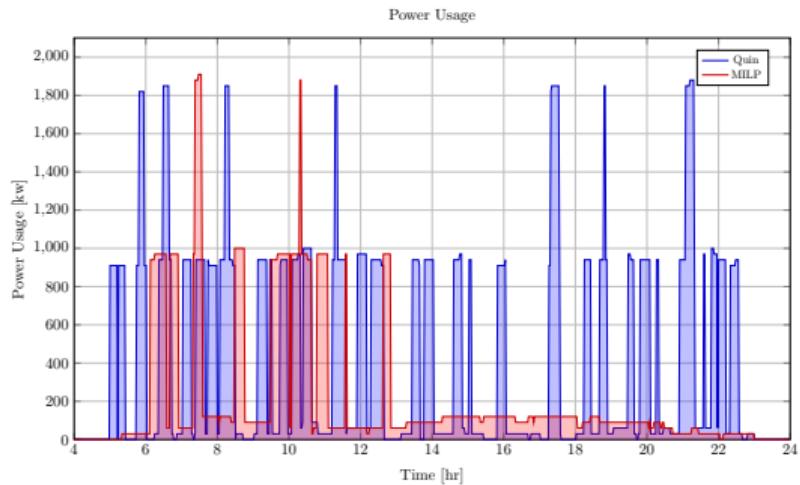
Schedule



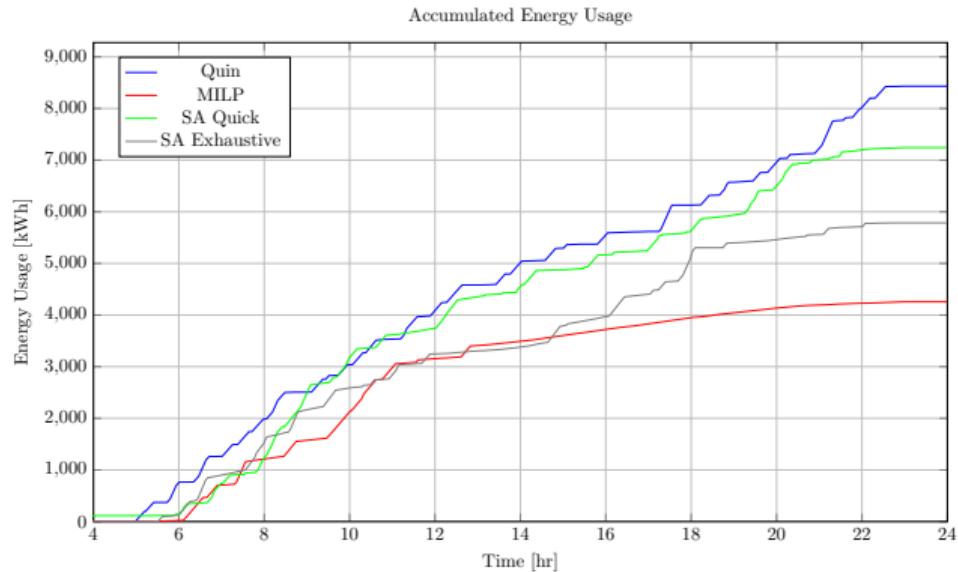
Schedule



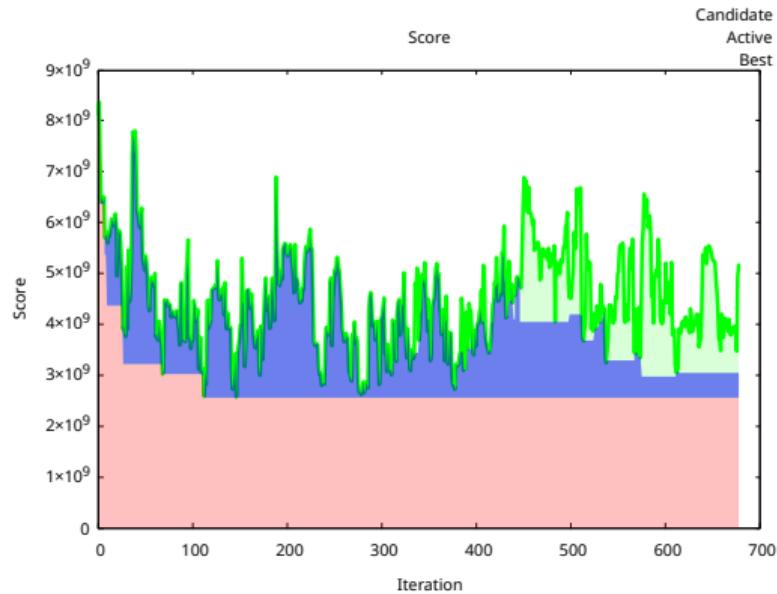
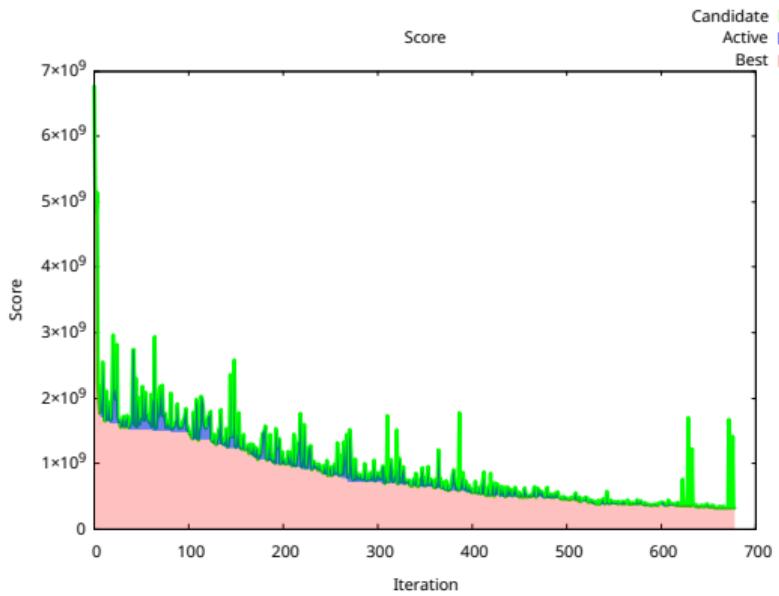
Power



Energy



How To Resolve This Problem?



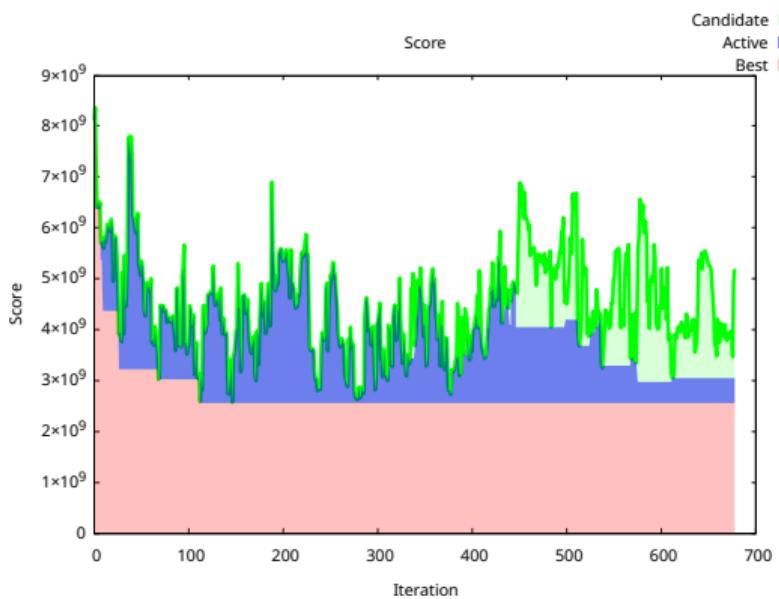
- ▶ Reverse search and weight the visit indices
- ▶ Be more aggressive in exploiting the best solution

- ▶ Candidate solutions diverge
- ▶ Hard time handling “difficult” routes

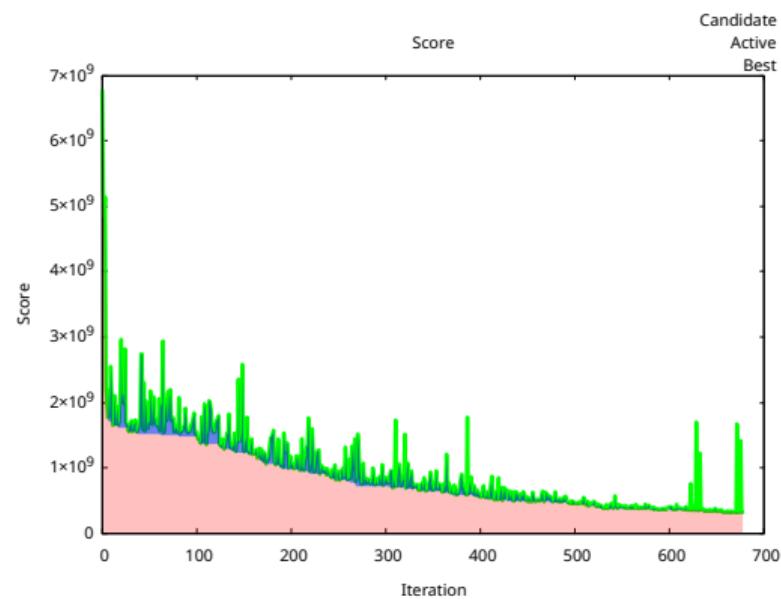
Score Convergence Comparison



Before Fix



After Fix



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Reforming mixed operation schedule for electric buses and traditional fuel buses by an optimal framework.
IET Intelligent Transport Systems, 15(10):1287–1303, Jul 2021.
-  Oliver Frendo, Nadine Gaertner, and Heiner Stuckenschmidt.
Open source algorithm for smart charging of electric vehicle fleets.
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-  Amra Jahic, Mina Eskander, and Detlef Schulz.
Preemptive vs. non-preemptive charging schedule for large-scale electric bus depots.
In *2019 IEEE PES Innovative Smart Grid Technologies Europe (ISGT-Europe)*.
IEEE, September 2019.
-  Daniel Mortensen, Jacob Gunther, Greg Droke, and Justin Whitaker.
Cost minimization for charging electric bus fleets.
World Electric Vehicle Journal, 14(12):351, December 2023.