A POSITION ALLOCATION PROBLEM APPROACH TO THE BATTERY ELECTRIC BUS CHARGING PROBLEM

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Outline

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Topic

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Introduction

The Position Allocation Problem Approach With Linear Battery Dynamic

The Simulated Annealing Approach With Linear Battery Dynamics

The Simulated Annealing Approach With Non-Linear Battery Dynamics

Problem Description

-	-	6:11 AM	6:16 AM	6:25 AM
6:30 AM	6:34 AM	6:41 AM	6:46 AM	6:55 AM
7:00 AM	7:04 AM	7:11 AM	7:16 AM	7:25 AM
7:30 AM	7:34 AM	7:41 AM	7:46 AM	7:55 AM
8:00 AM	8:04 AM	8:11 AM	8:16 AM	8:25 AM
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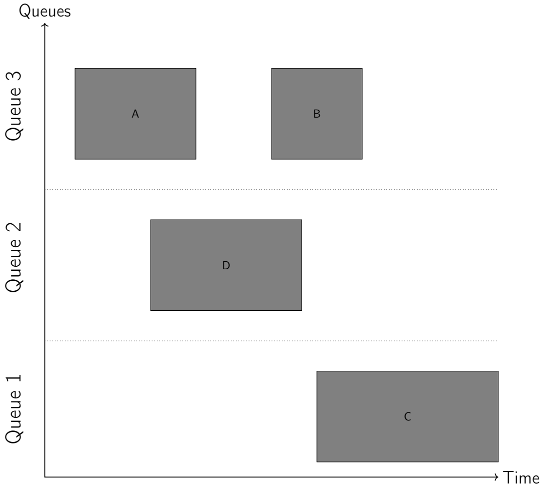


Brief State Of The Art

Ref	Consumption	Demand	Linear	Non-linear	Charger Min.	Battery Health
[1]	✓		✓			
[2]	✓	✓	✓			
[3]	✓	✓	✓	✓		
[4]	✓	✓	✓			
[5]	✓	✓	✓			
[6]	✓		✓			
[7]	✓			✓	✓	✓
[8]	✓		✓			
[9]	✓		✓		✓	
BPAP	✓		✓		✓	✓
SA BPAP	✓	✓	✓	✓	*	✓

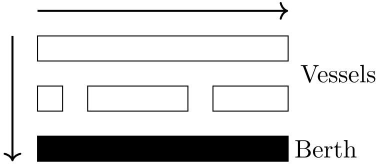
The Berth Allocation Problem[1]

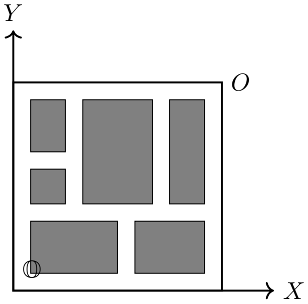
[1] https://www.mdpi.com/2077-1312/11/7/1280

The Berth Allocation Problem

- Vessels move down toward the quay
- Receive service
- Exit to the right

- A variant of the rectangle packing problem
- Solves the problem of optimally assigning incoming vessels to be serviced





The Position Allocation Problem

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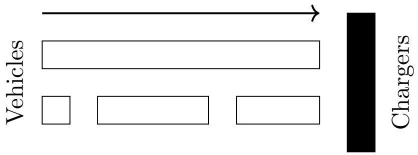
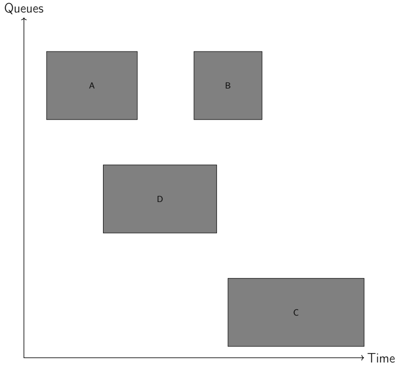
- Service flow is left to right
- Single charger type
- All arrivals are considered unique
- Service times are assumed to be known

- Discrete charger queues
- Multiple charger types
- Bus can have multiple visits
- Propagate battery SOC across visits
- Unknown charge times

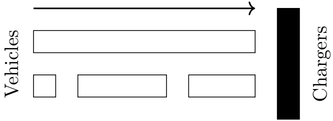


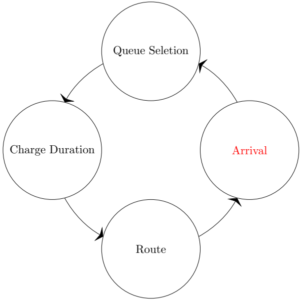


Topic

Requirements For BEB Implementation

- Charges must be able to be tracked
- Service time is unknown
- Accommodate different charger rates
- Minimize charger count
- Minimize consumption cost
- Encourage slow charger use for battery health





Mixed Integer Linear Program

$$\min \sum_{i=1}^{n_V} \sum_{q=1}^{n_Q} \left(w_{iq} m_q + g_{iq} \epsilon_q \right)$$

$$u_j - u_i - s_i - (\sigma_{ij} - 1)T \geq 0$$

$$v_j - v_i - (\psi_{ij} - 1)n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

$$\textstyle\sum_{q=1}^{n_Q} w_{iq} = 1$$

$$v_i = \textstyle\sum_{q=1}^{n_Q} q w_{iq}$$

$$s_i + u_i = d_i$$

$$a_i \leq u_i \leq (T - s_i)$$

$$d_i \leq \tau_i$$

$$\eta_i + \textstyle\sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i = \eta_{\gamma_i}$$

$$\eta_i + \textstyle\sum_{q=1}^{n_Q} g_{iq} r_q - \Delta_i \geq \nu_{\Gamma_i} \kappa_{\Gamma_i}$$

$$\eta_i + \textstyle\sum_{q=1}^{n_Q} g_{iq} r_q \leq \kappa_{\Gamma_i}$$

$$\eta_{\Gamma_b^0} = \alpha_{\Gamma_i} \kappa_{\Gamma_i}$$

$$\eta_{\Gamma_b^f} \geq \beta_{\Gamma_b} \kappa_{\Gamma_b}$$

Constraints

$$u_j - u_i - s_i - (\sigma_{ij} - 1)T \geq 0$$

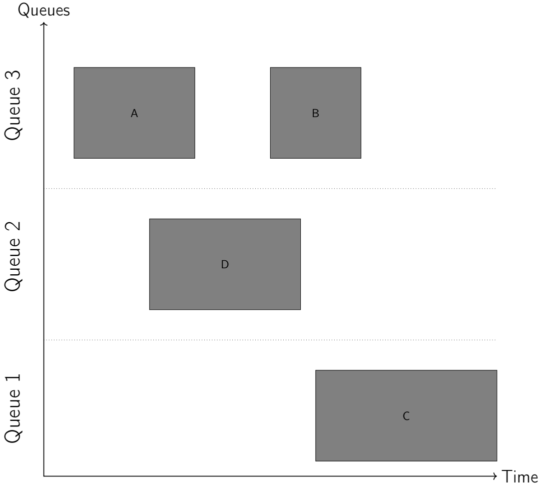
$$v_j - v_i - (\psi_{ij} - 1)n_Q \geq 1$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

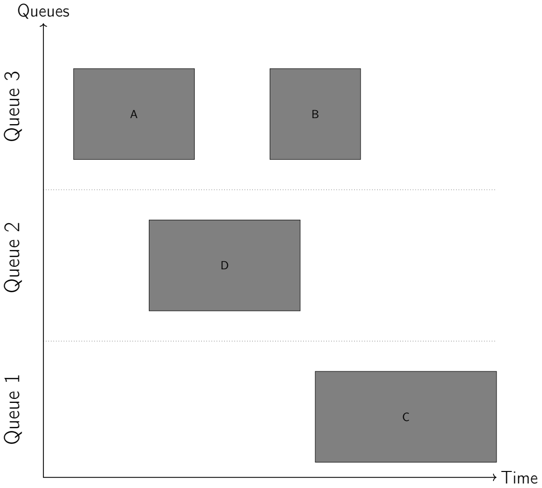
$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

- Used to ensure that gray rectangles do not overlap
- σ_{ij} establishes temporal ordering when active
- ψ_{ij} establishes spacial ordering when active

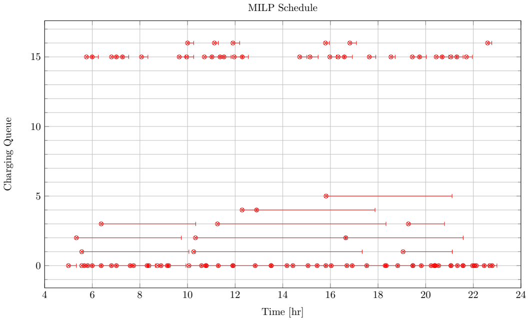


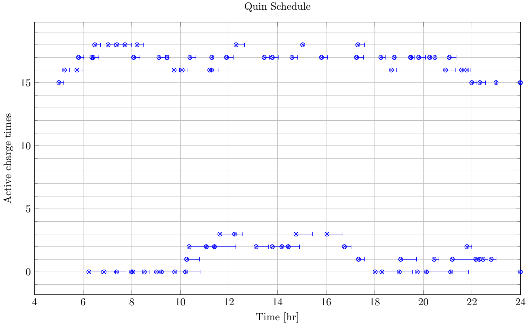
Parameters

Execution	2 hr
T	24 hr
n_V	338
n_A	35
α_i	90%
ν_i	25%
β_i	70%
m_q	1000q



Schedules





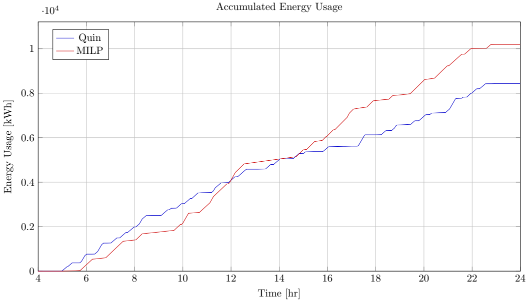
- Slow chargers utilized for frequently
- Fast chargers utilized more sparingly

- Slow chargers used for short durations
- Extensive use of fast chargers

SOC And Energy Use

	MILP	Qin
Mean	265.873	355.93
Min	97.04	0.000
Max	388	368.354

- SOC of Qin allowed to drop to 0
- PAP maintained minimum SOC and final SOC

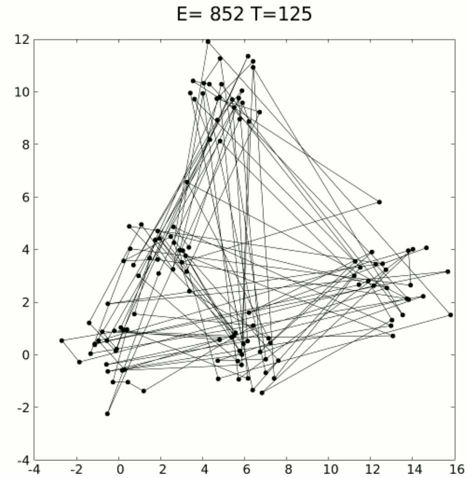


- PAP accumulated energy is larger due to minimum SOC constraints

Topic

Simulated Annealing[2]

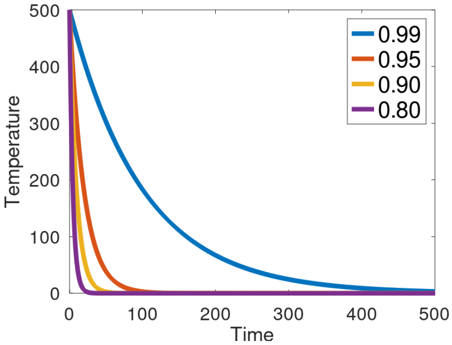
- A probabilistic technique for approximating the global optimum of a given function.
- Often applied to problems that contain many local solutions
- Three key components:
  - Cooling Schedule
  - Acceptance Criteria
  - Generation Mechanisms



[2] https://en.wikipedia.org/wiki/Simulated_annealing

Cooling Schedule

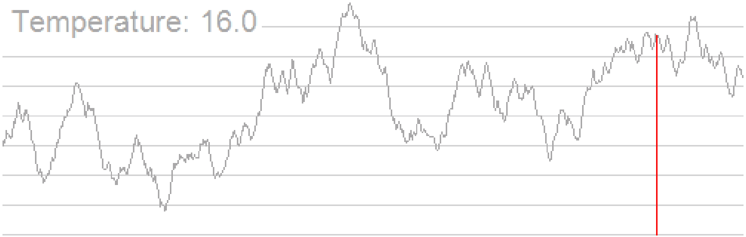
- The cooling equation models the rate at which the temperature decreases over time in the SA process.
- The temperature is high, SA encourages exploration. As the temperature decreases, exploitation is encouraged.

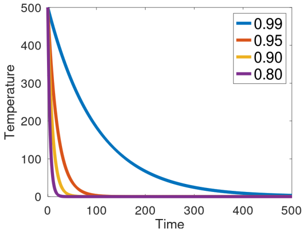


$$t_m = \beta t_{m-1}$$

Acceptance Criteria[3]

$$f(\mathbb{I}, \bar{\mathbb{I}}, t_m) = \begin{cases} 1 & \Delta E > 0 \\ e^{-\frac{\Delta E}{t_m}} & \text{otherwise} \end{cases}$$





[3] https://en.wikipedia.org/wiki/Simulated_annealing

Generation Mechanisms - Primitive Functions

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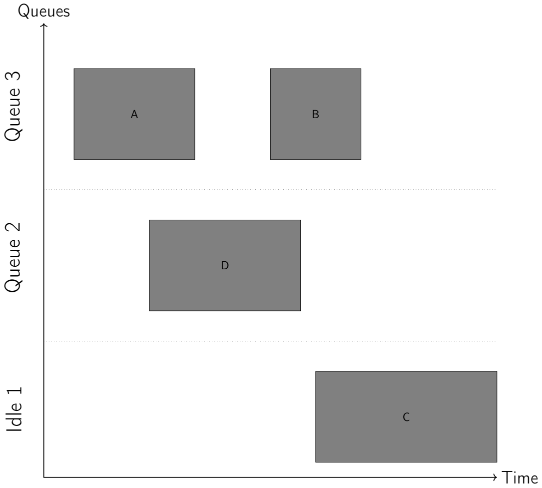
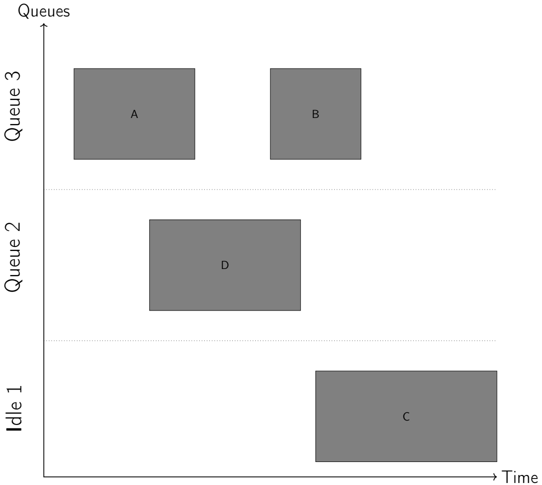
- New Visit: Move a bus from a wait queue to charge queue
- Slide Visit: Change the charge duration of a visit
- New Charger: Move a visit to a new charger
- Wait: Move a visit to its idle queue
- New Window: Execute Wait then New Visit primitives



Generation Mechanisms - Wrapper Functions

- Charge Schedule Generation: Iterate through each visit and execute New Visit
- Perturb Schedule: Randomly execute one of the primitives with a weighted distribution



Objective Function

$$J(\mathbb{I}) = z_d p_d + \sum_{i=1}^{n_V} \left[\epsilon_{q_i} r_{q_i} + z_p \phi_i(\eta_i - \nu_{b_i} \kappa_{b_i}) + z_c r_{q_i} s_i \right]$$

- Demand cost
  - $p_{T_p,h} = \frac{1}{T_p} \sum_{k=h-\frac{T_p}{dt}+1}^{h} p_k \ dt$
  - $p_{max} = \max\limits_{k \in [h-\frac{T_p}{dt}+1, h]} p_{T_p,h}$
  - $p_d = \max(p_{fix}, p_{max})$
- $\epsilon_{q_i} r_{q_i}$: Assignment Cost
- $z_p \phi_i(\eta_i - \nu_{b_i} \kappa_{b_i})$: Penalty Function
- $z_c r_{q_i} s_i$: Consumption Cost

Constraints

$$u_j - d_i - (\sigma_{ij} - 1)T \geq 0$$

$$q_j - q_i - 1 - (\psi_{ij} - 1)Q \geq 0$$

$$\sigma_{ij} + \sigma_{ji} \leq 1$$

$$\psi_{ij} + \psi_{ji} \leq 1$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1$$

$$s_i = d_i - u_i$$

$$\eta_{\xi_i} = \eta_i + r_{q_i} s_i - \Delta_i$$

$$\kappa_{b_i} \geq \eta_i + r_{q_i} s_i$$

$$a_i \leq u_i \leq d_i \leq e_i \leq \mathcal{T}$$

Parameters

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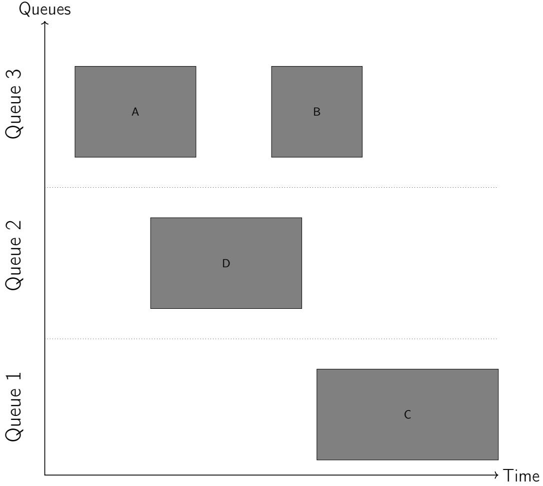
Model	Execution Time [s]
MILP	1900
Quick	1532.8
Heuristic	1916

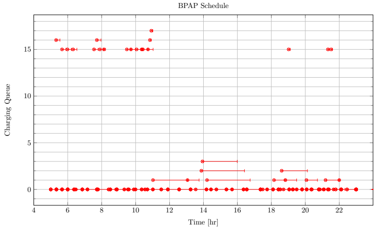
- $T_0 = 90000$
- $\beta = 0.997$
- $|t| = 3797$
- $n_K = 500$

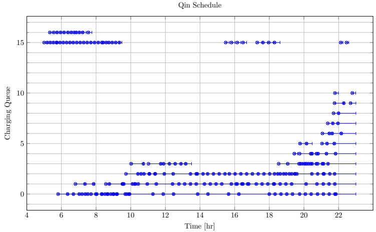


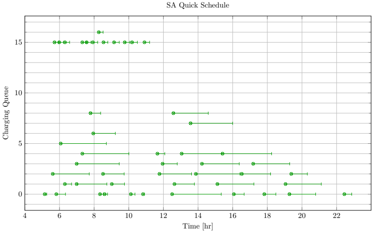
Schedule

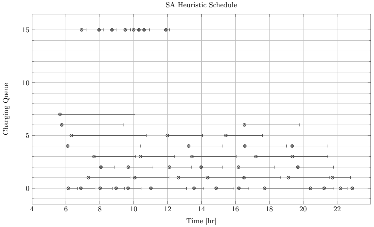
- Qin heavily utilizes fast charger
- BPAP emphasizes slow queues, but utilizes fast chargers readily

- SA techniques heavily utilize slow charging
- SA techniques utilize fast chargers more sparingly

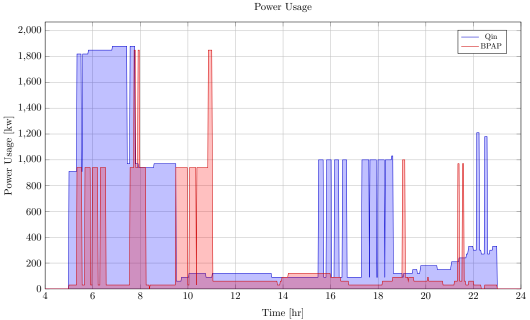


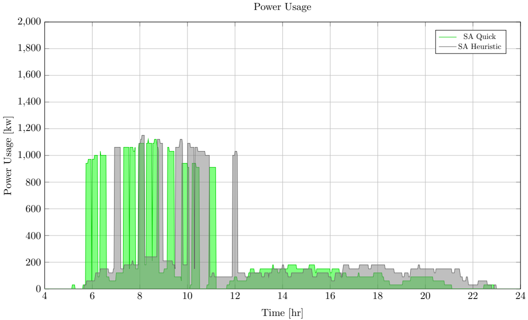






Power

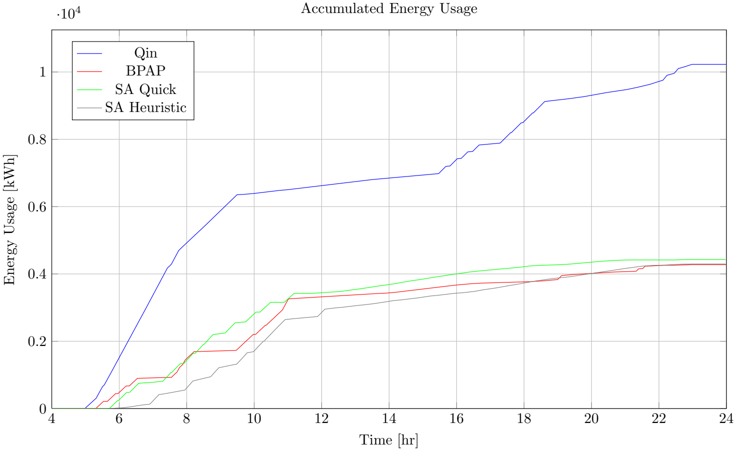




- Heuristic SA maintained the lowest demand peak
- Quick SA peak demand comparable to the PAP peak

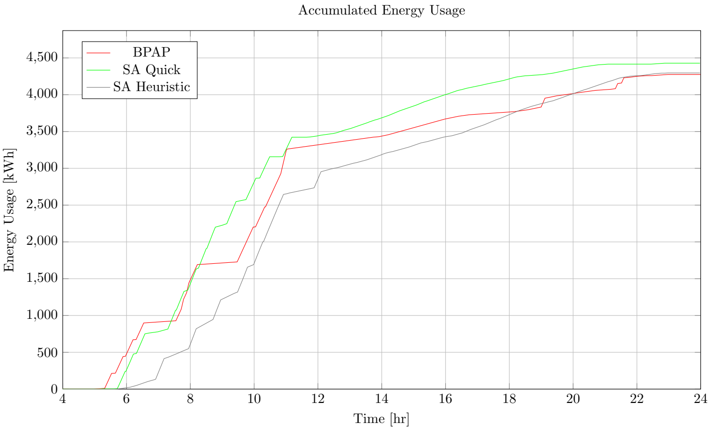
SOC and Energy

- SOC threshold deficits:
  - quick: 2.24 kWh
  - heuristic: 5.74 kWh
- Energy delta:
  - quick: 191.47 kWh
  - heuristic: 58.46 kWh
- Trade off of the lower peak demand



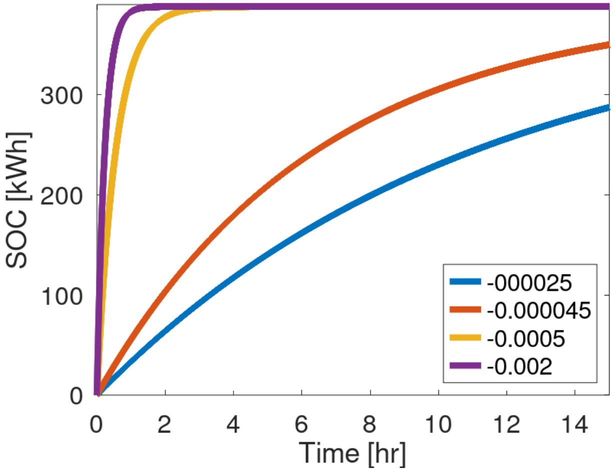
Scores and Energy

Schedule	Score
BPAP	18,500,000
Qin-Modified	34,578,526
Heuristic	11,673,937
Quick	11,234,577



Introduction

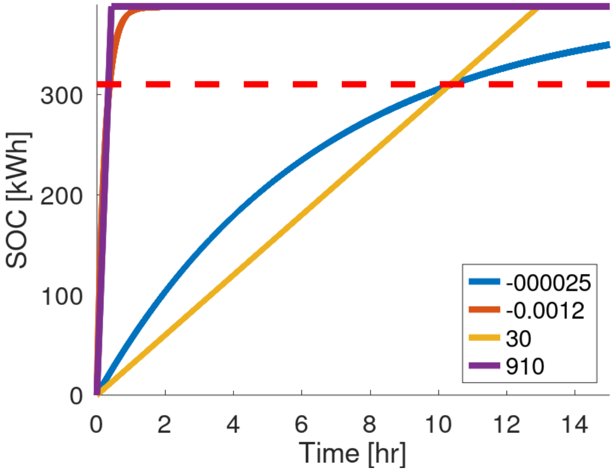
- Higher fidelity in approximating charge at high SOC
- Implemented in SA for simplicity



Non-Linear Battery Dynamics Model

$$\eta_{\xi_i} = \bar{a}_q \eta_i - \bar{b}_q \kappa_{b_i}$$

$$\bar{a}_q = e^{a_q dt} \quad \bar{b}_q = e^{a_q dt} - 1$$



Results

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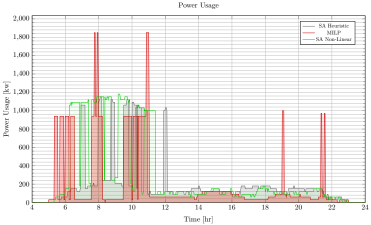
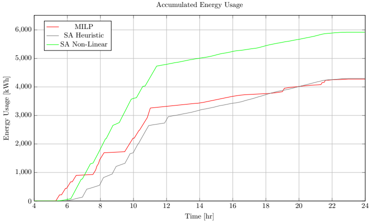
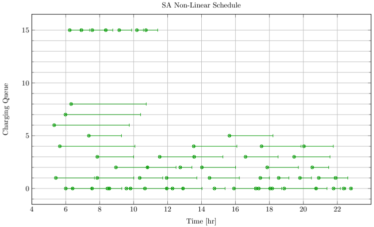
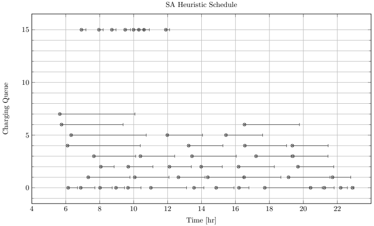
- Minimized demand power
- Larger energy consumption due to fast charger duration

- Similar schedule output, longer fast charger durations due to nonlinear model
- Minimum SOC of 85 kWh









Questions?

Appendix

Mixed Integer Linear Programming

$$
\begin{aligned}
\max\ J &= \sum_j c_j x_j + \sum_k d_k y_k \\
\text{subject to } & \sum_j a_{ij} x_j + \sum_k g_{ik} y_k \le b_i && (i = 1, 2, \ldots, m) \\
& x_j \ge 0 && (j = 1, 2, \ldots, n) \\
& y_k \in \mathbb{Z}^+ && (k = 1, 2, \ldots, n)
\end{aligned}
$$

- J: Objective function
- $x_j \in \mathbb{R}$ and $y_k \in \mathbb{Z}^+$: Decision Variables
- $c_j, d_k, a_{ij}, g_{ik}, b_i \in \mathbb{R}$: Parameters

SA Primitive Generators

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New Visit

Algorithm 0: New visit algorithm

Algorithm: New Visit

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
 $i \leftarrow \mathbb{S}_i$; `/* Extract visit index */`
 $a \leftarrow \mathbb{I}_{i.a}$; `/* Extract the arrivial time for visit` i `*/`
 $e \leftarrow \mathbb{I}_{i.e}$; `/* Extract the departure time for visit` i `*/`
 $q \leftarrow \mathbb{I}_{i.q}$; `/* Extract the current charge queue for visit` i `*/`
 $\bar{q} \leftarrow \mathcal{U}_Q$; `/* Select a random charging queue with a uniform distribution */`
 $C \leftarrow \mathcal{U}_{\mathbb{C}_q}$; `/* Select a random time slice from` $\mathbb{C}_q$ `*/`
 if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime(`C, i, q, a, e`)` $\notin \varnothing$ **then** `/* If there is time available in` C_q^j `*/`
  return $(i, (\bar{q}, \bar{u}, \bar{d}), \bar{C})$ `/* Return visit */`
 end
 return $(\varnothing)$; `/* Return nothing */`
end

Slide Visit

Algorithm 1: Slide Visit Algorithm

Algorithm: Slide Visit

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
    $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$`Purge`$(\mathbb{S})$; `/* Purge visit` i `from charger availibility matrix */`
    $C \leftarrow \bar{C}_{i.q_i}$; `/* Get the time availability of the purged visit */`
    `/* If there is time available in` C `*/`
    if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime`$(C, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \varnothing$ **then**
        return $(i, \mathbb{I}, (\mathbb{I}_{i.q_i}, \bar{u}, \bar{d}), \bar{C})$ `/* Return updated visit */`
    end
    return $(\varnothing)$; `/* Return nothing */`
end

New Charger

Algorithm 2: New Charger Algorithm

Algorithm: New Charger

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
    $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$ `Purge`$(\mathbb{S})$; `/* Purge visit` i `from charger availibility matrix */`
    $q \leftarrow \mathcal{U}_Q$; `/* Select a random charging queue with a uniform distribution */`
    if $(\bar{C}, \bar{u}, \bar{d}) \leftarrow$ `findFreeTime`$(\bar{\mathbb{C}}_{i.q}, \mathbb{S}_i, \mathbb{I}_q, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}) \notin \varnothing$ **then** `/* If there is time available in` C_q `*/`
        `/* Return visit, note` u `and` d `are the original inital/final charge times. */`
        return $(i, \mathbb{I}, (q, \mathbb{I}_{i.u}, \mathbb{I}_{i.d}), \bar{\mathbb{C}})$
    end
    return $(\varnothing)$; `/* Return nothing */`
end

Wait

Algorithm 3: Wait algorithm

Algorithm: Wait

Input: $\mathbb{S}$

Output: $\bar{\mathbb{S}}$

begin

  $(i, \mathbb{I}, \bar{\mathbb{C}}) \leftarrow$`Purge(`$\mathbb{S}$`);` `/* Purge visit` i `from charger availibility matrix */`

  $\bar{\mathbb{C}}'_{\mathbb{I}_{i.\Gamma_i}} \leftarrow \mathbb{C}' \cup \{[\mathbb{I}_{i.a}, \mathbb{I}_{i.e}]\}$; `/* Update the charger availability matrix for wait queue` $\bar{\mathbb{C}}_{i.q_i}$ `*/`

  return $(i, \mathbb{I}, (\mathbb{I}_{i.b}, \mathbb{I}_{i.a}, \mathbb{I}_{i.e}), \bar{\mathbb{C}})$ `/* Return visit */`

end

New Window

College of Engineering
UtahStateUniversity

Algorithm 4: New window algorithm

Algorithm: New Window

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
    $\bar{\mathbb{S}} \leftarrow$ `Wait`$(\mathbb{S})$; `/* Assign visit to its respective idle queue */`
    if $\bar{\bar{\mathbb{S}}} \leftarrow$ `NewVisit`$(\bar{\mathbb{S}}) \notin \varnothing$ **then** `/* Add visit` i `back in randomly */`
        return $\bar{\bar{\mathbb{S}}}$ `/* Return visit */`
    end
    return $(\varnothing)$; `/* Return nothing */`
end

SA Wrapper Functions

Charge Schedule Generation

Algorithm 5: Charge schedule generation algorithm

Algorithm: Candidate Solution Generator

Input: $\mathbb{S}$
Output: $\bar{\mathbb{S}}$

begin
  `/* Select an unscheduled BEB visit from a randomly indexed set of visits */`
  foreach $\mathbb{I}_i \in \mathbb{I}$ **do**
    $(i, \bar{\mathbb{I}}, \bar{\mathbb{C}}) \leftarrow$ `NewVisit(`$(\mathbb{I}_i, \mathbb{I}, \mathbb{C})$`);` `/* Assign the bus to a charger */`
  end
  return $(0, \bar{\mathbb{I}}, \bar{\mathbb{C}})$
end

Charge Schedule Perturbation

College of Engineering
UtahStateUniversity

Algorithm 6: Perturb schedule algorithm

Algorithm: Perturb Schedule

Input: $\mathbb{S}$

Output: $\bar{\mathbb{S}}$

begin

    $p \leftarrow [false; n_A]$; /* Create vector to track priority routes */

    $y^i \leftarrow [1.0; n_V]$; /* Create weight vector for index selection */

    /* Loop through the visits in reverse order */

    foreach $\mathbb{I}_i \leftarrow \mathbb{I}_{|\mathbb{I}|}$ *TO* $\mathbb{I}_1$ **do**

        /* If the current visit is part of a priority route */

        if $p_{\mathbb{I}_{i.b}} = true$ **then**

            $y^i_{\mathbb{I}_i} = y^i_{\mathbb{I}_{i.\xi}}$;

        end

        /* Else if the current visit's SOC does below the allowed threshold */

        else if $\mathbb{I}_{i.\eta} \leq \nu_{\mathbb{I}_{i.b}} \kappa_{\mathbb{I}_{i.b}}$ **then**

            $p_{\mathbb{I}_{i.b}} = true$; /* Indicate the current BEB's routes are to be prioritized */

            $y^i_{\mathbb{I}_i} = \kappa_{\mathbb{I}_{i.b}}(\nu_{\mathbb{I}_{i.b}} \kappa_{\mathbb{I}_{i.b}} - \mathbb{I}_{i.\eta})$; /* Calculate the weight of the current visit */

        end

    end

    $\mathbb{I}_i \leftarrow \mathcal{W}^{y^i}_{\mathbb{I}}$; /* Select an index with a weighted distribution */

    $i \leftarrow \mathbb{I}_i$; /* Extract visit index */

    $y^p \leftarrow [y^p_1, y^p_2, \ldots]$; /* Define the weight of each primitive generator */

    $PGF \leftarrow \mathcal{W}^{y^p}_{[1,n_G]}$; /* Select a generator function with weighted distribution */

    $\bar{\mathbb{S}} \leftarrow$ PGF($(i, \mathbb{I}, \mathbb{C})$); /* Excecute the generator function */

    return $(0, \bar{\mathbb{I}}, \bar{\mathbb{C}})$

end

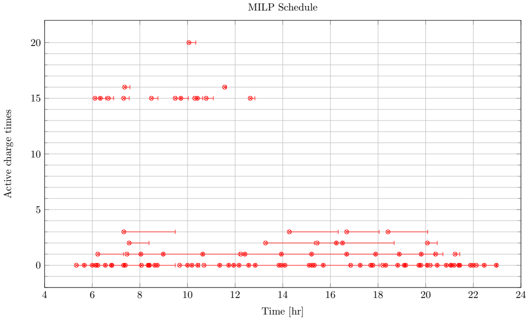
SA Thesis Results

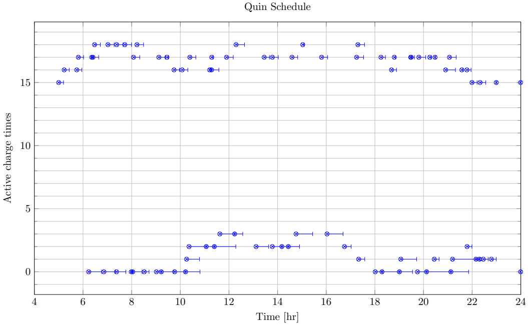
Parameters

Model	Execution Time [s]	Iteration [s]
MILP	3600	N/A
Quick	2275.25	0.25
Heuristic	3640.4	0.4

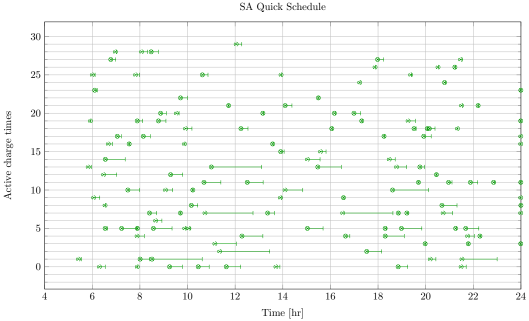
- $T_0 = 99999$
- $\beta = 0.999$
- $|t| = 3797$
- $n_K = 500$

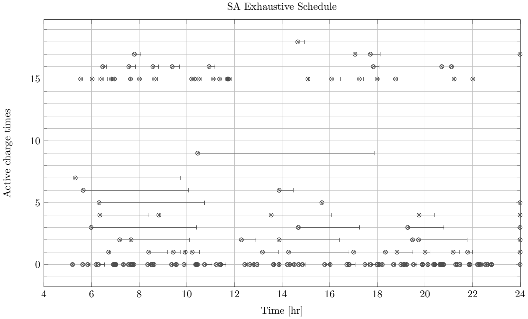
Schedule


MILP Schedule
Active charge times
Time [hr]
0
5
10
15
20
4
6
8
10
12
14
16
18
20
22
24

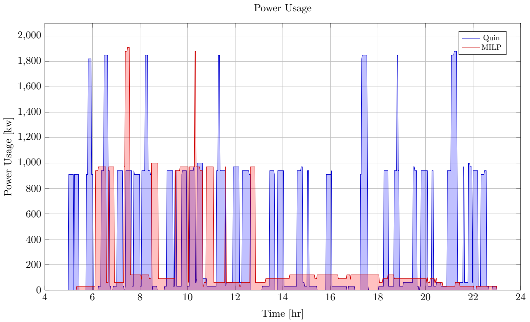

Quin Schedule
Active charge times
Time [hr]
0
5
10
15
4
6
8
10
12
14
16
18
20
22
24

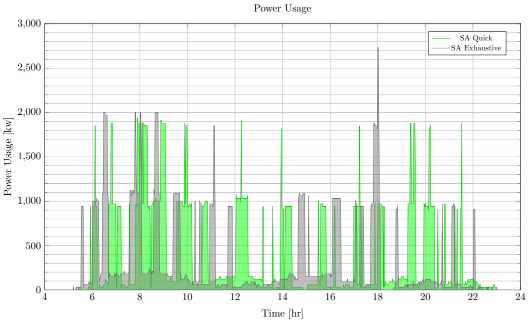
Schedule


SA Quick Schedule
Active charge times
Time [hr]
0
5
10
15
20
25
30
4
6
8
10
12
14
16
18
20
22
24

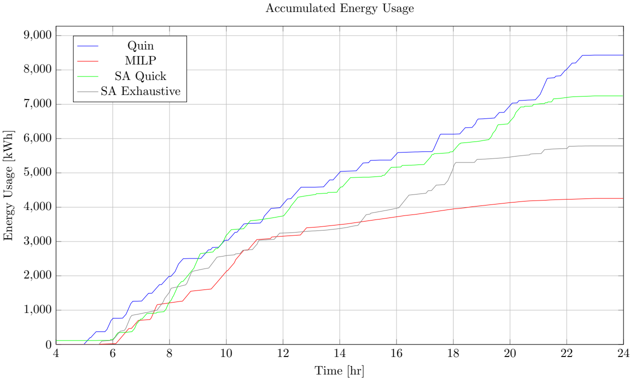

SA Exhaustive Schedule
Active charge times
Time [hr]
0
5
10
15
4
6
8
10
12
14
16
18
20
22
24

Power

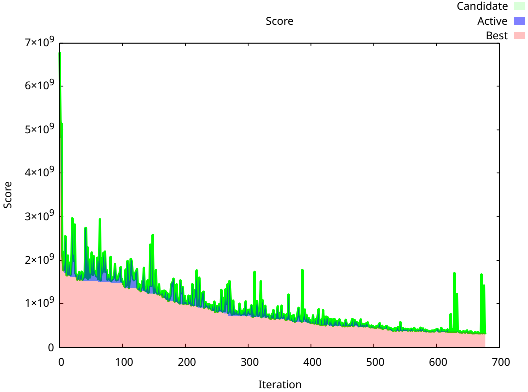

Power Usage
Quin
MILP
Power Usage [kw]
Time [hr]
2,000
1,800
1,600
1,400
1,200
1,000
800
600
400
200
0
4
6
8
10
12
14
16
18
20
22
24


Power Usage
SA Quick
SA Exhaustive
Power Usage [kw]
Time [hr]
3,000
2,500
2,000
1,500
1,000
500
0
4
6
8
10
12
14
16
18
20
22
24

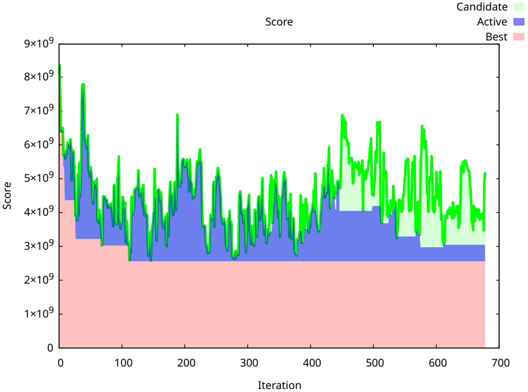
Energy


Accumulated Energy Usage
Quin
MILP
SA Quick
SA Exhaustive
Energy Usage [kWh]
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
Time [hr]
4
6
8
10
12
14
16
18
20
22
24

How To Resolve This Problem?



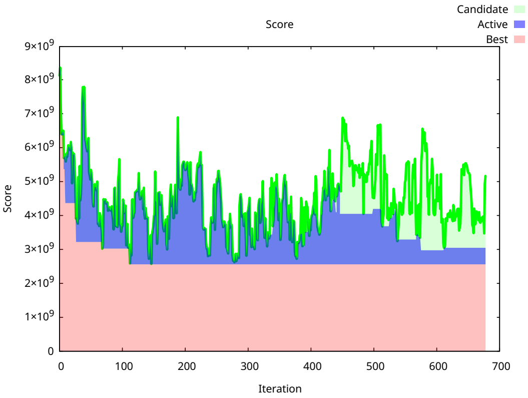
- Reverse search and weight the visit indices
- Be more aggressive in exploiting the best solution



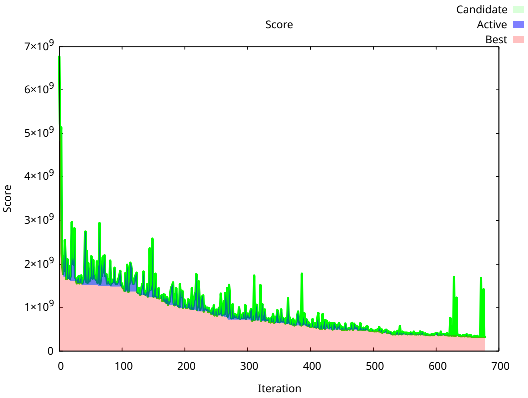
- Candidate solutions diverge
- Hard time handling "difficult" routes

Score Convergence Comparison

Before Fix



After Fix



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