

A Position Allocation Approach to the Scheduling of Battery Electric Bus Charging

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Abstract—A major challenge of incorporating electric vehicles into a bus fleet is the scheduling of buses to be charged while considering time constraints as well as battery health. This paper introduces a scheduling framework that utilizes both fast and slow charging assuming bus scheduling is fixed. Slow chargers are prioritized for battery health and fast chargers are utilized to satisfy time constraints. A Berth Allocation Problem (BAP) approach is modified and utilized to construct the Position Allocation Problem (PAP) Mixed Integer Linear Program (MILP) that optimally assigns incoming buses to chargers whilst taking into consideration battery dynamics of the buses. Results are present utilizing a randomly generated bus schedule for 40 buses.

Index Terms—Berth Allocation Problem (BAP), Position Allocation Problem (PAP), Mixed Integer Linear Program (MILP)

Battery Electric Bus (BEB) scheduling

I. INTRODUCTION

The public transportation system is crucial in any urban area; however, the increase awareness and concern of environmental impacts of petroleum based public transportation has driven an effort to reduce the pollutant footprint [1]–[4]. Particularly, the electrification of public bus transportation via battery power, i.e., battery electric buses (BEBs), has received significant attention [4]. Although the technology provides benefits beyond reduction in emissions such as lower driving costs, lower maintenance costs, and reduced vehicle noise, battery powered systems introduce new challenges such as larger upfront costs, and potentially several hour long “refueling” periods [2], [4]. Furthermore, the problem is exacerbated by the constraints of the transit schedule the fleet must adhere to, the limited amount of chargers available, as well as the adverse affects in the health of the battery due to fast charging [5]. This paper presents a continuous scheduling framework for a BEB fleet that shares limited fast and slow chargers. This framework takes into consideration linear charging dynamics and a fixed bus schedule while meeting a certain battery percentage threshold while remaining *while remaining ... what?*

Recent efforts have been made into solving scheduling and charging fleets as well as the infrastructure. Attention has been given to solving both problems simultaneously [6]–[9]. The added complexity of considering both the BEB charge scheduling and the infrastructure problems necessitates simplifications for sake of computation. First, only fast chargers are utilized in

*Intra sentence to PAP paragraph
Recent work has used the position allocation problem (PAP) as a means to schedule electric vehicle charging scheduling [edit].*

planning [6], [7], [9]–[14]. Second, significant simplifications to the charging models are made. Some approaches assume full charge [6], [9], [10], [13]. Others have assumed that the charge received is proportional to the time spent on the charger [11], [12], which can be a valid assumption when the battery state-of-charge (SOC) is below 80% charge [11].

The contribution of this work is a ~~Mixed integer~~ Linear Program (MILP) scheduling framework that considers bus schedules, ~~or~~ ~~around~~ ~~not~~ for battery dynamics, charge limits, and the availability of slow and fast chargers. The bus schedules are assumed to be fixed for the duration of the time horizon. The (MILP) is formed, and extends upon, the Position Allocation Problem (PAP) [15]. Linear charging dynamics are assumed, costs are assumed to be constant for the duration of the time horizon, and the amount of chargers is assumed to be constant. The MILP framework allows the addition and replacing of constraints. As such, battery dynamic constraints may be replaced with first order dynamic modeling and costs can be made dynamic. The solution of the problem provides the arrival time, selected charger (fast or slow), initial charge time, final charge time, and departure time from the station.

The PAP framework is constructed from the Berth Allocation Problem (BAP). The BAP solves the problem of allocating space for incoming vessels to be berthed. Each arriving vessel requires both time and space to be serviced and is assigned a berthing location. Vessels are lined up parallel to the berth to be serviced and are horizontally queued as shown in Fig 1a. The PAP utilizes this notion of queuing to reuse the BAP for queuing BEBs to be charged as shown in Figure 1b.

The remainder of the paper proceeds as follows: Section II the BAP is introduced with a MILP formulation example. Section III constructs the PAP and introduces battery dynamics into the MILP, and Section IV demonstrates an example of the PAP formulation. Section V provides some conclusions and potential future work. *The paper ends in section V with concluding remarks. The Position Allocation Problem*

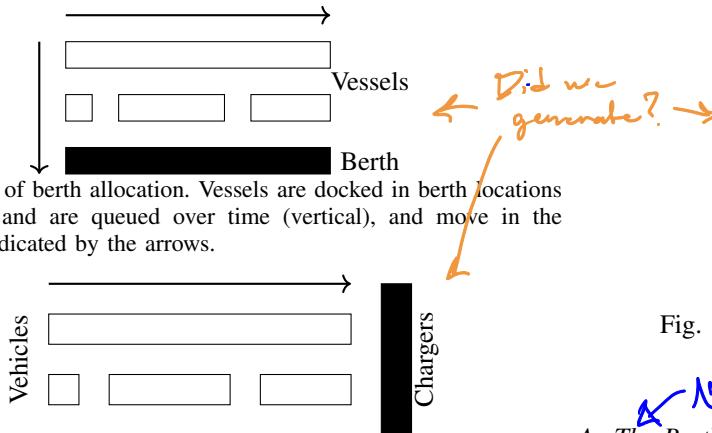
II. PRELIMINARIES

The BEB charge schedule formulation in this work builds upon the PAP. The PAP is a rectangle packing problem where a set of rectangles O are attempted to be optimally placed in a larger rectangle O' as shown in Fig 2. Both the set O and rectangle O' 's width and height are used to represent

TABLE I: Notation used throughout the paper

Variable	Description	Variable	Description
Input values			
A	Number of buses in use	I	Final index
M	An arbitrary very large upper bound value	N	Number of total visits
O	Bounding box for rectangle packing	\mathcal{O}	Set of rectangles to be packed in O
Q	Number of chargers	T	Time Horizon
Input variables			
Γ_i	Array of visit id's	α_i	Initial charge time for visit i
β_i	Final charge for bus i at the end of the time horizon	ϵ_q	Cost of using charger q per unit time
γ_i	Array of values indicating the next index visit i will arrive	κ_i	Battery capacity for bus i
λ_i	Discharge of visit over route i	ν	Minimum charge allowed on departure of visit i
τ_i	Time visit i must leave the station	ζ_i	Discharge rate for vehicle i
a_i	Arrival time of visit i	c_i	Detach time from charger for visit i
w_{iq}	Cost of a visit being assigned to charger q	r_q	Charge rate of charger q per unit time
Decision Variables			
δ_{ij}	$v_i < v_j = 1$ or $i \neq j = 0$	η_i	Initial charge for visit i
σ_{ij}	$u_i < u_j = 1$ or $i \neq j = 0$	c_i	detach time from charger for visit i
g_i	Linearization term for bilinear terms $g_i := p_i w_{iq}$	p_i	Amount of time spent on charger for visit i
u_i	Initial charge time of visit i	v_i	Assigned queue for visit i
w_{iq}	Vector representation of queue assignment		

Fig. 1: Comparison between BAP and PAP



(a) Example of berth allocation. Vessels are docked in berth locations (horizontal) and are queued over time (vertical), and move in the directions indicated by the arrows.

(b) Example of position allocation. Vehicles are placed in queues to be charged and move in the direction indicated by the arrow.

quantifiable values. The rectangle packing problem is a NP-hard problem that is a subset of the packing problem and can be used to describe many real life problems [16], [17]. In some of these problems, the dimensions of \mathcal{O} are held constant such as in the problem of packing modules on a chip, where the widths and height of the rectangles represent the physical width and heights of the modules [17]. Other problems, such as the BAP, allow one or both sides to be decision variables [18].

What is BAP?
The BAP is a form of the rectangle packing problem where the set of rectangles describe incoming vessels to be docked on a berth to be serviced as shown in Fig 1a [19]. The BAP is commonly formed as a Mixed Integer Linear Program (MILP) [18], [20]. This formulation utilizes the BAP to help determine which queue the bus should be placed on to be charged without violating time or space constraints. Notation is summarized in Table I.

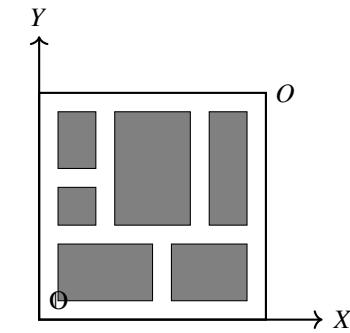


Fig. 2: Example of rectangle packing problem

A. The Berth Allocation Problem

The BAP solves the problem of optimally assigning incoming vessels to berth positions to be serviced (Fig 1a). The width and height of O represent the berth length S and time horizon T , respectively. Similarly, the width and height for \mathcal{O} represent the time spent to service vessel i and the space taken by docking vessel i , respectively. The vessel characteristics (length of the vessel, arrival time, handling time, desired departure time) are assumed to be known for all N vessels to be serviced. A representation of a BAP solution is shown in Fig 3.

The BAP objective is generally represented as minimizing some operational time for a given vessel i . The operational time may be chosen to minimize time of arrival to time of departure, time spent being serviced, or overall waiting time (i.e. $a_i \leq u_i$) [18], [20], [21]. The model must then constrain the vessel placement as to not allow overlap in time or space as well as disallowing discontinuities in arrival times to departure times. The following MILP describes the stated objective and constraints. The MILP is also shown in its entirety as it forms the foundation from which the PAP is constructed [15].

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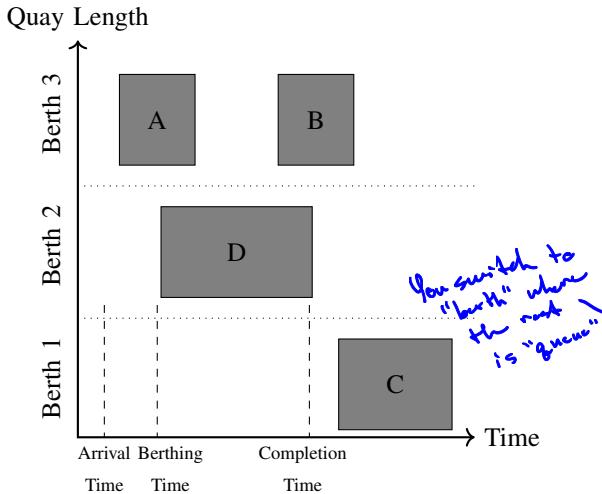


Fig. 3: The representation of the berth-time space

$$\min \sum_{i=1}^N (c_i - a_i) \quad (1)$$

Subject to the following constraints:

$$u_j - u_i - p_i - (\sigma_{ij} - 1)T \geq 0 \quad (2a)$$

$$v_j - v_i - s_i - (\delta_{ij} - 1)S \geq 0 \quad (2b)$$

$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \quad (2c)$$

$$\sigma_{ij} + \sigma_{ji} \leq 1 \quad (2d)$$

$$\delta_{ij} + \delta_{ji} \leq 1 \quad (2e)$$

$$p_i + u_i = c_i \quad (2f)$$

$$a_i \leq u_i \leq (T - p_i) \quad (2g)$$

$$\sigma_{ij} \in \{0, 1\}, \delta_{ij} \in \{0, 1\} \quad (2h)$$

Where, for this problem, the following are constants

- S : berth length *Not in table 1*
- T : time horizon
- N : number of incoming vessels
- p_i : charging time for vessel i ; $\forall 1 \leq i \leq N$
- s_i : size of vehicle i ; $\forall 1 \leq i \leq N$
- a_i : arrival time of vessel i ; $\forall 1 \leq i \leq N$

and the following are decision variables

- u_i : starting time of service for vessel i ; $\forall 1 \leq i \leq N$
- v_i : berth position i ; $\forall 1 \leq i \leq N$
- c_i : departure time for vessel i ; $\forall 1 \leq i \leq N$
- σ_{ij} : $u_i < u_j$; $\forall 1 \leq i < j \leq N$
- δ_{ij} : $v_i < v_j$; $\forall 1 \leq i < j \leq N$

The objective function (1) minimizes the time spent to service each vessel by minimizing over the difference between the departure time (c) and arrival time (a).

Constraints 2a-2e are the “queuing constraints”. They are used to prevent overlapping over both space and time as shown in Fig 4. In terms of the BAP, constraint (2a) states that the

starting service time (u) for vessel j must be greater than the starting time of vessel i plus its service time (p). The last term utilizes the Big-M notation to activate or deactivate the constraint. Specifically, (2a) is active if both i and j are to visit the same berth space ($v_i = v_j$). A value of $\sigma_{ij} = 1$ will activate the constraint to ensure that i is complete before j is allowed to begin being serviced. If $\sigma = 0$, then the constraint is of the form $T + u_j + p_j > u_i$ rendering the constraint “inactive” because u_i cannot be larger than $T + u_j + p_j$.

In a similar fashion, δ_{ij} effectively determines whether the vessel will be located in adjacent berths at the same time ($v_i = v_j$). If $\delta_{ij} = 1$ then (2b) is rendered active and vessels i and j must be in adjacent berthing locations.

Constraints 2c-2e are used to establish queuing by providing a relationship between queuing variables (u and v). Constraint (2e) states that one of the following is true: $u_i + p_i < u_j$ ($\sigma_{ij} = 1$) or $u_j < u_i + p_i$ ($\sigma_{ji} = 1$), and $v_i + s_i < v_j$ ($\delta_{ij} = 1$) or $v_j < v_i + s_i$ ($\delta_{ji} = 1$). Constraints (2d) and (2e) enforce consistency, i.e. $u_i + p_i < u_j$ and $u_j < u_i + p_i$ cannot be simultaneously. In a similar manner, $v_i + s_i < u_j$ and $u_j < v_i + s_i$ cannot be true simultaneously. This enforces a relationship between vessels: either one is before the other temporally or they are in different queues.

The last constraints enforce continuity for each vessel. Constraint (2f) states that the service start time (u) plus the time to service vessel i (p) must equal the departure time (c). Constraint (2g) enforces the arrival time (a) must be less than or equal to the service start time (u) which must also be less than or equal to the latest time the vessel may begin to be serviced to stay within the time horizon. Constraint (2h) defines the set of values σ and δ .

This formulation forms the basis of the PAP; however there will be some slight differences in the way the variables are perceived. Starting service time (u) is now the starting charge time and the berth location (v) is now the queue for charger (q). Another difference is that (p) will now become a decision variable which will be driven by the battery dynamic constraints.

In addition, the BAP has characteristics that can be leveraged and others that must be addressed to fit the desired model. An inconvenient property is that each vessel i ; $\forall 1 \leq i \leq N$ is assumed to be a new (i.e. unique) visit. This becomes a problem for accounting for battery dynamics when busses revisit after completing a route. Each bus visit must then be associated with a specific bus ID. However, due to the nature of the BAP, little effort is required to convert the problem from serial berth docking to parallel charge queuing. The results may be vectorized to achieve this goal [15]. Furthermore, this formulation allows S and T to be continuous or discrete [18], [20]. Because of this flexibility, the berth is discretized to accommodate Q chargers while time is kept continuous as shown in Fig 3.

III. PROBLEM FORMULATION

The MILP is formulated with two sets of constraints: queuing constraints and battery dynamics constraints. This

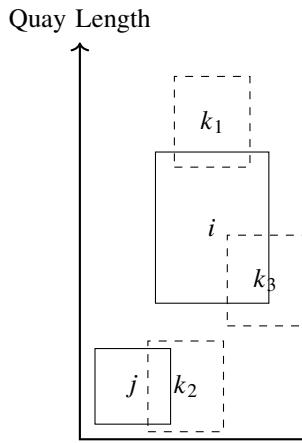


Fig. 4: Examples of different methods of overlapping. Space overlap: $v_{k_1} < v_i + s_i \therefore \delta_{k_1 i} = 0$. Time overlap $u_{k_1} < u_j + p_j \therefore \sigma_{k_2 j} = 0$. Both space and time overlap $\sigma_{k_3 i} = 0$ and $\delta_{k_3 j} = 0$.

section will build off and modify the previous formulation ((1) and (2)) and progressively construct the battery dynamic constraints to create the Position Allocation MILP. All notation is defined in Table I.

A. Queuing Constraints

Consider the following set of constraints:

$$u_i - u_j - p_j - (\sigma_{ij} - 1)T \geq 0 \quad (3a)$$

$$v_i - v_j - s_j - (\delta_{ij} - 1)Q \geq 0 \quad (3b)$$

$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \quad (3c)$$

$$\sigma_{ij} + \sigma_{ji} \leq 1 \quad (3d)$$

$$\delta_{ij} + \delta_{ji} \leq 1 \quad (3e)$$

$$p_i + u_i = c_i \quad (3f)$$

$$a_i \leq u_i \leq (T - p_i) \quad (3g)$$

$$c_i \leq \tau_i \quad (3h)$$

$$\sigma_{ij} \in \{0, 1\}, \delta_{ij} \in \{0, 1\} \quad (3i)$$

$$v_i \in \{1, 2, \dots, Q\} \quad (3j)$$

$$(3k)$$

Constraints (3a)-(3g) are the same as previously described in Section II, except charging time p is now a decision variable. Additionally, starting charge time u , charge duration p , and detach time a are continuous and charging queue, v , discrete, as represented in (3j). Constraint (3h) states that the ending charge time c must be less than or equal to the required departure time from the station τ .

The constraints (3) as stated are flawed. If the objective function given in (1) were minimized while being subject to the constraints in (3), p being defined as a decision variable would naturally cause the solution to converge to 0. This effectively minimizes the objective $c_i - a_i$, but also would imply no charging should ever be done. To remedy this, battery dynamic constraints will be introduced.

Using purely the constraints in (3) with the objective in (1) would result in c_i being chosen as small as possible by employing $p_i = 0, u_i = c_i$. Thus, the vehicles would not charge. Furthermore, it does not consider any recharging of the BEB to the charging stations.

B. Battery Dynamic Constraints

The battery dynamic constraints are used to drive time spent on the charger p as well define initial, final, and intermediate bus charges for visit i . Consider the following MILP:

$$\sum_{i=1}^N \sum_{q=1}^Q (w_i m_q + g_i \epsilon_q) \quad (4)$$

Subject to the following constraints: For battery charge dynamics:
the constraints in (3) as well as

$$\eta_i = \alpha \kappa \Gamma_i \quad (5a)$$

$$\eta_i + \sum_{q=1}^Q g_{iq} r_q - \lambda_i = \eta_{\gamma_i} \quad (5b)$$

$$\eta_i + \sum_{q=1}^Q g_{iq} r_q - \lambda_i \geq \nu \kappa \Gamma_i \quad (5c)$$

$$\eta_i + \sum_{q=1}^Q g_{iq} r_q \leq \kappa \Gamma_i \quad (5d)$$

$$\eta_i + \sum_{q=1}^Q g_{iq} r_q - \lambda_i \geq \beta \kappa \Gamma_i \quad (5e)$$

$$p_i + (1 - w_{iq})M \geq g_{iq} \quad (5f)$$

$$p_i \leq g_{iq} \quad (5g)$$

$$M w_{iq} \geq g_{iq} \quad (5h)$$

$$0 \leq g_{iq} \quad (5i)$$

$$\sum_{q=1}^Q q w_{iq} = v_i \quad (5j)$$

$$\sum_{q=1}^Q w_{iq} = 1 \quad (5k)$$

$$w_{iq} \in \{0, 1\} \quad (5l)$$

Constraints (5a)-(5e) provide initialization and terminal conditions as well as intermediate constraints to provide continuity in vehicle charges. Constraint (5a) states the first arrival for each bus is initialized with a charge of $(\alpha \kappa \Gamma_i)$. Constraint (5b) defines that the previous charge (η) plus the charging done by charger (q) minus the discharge amount due to completing route (λ). Constraints (5c) is similar to (5b), except it states that the charge after being charged by (q) and the discharge from route (λ) must be greater than some percentage of its capacity (κ). Constraint (5d) states that the charging done for visit i cannot be greater than the capacity of the battery (κ). Constraint (5e) states that the last visit for each vehicle must have a minimum charge of $(\beta \kappa \Gamma_i)$. *Why? Meet end of day reg.*

The set of constraints (5f)-(5i) are used to alleviate non-linearity introduced by the bilinear term $g_{iq} := w_{iq} p_i$. This is accomplished using Big-M notation [22]. Constraints (5f) and (5g) state that if $w_{iq} = 1$ then the two equations will take the form of $p_i \leq g_{eq}$ and $p_i \geq g_{iq}$ effectively stating $p_i = g_{iq}$, if $w_{iq} = 0$ then the two equations will take the form

of $p_i \leq g_{eq}$ and $p_i \geq g_{iq} - M$ which deactivates the constraint. In a similar fashion, (5h) and (5i) state if $w_{iq} = 0$ then the constraints take the form $0 \geq g_{iq}$ and $0 \leq g_{iq}$ directly implying $0 = g_{iq}$. If $w_{iq} = 1$ then the constraints take the form $M \leq g_{iq}$ and $0 \geq g_{iq}$ disabling the constraint. Putting the two sets of constraints together defines the linearized representation of $w_{iq} p_i$.

The set of constraints (5j)-(5k) define the linking constraint between the queuing constraint v and w (i.e $1 * w_{iq} + 2 * w_{iq} + \dots = v_i$) and enforces that only a single w may be selected per visit, respectively. The last constraint (5l) defines the set of values for w .

IV. EXAMPLE

An example will now be presented to demonstrate the utility of the developed MILP. A description of the scenario is first presented followed by results.

A. Scenario

The given example utilizes $A = 40$ buses with $N = 220$ returns to the station divided between the A buses. Each bus has a 388 KWh battery that is required to stay above 25% charge (97 KWh) to maintain battery health, and the bus is assumed to start with 90% charge (349 KWh). Additionally, each bus is required to end with a minimum charge of 95% (349 KWh). Planning is done over a 24 hour time horizon. $Q = 9$ chargers are utilized where five of the chargers are slow charging (100 KWh) and four are fast charging (400 KWh). The slow chargers take longer, but are safer for the battery than the fast charger. Therefore, the slow chargers are modeled with a lower cost than the fast charger for both assigning (m) and utilization (ϵ) in the objective (4) as to penalize utilization of the fast chargers.

The bus schedules are randomly generated. It is assumed that each bus has no more than 30 minutes between route departures. Bus route times are completely random and may vary between $\frac{a_i + a_{\gamma_i}}{2}$ to a_{γ_i} i.e. the route time may be anywhere between the average time between the current and next arrival to the next arrival time. The discharge is assumed to be linear and is calculated via $\lambda_i = \text{rand}(\frac{a_i + a_{\gamma_i}}{2}, a_{\gamma_i}) \zeta \gamma_i$ where $1 \leq i \leq N$.

The optimization is performed using the Gurobi MILP solver [23] on a machine running a quad-core Intel i7-9700 4.7 GHz processor.

B. Results

The schedule generated by the MILP is shown in Fig 6. The top graph indicates the slow charger usage, and the bottom indicates the fast charger usage. Although $Q = 9$ chargers were used, Fig 6 shows that only five chargers were utilized. The MILP formalization will, in addition to utilizing the slow chargers as much as possible, use only the minimum required amount of chargers. This behavior is due to the rectangle packing constraints (3).

Each color in Fig 6 is used to identify a new bus visit. It is noted that the overlaps in Fig 6 indicate the waiting time

What is the difference between health and deactivate?

Is this an integer sum?

of vehicle j whilst vehicle i charges. This is recognized by viewing the vertical bars. These bars indicate the time bus i is set to charge. The area before indicate waiting time and the area after indicates the time spent on the charger.

Fig 5 depicts the charge for every bus for bus over the time horizon. Every vehicle begins at 90% charge, finishes at 95% charge, and never goes below 25% in the intermediate arrivals as stated in the constraints (5). The total power used is summarized in Fig 7 which is representative of the linear battery constraints modeled.

Not sure that total power is helpful

V. CONCLUSION

This work developed a MILP scheduling framework that optimally assigns slow and fast chargers to a BEB bus fleet assuming a constant schedule. The BAP was introduced with an example formulation and was then compared to the PAP. The PAP constructed on the BAP to allow the time spent on the charger (p) to be a decision variable. Because the original PAP required service time (p) to be given, linear battery dynamics were introduced to drive charging times. Additional constraints were also introduced to provide limits and continuity for the battery dynamics.

An example was presented that demonstrated the ability of the formulation to utilize slow chargers when possible and fast chargers when necessary. The example also demonstrated that if a surplus of chargers are given, the minimum amount of chargers will be utilized. This behavior is given by the packing constraints. The battery dynamic constraints to limit maximum and minimum charges was also observed. Future work may expand on this model by introducing first order battery dynamics to increase fidelity of the battery dynamic model.

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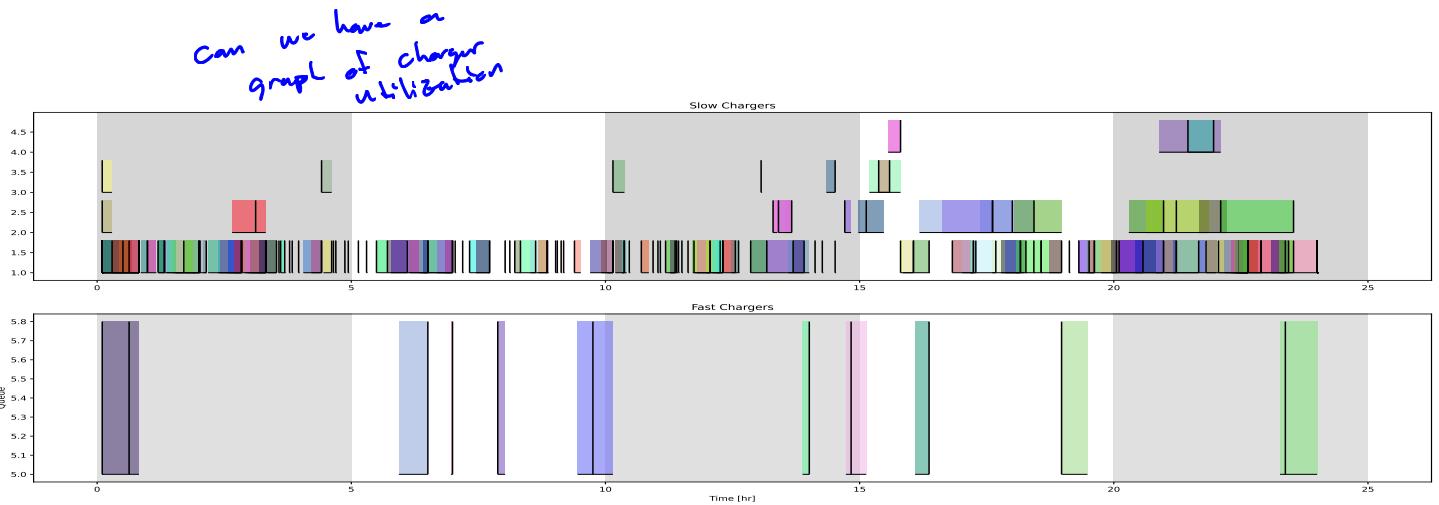


Fig. 5: Bus schedule generated with PAP MILP. Each color indicates a new visit. The vertical bars indicate the time the vehicles are set to charge, the area before said bars indicate the waiting times for each visit $1 \leq i \leq N$. Similarly, the area after the bars indicate the time spent on the charger.

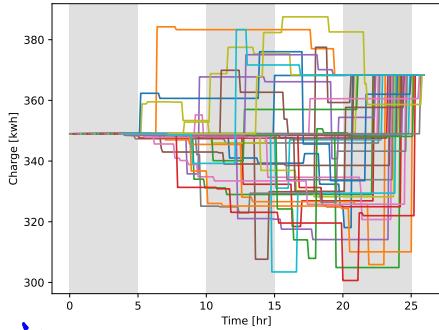


Fig. 6: Charges for each bus.
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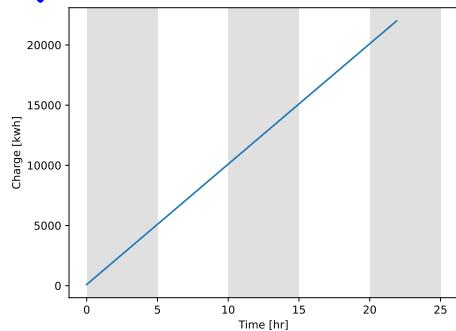


Fig. 7: Total power used.

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