A Position Allocation Approach to the Scheduling of Battery Electric Bus Charging

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***Abstract*—[To do: Copied] A major challenge to adopting battery electric buses into bus fleets is the scheduling of the battery charging while considering route timing constraints, battery charge, and battery health. This work develops a scheduling framework to balance the use of slow and fast chargers assuming the bus routes and charger locations are fixed. Slow chargers are utilized when possible for sake of battery health and fast chargers are used when needed to accommodate timing constraints and ensure a sufficient charge for route execution. A directed, acyclic graph is used to model the available charge times for buses that periodically return to a depot for charging. A constrained network flow Mixed-Integer Linear Program (MILP) problem is formulated to optimize the scheduling of chargers as well as to determine the number of chargers required to meet charging thresholds. Results are presented using a randomly generated bus schedule for thirty buses and demonstrate the ability of the planner to consider peak time charging costs while planning with fixed and variable numbers of chargers.**

***Index Terms*—**

I. Introduction

The public transportation system is crucial in any urban area; however, the increase awareness and concern of environmental impacts of petroleum based public transportation has driven an effort to reduce the pollutant footprint [1]–[4]. Particularly, the electrification of public bus transportation via battery power, i.e., battery electric buses (BEBs), has received significant attention [4]. Although the technology provides benefits beyond reduction in emissions such as lower driving costs, lower maintenance costs, and reduced vehicle noise, battery powered systems introduce new challenges such as larger upfront costs, and potentially several hour long "refueling" periods [2], [4]. Furthermore, the problem is exacerbated by the constraints of the transit schedule the fleet must adhere to, the limited amount of chargers available, as well as the adverse affects in the health of the battery due to fast charging [5]. This paper presents a continuous scheduling framework for a BEB fleet that shares limited fast and slow chargers. This framework takes into consideration linear charging dynamics and a fixed bus schedule while meeting a certain battery percentage threshold while remaining.

Recent efforts have been made into solving scheduling and charging fleets as well as the infrastructure. Attention has been given to solving both problems simultaneously [6]–[9]. The added complexity of considering both the BEB charge scheduling and the infrastructure problems necessitates simplifications for sake of computation. First, only fast chargers are utilized in planning [6], [7], [9]–[14]. Second, significant simplifications to the charging models are made. Some approaches assume full charge [6], [9], [10], [13]. Others have assumed that the charge received is proportional to the time spent on the charger [11], [12], which can be a valid assumption when the battery state-of-charge (SOC) is below 80% charge [11].

The contribution of this work is a Mixed Integer Linear Program (MILP) scheduling framework that considers bus schedules, charging battery dynamics, charge limits, and availability of slow and fast chargers. The bus schedules are assumed to be fixed for the duration of the time horizon. The linear program is formed, and extends upon, the Position Allocation Problem (PAP) [15]. Linear charging dynamics are assumed, costs are assumed to be constant for the duration of the time horizon, and the amount of chargers is assumed to be constant. The MILP framework allows the addition and replacing of constraints. As such, battery dynamic constraints may be replaced with first-order dynamic modeling and costs can be made dynamic. The solution of the problem provides the arrival time, selected charger (fast or slow), initial charge time, final charge time, and departure time from the station.

The PAP framework is constructed from the Berth Allocation Problem (BAP). The BAP solves the problem of allocating space for incoming vessels to be berthed. Each arriving vessel requires both time and space to be serviced and is assigned a berthing location. Vessels are lined up parallel to the berth to be serviced and are horizontally queued as shown in Fig 1a. The PAP utilizes this notion of queuing to re-use the BAP for queuing BEBs to be charged as shown in Figure 1b.

The remainder of the paper proceeds as follows. To do: List out sections

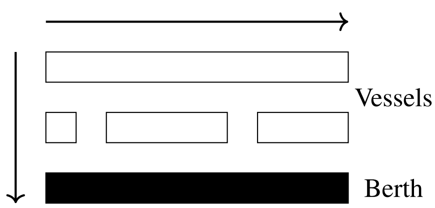
II. Preliminaries

The BEB charge schedule formulation in this work builds upon the PAP. The PAP is a rectangle packing problem where a set of rectangles $\mathbb{O}$ are attempted to be optimally placed in a larger rectangle O as shown in Fig 2. Both the set $\mathbb{O}$ and rectangle O's width and height are used to represent

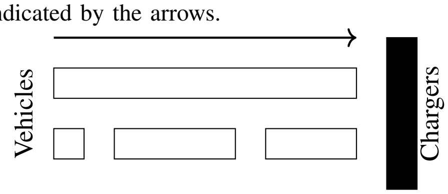
TABLE I: Notation used throughout the paper

Variable	Description	Variable	Description
Input values			
A	Number of buses in use	I	Final index
M	An arbitrary very large upper bound value	N	Number of total visits
O	Bounding box for rectangle packing	$\mathbb{O}$	Set of rectangles to be packed in O
Q	Number of chargers	T	Time Horizon
Input variables			
Γ_i	Array of visit id's	α_i	Initial charge time for visit i
β_i	Final charge for bus i at the end of the time horizon	ϵ_q	Cost of using charger q per unit time
γ_i	Array of values indicating the next index visit i will arrive	κ_i	Battery capacity for bus i
λ_i	Discharge of visit over route i	ν	Minimum charge allowed on departure of visit i
τ_i	Time visit i must leave the station	a_i	Arrival time of visit i
c_i	Detach time from charger for visit i	m_q	Cost of a visit being assigned to charger q
r_q	Charge rate of charger q per unit time		
Decision Variables			
δ_{ij}	$v_i < v_j = 1$ or $i \neq j = 0$	η_i	Initial charge for visit i
σ_{ij}	$u_i < u_j = 1$ or $i \neq j = 0$	c_i	detach time from charger for visit i
g_i	Linearization term for bilinear terms $g_i := p_i w_{iq}$	p_i	Amount of time spent on charger for visit i
u_i	Initial charge time of visit i	v_i	Assigned queue for visit i
w_{iq}	Vector representation of queue assignment		

Fig. 1: Comparison between BAP and PAP



(a) Example of berth allocation. Vessels are docked in berth locations (horizontal) and are queued over time (vertical), and move in the directions indicated by the arrows.



(b) Example of position allocation. Vehicles are placed in queues to be charged and move in the direction indicated by the arrow.

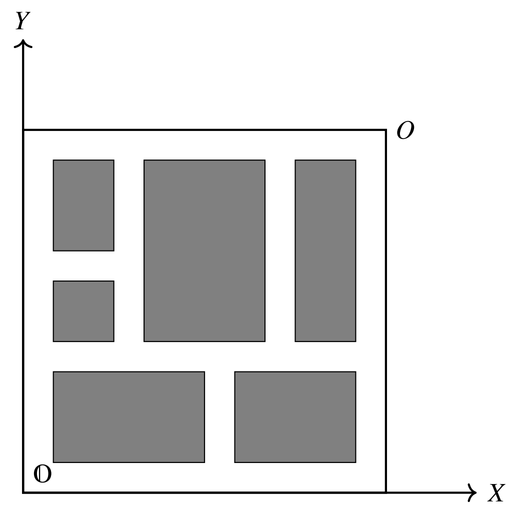


Fig. 2: Example of rectangle packing problem

quantifiable values. The rectangle packing problem is a NP-hard problem that is a subset of the packing problem and can be used to describe many real life problems [16], [17]. In some of these problems, the dimensions of $\mathbb{O}$ are held constant such as in the problem of packing modules on a chip, where the widths and height of the rectangles represent the physical width and heights of the modules [17]. Other problems, such as the BAP, allow one or both sides to be decision variables [18].

The BAP is a form of the rectangle packing problem where the set of rectangles describe incoming vessels to a be docked on a berth to be serviced as shown in Fig 1a [19]. The BAP is commonly formed as a Mixed Integer Linear Program (MILP) [18], [20]. This formulation utilizes the BAP to help determine which queue the bus should be placed on to be charged without violating time or space constraints. Notation is summarized in Table I.

A. *The Berth Allocation Problem*

The BAP solves the problem of optimally assigning incoming vessels to berth positions to be serviced (Fig 1a). The width and height of O represent the berth length S and time horizon T, respectively. Similarly, the width and height for $\mathbb{O}$ represent the time spent to service vessel i and the space taken by docking vessel i, respectively. The vessel characteristics (length of the vessel, arrival time, handling time, desired departure time) are assumed to be known for all N vessels to be serviced. A representation of a BAP solution is shown in Fig 3.

The BAP objective is generally represented as minimizing some operational time for a given vessel i. The operational time may be chosen to minimize time of arrival to time of departure, time spent being serviced, or overall waiting time (i.e. $a_i \leq u_i$) [18], [20], [21]. The model must then constrain the vessel placement as to not allow overlap in time or space as

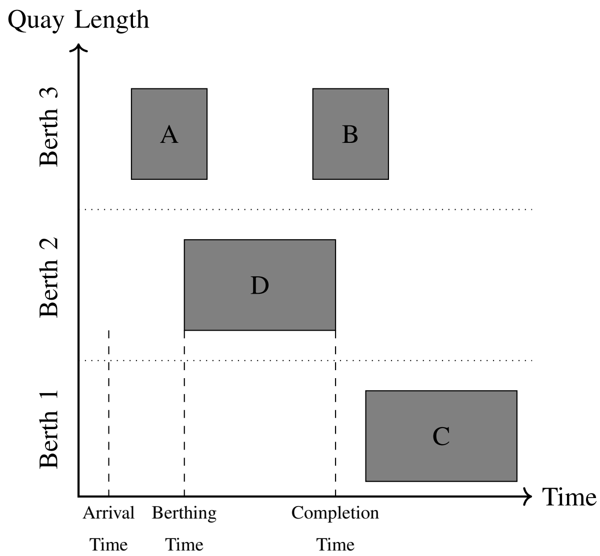


Fig. 3: The representation of the berth-time space

well as disallowing discontinuities in arrival times to departure times. The followng MILP describes the stated objective and constraints. The MILP is also shown in its entirety as it forms the foundation from which the PAP is constructed [15].

$$min \sum_{i=1}^{N}(c_i - a_i) \tag{1}$$

Subject to the following constraints:

$$u_j - u_i - p_i - (\sigma_{ij} - 1)T \geq 0 \tag{2a}$$
$$v_j - v_i - s_i - (\delta_{ij} - 1)S \geq 0 \tag{2b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{2c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{2d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{2e}$$
$$p_i + u_i = c_i \tag{2f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{2g}$$
$$\sigma_{ij} \in \{0,1\},\ \delta_{ij} \in \{0,1\} \tag{2h}$$

Where for this problem the following are constants

- S : berth length
- T : time horizon
- N : number of incoming vessels
- p_i : charging time for vessel $i\ ;\forall 1 \leq i \leq N$
- s_i : size of vehicle $i\ ;\forall 1 \leq i \leq N$
- a_i : arrival time of vessel $i\ ;\forall 1 \leq i \leq N$

and the following are decision variables

- u_i : starting time of service for vessel $i\ ;\forall 1 \leq i \leq N$
- v_i : berth position $i\ ;\forall 1 \leq i \leq N$
- c_i : departure time for vessel $i\ ;\forall 1 \leq i \leq N$
- σ_{ij} : $u_i < u_j\ ;\forall 1 \leq i < j \leq N$
- δ_{ij} : $v_i < v_j\ ;\forall 1 \leq i < j \leq N$

The objective function (1) minimizes the time spent to service each vessel by minimizing over the difference between the departure time (c) and arrival time (a).

Constraints 2a-2e are the "queuing constraints". They are used to prevent overlapping over both space and time as shown in Fig 4. In terms of the BAP, constraint (2a) states that the starting service time (u) for vessel j must be greater than the starting time of vessel i plus its service time (p). The last term utilizes the Big-M notation to activate or deactivate the constraint. Specifically, (2a) is active if both i and j are visiting the same berth ($u_i = u_j$). A value of $\sigma_{ij} = 1$ will activate the constraint to ensure that i is complete before j is allowed to begin being serviced. If $\sigma = 0$, then the constraint is of the form $T + u_j + p_j > u_i$ rendering the constraint "inactive" because u_i cannot be larger than $T + u_j + p_j$.

In a similar fashion, δ_{ij} effectively determines whether the vessel will be located in adjacent berths. If $\delta_{ij} = 1$ then (2b) is rendered active and vessels i and j must be in adjacent berthing locations.

Constraints 2c-2e are used to establish queuing by providing a relationship between queuing variables (u and v). Constraint (2e) states that one of the following is true: $u_i < u_j \iff \sigma_{ij} = 1$ or $u_j < u_i \iff \sigma_{ji} = 1$, and $v_i < v_j \iff \delta_{ij} = 1$ or $v_j < v_i \iff \delta_{ji} = 1$. Constraints (2d) and (2e) enforce consistency, i.e. $u_i < u_j$ and $u_j < u_i$ cannot be simultaneously true. This enforces a relationship between vessels: either one is before the other temporally or they are in different queues.

The last constraints enforce continuity for each vessel. Constraint (2f) states that the service start time (u) plus the time to service vessel i (p) must equal the departure time (c). Constraint (2g) enforces the arrival time (a) must be less than or equal to the service start time (u) which must also be less than or equal to the latest time the vessel may begin to be serviced to stay within the time horizon. Constraint (3i) defines the set of values σ and δ.

This formulation forms the basis of the PAP; however there will are some slight differences in the way the variables are perceived. Starting service time u is now the starting charge time and the berth location v is now the queue for charger q. Another difference is that p will now become a decision variable which will be driven by the battery dynamic constraints.

In addition, the BAP has characteristics that can be leveraged and others that must be addressed to fit the desired model. An inconvenient property is that each vessel $i\ ;\forall 1 \leq i \leq N$ is assumed to be a new (i.e. unique) visit. This becomes a problem for accounting for battery dynamics when busses revisit after completing a route. Each bus visit must then be associated with a specific bus ID. However, due to the nature of the BAP, little effort is required to convert the problem from serial berth docking to parallel charge queuing. The results may be vectorized to achieve this goal [15]. Furthermore, this formulation allows S and T to be continuous or discrete [18], [20]. Because of this flexibility, the berth is discretized to accommodate Q chargers while time is kept continuous as shown in Fig 3.

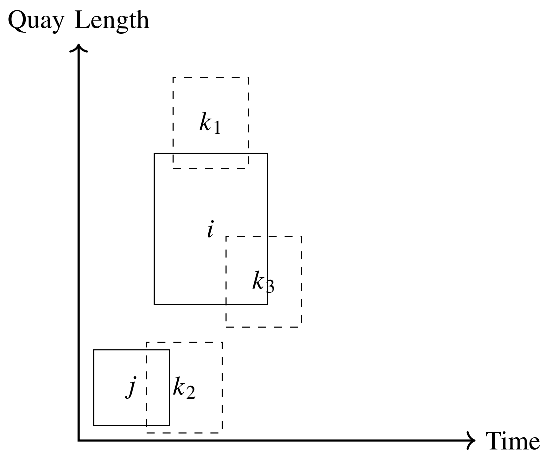


Fig. 4: Examples of different methods of overlapping. Space overlap: $v_{k_1} < v_i + s[i] \therefore \delta_{k_1 i} = 0$. Time overlap $u_{k_1} < u_j + p[j] \therefore \sigma_{k_2 j} = 0$. Both space and time overlap $\sigma_{k_3 i} = 0$ and $\delta_{k_3 j} = 0$.

III. Problem Formulation

The MILP is formulated with two sets of constraints: queuing constraints and battery dynamics constraints. This section will build off and modify the previous formulation ((1) and (2)) and progressively construct the battery dynamic constraints to create the Position Allocation MILP. All notation is defined in Table I.

A. Queuing Constraints

Consider the following set of constraints:

$$u_i - u_j - p_j - (\sigma_{ij} - 1)T \geq 0 \tag{3a}$$
$$v_i - v_j - s_j - (\delta_{ij} - 1)S \geq 0 \tag{3b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{3c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{3d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{3e}$$
$$p_i + u_i = c_i \tag{3f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{3g}$$
$$c_i \leq \tau_i \tag{3h}$$
$$\sigma_{ij} \in \{0, 1\},\ \delta_{ij} \in \{0, 1\} \tag{3i}$$
$$v_i \in \{1, 2, ..., Q\} \tag{3j}$$
$$\tag{3k}$$

Constraints (3a)-(3g) are the same as previously described in section II, except charging time p is now a decision variable. Additionally, starting charge time u is continuous and charging queue v discrete as represented in (3j). Constraint (3h) states that the ending charge time c must be less than or equal to the required departure time from the station τ.

The constraints (3) as stated are flawed. If the objective function given in (1) were minimized while being subject to the constraints in (3), p being defined as a decision variable would naturally cause the solution to converge to 0. This effectively minimizes the objective $c_i - a_i$, but also would imply no charging should ever be done. To remedy this, battery dynamic constraints will be introduced.

B. Battery Dynamic Constraints

The battery dynamic constraints are used to drive time spent on the charger p as well define initial, final, and intermediate bus charges for visit i. Consider the following MILP:

$$\sum_{i=1}^{N} \sum_{q=1}^{Q} \left(w_i m_q + g_i \epsilon_q \right) \tag{4}$$

Subject to the following constraints:

$$\eta_i = \alpha \kappa_{\Gamma_i} \tag{5a}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q - \lambda_i = \eta_{\gamma_i} \tag{5b}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q - \lambda_i \geq \nu \kappa_{\Gamma_i} \tag{5c}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q \leq \kappa \tag{5d}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q - \lambda_i \geq \beta \kappa_{\Gamma_i} \tag{5e}$$
$$p_i + (1 - w_{iq})M \geq g_{iq} \tag{5f}$$
$$p_i \leq g_{iq} \tag{5g}$$
$$M w_{iq} \geq g_{iq} \tag{5h}$$
$$0 \leq g_{iq} \tag{5i}$$
$$\sum_{q=1}^{Q} q w_{iq} = v_i \tag{5j}$$
$$\sum_{q=1}^{Q} w_{iq} = 1 \tag{5k}$$
$$w_{iq} \in \{0, 1\} \tag{5l}$$

Constraints (5a)-(5e) provide initialization and terminal conditions as well as intermediate constraints to provide continuity in vehicle charges. Constraint (5a) states the first arrival for each bus is initialized with a charge of $\alpha \kappa_{\Gamma_i}$. Constraint (5b) defines that the previous charge η plus the charging done by charger q minus the discharge amount due to completing route λ. Constraints (5c) is similar to (5b), except it states that the charge after being charged by q and the discharge from route λ must be greater than some percentage of its capacity κ. Constraint (5d) states that the charging done for visit i cannot be greater than the capacity of the battery κ. Constraint (5e) states that the last visit for each vehicle must have a minimum charge of $\beta \kappa_{\Gamma_i}$.

The set of constraints (5f)-(5i) are used to alleviate non-linearity introduced by the bilinear term $g_{iq} := w_{iq} p_i$. This is accomplished using Big-M notation [22]. Constraints (5f) and (5g) state that if $w_{iq} = 1$ then the two equations will take the form of $p_i \leq g_{eq}$ and $p_i \geq g_{iq}$ effectively stating $p_i = g_{iq}$, if $w_{iq} = 0$ then the two equations will take the form of $p_i \leq g_{eq}$ and $p_i \geq g_{iq} - M$ which deactivates the constraint. In a similar fashion, (5h) and (5i) state if $w_{iq} = 0$ then the constraints take the form $0 \geq g_{iq}$ and $0 \leq g_{iq}$ directly implying $0 = g_{iq}$. If $w_{iq} = 1$ then the constraints take the form $M \leq g_{iq}$ and $0 \geq g_{iq}$ disabling the constraint. Putting the two sets of constraints together defines the linearized representation of $w_{iq} p_i$.

The set of constraints (5j)-(5k) define the linking constraint between the queuing constraint v and w (i.e $1 * w_{iq} + 2 * w_{iq} + \cdots = v_i$) and enforces that only a single w may be selected per visit, respectively. The last constraint (5l) defines the set of values that w may be.

IV. Example

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