A Position Allocation Approach to the Scheduling of Battery Electric Bus Charging

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ABSTRACT

Robust charging schedules for an increasing interest of battery electric bus (BEB) fleets is a critical component to a successful adoption. In this paper, a BEB charging scheduling framework that considers spatiotemporal schedule constraints, route schedules, fast and slow charging, and battery charging dynamics is modeled as a mixed integer linear program (MILP). The MILP is modeled after the berth allocation problem (BAP) in a modified form known as the position allocation problem (PAP). Linear battery dynamics are included to model the charging and discharging of buses while at the station and during their routes, respectively. The optimization coordinates BEB charging to ensure each BEB has sufficient charge while using slow chargers where possible for sake of battery health. The model also minimizes the total number of chargers utilized and prioritizes slow chargers. The model validity is demonstrated with a set of routes sampled from Utah Transit Authority (UTA) for 35 buses and 338 visits to the charging station. The model is also compared to a heuristic based algorithm, referred to as the Quin-Modified method. The results presented show that the slow chargers are more readily selected and the charging and spatiotemporal constraints are met while considering the battery dynamics, minimizing the charger count, and consumption cost.

Keywords: Berth Allocation Problem (BAP), Position Allocation Problem (PAP), Mixed Integer Linear Program (MILP), Battery Electric Bus (BEB), Scheduling at least 5 are mandatory.

1 INTRODUCTION

The public transportation system is crucial in any urban area; however, the increased awareness and concern of environmental impacts of petroleum based public transportation has driven an effort to reduce the pollutant footprint [4, 23, 6, 11]. Particularly, the electrification of public bus transportation via battery power, i.e., battery electric buses (BEBs), has received significant attention [11]. Although the technology provides benefits beyond reduction in emissions, such as lower driving costs, lower maintenance costs, and reduced vehicle noise, battery powered systems introduce new challenges such as larger upfront costs, and potentially several hours long "refueling" periods [23, 11]. Furthermore, the problem is exacerbated by the constraints of the transit schedule to which the fleet must adhere, the limited amount of chargers available, and the adverse affects in the health of the battery due to fast charging [13]. This paper presents a continuous scheduling framework for a BEB fleet that shares limited fast and slow chargers. This framework takes into consideration linear charging dynamics and a fixed bus schedule while meeting a certain battery charge threshold throughout the day.

Many recent efforts have been made to simultaneously solve the problems of route scheduling, and charging fleets and determining the infrastructure upon which they rely, e.g., [22, 18, 9, 21]. Several simplifications are made to make these problems computationally feasible To do: citations. These simplifications to the charge scheduling model include utilizing only fast chargers while planning [22, 18, 21, 25, 12, 24, 20, 16]. If slow chargers are used, they are only employed at the depot and not the station [8, 19]. Some approaches also simplify by assuming a full charge is always achieved [22, 21, 25, 20]. Others have assumed that the charge received is proportional to the time spent on the charger [12, 24], which can be a valid assumption when the battery state-of-charge (SOC) is below 80% charge [12].

This work builds upon the Position Allocation Problem [15], a modification of the well studied Berth Allocation Problem (BAP), as a means to schedule the charging of electric vehicles [1, 5, 10]. The BAP is a continuous time model that solves the problem of allocating space for incoming vessels to be berthed. Each arriving vessel requires both time and space to be serviced and is assigned a berthing location [10]. Vessels are lined up parallel to the berth to be serviced and are horizontally queued as shown in Fig 1. The PAP utilizes this notion of queuing for scheduling vehicles to be charged, as shown in Fig 2. The PAP is formulated as a rectangle packing problem by assuming that vehicle charging will take a fixed amount of time, the amount of vehicles that can charge is limited by the physical width of the vehicles, and each vehicle visits the charger a single time [15].

The main contribution of this work is the extension of the PAP novel approach to BEB charger scheduling. This includes modeling and incorporation of a proportional charging model into the MILP framework, consideration of multiple charger types, and inclusion of the route schedule for each bus. The last contribution is of importance because both the BAP and PAP consider each arrival to be unique; thus, a method of tracking buses must be implemented. The result is a MILP formulation that coordinates charging times and charger type for every visit that each bus makes to the station while considering a dynamic charge model and scheduling constraints.

To do: Double check that this is still accurate The remainder of the paper proceeds as follows: In Section 2, the PAP is introduced with a formulation of the resulting MILP. Section 3 constructs the MILP for BEB scheduling, including modifications to the PAP queuing constraints and development of a dynamic charging model. Section 4 demonstrates an example of using the formulation to coordinate 35 buses over 338 total visits to the station. The paper ends in Section 5 with concluding remarks.

2 THE POSITION ALLOCATION PROBLEM

This section provides a brief overview of the BAP and a detailed formulation of PAP as presented in [15].

2.1 Overview of BAP

The BAP is a rectangle packing problem where a set of rectangles, $\mathbb{O}$, are attempted to be optimally placed in a larger rectangle, O, as shown in Fig 3. The rectangle packing problem is an NP-hard problem that can be used to describe many real life problems [3, 14]. In some of these problems, the dimensions of $\mathbb{O}$ are held constant such as in the problem of packing modules on a chip, where the widths and height of the rectangles represent the physical width and heights of the modules [14]. Other problems, such as the BAP can allow one side of the rectangle to vary depending on its assigned position (e.g. the handling time is dependent on the berth) [1].

The BAP solves the problem of optimally assigning incoming vessels to berth positions to be serviced (Fig 1). The width and height of O represent the berth length S and time horizon T, respectively. Similarly, the width and height for $\mathbb{O}$ represent the time spent to service vessel i and the space taken by docking vessel i, respectively. The vessel characteristics (length of the vessel, arrival time, handling time, desired departure time) are assumed to be known for all N vessels to be serviced. A representation of a BAP solution is shown in Fig 4.

2.2 The PAP Formulation

The BAP forms the basis of the PAP; however, there are some differences in the way the variables are perceived. For the i^{th} visit, starting service time, u_i, is now the starting charge time, the berth location, v_i, is now the charger queue for assignment, and the service time, p_i, is now the time to charge. There are also a few key concepts about how the system is modeled. The PAP models the set of chargers as one continuous line; that is, the natural behavior of the PAP model is to allow vehicles to be queued anywhere along $[0, S]$. Similarly, the charge times are continuous and can be placed anywhere on the time horizon, $[0, T]$, as long as the allocated times do not interfere with other scheduled charge times. The PAP utilizes a number of parameters. The following parameters are constants.

- S : Size of the vehicle
- T : time horizon

- n_V : total number of incoming vehicles
- p_i : charging time for vehicle i; $1 \le i \le n_N$
- s_i : width of vehicle i; $1 \le i \le n_N$
- a_i : arrival time of vehicle i; $1 \le i \le n_N$

These constants define the problem bounds. The following list provides a series of decision variables used in the formulation.

- u_i : starting charge time for vehicle i; $1 \le i \le n_N$
- v_i : assigned charge queue for vehicle i; $1 \le i \le n_N$
- c_i : departure time for vehicle i; $1 \le i \le n_N$
- σ_{ij} : binary variable that determines ordering of vehicles i and j in time
- δ_{ij} : binary variable that determines relative position of vehicles i and j when charging simultaneously

To determine the values for each of these decision variables, a MILP is formulated in [15] and shown here for completeness.

$$\min \sum_{i=1}^{N} (c_i - a_i) \tag{1}$$

Subject to:

$$u_j - u_i - p_i - (\sigma_{ij} - 1)T \ge 0 \tag{2a}$$
$$v_j - v_i - s_i - (\delta_{ij} - 1)S \ge 0 \tag{2b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \ge 1 \tag{2c}$$
$$\sigma_{ij} + \sigma_{ji} \le 1 \tag{2d}$$
$$\delta_{ij} + \delta_{ji} \le 1 \tag{2e}$$
$$p_i + u_i = c_i \tag{2f}$$
$$a_i \le u_i \le (T - p_i) \tag{2g}$$
$$\sigma_{ij} \in \{0, 1\},\ \delta_{ij} \in \{0, 1\} \tag{2h}$$
$$v_i \in [0\ S] \tag{2i}$$

The objective function (1) minimizes the time spent to service each vehicle by minimizing over the sum of differences between the departure time, c_i, and arrival time, a_i. i.e., It seeks to get each vehicle charged and on its way as quickly as possible.

Constraints 2a-2e are used to ensure that individual rectangles do not overlap. For the PAP, they ensure that two vehicles charging simultaneously are at different positions and, similarly, two vehicles that have overlapping positions do not overlap temporally. Constraint (2a) establishes temporal ordering when active ($\sigma_{ij} = 1$). Similarly, when $\delta_{ij} = 1$ in (2b) then spatial ordering is established. Constraints 2c-2e enforce that spatial and/or temporal ordering is established between each possible queue/vehicle pair. Constraints (2d) and (2e) enforce consistency. For example, (2d) enforces that vehicle i cannot come before vehicle j and vehicle j simultaneously come before vehicle i.

The last constraints force relationships between arrival time, charge start time, and departure time. Constraint (2f) states that the service start time, u_i, plus the time to service vehicle i, p_i, must equal the departure time, c_i. Constraint (2g) enforces the arrival time, a_i, to be less than or equal to the service start time, u_i, which in turn must be less than or equal to the latest time the vehicle may begin to be serviced to stay within the time horizon. Constraint (2h) ensures that σ_{ij} and δ_{ij} are binary. Constraint (2i) ensures that the assigned value of v_i is a valid charging position.

3 A RECTANGLE PACKING FORMULATION FOR BEB CHARGING

Applying the PAP to BEB charging requires four fundamental changes. The first is that the time that a BEB spends charging is allowed to vary. Thus, p_i becomes a variable of optimization. Second, in the PAP each charging visit is assumed to be a different vehicle. For the BEB charging problem, each bus may make multiple visits to the station throughout the day and the resulting charge for a bus at a given time is dependent upon each of the prior visits made. Third, in the PAP, the charger is one continuous bar with vehicle width effectively restricting the number of vehicles charging simultaneously. For the BEB, it is assumed that a discrete number of chargers exist. Moreover, it is assumed that these chargers can charge the vehicles at a different rates. The fourth fundamental change is related to the first three. The charge of each bus must be tracked in the optimization to ensure that charging across multiple visits is sufficient to allow each bus to execute its route throughout the day.

To do: make sure this is still accurate The discussion of the four changes are separated into two sections. Section 3.1 discusses the changes in the spatial-temporal constraint formulation to form a queuing constraint. Section 3.2 then discusses the addition of the bus charge management. This section ends with a brief discussion of a modified objective function and the statement of the full problem in Section 3.3. The notation is explained throughout and summarized in Table 1.

3.1 Queuing Constraints

The queuing constraints ensure that the busses entering the queues for charging are assigned in a feasible manner as they come into the station. There are three sets to differentiate between different entities. $\mathbb{B} = \{1, ..., n_B\}$ is the set of bus indices with index b used to denote an individual bus, $\mathbb{Q} = \{1, ..., n_Q\}$ is the set of queues with index q used to denote an individual queue, and $\mathbb{V} = \{1, ..., n_V\}$ is a set of visits to the station with i, j used to refer to individual visits. The mapping $\Gamma : \mathbb{V} \to \mathbb{B}$ is used to map a visit index to a bus index with the shorthand Γ_i used to refer to the bus index for visit i.

Most variables are now defined in terms of a visit. Two separate visits could correspond to different buses or visits by the same bus. The PAP spatial variable, s_i, is removed and v_i is made to be an integer corresponding to which queue visit i will be using. Thus, when $\delta_{ij} = 1$, vehicle i is queued to a charger that has a larger index than the charger that vehicle j is queued, i.e., $v_i - v_j \geq 1$. The variable S is likewise replaced with n_Q. Note that $n_Q = n_B + n_C$, where n_B is the number of busses and n_C is the number of chargers. The rationale for having more queues than chargers is to allow buses to sit idle instead of requiring the bus to charge at each visit. The modified queuing constraints can be written as follows.

$$v_i - v_j - (\delta_{ij} - 1)n_Q \geq 1 \tag{3a}$$
$$c_i \leq \tau_i \tag{3b}$$
$$p_i \geq 0 \tag{3c}$$
$$v_i \in \mathbb{Q} \tag{3d}$$

To do: (3c) can probabably be removed

Constraint (3a) is nearly identical to (2b), but rather than viewing the charger as a continuous strip of length S, it is discretized into n_Q queues a width of unit length one. A BEB is also assigned a unit length of one which is reflected in (3a) by $\cdot \geq 1$. Constraint (3b) ensures that the time the BEB is detatched from the charger is before its departure time. Constraint (3d) defines the integer set of indices for queues for v_i.

3.2 Battery Charge Dynamic Constraints

Battery dynamic constraints are now introduced to relate busses to visits and guarantee that buses have sufficient time to charge. Two constraints are enforced on the bus charge: busses must always have sufficient charge to execute their respective routes and each bus must end the day with a specific charge threshold, preparatory to execution for the next day.

The charge at the beginning of visit i is denoted as η_i. As a charge on the bus is dependent upon the visits that bus makes to the station, the mapping $\Upsilon : \mathbb{V} \to \mathbb{V} \bigcup \{\varnothing\}$ is used to determine the next visit that corresponds to the same bus, with Υ_i being shorthand notation. Thus, Γ_i and Γ_{Υ_i} would both map to the same bus index as long as Υ_i is not the null element, $\varnothing$. That is, Γ_{Υ_i} where $\Upsilon_i = 0$ indicates that there are no future visits for bus i.

To drive time spent on the charger, p_i, as well as define initial, final, and intermediate bus charges for each visit i, the sets for initial and final visits must be defined. Let the mapping of the first visit by each bus be denoted as $\Gamma_i^0 : \mathbb{V} \rightarrow \mathbb{B}$. The indexed value of Γ_i^0 represents the index for the first visit of bus b or the null element, $\varnothing$. Similarly, let $\Gamma_i^f : \mathbb{V} \rightarrow \mathbb{B}$ contain the indexes for the final visit of each bus b or the null element. The initial and final bus charge percentages, α and β, can then be represented by the constraint equations $\eta_{\Gamma_i^0} = \alpha\kappa_{\Gamma_i^0}$ and $\eta_{\Gamma_i^f} = \beta\kappa_{\Gamma_i^f}$, respectively. The intermediate charges mustbe determined at solve time.

It is assumed that the charge received is proportional to the time spent charging. The charge rate for charger q is denoted as r_q. Note that a value of $r_q = 0$ corresponds to a queue where no charging occurs. A bus in such a queue is simply waiting for the departure time. The queue indices are ordered such that the final n_B queues have $r_q = 0$ to allow an arbitrary number of buses to sit idle at any given moment in time. The amount of discharge between visits i and Υ_i, the next visit of the same bus, is denoted as λ_i. If visit i occurred at charger q, the charge of the bus coming into visit Υ_i would be $\eta_{\Upsilon_i} = \eta_i + p_i r_q - \lambda_i$.

The binary decision variable w_{iq} is introduced to determine whether visit i uses charger q. This allows the charge of the bus coming into visit Υ_i to be written in summation form as

$$\eta_{\Upsilon_i} = \eta_i + \sum_{q=1}^{n_Q} p_i w_{iq} r_q - \lambda_i \tag{4a}$$

$$\sum_{q=1}^{n_Q} w_{iq} = 1 \tag{4b}$$

$$w_{iq} \in \{0, 1\} \tag{4c}$$

The choice of queue for visit i, becomes a slack variable and is defined in terms of w_{iq} as

$$v_i = \sum_{q=1}^{n_Q} q w_{iq} \tag{5}$$

Maximum and minimum values for the charges are included to ensure that the battery is not overcharged and to guarantee sufficient charge for subsequent visits. The upper and lower battery charge bounds for bus b are κ_b and ν_b, respectively. As η_i corresponds to the charge at the beginning of the visit, the upper bound constraint must also include the charge received during the visit as follows.

$$\eta_i + \sum_{q=1}^{n_Q} p_i w_{iq} r_q \leq \kappa_{\Gamma_i} \tag{6a}$$

$$\eta_i \geq \nu_{\Gamma_i} \kappa_{\Gamma_i} \tag{6b}$$

Note that the term $p_i w_{iq}$ is a bilinear term. A standard way of linearizing a bilinear term that contains an integer variable is by introducing a slack variable with an either/or constraint [2, 17]. Allowing the slack variable g_{iq} to be equal to $p_i w_{iq}$, g_{iq} can be defined as

$$g_{iq} = \begin{cases} p_i & w_{iq} = 1 \\ 0 & w_{iq} = 0 \end{cases}. \tag{7}$$

Equation (7) can be expressed as a mixed integer constraint using big-M notation with the following four constraints.

$$\begin{aligned}
p_i - (1 - w_{iq})M &\leq g_{iq} && \text{(8a)} \\
p_i &\geq g_{iq} && \text{(8b)} \\
Mw_{iq} &\geq g_{iq} && \text{(8c)} \\
0 &\leq g_{iq} && \text{(8d)}
\end{aligned}$$

where M is a large value. If $w_{iq} = 1$ then (8a) and (8b) become $p_i \leq g_{iq}$ and $p_i \geq g_{iq}$, forcing $p_i = g_{iq}$ with (8c) being inactive. If $w_{iq} = 0$, (8a) is inactive and (8c) and (8d) force $g_{iq} = 0$.

3.3 The BEB Charging Problem

The goal of the MILP is to utilize chargers as little as possible to reduce energy costs with the fast charging being penalized more to reduce battery damage. Thus, an assignment cost m_q and usage cost ϵ_q are associated with each charger, q. These weights can be adjusted based on charger type or time of day that the visit occurs. The assignment term takes the form $w_{iq}m_q$, and the usage term takes the form $g_{iq}\epsilon_q$. The resulting BEB charging problem is defined in Eq 9.

$$\min \sum_{i=1}^{N} \sum_{q=1}^{n_Q} \Big(w_{iq}m_q + g_{iq}\epsilon_q \Big) \tag{9}$$

Subject to the constraints

$$\begin{aligned}
u_i - u_j - p_j - (\sigma_{ij} - 1)T &\geq 0 && \text{(10a)} \\
v_i - v_j - (\delta_{ij} - 1)n_Q &\geq 1 && \text{(10b)} \\
\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} &\geq 1 && \text{(10c)} \\
\sigma_{ij} + \sigma_{ji} &\leq 1 && \text{(10d)} \\
\delta_{ij} + \delta_{ji} &\leq 1 && \text{(10e)} \\
p_i + u_i &= c_i && \text{(10f)} \\
a_i \leq u_i &\leq (T - p_i) && \text{(10g)} \\
c_i &\leq \tau_i && \text{(10h)} \\
\eta_{\Gamma_i^0} &= \alpha\kappa_{\Gamma_i^0} && \text{(10i)} \\
\eta_i + \sum_{q=1}^{n_Q} g_{iq}r_q - \lambda_i &= \eta_{\gamma_i} && \text{(10j)} \\
\eta_i + \sum_{q=1}^{n_Q} g_{iq}r_q - \lambda_i &\geq \nu\kappa_{\Gamma_i} && \text{(10k)} \\
\eta_i + \sum_{q=1}^{n_Q} g_{iq}r_q &\leq \kappa_{\Gamma_i} && \text{(10l)} \\
\eta_{\Gamma_i^f} &\geq \beta\kappa_{\Gamma_i^f} && \text{(10m)} \\
p_i - (1 - w_{iq})M &\leq g_{iq} && \text{(10n)} \\
p_i &\geq g_{iq} && \text{(10o)} \\
Mw_{iq} &\geq g_{iq} && \text{(10p)} \\
0 &\leq g_{iq} && \text{(10q)} \\
v_i &= \sum_{q=1}^{n_Q} q w_{iq} && \text{(10r)} \\
\sum_{q=1}^{n_Q} w_{iq} &= 1 && \text{(10s)} \\
w_{iq} &\in \{0, 1\} && \text{(10t)} \\
\sigma_{ij} \in \{0, 1\},\ \delta_{ij} &\in \{0, 1\} && \text{(10u)} \\
v_i &\in \mathbb{Q} && \text{(10v)} \\
i &\in \mathbb{V} && \text{(10w)} \\
q &\in \mathbb{Q} && \text{(10x)}
\end{aligned}$$

Constraints (10a)-(10h) are reiterations of the queuing constraints in (3). Constraints (10i)-(10m) provide the battery charge constraints. Constraints (10n) through (10q) define the charge gain of every visit/queue pairing. The last constraints (10t)-(10x) define the sets of valid values for each variable.

4 EXAMPLE

An example will now be presented to demonstrate the utility of the developed MILP charge scheduling techique. A description of the scenario is first presented followed a description of an alternative heuristic based planning strategy called Quin-Modified which is used as a comparison to the MILP PAP. Results are then presented for each of planning strategies are presented, analyzed, and discussed.

4.1 BEB Scenario

To display the capabilities of the model, an example scenario is presented. The scenario was ran over a time horizon of $T = 24$ hours, utilizes $A = 35$ buses with $N = 338$ visits to the station divided between the A buses. Each bus has a 388 KWh battery that is required to stay above 25% charge (3589 KWh) to maintain battery health, and the bus is assumed to begin the working day with 90% charge (349.2 KWh). Additionally, each bus is required to end the day with a minimum charge of 70% (271.6 KWh). Each bus is assumed to discharge at a rate of 30 KWh. Note that there are many factors that play a factor in the rate of discharge; however, for the sake of simplicity an average rate is used. $n_C = 30$ chargers are utilized where 15 of the chargers are slow charging (30 KWh) and 15 are fast charging (911 KWh).

To encourage the MILP PAP problem to utilize the fewest number of chargers, the value of m_q in the objective function, (9), is $\forall q \in Q; m_q = 1000q$. The charge duration scalar, ϵ_q, is defined as $\epsilon_q = r_q$ to create a consumption cost term, $g_{iq}\epsilon_q$. This is utilized to also encourage the model to minimize active charger times, especially for the fast chargers.

Another heuristic-based optimization strategy, referred to as Quin-Modified, is also employed as a means of comparison with the results of the MILP PAP. The Quin-Modified strategies is a based on the threshold strategy of [16]. The strategy has been modified slightly to accommodate the case of multiple charger types and without exhaustive search for the best charger type. The heuristic is based on a set of rules that revolve around the initial charge of the bus at visit i. There are three different thresholds, low (60%), medium (75%), and high (90%). Buses below the low threshold are prioritized to fast chargers then are allowed to utilize slow chargers if no fast chargers are available. Buses between the low and medium threshold prioritize slow chargers first and utilize fast chargers only if no slow chargers are available. Buses above the medium threshold and below high will only be assigned to slow chargers. Buses above the high threshold will not be charged. Once a bus has been assigned to a charger, it remains on the charger for the duration of the time it is at the station, or it reaches 90% charge, whichever comes first.

The schedule is sampled from the Utah Transit Authority (UTA) To do: is it? bus routing data that occurs over a 24-hour time period. The total number of constraints resulted in 11830 continuous and 238628 integer/binary constraints. The optimization was performed using the Gurobi MILP solver [7] on a machine running an AMD Ryzen 9 5900X 12 - Processor (24 core) at 4.95GHz. The solver was allowed to run for 14400 seconds and did not converge to the optimal result with a gap 52.5% .

4.2 Results

The schedule generated by the Quin-Modified strategy and the MILP PAP is shown in Fig 6a and Fig 6b, respectively. The x-axis represents the time in hours. The y-axis represents the assigned charger. Points between zero and 14 are active times for slow chargers, and points between the range of 14 and 29 are active times for fast chargers. The filled symbol represents the starting charge time for a bus b with the line to the vertical tick signifying the region of time the charger is active. The line connecting points represent the charge sequence for a bus. Each color in Fig 6a and 6b are used to identify the bus assigned.

The first observation is in the choice of preferred chargers between the Quin-Modified and MILP scheduler. The Quin-Modified schedule uses at most 4 fast chargers and 3 slow whereas the MILP schedule uses at most 2 fast chargers and 8 slow, the eighth charger actually being the ninth queue, but that is due to the non-optimal results. Both the Quin-Modifed and MILP schedule used the fast chargers in short bursts (0.2-0.5 hours). The main difference lies in the utilization strategy of the slow chargers. The Quin-Modified, for the most part, opted for shorter bursts for the slow chargers (0.3-0.7 hour), most heavily placed on the first slow charger. The MILP also used these shorter charge times on the first slow charger; however, the schedule was able to recognize the bus routes that had longer durations at the station and could choose the lower cost option, slow charging, when available and of lower cost. Although one the MILP's objectives is to minimize the amount of chargers used, the Quin-Modified ended up using 7 chargers while the MILP used 8. The reason for this is the construction of the objective function that is attempting to optimize over the consumption cost, total number of chargers, and charger type. Hence, the objective function found it more efficient to utilize an extra slow charger for a longer duration than to add another fast charger for a short duration. Although both schedules generated are valid, no comparison of the quality the schedule can be made.

Figs 7a and 7b depicts the charge for every bus over the time horizon. Every vehicle begins at 90% charge, finishes at 70% charge in the MILP PAP schedule, and never goes below 25% in the intermediate arrivals as stated in the constraints (10). There is no guarantee for this in the Quin-Modified strategy which can be seen by some of the bus charges reaching negative values and the distribution of final charges. The only sense of guarantee that the Quin-Modified supplies is its predictability within the intermediate visits because of the heuristic nature (i.e. if the charge the low threshold, a fast charger will be prioritized) whereas MILP places a bus in the queue that "makes sense" in respect to the larger picture. The MILP PAP does not have an obvious sense of decision-making due to its weighted decisions that are affected by the accumulation of decisions made prior.

Another important measure for the chargers is to compare the amount of power and energy consumed. Fig 8 depicts the power consumption throughout the time horizon. It can easily be seen that the Quin-Modifed power consumption is steadily less than the MILP schedule. This can be accounted for by the MILP's constraints to keep the bus charges above 25% and to reach the 70% charge at the end of the working day. It is also important to note that largest peak for the Quin-Modified schedule verses the lack of any real peak for the MILP PAP schedule. Although the MILP PAP had firmer constraints than the Quin-Modified algorithm, it maintained a steady power consumption profile throughout the time horizon. Along a similar vein, the accumulated energy consumed is shown in Fig 9. The MILP schedule is more efficient up until about hour 11. Again, this can be accounted for by the fact the MILP is accommodating the extra constraints. Even with these constraints, MILP PAP consumes about $1 \cdot 10^5$ Kwh more than the Quin-Modified. The overlap of the MILP PAP can be accounted for by Fig 10a and 10b. At the 11^{th} hour, a spike in slow chargers can be seen in an attempt to keep the bus charges above the 25% charge and preparing to meet the final 70% constraint.

5 CONCLUSION

This work developed a MILP scheduling framework that optimally assigns slow and fast chargers to a BEB bus fleet assuming a constant schedule. The BAP was introduced with an example formulation and was then compared to the PAP. The PAP constructed on the BAP to allow the time spent on the charger, p_i, to be a decision variable. Because the original PAP required service time, p_i, to be given, linear battery dynamics were introduced to drive charging times. Additional constraints were also introduced to provide limits for the battery dynamics.

An example for the MILP PAP formulation was then presented and compared to a heuristic based schedule, referred to as Quin-Modified. The MILP PAP optimization was run for 14400 seconds to a non-optimal solution with a gap of 52.5% . The Quin-Modified schedule more heavily utilized the fast chargers and used slow chargers more as a supplement. In sharp contrast, the MILP PAP schedule heavily leaned on the slow chargers and supplemented fast chargers as needed. This can be quantified by the four fast and 3 slow chargers utilized by the Quin-Modified schedule whereas the MILP PAP schedule utilized 8 slow chargers and 2 fast chargers. More importantly, the MILP PAP schedule utilized approximately $1 \cdot 10^5$ Kwh more than the Quin-Modified, but the charges remained above the constrained minimum charge of 25% , and charged all the buses to 70% at the end of the working day. The Quin-Modified schedule, on the other hand, failed to keep all the bus charges above 0% throughout the time horizon.

Further fields of interest are to utilize the formulation (Eq (9) and (10)) with nonlinear battery dynamics, calculation and utilization of the demand and consumption cost in the objective function, and utilizing this formulation in a metaheuristic solver. Furthermore, “fuzzifying” the initial and final charge times is of interest to allow flexibility in the arrival and departure times.

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FIGURE CAPTIONS

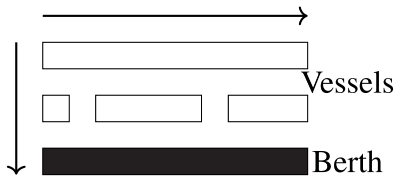

Figure 1. Example of berth allocation. Vessels are docked in berth locations (horizontal) and are queued over time (vertical). The vertical arrow represents the movement direction of queued vessels and the horizontal arrow represents the direction of departure.

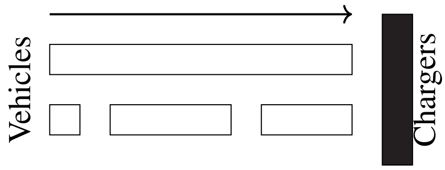

Figure 2. Example of position allocation. Vehicles are placed in queues to be charged and move in the direction indicated by the arrow.

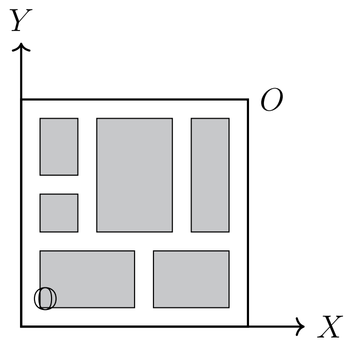

Figure 3. Example of rectangle packing problem

Table 1. Notation used throughout the paper

Variable	Description
Input values	
n_B	Number of buses
M	An arbitrary very large upper bound value
n_V	Number of total visits
n_Q	Number of queues
n_C	Number of chargers
$\mathbb{V}$	Set of visit indices, $\mathbb{V} = \{1, ..., n_V\}$
$\mathbb{B}$	Set of bus indices, $\mathbb{B} = \{1, ..., n_B\}$
$\mathbb{Q}$	Set of queue indices, $\mathbb{Q} = \{1, ..., n_Q\}$
i, j	Indices used to refer to visits
b	Index used to refer to a bus
q	Index used to refer to a queue
Problem definition parameters	
Γ	$\Gamma : \mathbb{V} \to \mathbb{B}$ with Γ_i used to denote the bus for visit i
α_i	Initial charge percentage time for visit i
β_i	Final charge percentage for bus i at the end of the time horizon
ϵ_q	Cost of using charger q per unit time
Υ	$\Upsilon : \mathbb{V} \to \mathbb{V}$ mapping a visit to the next visit by the same bus with Υ_i being the shorthand.
κ_b	Battery capacity for bus b
λ_i	Discharge of visit over route i
ν_b	Minimum charge allowed for bus b
τ_i	Time visit i must depart the station
ζ_b	Discharge rate for bus b
a_i	Arrival time of visit i
i_0	Indices associated with the initial arrival for every bus in A
i_f	Indices associated with the final arrival for every bus in A
m_q	Cost of a visit being assigned to charger q
r_q	Charge rate of charger q per unit time
Decision Variables	
δ_{ij}	Binary variable determining temporal ordering of vehicles i and j
η_i	Initial charge for visit i
σ_{ij}	Binary variable determining the queue ordering between vehicles i and j
c_i	Ending charge time for visit i
g_{iq}	The charge gain for visit i from charger q
p_i	Amount of time spent on charger for visit i
u_i	Starting charge time of visit i
v_i	Assigned queue for visit i
w_{iq}	Binary assignment variable for visit i to queue q

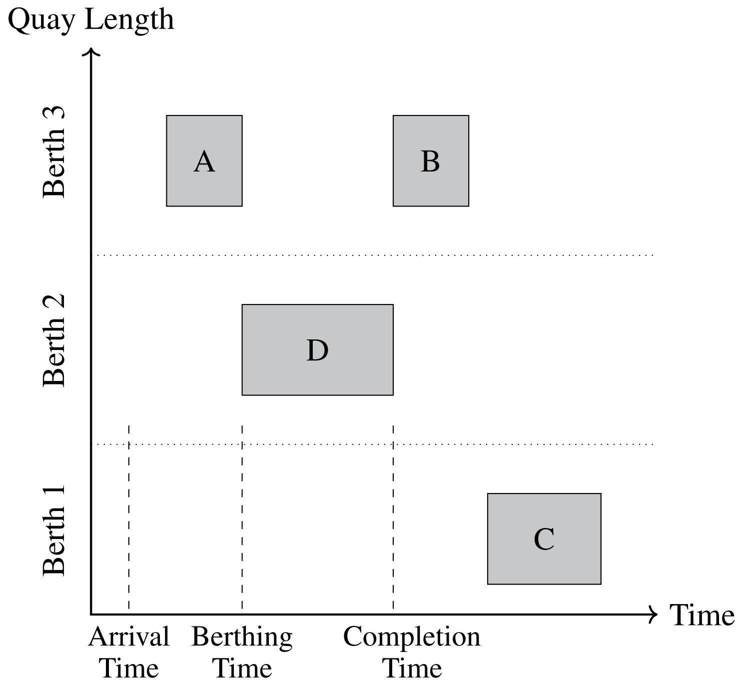

Figure 4. The representation of the berth-time space

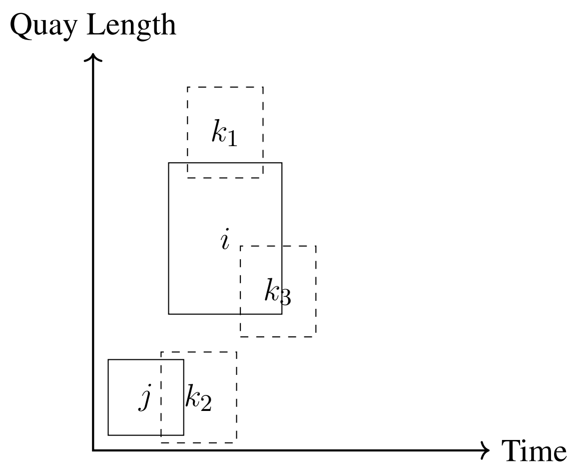

Figure 5. Examples of different methods of overlapping. Space overlap: $v_{k_1} < v_i + s_i \therefore \delta_{k_1 i} = 0$. Time overlap $u_{k_1} < u_j + p_j \therefore \sigma_{k_2 j} = 0$. Both space and time overlap $\sigma_{k_3 i} = 0$ and $\delta_{k_3 j} = 0$.

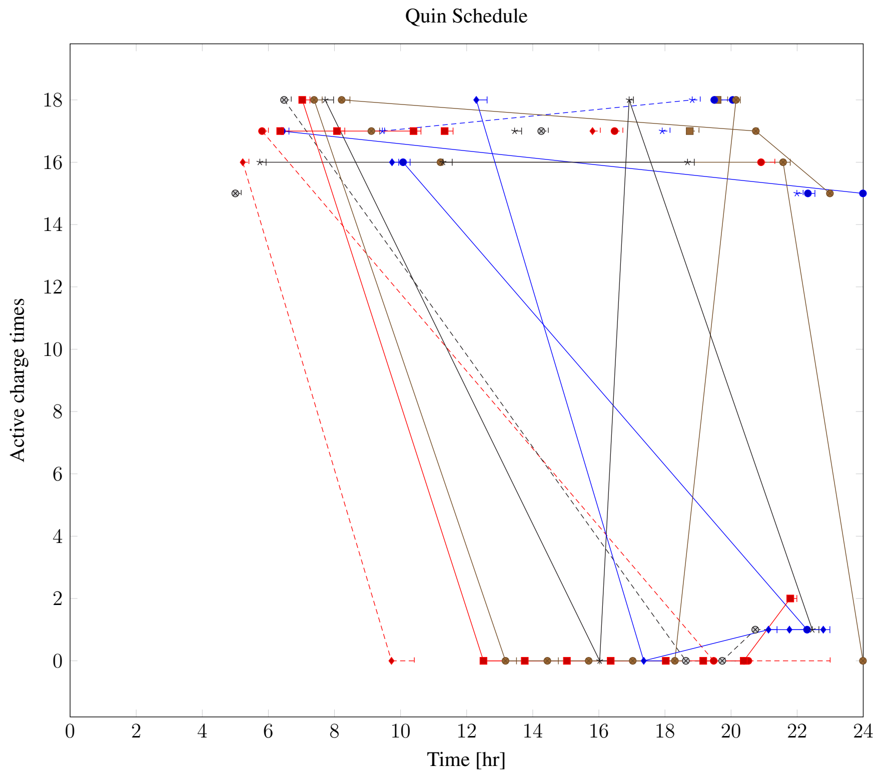

Figure 6a. Charging schedule generated by Quin Modified algorithm.

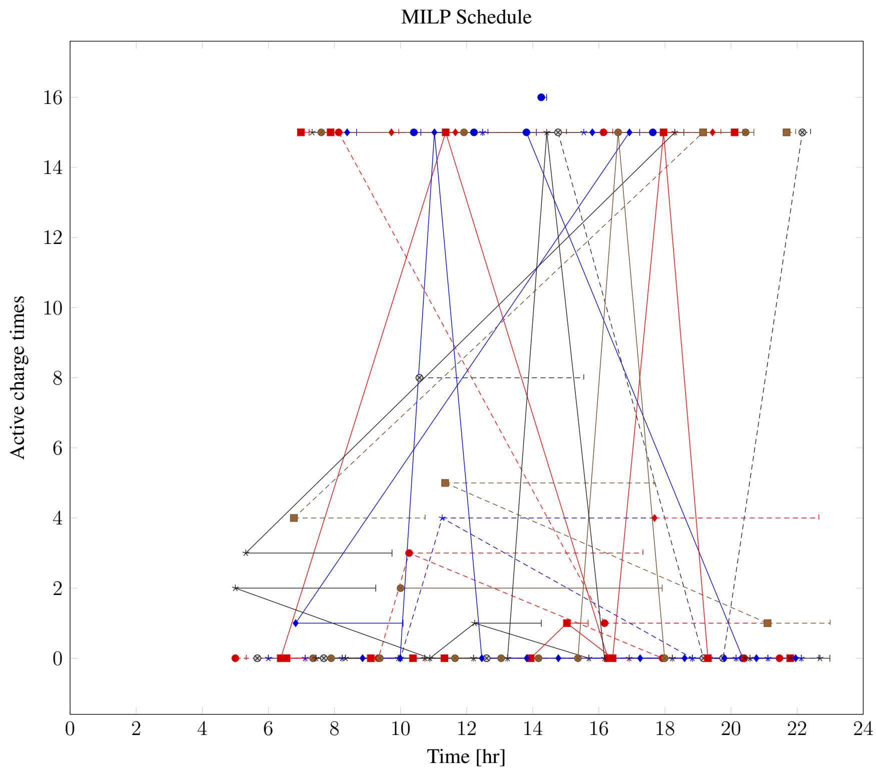

Figure 6b. Charging schedule generated by MILP PAP algorithm.

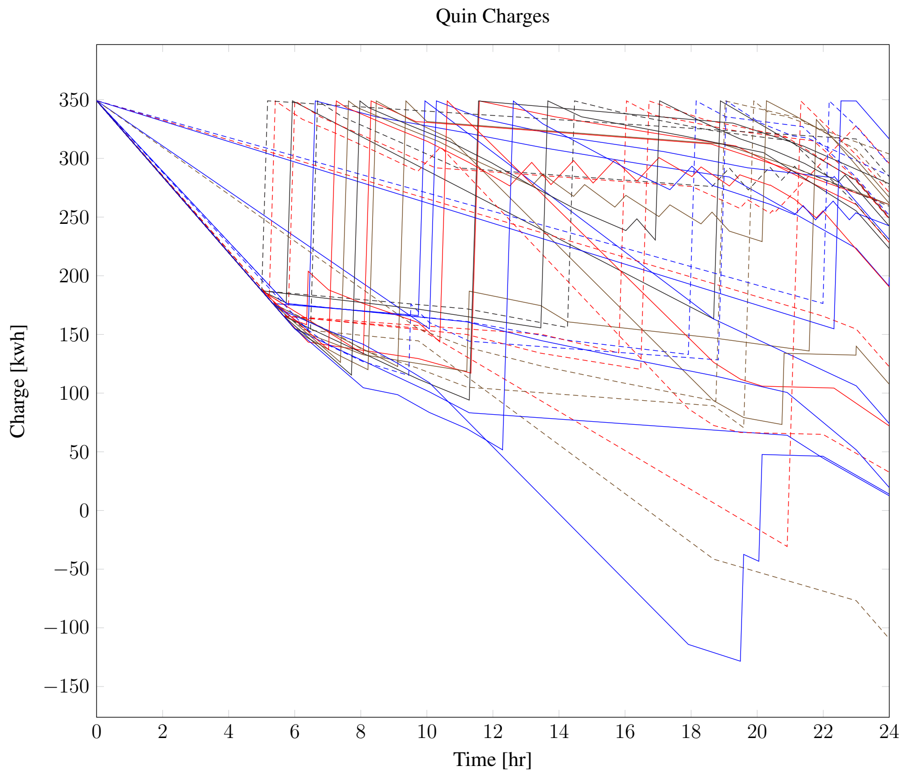

Figure 7a. Bus charges for the Quin Modified charging schedule. The charging scheme of the Quin charger is more predictable during the working day.

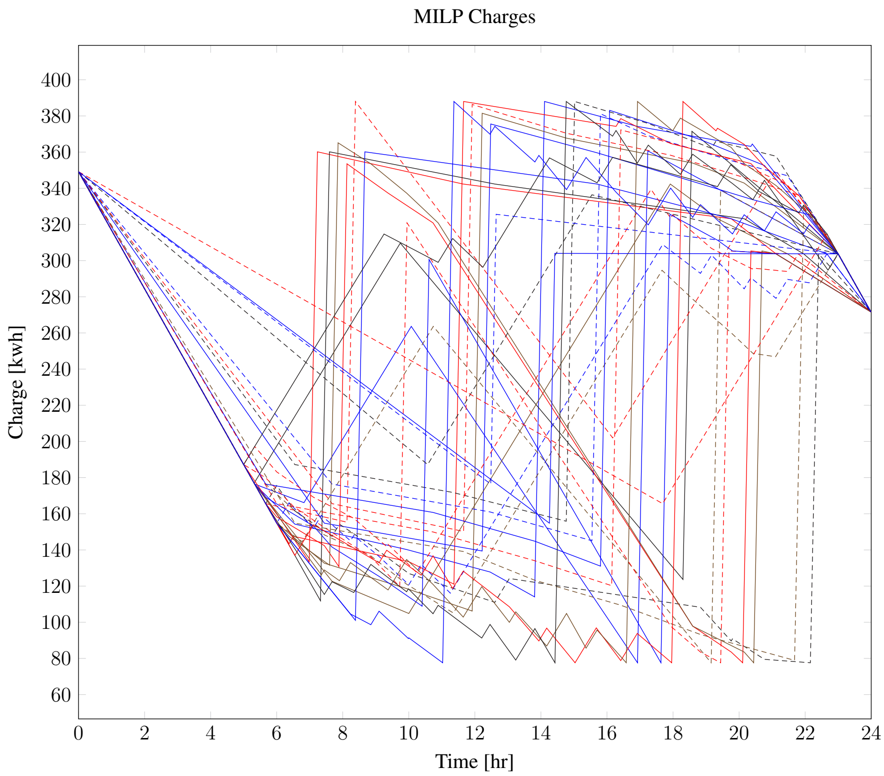

Figure 7b. The bus charges for the MILP PAP charging schedule. The MILP model allows for guarantees of minimum/maximum changes during the working day as well as charges at the end of the day.

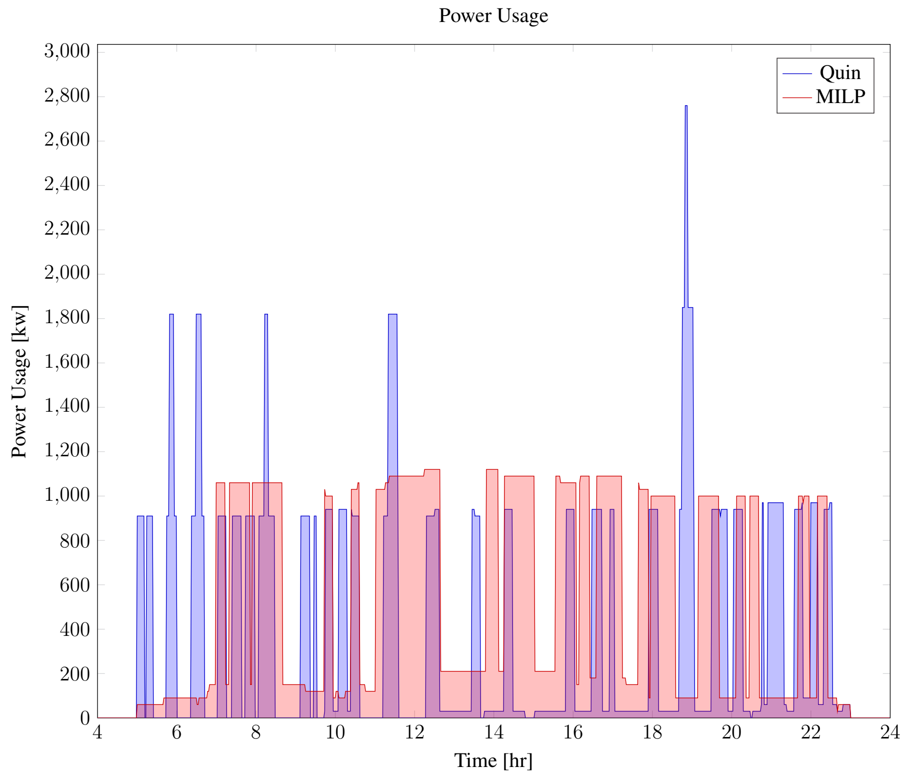

Figure 8. Total accumulated energy consumed by the Quin-Modified and MILP schedule throughout the time horizon.

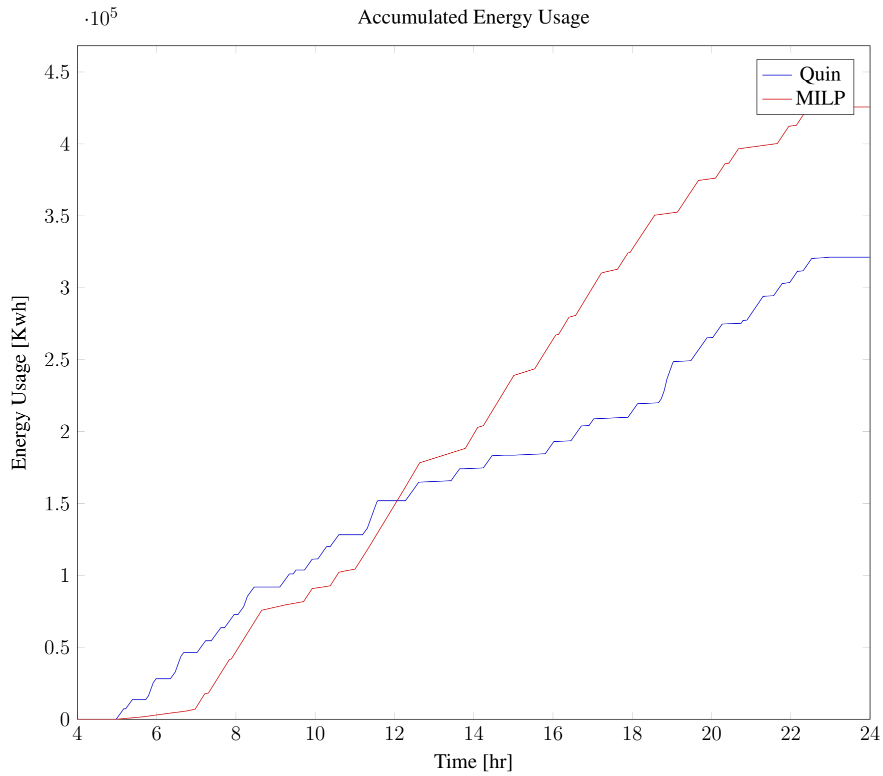

Figure 9. Amount of power consumed by Quin-Modified and MILP schedule over the time horizon.

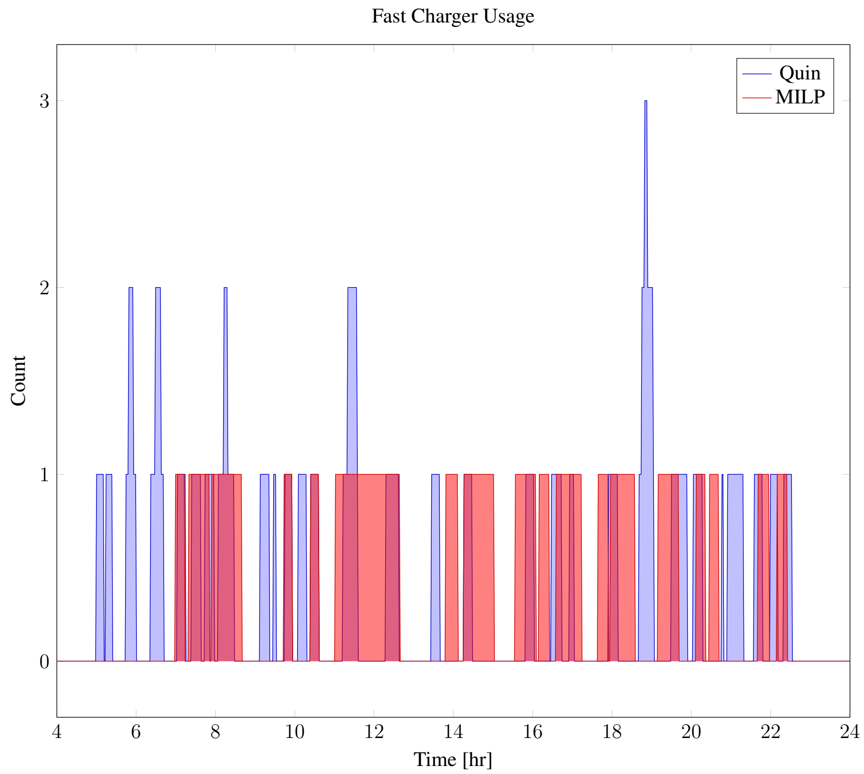

Figure 10a. Number of fast chargers for Quin and MILP PAP.

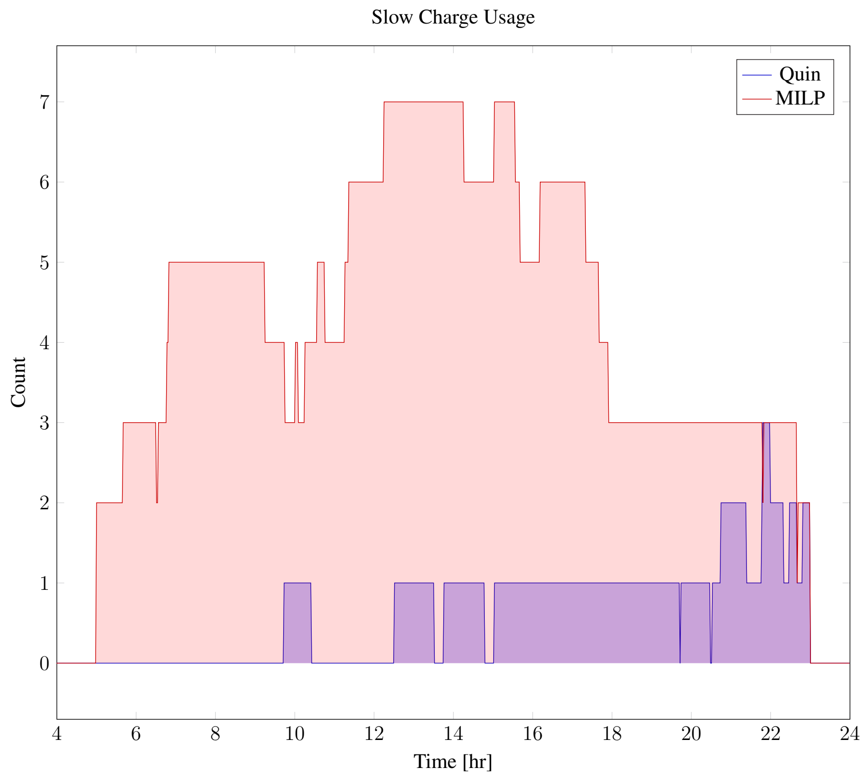

Figure 10b. Number of slow chargers for Quin and MILP PAP.