A Position Allocation Approach to the Scheduling of Battery Electric Bus Charging

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***Abstract*—Dependable charging schedules for an increasing interest of battery electric bus (BEB) fleets is a critical component to a successful adoption. In this paper, a BEB charging scheduling framework that considers spatiotemporal schedule constraints, route schedules, fast and slow charging, and battery charging dynamics is modeled as a mixed integer linear program (MILP). The MILP is modeled after the berth allocation problem (BAP) in a modified form known as the position allocation problem (PAP). Linear battery dynamics are included to model the charging and discharging of buses while at the station and during their routes, respectively. The model optimally assigns a predefined set of routes for a BEB fleet to queues where slow chargers are preferred for battery health and meeting bus route constraints. The model validity is demonstrated with a randomly generated set of routes for 40 buses and 220 visits to the charging station. The results show that the slow chargers are more readily selected, the charging and spatiotemporal constraints are met, ~~as well as~~ while the battery dynamics ~~properly being propagated~~.**

and considering

***Index Terms*—Berth Allocation Problem (BAP), Position Allocation Problem (PAP), Mixed Integer Linear Program (MILP), Battery Electric Bus (BEB), Scheduling**

I. Introduction

The public transportation system is crucial in any urban area; however, the increased awareness and concern of environmental impacts of petroleum based public transportation has driven an effort to reduce the pollutant footprint [1]–[4]. Particularly, the electrification of public bus transportation via battery power, i.e., battery electric buses (BEBs), has received significant attention [4]. Although the technology provides benefits beyond reduction in emissions, such as lower driving costs, lower maintenance costs, and reduced vehicle noise, battery powered systems introduce new challenges such as larger upfront costs, and potentially several hours long "refueling" periods [2], [4]. Furthermore, the problem is exacerbated by the constraints of the transit schedule to which ~~that~~ the fleet must adhere ~~to~~, the limited amount of chargers available, as well as the adverse affects in the health of the battery due to fast charging [5]. This paper presents a continuous scheduling framework for a BEB fleet that shares limited fast and slow chargers. This framework takes into consideration linear charging dynamics and a fixed bus schedule while meeting a certain battery percentage threshold for intermediate and final charges.

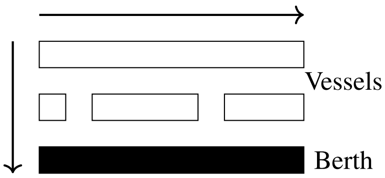
Recent research efforts have been made into solving the problem of scheduling and charging fleets as well as determining the infrastructure that they rely upon. Much ~~A~~attention has been given to solving both problems simultaneously [6]–[9]. The added complexity of considering both the BEB charge scheduling and the infrastructure problems necessitates simplifications for sake of computation. First, only fast chargers are utilized in planning [6], [7], [9]–[14]. Second, significant simplifications to the charging models are made. Some approaches assume full charge [6], [9], [10], [13]. Others have assumed that the charge received is proportional to the time spent on the charger [11], [12], which can be a valid assumption when the battery state-of-charge (SOC) is below 80% charge [11].

Furthermore, the above described research shows a need for in determining a system that outputs an optimal charging schedule. In [6], [7], [9], initial interest is demonstrated in this topic. [7] uses simulation alongside an optimization strategy to identify charging station locations and average stop times to supply enough charge for BEBs to complete their routes. [6] utilizes a network flow approach to optimize the deployment strategy of BEBs; however, the focus is primarily on replacing diesel or CNG buses. In [8], the focus is on developing a strategy to optimally charge batteries for electric vehicles considering different sources of battery degradation. [9], who also touches on the need for further research in the recharging problem, addresses a similar problem as this paper where the overall objective is to minimize the annual total electric bus recharging system operating costs. Similarly to other works, such as [6], discrete network flow approaches are utilized. Although effective, having to discretize the system introduces variables scaled proportionally to the fidelity of the discretization of the model. This work attempts to remedy this by utilizing the Position Allocation Problem (PAP) which utilizes a continuous model to describe the system.

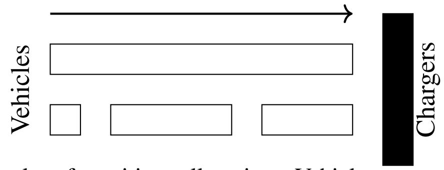
word smith

This work builds upon ~~The only recent work for scheduling and charging, from which this paper stems its basis, has used~~ the Position Allocation Problem, a modification of the Berth Allocation Problem (BAP), as a means to schedule the charging of electric vehicles [15]. T~~he PAP framework is constructed from the Berth Allocation Problem (BAP) [15]~~. The BAP solves the problem of allocating space for incoming vessels to be berthed. Each arriving vessel requires both time and space to be serviced and is assigned a berthing location [16]. Vessels are lined up parallel to the berth to be serviced and are horizontally queued as shown in Fig 1a. The PAP utilizes this

Fig. 1: Comparison between BAP and PAP



(a) Example of berth allocation. Vessels are docked in berth locations (horizontal) and are queued over time (vertical). The vertical arrow represents the movement direction of queued vessels and the horizontal arrow represents the direction of departure.



(b) Example of position allocation. Vehicles are placed in queues to be charged and move in the direction indicated by the arrow.

word-smith

notion of queuing to reuse the BAP for queuing vehicles to be charged as shown in Fig 1b. The PAP assumes that the time to charge is given and defines the full charge time. Additionally, a single, continuous charger is assumed [15].

The contribution of this work is a BEB charger scheduling framework that considers a fixed route schedules for each bus, a proportional model for battery dynamics, charge limits, and the availability of slow and fast chargers. ~~The bus schedules are assumed to be fixed for the duration of the time horizon~~. A solution MILP is formed as a modified PAP [15], where the solution of the problem provides the arrival time, selected charger (fast or slow), initial charge time, final charge time, and departure time from the station.

repetitive

This paper expands upon the PAP by allowing charge times to be dynamically chosen, considers ~~including~~ a linear battery dynamic model, allows ~~allowing~~ multiple charger types (fast, and slow), as well as the ability to link consecutive visits to specific vehicles to accommodate revisiting BEBs.

The remainder of the paper proceeds as follows: In Section II, the BAP is introduced with a MILP formulation example. Section III constructs the PAP and introduces battery dynamics into the MILP, and Section IV demonstrates an example of the PAP formulation and its ability to optimally assign vehicles while considering linear battery dynamics and meeting scheduling constraints. The paper ends in Section V with concluding remarks.

revisit

Do we want to still call it PAP?

Section{Overview of BAP}

II. The Position Allocation Problem

The BEB charge schedule formulation in this work builds upon the PAP, which, in turn, builds upon the BAP. This section provides a brief overview of the BAP problem and a detailed formulation of PAP as presented in [15]. The BAP is a rectangle packing problem where a set of rectangles ($\mathbb{O}$) are attempted to be optimally placed in a larger rectangle (O) as shown in Fig 2. ~~Both the set $\mathbb{O}$ and rectangle O's width and height are used to represent quantifiable values.~~ The rectangle packing problem is an NP-hard problem that can be used to describe many

I think this is a replacement, but I'm not certain what is meant by "quantifiable values"

The width and height of each rectangle is considered fixed.

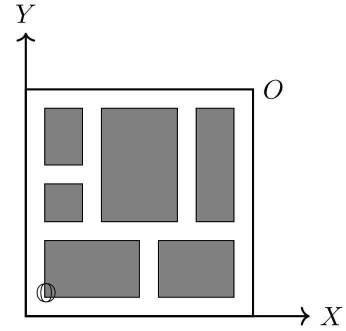


Fig. 2: Example of rectangle packing problem

real life problems [17], [18]. In some of these problems, the dimensions of $\mathbb{O}$ are held constant such as in the problem of packing modules on a chip, where the widths and height of the rectangles represent the physical width and heights of the modules [18]. Other problems, such as the BAP in some instances, allow one side of the rectangle to vary depending on its assigned position (i.e. the handling time is dependent on the berth) [19].

the BAP solves the problem of optimally assigning incoming vessels to berth positions to be serviced (Fig 1a). The width and height of O represent the berth length S and time horizon T, respectively. Similarly, the width and height for $\mathbb{O}$ represent the time spent to service vessel i and the space taken by docking vessel i, respectively. The vessel characteristics (length of the vessel, arrival time, handling time, desired departure time) are assumed to be known for all N vessels to be serviced. A representation of a BAP solution is shown in Fig 3.

The BAP objective is generally represented as minimizing some operational time for a given vessel i. The operational time may be chosen to minimize the difference between arrival and departure times, time spent being serviced, or overall waiting time [19]–[21]. The model must then constrain the vessel placement as to not allow overlap spatially or temporally.

\section{The PAP formulation}

Mention that PAP considers a continuous charger width, not actual individual chargers

The BAP formulation forms the basis of the PAP; however, there are some minor ~~slight~~ differences in the way the variables are perceived. For the i^{th} visit, starting service time, u_i, is now the starting charge time, the berth location, v_i, is now the charger queue for assignment, and the service time, p_i, is now the time to charge. The following MILP describes the stated objective and constraints. The MILP is shown in its entirety as it forms the foundation from which the PAP is constructed [15].

$$\min \sum_{i=1}^{N} (c_i - a_i) \tag{1}$$

Subject to the following constraints:

Should change to be "subject to"

BAP specific variables S, $\mathbb{O}$, O

Indices i, j : visit indices
b : bus

TABLE I: Notation used throughout the paper

Variable	Description	Variable	Description
Input values		C	Number of chargers
A	Number of buses ~~in use~~	M	An arbitrary very large upper bound value
N	Number of total visits	O	Bounding box for rectangle packing
$\mathbb{O}$	Set of rectangles to be packed in O	Q	Number of chargers
S	Length of charger	T	Time horizon
~~Input variables~~ Problem definition parameters			
Γ	Array of ID's for each visit i	α_i	? Initial charge percentage time for visit i
β_b	Final charge percentage for bus b at the end of the time horizon	ϵ_q	Cost of using charger q per unit time
γ_i	~~Array of values indicating the next index visit i will arrive~~ Mapping from visit i to the next visit index for same bus	κ_b	Battery capacity for bus b
λ_i	Discharge of visit over route i	ν	Minimum charge allowed on departure of visit i
τ_i	Time visit i must depart the station	ζ_b	Discharge rate for ~~vehicle~~ bus b
a_i	Arrival time of visit i	i_{0b}	~~Indices~~ Visit index associated with the initial ~~arrival for every bus in A~~ visit of bus b
i_{fb}	~~Indices associated with the final arrival for every bus in A~~ Visit index for final visit by bus b	m_q	Cost of a visit being assigned to charger q
r_q	Charge rate of charger q per unit time	s_q	~~Length~~ width of vehicle i
Decision Variables			
δ_{ij}	If $v_i + s_i \leq v_j$ ($\delta_{ij} = 1$) otherwise $\delta = 0$	η_i	Initial charge for visit i
σ_{ij}	If $u_i + p_i \leq u_j$ ($\sigma_{ij} = 1$) otherwise $\sigma = 0$	c_i	~~Detach time from charger~~ Ending charge for visit i
g_i	Linearization term for bilinear terms $g_i := p_i w_{iq}$	p_i	Amount of time spent on charger for visit i
u_i	~~Initial~~ Starting charge time of visit i	v_i	Assigned queue for visit i
w_{iq}	$w_{iq} = 1$ if bus visit i is assigned to charger q		

update

This figure doesn't help me understand BAP. It confuses me

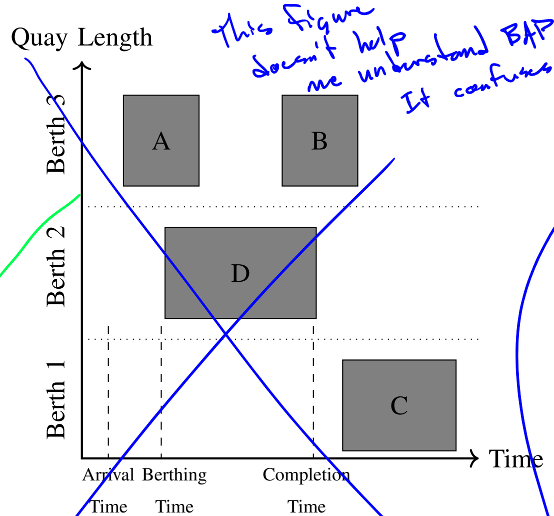


Fig. 3: The representation of the berth-time space

$$u_j - u_i - p_i - (\sigma_{ij} - 1)T \geq 0 \tag{2a}$$
$$v_j - v_i - s_i - (\delta_{ij} - 1)S \geq 0 \tag{2b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{2c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{2d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{2e}$$
$$p_i + u_i = c_i \tag{2f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{2g}$$
$$\sigma_{ij} \in \{0,1\},\ \delta_{ij} \in \{0,1\} \tag{2h}$$
$$v_i \in [0, S] \tag{2i}$$

Introduce variables

Move to before MILP

Where, for this problem, the following are constants

- S : charger length
- T : time horizon
- N : number of incoming vehicles

- p_i : charging time for vehicle i; $1 \leq i \leq N$
- s_i : ~~size~~ width of vehicle i; $1 \leq i \leq N$
- a_i : arrival time of vehicle i; $1 \leq i \leq N$

and the following are decision variables

- u_i : starting time of service for vehicle i; $1 \leq i \leq N$
- v_i : charger queue i; $1 \leq i \leq N$
- c_i : departure time for vehicle i; $1 \leq i \leq N$
- σ_{ij} : $u_i + p_i < u_j$; $1 \leq i, j \leq N$; $i \neq j$
- δ_{ij} : $v_i + s_i < v_j$; $1 \leq i, j \leq N$; $i \neq j$?

The objective function (1) minimizes the time spent to service each vehicle by minimizing over the sum of differences between the departure time, c_i, and arrival time, a_i. i.e., It wants to get each vehicle charged and on its way as quickly as possible.

Constraints 2a-2e are the "queuing constraints". They are used to ~~prevent overlapping in both space and time~~ ensure the rectangles do not overlap, i.e., if vehicle are in the same queue ... Need better description. as shown in Fig 3 (That was BAP). ~~In terms of the BAP,~~ Constraint (2a) states that the starting ~~service~~ charge time for ~~vessel~~ vehicle j, u_j, must be greater than the starting time of ~~vessel~~ vehicle i, u_i, plus its service time, p_i. Not true. only if the constraint is active. The ~~last term~~ $(\sigma_{ij} - 1)T$ term utilizes big-M ~~notation~~ formulation to activate or deactivate the constraint. A value of $\sigma_{ij} = 1$ will activate the constraint to ensure that i is complete before j is allowed to begin being serviced. If $\sigma_{ij} = 0$, then the constraint is of the form $T + u_j + p_j > u_i$ rendering the constraint "inactive" because u_i cannot be larger than $T + u_j + p_j$. ~~This~~ A choice of $\sigma_{ij} = 0$ effectively allows the charging windows of the vehicle to overlap.

In a similar fashion, δ_{ij} determines whether the vehicles will be charging in the same queue. If $\delta_{ij} = 1$ then (2b) is rendered active and vehicle i and j must be charging in different queues. If $\delta_{ij} = 0$ then the constraint is deactivated and the vehicle queue assignments may be the same.

Constraints 2c-2e are used to establish queuing by providing a relationship between queuing variables. Constraint (2c) states that one of the following is true for each vehicle pair: $u_i + p_i < u_j$ ($\sigma_{ij} = 1$), $u_j < u_i + p_i$ ($\sigma_{ji} = 1$), $v_i + s_i < v_j$ ($\delta_{ij} = 1$), $v_j < v_i + s_i$ ($\delta_{ji} = 1$). Constraints (2d) and (2e) enforce consistency, i.e. $u_i + p_i < u_j$ and $u_j < u_i + p_i$ cannot hold true simultaneously. Similarly, $v_i + s_i < u_j$ and $u_j < v_i + s_i$

reword

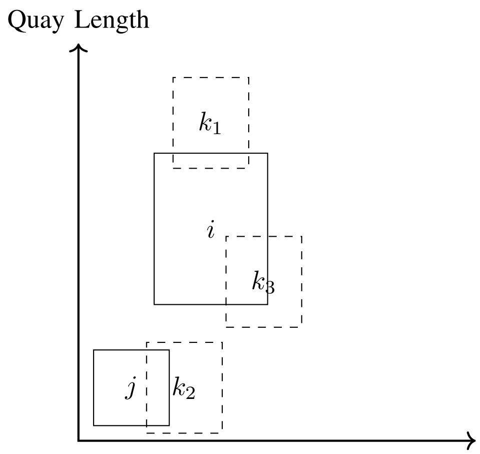


Fig. 4: Examples of different methods of overlapping. Space overlap: $v_{k_1} < v_i + s_i \therefore \delta_{k_1 i} = 0$. Time overlap $u_{k_1} < u_j + p_j \therefore \sigma_{k_2 j} = 0$. Both space and time overlap $\sigma_{k_3 i} = 0$ and $\delta_{k_3 j} = 0$.

cannot be true simultaneously. This enforces a relationship between vessels: either one is before the other temporally or they are in different queues.

The last constraints force relationships between arrival time, charge start time, and departure time. Constraint (2f) states that the service start time, u_i, plus the time to service vessel i, p_i, must equal the departure time, c_i. Constraint (2g) enforces the arrival time, a_i, to be less than or equal to the service start time, u_i, which in turn must be less than or equal to the latest time the vessel may begin to be serviced to stay within the time horizon. Constraint (2h) ensures that σ_{ij} and δ_{ij} are binary. Constraint (2i) ensures that the assigned value of v_i is a valid charging position.

Applying the PAP to BEB charging requires ~~two~~ Four fundamental, but significant changes. The first is that ~~the BEB may not charge for a set amount of time~~ the time that a BEB spends charging is allowed to vary. Thus, p_i becomes a variable of optimization ~~and the resulting charge of each battery must be tracked~~. Second, in the PAP each ~~vessel~~ visit, i, is assumed to be a different vehicle. ~~This is a problem when accounting for battery dynamics when buses revisit after completing a route.~~ For the BEB problem, each bus may make multiple visits to the station throughout the day and the resulting charge is dependent upon each visit made.

III. Problem Formulation

The discussion of the four changes are separated into two sections. The first discusses the changes in the queuing constraints while the second discusses the addition of the bus charge management. Section III.C then addresses the objective function and Section III.D states the complete problem.

~~The MILP is formulated with two sets of constraints: queuing constraints and battery dynamics constraints. This section will build off and modify the PAP formulation in (1) and (2) to progressively construct the battery dynamic constraints to create the Position Allocation MILP.~~ All notation is defined in Table I.

A. Queuing Constraints

Instead of referring to a vehicle or bus arriving at a station, each variable will instead be associated with a visit. Visits i and j could correspond to different buses or visits by the same bus.

~~Consider the following set of constraints:~~

The third change is formulaicaly simple, but fundamentally essential. In the PAP, the charger is one continuous bar with with vehicle width effectively restricting the number of vehicles charging simultaneously. For the BEB, it is assumed that a specific number of chargers exist and these chargers can charge the vehicle at a different rate.

Instead of each vehicle having a width, the spatial variable s_i is removed and v_i is made to be an integer corresponding to which queue visit i will be using. Thus, when $\delta_{ij} = 1$, the visits must be at different chargers, i.e. $v_i - v_j \geq 1$. The variable S is likewise replaced with Q, where Q corresponds to the number of queues. Note that Q is the number of chargers plus the number of individual lanes, as will be described in Section III.B.

The modified queuing constraints can be written as follows.

$$u_i - u_j - p_j - (\sigma_{ij} - 1)T \geq 0 \tag{3a}$$
$$v_i - v_j - \cancel{s_j} - (\delta_{ij} - 1)Q \geq 1 \tag{3b}$$
red
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{3c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{3d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{3e}$$
$$p_i + u_i = c_i \tag{3f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{3g}$$
$$c_i \leq \tau_i \tag{3h}$$
$p_i \geq 0$
$$\sigma_{ij} \in \{0, 1\},\ \delta_{ij} \in \{0, 1\} \tag{3i}$$
$$v_i \in [1, 2, ..., Q] \tag{3j}$$
red

The fourth fundamental change is related to the first three. The charge of each bus must be tracked to ensure that charging across multiple visits is sufficient to allow each bus to execute its route throughout the day.

Constraints (3a)-(3g) and (3i) are ~~the same as previously described~~ nearly identical to those described in Section II with the sole change in (3b) being described above to conform to a queue. ~~Additionally, the charging queue, $v_i \in Q$ is discrete as represented in Fig 3. This is to say, rather than using a continuous charger of length S, it is discretized into Q available chargers.~~ Constraint (3h) is added to ensure ~~states~~ that the ending charge time, c_i, must be less than or equal to the required departure time from the station, τ_i. This enables the bus schedules to be considered. Finally, (3j) enforces v_i to be an integer.

Using purely the constraints in (3) with the objective in (1) would result in c_i being chosen as small as possible by employing $p_i = 0$, $u_i = c_i$. Thus, the vehicles would not charge. Furthermore, it does not encode any revisiting of the BEB to the charging station. To remedy this, battery dynamic constraints are introduced.

B. Battery Charge Dynamic Constraints

The charge at the beginning of visit i is denoted η_i

Need to introduce variables to make readable

The ~~battery~~ charge dynamic constraints are used to ~~drive time spent on the charger, p_i, as well as define initial, final, and intermediate bus charges for each visit i.~~ ensure sufficient time is spent charging to allow buses to execute their routes and end the day with a specific charge threshold. The initial and final bus charges are predefined and are represented by the equations $\eta_{i_0} = \alpha_{i_0}\kappa_{i_0}$ and $\eta_{i_f} = \beta_{i_f}\kappa_{i_f}$, respectively, where α_{i_0} and β_{i_f} are percentages of the battery capacity for the first and final visits for each bus, b, respectively. ~~The intermediate charges must be determined at solve time.~~ Not sure what this means.

need to define with last index

This notation doesn't make sense

To accomplish such a task, each arrival i must be associated with an initial charge, η_i, which represents the charge received from the previous visit minus the discharge observed while on route. However, because the ~~PAP~~ assumes each arrival is unique, bus visits must be associated with a unique bus ID to appropriately initialize charges for each visit. This visit/ID pair is represented as vector denoted by Γ, where the index represents visit i and the value is the bus ID. To propagate charges forward, another term γ is introduced to represent the next index bus i arrives. For example, assume $A = 3$ buses, $N = 5$ visits, and $\Gamma = [0, 1, 2, 0, 1]$. Γ informs us that the first and fourth visits correspond to bus 1 and the second and fifth visits correspond to bus 2. As γ maps from one visit to the next, it would take the value $\gamma = [3, 4, -1, -1, -1]$, where 0-indexing is assumed and -1 represents no further visits for bus Γ_i. Add in i_0

re-cite the 80% threshold

It is assumed that the charge received is proportional to the time spent charging. The charge rate for charger q is denoted as r_q, the charge at visit i is η_i, and the amount of discharge between visit i and the next visit of the same bus, λ_{γ_i}. If visit i occured at charger q, the charge of the bus coming into visit γ_i would be

$$\eta_{\gamma_i} = \eta_i + p_i r_q - \lambda_{\gamma_i}. \tag{4}$$

The binary decision variable w_{iq} is introduced to determine whether visit i uses charger q. This allows the charge of the bus coming into visit γ_i to be written in summation form as

Do we relate v_i to w_{iq}? A Yes. In 9(r). Should include here.

$$\eta_{\gamma_i} = \eta_i + \sum_{q=1}^{Q} p_i w_{iq} r_q - \lambda_i \tag{5a}$$

$$w_{iq} \in \{0, 1\} \tag{5b}$$

move here

Maximum and minimum values for the charges are to be placed to ensure the battery is not overcharged and to guarantee sufficient charge for subsequent visits. Assuming each bus has its own upper bound, the upper bound for the bus corresponding to visit i is κ_{Γ_i}. The lower bound is assumed to be a percentage of the upper bound, and is written as $\nu\kappa_{\Gamma_i}$. The upper and lower bound constraints are written using the charge as

$$\eta_i + \sum_{q=1}^{Q} p_i w_{iq} r_q \leq \kappa_{\Gamma_i} \tag{6a}$$

$$\eta_i + \sum_{q=1}^{Q} p_i w_{iq} r_q - \lambda_i \geq \nu\kappa_{\Gamma_i} \tag{6b}$$

Note that the term $p_i w_{iq}$ is a bilinear term (two decision variables being multiplied together) which is nonlinear [22]. A standard way of linearizing a bilinear term that contains a continuous and integer term is by introducing a slack variable ~~new term, an either/or constraint,~~ with an either/or constraint ~~and utilizing big-M notation~~ [22], [23]. Introducing the slack variable g_{iq} to be equal to $p_i w_{iq}$, g_{iq} can be defined using the following constraints

Backwards

$g_{iq} = p_i w_{iq} = \begin{cases} p_i & w_{iq}=1 \\ 0 & w_{iq}=0 \end{cases}$

$g_{iq} \geq p_i - (1-w_{iq})M$

$w_{iq}=1$ $\quad$ $w_{iq}=0$

$$p_i + (1 - w_{iq})M \geq g_{iq} \quad g_{iq} \leq p_i \quad g_{iq} \leq M \tag{7a}$$

$g_{iq} \leq p_i$

$$p_i \leq g_{iq} \quad p_i \leq g_{iq} \quad p_i \leq g_{iq} \leftarrow \text{forces } p_i = 0 \tag{7b}$$

$$M w_{iq} \geq g_{iq} \quad g_{iq} \leq M \tag{7c}$$

$$0 \leq g_{iq} \quad 0 \leq g_{iq} \tag{7d}$$

(7a) is then inactive.

where M is a large value. ~~defined sufficiently large such that for constraints (7a) and (7b)~~ If $w_{iq} = 1$ then ~~the two~~ (7a) and (7b) equations will take the form of $p_i \leq g_{iq}$ and $p_i \geq g_{iq}$ effectively, stating $p_i = g_{iq}$. If $w_{iq} = 0$ then (7a) & (7b) are inactive and (7c) and (7d) ~~the two equations will take the form of $p_i \leq g_{iq}$~~ ← would force $p_i=0$ ~~and $p_i \geq g_{iq} - M$ which deactivates the constraint. Similarly, (7c) and (7d) state if $w_{iq} = 0$ then the constraints~~ take the form $0 \geq g_{iq}$ and $0 \leq g_{iq}$, forcing ~~directly implying~~ $0 = g_{iq}$ $=0$. ~~If $w_{iq} = 1$ then the constraints take the form $M \leq g_{iq}$ and $0 \leq g_{iq}$ deactivating the constraint. Putting the two sets of constraints together defines the linear representation of the bilinear term $w_{iq} p_i$.~~

C. The Objective Function

~~The last consideration is the objective function.~~ The goal of the MILP is to utilize ~~slow chargers as much as possible.~~ charges as little as possible (i.e, reduce energy costs) with fast charging penalized greater to reduce battery damage. Thus, an assignment cost m_q and usage cost ϵ_q are associated with each charger, q. The cost for both the assignment and utilization of slow chargers is less than that of the fast chargers. The objective function has an assignment term, $w_{iq} m_q$, which is non-zero if charger q is used for visit i. Similarly, a usage term $g_{iq}\epsilon_q$ is non-zero only if charge is recieved for visit i at charger q. The resulting objective is defined in Eq 8. The assignment cost, $w_{iq} m_q$, and the usage cost, $g_{iq}\epsilon_q$, are summed over each visit, i, and charger, q.

Why do we need this term?

$$\min \sum_{i=1}^{N} \sum_{q=1}^{Q} \left(w_{iq} m_q + g_{iq}\epsilon_q \right) \tag{8}$$

D. The BEB Charging Problem

The resulting problem to be optimized is outlined in Eq 9. Constraints (9a)-(9h) are reiterations of the queuing constraints in (3). Constraints (9i)-(9m) provide initialization and terminal conditions as well as intermediate constraints to provide continuity in vehicle charges. Constraint (9i) states the first arrival for each bus is initialized with a charge of $\alpha\kappa_{\Gamma_i}$. Constraints (9j) and (9k) define the battery charge dynamics using (6a) and (6b) with g_{iq}. ~~defines that the previous charge, η_i, plus the charging done by charger q minus the discharge amount due to completing route, λ_i. This constraint is only used with indices with valid next visit values (i.e. $\gamma_i \neq -1$). Constraint (9k) is similar to (9j), except it states that the initial charge, η_i, plus the charge from q and the discharge from route, λ_i, must be greater than some percentage of its capacity, κ_{Γ_i}, in order to guarantee sufficient charge to complete the next route.~~ Constraint (9l) states that the charging done for visit i cannot be greater than the capacity of the battery, κ_{Γ_i}. Constraint (9m) states that the last visit for each vehicle must have a minimum charge of $\beta\kappa_{\Gamma_i}$, ~~Constraint (9n) is included to~~ guaranteeing a minimum initial charge for the next working day.

The constraints (9o)-(9q) represent the linear form of the bilinear term the $g_{iq} = p_i w_{iq}$. The set of constraints (9r)-(9s) define the linking constraint between the queuing constraint and w_{iq} (i.e $1w_{i1} + 2w_{i2} + \cdots + Qw_{iQ} = v_i$) and enforces that only a single w may be selected per visit, respectively. The last constraints (9t)-(9x) defines the sets of valid values for each variable.

$$u_i - u_j - p_j - (\sigma_{ij} - 1)T \geq 0 \tag{9a}$$
$$v_i - v_j - s_j - (\delta_{ij} - 1)Q \geq 0 \tag{9b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{9c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{9d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{9e}$$
$$p_i + u_i = c_i \tag{9f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{9g}$$
$$c_i \leq \tau_i \tag{9h}$$
$$\eta_{i_0} = \alpha_{i_0} \kappa_{\Gamma_{i_0}} \tag{9i}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q - \lambda_i = \eta_{\gamma_i} \tag{9j}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q - \lambda_i \geq \nu \kappa_{\Gamma_i} \tag{9k}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq} r_q \leq \kappa_{\Gamma_i} \tag{9l}$$
Confusing
$$\eta_{i_f} \geq \beta_{i_f} \kappa_{\Gamma_{i_f}} \tag{9m}$$
Backwards
$$p_i + (1 - w_{iq})M \geq g_{iq} \tag{9n}$$
$$p_i \leq g_{iq} \tag{9o}$$
$$M w_{iq} \geq g_{iq} \tag{9p}$$
$$0 \leq g_{iq} \tag{9q}$$
$$\sum_{q=1}^{Q} q w_{iq} = v_i \tag{9r}$$
$$\sum_{q=1}^{Q} w_{iq} = 1 \tag{9s}$$
$$w_{iq} \in \{0, 1\} \tag{9t}$$
$$\sigma_{ij} \in \{0, 1\}, \ \delta_{ij} \in \{0, 1\} \tag{9u}$$
$$v_i \in [1, ..., Q] \tag{9v}$$
$$i \in [1, ..., N] \tag{9w}$$
$$q \in [1, ..., Q] \tag{9x}$$

IV. Example

An example will now be presented to demonstrate the utility of the developed MILP. A description of the scenario is first presented followed by results.

A. Scenario

The given exampled utilizes $A = 40$ buses with $N = 220$ visits to the station divided between the A buses. Each bus has a 388 KWh battery that is required to stay above 25% charge (97 KWh) to maintain battery health, and the bus is assumed to begin the working day with 90% charge (349 KWh). Additionally, each bus is required to end the day with a minimum charge of 95% (368 KWh). Planning is done over a 24-hour time horizon. $Q = 9$ chargers are utilized where five of the chargers are slow charging (100 KWh) and four are fast charging (400 KWh). As discussed in Section III-C, the slow chargers take longer, but are better for battery health than the fast chargers. Therefore, the slow chargers are modeled with a lower cost than the fast chargers for both assignment, $m_q = r_q$, and utilization, $\epsilon_q = r_q$, in the objective, (8).

The bus schedules are randomly generated. It is assumed that each bus has no more than 30 minutes between route departures. Bus routes are created by generating a random list of arrival times, assigning bus ID's to each visit, then generating route duration. They may vary anywhere from a minimum of the average time between the current and next arrival to a maximum of the next arrival time (i.e. $\frac{a_i + a_{\gamma_i}}{2}$ to a_{γ_i}). The discharge is assumed to be linear and is calculated via $\lambda_i = \text{rand}(\frac{a_i + a_{\gamma_i}}{2}, a_{\gamma_i})\zeta_{\gamma_i}$ where $1 \leq i \leq N$ and ζ_i is the discharge rate for each bus.

The optimization was performed using the Gurobi MILP solver [24] on a machine running a quad-core Intel i7-9700 4.7 GHz processor. The optimizer ran for 5 seconds to completion to produce the optimal solution.

B. Results

The schedule generated by the MILP is shown in Fig 5. The top graph indicates the slow charger usage, and the bottom indicates the fast charger usage. Although $Q = 9$ chargers were used, Fig 5 shows that only five chargers were utilized.

Each color in Fig 5 is used to identify the bus ID assigned. It is noted that the overlaps in Fig 5 indicate the waiting time of vehicle j while vehicle i charges. This is recognized by viewing the vertical bars. These bars indicate the time bus i is set to charge. The area before indicate waiting time and the area after indicates the time spent on the charger. Note that, based upon vehicles needs, bus i may arrive before bus j, but wait until after bus j charges before starting its charge (Fig 6). This is an unintuitive schedule caused by the cost of assignment parameter in the objective function, m_q. This type of constraint would not be achieved through a greedy scheduling algorithm.

Fig 8 depicts the charge for every bus over the time horizon. Every vehicle begins at 90% charge, finishes at 95% charge, and never goes below 25% in the intermediate arrivals as stated in the constraints (9). Fig 7 represents the usage of each charger over the time horizon. The maximum amount of slow chargers used at any given time is three and only one fast charger is utilized at a time.

The pattern, for the most part, in Fig 5 is to populate the first charger and supply the other chargers as needed. Where this pattern primarily breaks down is around hour 15. Having moved each bus one charger down would have resulted in the same cost. This behavior is due to a lack of cost for the amount of total chargers being utilized. Adding a cost in the objective function to minimize the total amount of unique chargers would resolve this problem by effectively attempting to "pack" the chargers down reducing the peak of Fig 7.

Although this formulation effectively discourages the use of fast chargers to utilize the more cost-effective slow charges more readily, there is no consideration for the cost consumption and demand cost. Consumption cost accounts for peak use of power in the system and demand cost accounts for the total energy used. These metrics are used to calculate the monetary cost of the system. Calculating the demand cost would create more bilinear terms as the objective function would have to sum over $\sum_{i=1}^{N}\sum_{q=1}^{Q} w_{iq} r_q(p_i)$, creating yet another linearization term. Furthermore, the demand cost would require discretization of the system. The demand utilizes the peak energy consumption over 15 minute intervals and uses the largest peak as the rate to charge. This would require tracking of when a charger is active and including another variable to enable and disable $w_{iq} r_q$. This example displays the limitation of the MILP solver as it adds effort and complexity to the system as a trade-off for calculating the optimal schedule. Other metaheuristic solvers, such as simulated annealing (SA), would allow the problem to be solved without having to linearize the system. To do: get the citation for this

V. Conclusion

This work developed a MILP scheduling framework that optimally assigns slow and fast chargers to a BEB bus fleet assuming a constant schedule. The BAP was introduced with an example formulation and was then compared to the PAP. The PAP constructed on the BAP to allow the time spent on the charger, p_i, to be a decision variable. Because the original PAP required service time, p_i, to be given, linear battery dynamics were introduced to drive charging times. Additional constraints were also introduced to provide limits for the battery dynamics.

An example was presented that demonstrated the ability of the formulation to optimally select slow and fast chargers to meet the requirements of the time schedule and the charge consumed during the bus routes. It was also observed that the formulation was able to successfully create a charging schedule without creating conflicts (Fig 5). The charge for each bus was tracked in Fig 8 and shows that the schedule maintained the charges between the maximum charge, 100%, and the minimum charge, 25%.

Limitations were demonstrated in the lack of objective to limit the total amount of chargers utilized and to calculate the demand and consumption costs. Further fields of interest are to utilize the formulation (Eq (8) and (9)) with nonlinear battery dynamics, calculation and utilization of the demand and consumption cost in the objective function, and utilizing this formulation in a metaheuristic solver.

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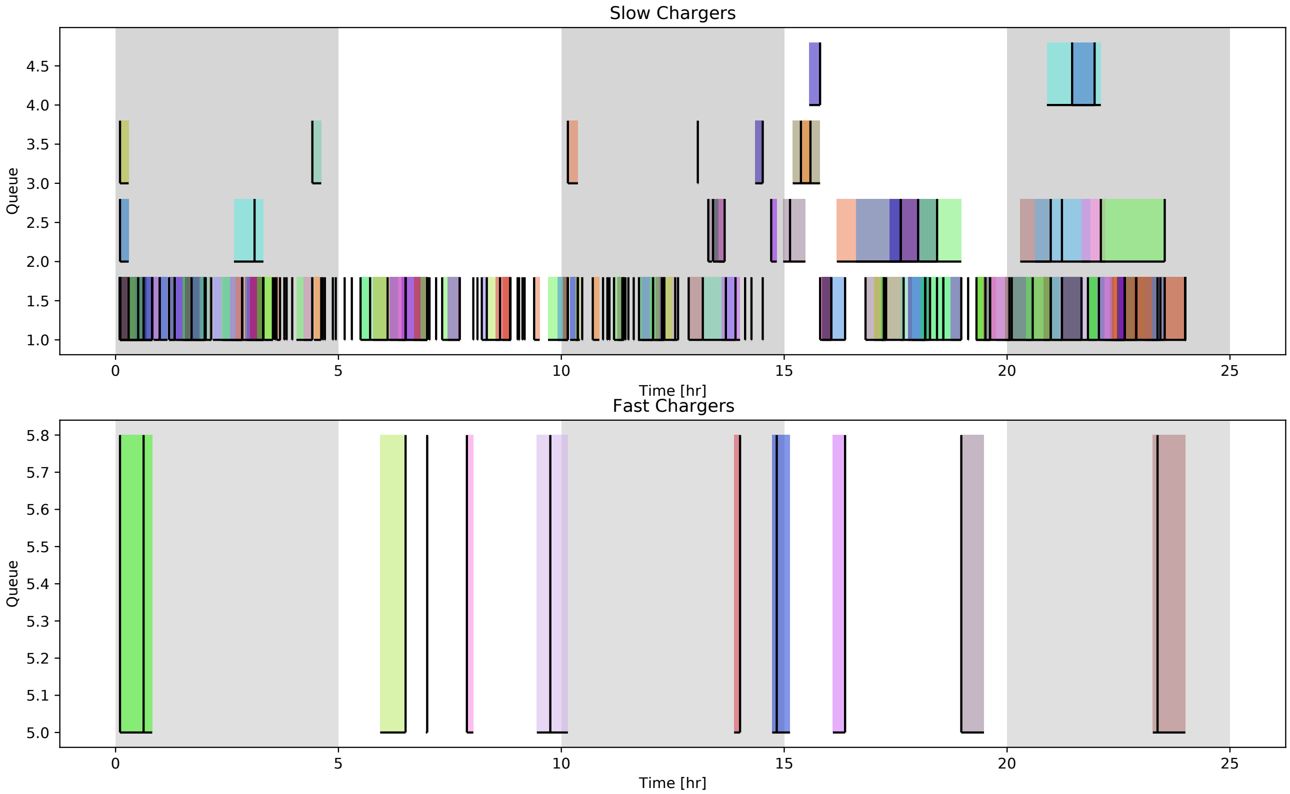

Fig. 5: Bus schedule generated with PAP MILP. Each color is assigned to a specific bus ID. The vertical bars indicate the time the vehicles are set to charge, the area before said bars indicate the waiting times for each visit $1 \leq i \leq N$. Similarly, the area after the bars indicate the time spent on the charger.

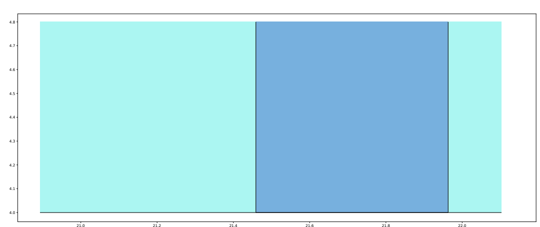

Fig. 6: An example, from Fig 5, of a bus i arriving before j, having i wait for j to arrive, and charge i after j is detached.

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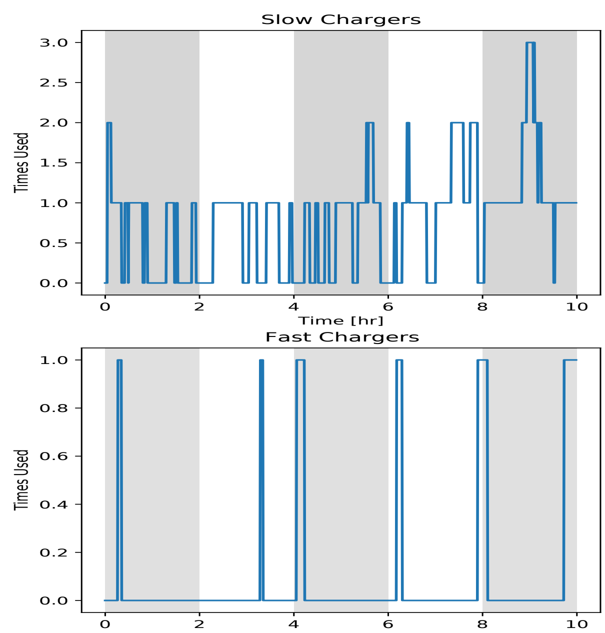

Fig. 7: Amount of slow and fast chargers used at any given time.

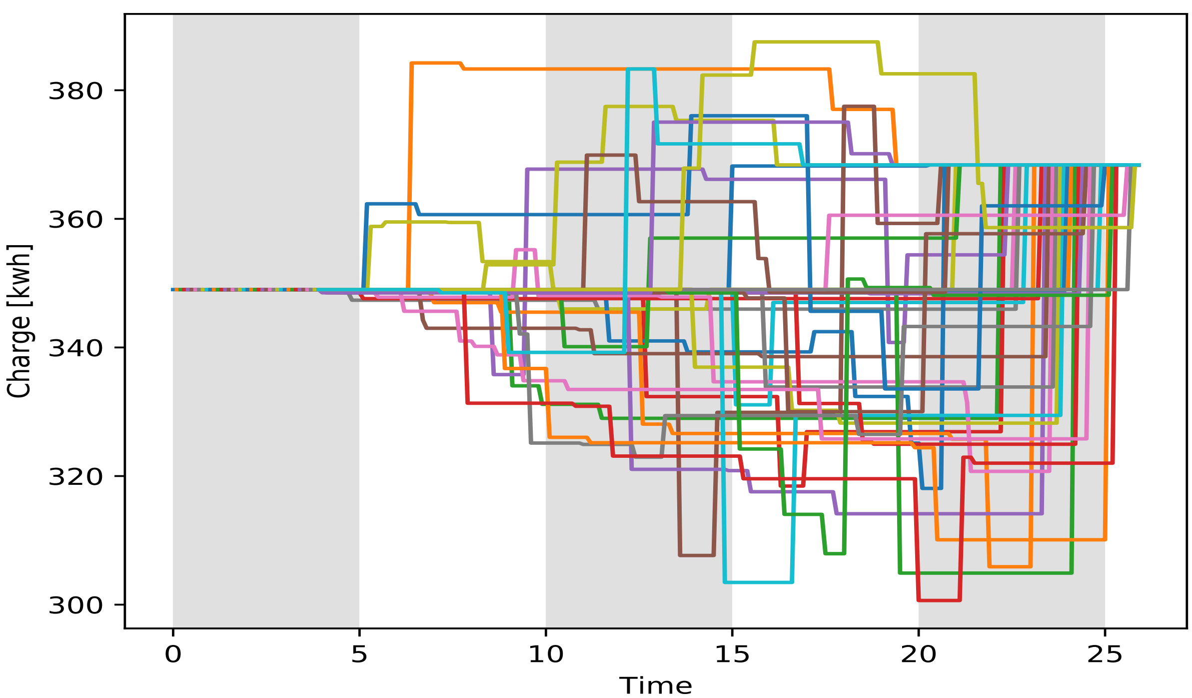

Fig. 8: Charge for each bus over the time horizon.