A Position Allocation Approach to the Scheduling of Battery Electric Bus Charging

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***Abstract*—This paper introduces an electric bus scheduling framework that utilizes both fast and slow charging assuming a fixed route schedule. Slow chargers are prioritized for battery health and fast chargers are utilized to satisfy time constraints. A Berth Allocation Problem (BAP) approach is modified and utilized to construct the Position Allocation Problem (PAP) Mixed Integer Linear Program (MILP) that optimally assigns incoming buses to chargers while taking into consideration battery dynamics of the buses. Results are present utilizing a randomly generated bus schedule for 40 buses.**

***Index Terms*—Berth Allocation Problem (BAP), Position Allocation Problem (PAP), Mixed Integer Linear Program (MILP), Battery Electric Bus (BEB), Scheduling**

I. Introduction

The public transportation system is crucial in any urban area; however, the increased awareness and concern of environmental impacts of petroleum based public transportation has driven an effort to reduce the pollutant footprint [1]–[4]. Particularly, the electrification of public bus transportation via battery power, i.e., battery electric buses (BEBs), has received significant attention [4]. Although the technology provides benefits beyond reduction in emissions such as lower driving costs, lower maintenance costs, and reduced vehicle noise, battery powered systems introduce new challenges such as larger upfront costs, and potentially several hour long "refueling" periods [2], [4]. Furthermore, the problem is exacerbated by the constraints of the transit schedule that the fleet must adhere to, the limited amount of chargers available, as well as the adverse affects in the health of the battery due to fast charging [5]. This paper presents a continuous scheduling framework for a BEB fleet that shares limited fast and slow chargers. This framework takes into consideration linear charging dynamics and a fixed bus schedule while meeting a certain battery percentage threshold for intermediate and final charges.

Recent research efforts have been made into solving the problem of scheduling and charging fleets as well as determining the infrastructure that they rely upon. Attention has been given to solving both problems simultaneously [6]–[9]. The added complexity of considering both the BEB charge scheduling and the infrastructure problems necessitates simplifications for sake of computation. First, only fast chargers are utilized in planning [6], [7], [9]–[14]. Second, significant simplifications to the charging models are made. Some approaches assume full charge [6], [9], [10], [13]. Others have assumed that the charge received is proportional to the time spent on the charger [11], [12], which can be a valid assumption when the battery state-of-charge (SOC) is below 80% charge [11].

Recent work, for scheduling and charging, has used the Position Allocation Problem (PAP) as a means to schedule the charging of electric vehicles [15]. The PAP framework is constructed from the Berth Allocation Problem (BAP) [15]. The BAP solves the problem of allocating space for incoming vessels to be berthed. Each arriving vessel requires both time and space to be serviced and is assigned a berthing location [16]. Vessels are lined up parallel to the berth to be serviced and are horizontally queued as shown in Fig 1a. The PAP utilizes this notion of queuing to reuse the BAP for queuing vehicles to be charged as shown in Fig 1b. The PAP assumes that the time to charge is given and defines the full charge time. Additionally, a single, continuous charger is assumed [15].

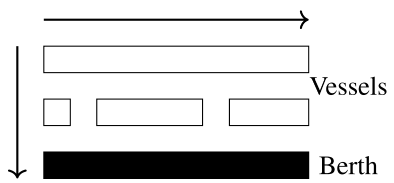
The contribution of this work is a BEB charger scheduling framework that considers route schedules, a proportional model for battery dynamics, charge limits, and the availability of slow and fast chargers. The bus schedules are assumed to be fixed for the duration of the time horizon. A MILP is formed as a modified PAP [15] where the solution of the problem provides the arrival time, selected charger (fast or slow), initial charge time, final charge time, and departure time from the station. This paper expands upon the PAP by allowing charge times to be dynamically chosen, including a linear battery dynamic model, allowing multiple charger types (fast, and slow), as well as the ability to link consecutive visits to specific vehicles to accommodate revisiting BEBs.

The remainder of the paper proceeds as follows: Section II the BAP is introduces with a MILP formulation example. Section III constructs the PAP and introduces battery dynamics into the MILP, and Section IV demonstrates an example of the PAP formulation and its ability to optimally assigning vehicles while considering battery dynamics and meeting scheduling constraints. The paper ends in Section V with concluding remarks.

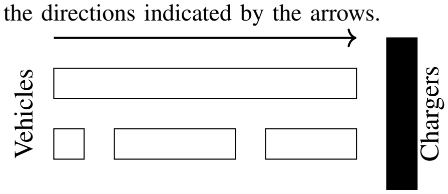
TABLE I: Notation used throughout the paper

Variable	Description	Variable	Description
Input values			
A	Number of buses in use	M	An arbitrary very large upper bound value
N	Number of total visits	O	Bounding box for rectangle packing
$\mathbb{O}$	Set of rectangles to be packed in O	Q	Number of chargers
S	Length of charger	T	Time Horizon
Input variables			
Γ_i	Array of visit ID's	α_i	Initial charge percentage time for visit i
β_i	Final charge percentage for bus i at the end of the time horizon	ϵ_q	Cost of using charger q per unit time
γ_i	Array of values indicating the next index visit i will arrive	κ_i	Battery capacity for bus i
λ_i	Discharge of visit over route i	ν	Minimum charge allowed on departure of visit i
τ_i	Time visit i must leave the station	ζ_i	Discharge rate for vehicle i
a_i	Arrival time of visit i	m_q	Cost of a visit being assigned to charger q
r_q	Charge rate of charger q per unit time	s_q	Length of vehicle i
Decision Variables			
δ_{ij}	If $v_i + s_i \leq v_j$ ($\delta = 1$) otherwise $\delta = 0$	η_i	Initial charge for visit i
σ_{ij}	If $u_i + p_i \leq u_j$ ($\sigma = 1$) otherwise $\sigma = 0$	c_i	Detach time from charger for visit i
g_i	Linearization term for bilinear terms $g_i := p_i w_{iq}$	p_i	Amount of time spent on charger for visit i
u_i	Initial charge time of visit i	v_i	Assigned queue for visit i
w_{iq}	Vector representation of queue assignment v		

Fig. 1: Comparison between BAP and PAP



(a) Example of berth allocation. Vessels are docked in berth locations (horizontal) and are queued over time (vertical), and move in the directions indicated by the arrows.



(b) Example of position allocation. Vehicles are placed in queues to be charged and move in the direction indicated by the arrow.

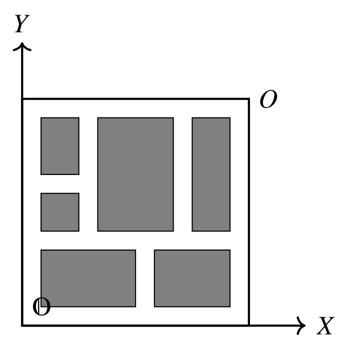


Fig. 2: Example of rectangle packing problem

II. The Position Allocation Problem

The BEB charge schedule formulation in this work builds upon the PAP. The PAP is a rectangle packing problem where a set of rectangles ($\mathbb{O}$) are attempted to be optimally placed in a larger rectangle (O) as shown in Fig 2. Both the set $\mathbb{O}$ and rectangle O's width and height are used to represent quantifiable values. The rectangle packing problem is an NP-hard problem that can be used to describe many real life problems [17], [18]. In some of these problems, the dimensions of $\mathbb{O}$ are held constant such as in the problem of packing modules on a chip, where the widths and height of the rectangles represent the physical width and heights of the modules [18]. Other problems, such as the BAP, allow one or both sides of each rectangle to be decision variables [19].

The BAP solves the problem of optimally assigning incoming vessels to berth positions to be serviced (Fig 1a). The width and height of O represent the berth length S and time horizon T, respectively. Similarly, the width and height for $\mathbb{O}$ represent the time spent to service vessel i and the space taken by docking vessel i, respectively. The vessel characteristics (length of the vessel, arrival time, handling time, desired departure time) are assumed to be known for all N vessels to be serviced. A representation of a BAP solution is shown in Fig 3.

The BAP objective is generally represented as minimizing some operational time for a given vessel i. The operational time may be chosen to minimize the difference between arrival and departure times, time spent being serviced, or overall waiting time [19]–[21]. The model must then constrain the vessel placement as to not allow overlap in time or space.

The BAP formulation forms the basis of the PAP; however there are some slight differences in the way the variables are perceived. For the i^{th} visit, starting service time, u_i is now the starting charge time, the berth location, v_i is now the charger queue for assignment, and the service time, p_i, is now the time to charge. The following MILP describes the stated objective and constraints. The MILP is also shown in its entirety as it forms the foundation from which the PAP is constructed [15].

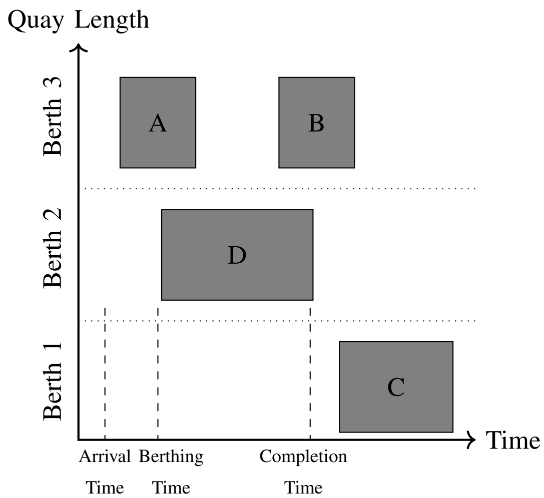


Fig. 3: The representation of the berth-time space

$$min \sum_{i=1}^{N}(c_i - a_i) \tag{1}$$

Subject to the following constraints:

$$u_j - u_i - p_i - (\sigma_{ij} - 1)T \geq 0 \tag{2a}$$
$$v_j - v_i - s_i - (\delta_{ij} - 1)S \geq 0 \tag{2b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{2c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{2d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{2e}$$
$$p_i + u_i = c_i \tag{2f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{2g}$$
$$\sigma_{ij} \in \{0, 1\},\ \delta_{ij} \in \{0, 1\} \tag{2h}$$

Where, for this problem, the following are constants

- S : charger length
- T : time horizon
- N : number of incoming vehicles
- p_i : charging time for vehicle i; $1 \leq i \leq N$
- s_i : size of vehicle i; $1 \leq i \leq N$
- a_i : arrival time of vehicle i; $1 \leq i \leq N$

and the following are decision variables

- u_i : starting time of service for vehicle i; $1 \leq i \leq N$
- v_i : charger queue i; $1 \leq i \leq N$
- c_i : departure time for vehicle i; $1 \leq i \leq N$
- σ_{ij} : $u_i + p_i < u_j$; $1 \leq i, j \leq N$; $i \neq j$
- δ_{ij} : $v_i + s_i < v_j$; $1 \leq i, j \leq N$; $i \neq j$

The objective function (1) minimizes the time spent to service each vehicle by minimizing over the sum of differences between the departure time, c_i, and arrival time, a_i.

Constraints 2a-2e are the "queuing constraints". They are used to prevent overlapping in both space and time as shown in Fig 4. In terms of the BAP, constraint (2a) states that the starting service time for vessel j, u_j, must be greater than the starting time of vessel i, u_i, plus its service time, p_i. The last term utilizes the Big-M notation to activate or deactivate the constraint. A value of $\sigma_{ij} = 1$ will activate the constraint to ensure that i is complete before j is allowed to begin being serviced. If $\sigma = 0$, then the constraint is of the form $T + u_j + p_j > u_i$ rendering the constraint "inactive" because u_i cannot be larger than $T + u_j + p_j$. This effectively allows the charging windows of the vehicle to overlap.

In a similar fashion, δ_{ij} determines whether the vehicles will be charging in the same queue. If $\delta_{ij} = 1$ then (2b) is rendered active and vehicle i and j must be charging in different queues. If $\delta_{ij} = 0$ then the constraint is deactivated and the vehicle queue assignments may be the same.

Constraints 2c-2e are used to establish queuing by providing a relationship between queuing variables. Constraint (2e) states that one of the following is true for each vehicle pair: $u_i + p_i < u_j$ ($\sigma_{ij} = 1$), $u_j < u_i + p_i$ ($\sigma_{ji} = 1$), $v_i + s_i < v_j$ ($\delta_{ij} = 1$), $v_j < v_i + s_i$ ($\delta_{ji} = 1$). Constraints (2d) and (2e) enforce consistency, i.e. $u_i + p_i < u_j$ and $u_j < u_i + p_i$ cannot hold true simultaneously. In a similar manner, $v_i + s_i < u_j$ and $u_j < v_i + s_i$ cannot be true simultaneously. This enforces a relationship between vessels: either one is before the other temporally or they are in different queues.

The last constraints force relationships between arrival time, charge start time, and departure time. Constraint (2f) states that the service start time, u_i, plus the time to service vessel i, p_i, must equal the departure time, c_i. Constraint (2g) enforces the arrival time, a_i, must be less than or equal to the service start time, u_i, which in turn must be less than or equal to the latest time the vessel may begin to be serviced to stay within the time horizon. Constraint (2h) ensures that σ_{ij} and δ_{ij} are binary.

Applying the PAP to BEB charging requires two fundamental, but significant changes. The first is that the BEB may not charge for a set amount of time. Thus, p_i becomes a variable of optimization and the resulting charge of each battery must be tracked. Second, in the PAP each vessel i; $\forall 1 \leq i \leq N$ is assumed to be a different vehicle. This is a problem when accounting for battery dynamics when buses revisit after completing a route.

III. Problem Formulation

The MILP is formulated with two sets of constraints: queuing constraints and battery dynamics constraints. This section will build off and modify the previous formulation ((1) and (2)) and progressively construct the battery dynamic constraints to create the Position Allocation MILP. All notation is defined in Table I.

A. Queuing Constraints

Consider the following set of constraints:

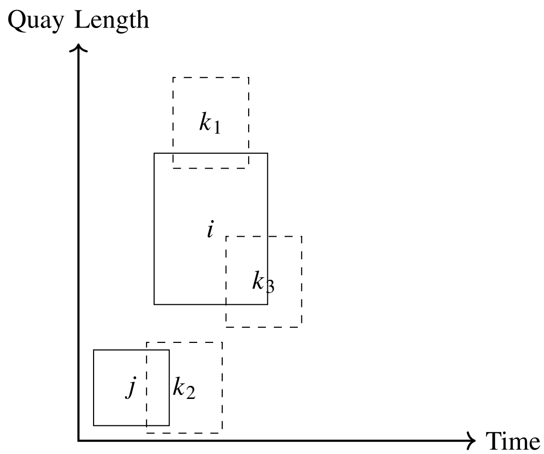


Fig. 4: Examples of different methods of overlapping. Space overlap: $v_{k_1} < v_i + s_i \therefore \delta_{k_1 i} = 0$. Time overlap $u_{k_1} < u_j + p_j \therefore \sigma_{k_2 j} = 0$. Both space and time overlap $\sigma_{k_3 i} = 0$ and $\delta_{k_3 j} = 0$.

$$u_i - u_j - p_j - (\sigma_{ij} - 1)T \geq 0 \tag{3a}$$
$$v_i - v_j - s_j - (\delta_{ij} - 1)Q \geq 0 \tag{3b}$$
$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1 \tag{3c}$$
$$\sigma_{ij} + \sigma_{ji} \leq 1 \tag{3d}$$
$$\delta_{ij} + \delta_{ji} \leq 1 \tag{3e}$$
$$p_i + u_i = c_i \tag{3f}$$
$$a_i \leq u_i \leq (T - p_i) \tag{3g}$$
$$c_i \leq \tau_i \tag{3h}$$
$$\sigma_{ij} \in \{0,1\},\ \delta_{ij} \in \{0,1\} \tag{3i}$$
$$v_i \in \{1, 2, ..., Q\} \tag{3j}$$

Constraints (3a)-(3g) and (3i) are the same as previously described in Section II. Additionally, the charging queue, v_i is discrete as represented in Fig 3. Constraint (3h) states that the ending charge time, c_i, must be less than or equal to the required departure time from the station, τ_i.

Using purely the constraints in (3) with the objective in (1) would results in c_i being chosen as small as possible by employing $p_i = 0$, $u_i = c_i$. Thus, the vehicles would not charge. Furthermore, it does not encode any revisiting of the BEB to the charging station. To remedy this, battery dynamic constraints are introduced.

B. Battery Dynamic Constraints

The battery dynamic constraints are used to drive time spent on the charger, p_i, as well as define initial, final, and intermediate bus charges for each visit i. The initial and final bus charges are predefined and are represented by the equations $\eta = \alpha\kappa$ and $\eta = \beta\kappa$, respectively, where α and β are percentages of the battery capacity. The intermediate charges must be determined at solve time.

To accomplish such a task, each arrival i must be associated with a initial charge, η_i, which represents the charge received from the previous visit minus the discharge observed while on route. However, because the PAP assumes each arrival is unique, bus visits must be associated with a unique bus ID to appropriately initialize charges for each visit. This visit/ID pair is represented as vector denoted by Γ, where the index represents visit i and the value is the ID. To propagate charges forward, another term γ is introduced to represent the next index bus i arrives. For example, assume $A = 3$ buses, $N = 5$ visits, and $\Gamma = [1, 2, 3, 1, 2]$. Γ informs us that the first and fourth visits correspond to bus 1 and the second and fifth visits correspond to bus 2. As γ maps from one visit to the next, it would take the value $\gamma = [3, 4, -1, -1, -1]$, where 0-indexing is assumed and -1 represents no further visits for bus Γ_i.

It is assumed that the charge received is proportional to the time spent charging. The charge rate for charger q is denoted as r_q, the charge at visit i as η_i, and the amount of discharge between visit i and the next visit of the same bus, γ_i. If visit i occured at charger q, the charge of the bus coming into visit γ_i would be

$$\eta_{\gamma_i} = \eta_i + p_i r_q - \lambda_i. \tag{4}$$

The binary decision decision variable w_{iq} is introduced to determine whether visit i uses charger q. This allows the charge of the bus coming into visit γ_i to be written in summation form as

$$\eta_{\gamma_i} = \eta_i + \sum_{q=1}^{Q} p_i w_{iq} r_q - \lambda_i \tag{5a}$$
$$w_{iq} \in \{0,1\} \tag{5b}$$

Maximum and minimum values for the charges are to be placed to ensure the battery is not overcharged and to guarantee sufficient charge for subsequent visits. Assuming each bus has its own upper bound, the upper bound for the bus corresponding to visit i is κ_i. The lower bound is assumed to be a percentage of the upper bound, and is written as $\nu\kappa_{\Gamma_i}$. The upper and lower bound constraints are written using the charge as

$$\eta_i + \sum_{q=1}^{Q} p_i w_{iq} r_q \leq \kappa_{\Gamma_i} \tag{6a}$$
$$\eta_i + \sum_{q=1}^{Q} p_i w_{iq} r_q - \lambda_i \geq \nu\kappa_{\Gamma_i} \tag{6b}$$

Note that the term $p_i w_{iq}$ is a bilinear term (two decision variables being multiplied together) which is nonlinear [22]. A standard way of linearizing a bilinear term that contains a continuous and integer term is by introducing a new term, an either/or constraint, and utilizing Big-M notation [22], [23]. Introducing the slack variable g_{iq} to be equal to $p_i w_{iq}$, g_{iq} can be define using the following constraints

$$p_i + (1 - w_{iq})M \geq g_{iq} \tag{7a}$$
$$p_i \leq g_{iq} \tag{7b}$$
$$Mw_{iq} \geq g_{iq} \tag{7c}$$
$$0 \leq g_{iq} \tag{7d}$$
$$\tag{7e}$$

where M is defined sufficiently large such that for constraints (7a) and (7b) if $w_{iq} = 1$ then the two equations will take the form of $p_i \leq g_{iq}$ and $p_i \geq g_{iq}$ effectively, stating $p_i = g_{iq}$. If $w_{iq} = 0$ then the two equations will take the form of $p_i \leq g_{eq}$ and $p_i \geq g_{iq} - M$ which deactivates the constraint. In a similar fashion, (7c) and (7d) state if $w_{iq} = 0$ then the constraints take the form $0 \geq g_{iq}$ and $0 \leq g_{iq}$ directly implying $0 = g_{iq}$. If $w_{iq} = 1$ then the constraints take the form $M \leq g_{iq}$ and $0 \geq g_{iq}$ deactivating the constraint. Putting the two sets of constraints together defines the linear representation of the bilinear term $w_{iq}p_i$.

The last consideration is the objective function. The goal of the MILP is to utilize slow chargers as much as possible. Thus, an assignment cost m_q and usage cost ϵ_q are associated with each charger, q. The cost for both the assignment and utilization of slow chargers is less than that of the fast chargers. The objective function has an assignment term, $w_{iq}m_q$, which is non-zero if charger q is used for visit i. Similarly, a usage term $g_{iq}\epsilon_q$ is non-zero only if charge is recieved for visit i at charger q. The resulting objective is defined as

$$\sum_{i=1}^{N} \sum_{q=1}^{Q} \left(w_{iq}m_q + g_{iq}\epsilon_q \right) \tag{8}$$

The optimization is performed to the constraints in (3) as well as the following constraints for battery charger dynamics:

$$\eta_i = \alpha\kappa_{\Gamma_i} \tag{9a}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq}r_q - \lambda_i = \eta_{\gamma_i} \tag{9b}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq}r_q - \lambda_i \geq \nu\kappa_{\Gamma_i} \tag{9c}$$
$$\eta_i + \sum_{q=1}^{Q} g_{iq}r_q \leq \kappa_{\Gamma_i} \tag{9d}$$
$$\eta_i \geq \beta\kappa_{\Gamma_i} \tag{9e}$$
$$p_i + (1 - w_{iq})M \geq g_{iq} \tag{9f}$$
$$p_i \leq g_{iq} \tag{9g}$$
$$Mw_{iq} \geq g_{iq} \tag{9h}$$
$$0 \leq g_{iq} \tag{9i}$$
$$\sum_{q=1}^{Q} qw_{iq} = v_i \tag{9j}$$
$$\sum_{q=1}^{Q} w_{iq} = 1 \tag{9k}$$
$$w_{iq} \in \{0, 1\} \tag{9l}$$

Constraints (9a)-(9e) provide initialization and terminal conditions as well as intermediate constraints to provide continuity in vehicle charges. Constraint (9a) states the first arrival for each bus is initialized with a charge of $\alpha\kappa_{\Gamma_i}$. Constraint (9b) defines that the previous charge, η_i, plus the charging done by charger q minus the discharge amount due to completing route, λ_i. Constraint (9c) is similar to (9b), except it states that the initial charge, η_i, plus the charge from q and the discharge from route, λ_i, must be greater than some percentage of its capacity, κ_{Γ_i}, in order to guarantee sufficient charge to complete the next route. Constraint (9d) states that the charging done for visit i cannot be greater than the capacity of the battery, κ_{Γ_i}. Constraint (9e) states that the last visit for each vehicle must have a minimum charge of $\beta\kappa_{\Gamma_i}$. Constraint (9e) is included to guarantee a minimum initial charge for the next working day.

The constraints (9g)-(9i) represent the linear form of the bilinear term the $g_{iq} := p_i w_{iq}$. The set of constraints (9j)-(9k) define the linking constraint between the queuing constraint and w_{iq} (i.e $1w_{i1} + 2w_{i2} + \cdots + Qw_{iQ} = v_i$) and enforces that only a single w may be selected per visit, respectively. The last constraint (9l) defines w_{iq} to be binary.

IV. Example

An example will now be presented to demonstrate the utility of the developed MILP. A description of the scenario is first presented followed by results.

A. Scenario

The given exampled utilizes $A = 40$ buses with $N = 220$ returns to the station divided between the A buses. Each bus has a 388 KWh battery that is required to stay above 25% charge (97 KWh) to maintain battery health, and the bus is assumed to begin the working day with 90% charge (349 KWh). Additionally, each bus is required to end the day with a minimum charge of 95% (368 KWh). Planning is done over a 24 hour time horizon. $Q = 9$ chargers are utilized where five of the chargers are slow charging (100 KWh) and four are fast charging (400 KWh). The slow chargers take longer, but are safer for the battery than the fast charger. Therefore, the slow chargers are modeled with a lower cost than than the fast charger for both assignment, $m_q = r_q$, and utilization, $\epsilon_q = r_q$, in the objective, (8).

The bus schedules are randomly generated. It is assumed that each bus has no more than 30 minutes between route departures. Bus route durations are completely random. They may vary anywhere from a minimum of the average time between the current and next arrival to a maximum of the next arrival time (i.e. $\frac{a_i+a_{\gamma_i}}{2}$ to a_{γ_i}). The discharge is assumed to be linear and is calculated via $\lambda_i = rand(\frac{a_i+a_{\gamma_i}}{2}, a_{\gamma_i})\zeta_i\gamma_i$ where $1 \leq i \leq N$ and ζ_i is the discharge rate for each bus.

The optimization was performed using the Gurobi MILP solver [24] on a machine running a quad-core Intel i7-9700 4.7 GHz processor. The optimizer ran for 5 seconds to produce the optimal solution.

B. Results

The schedule generated by the MILP is shown in Fig 5. The top graph indicates the slow charger usage, and the bottom indicates the fast charger usage. Although $Q = 9$ chargers were used, Fig 5 shows that only five chargers were utilized.

Each color in Fig 5 is used to identify the bus ID assigned. It is noted that the overlaps in Fig 5 indicate the waiting time of vehicle j while vehicle i charges. This is recognized by viewing the vertical bars. These bars indicate the time bus i is set to charge. The area before indicate waiting time and the area after indicates the time spent on the charger. Note that, based upon vehicles needs, bus i may arrive before bus j, but wait until after bus j charges before starting its charge (Fig 6). This is a very unintuitive schedule that would not be achieved through a greedy scheduling algorithm.

Fig 7 depicts the charge for every bus over the time horizon. Every vehicle begins at 90% charge, finishes at 95% charge, and never goes below 25% in the intermediate arrivals as stated in the constraints (9). Fig 8 represents the usage of each charger over time time horizon. The maximum amount of slow chargers used at any given time is three and only one fast charger is utilized at a time.

V. Conclusion

This work developed a MILP scheduling framework that optimally assigns slow and fast chargers to a BEB bus fleet assuming a constant schedule. The BAP was introduced with an example formulation and was then compared to the PAP. The PAP constructed on the BAP to allow the time spent on the charger, p_i, to be a decision variable. Because the original PAP required service time, p_i, to be given, linear battery dynamics were introduced to drive charging times. Additional constraints were also introduced to provide limits for the battery dynamics. An example was presented that demonstrated the ability of the formulation to utilize slow chargers when possible and fast chargers when necessary. The battery dynamic constraints to limit maximum and minimum charges was also observed.

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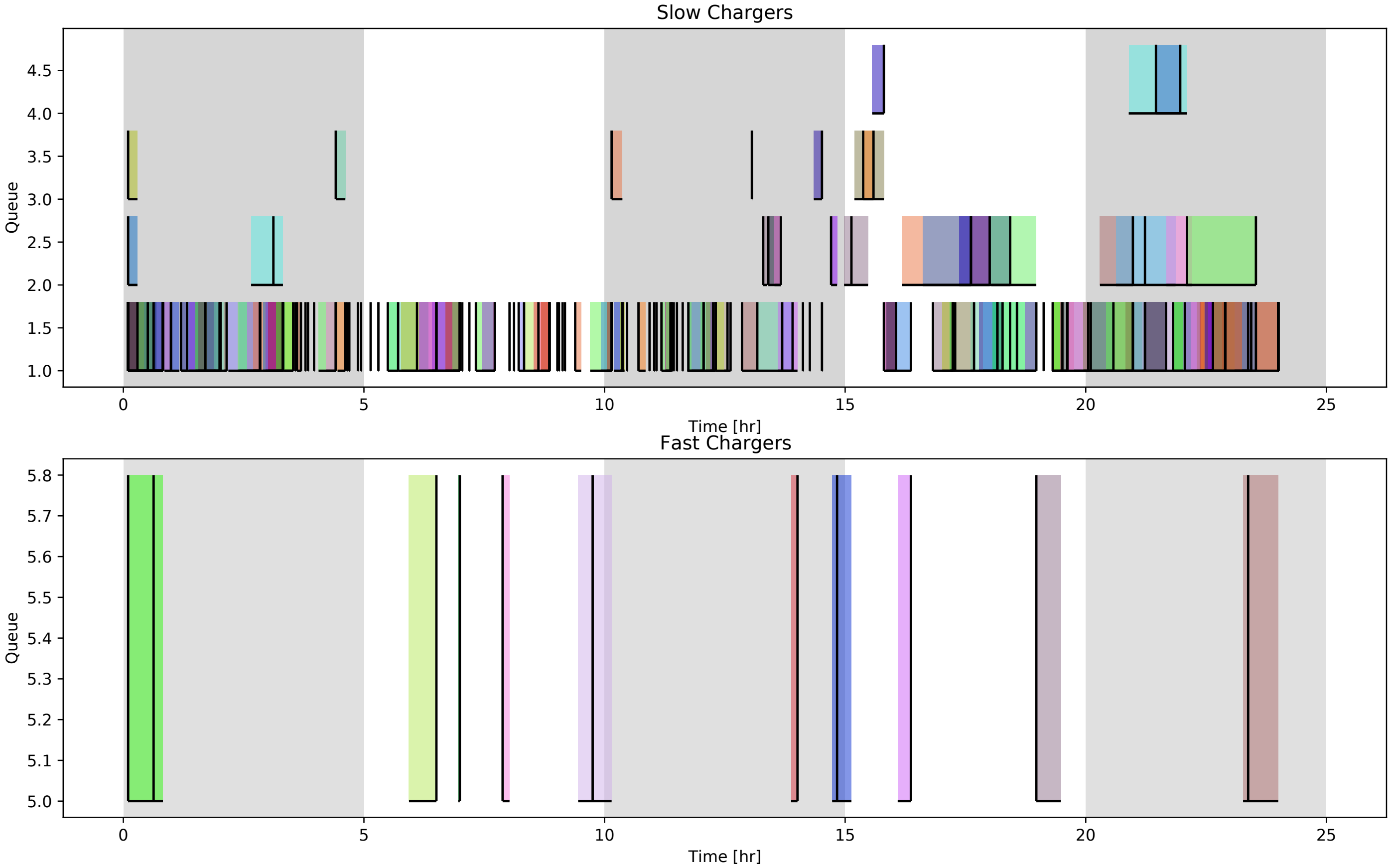

Fig. 5: Bus schedule generated with PAP MILP. Each color is assigned to a specific bus ID. The vertical bars indicate the time the vehicles are set to charge, the area before said bars indicate the waiting times for each visit $1 \leq i \leq N$. Similarly, the area after the bars indicate the time spent on the charger.

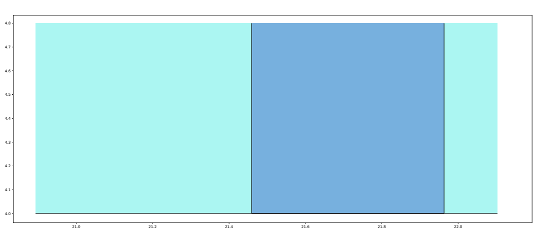

Fig. 6: Example of a bus i arriving before j, having i wait for j to arrive, and charge i after j is detached.

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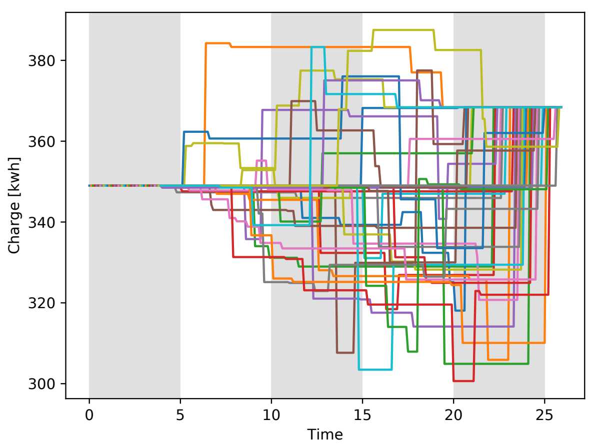

Fig. 7: Charge for each bus over the time horizon.

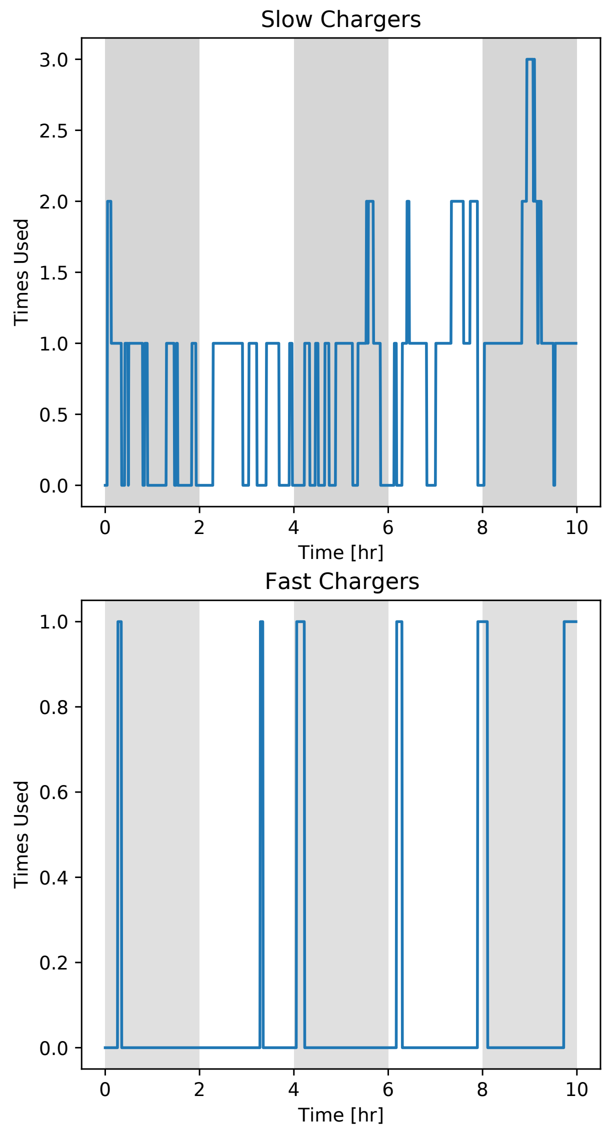

Fig. 8: Amount of slow and fast chargers used at any given time.