

# Bus Charging Schedule Simulated Annealing with MILP Constraints

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November 1, 2022

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## 1 Introduction

This document outlines the simulated annealing (SA) approach to the bus charging scheduling problem utilizing Mixed Integer Linear Programming (MILP) constraints as the method of determining feasible charging schedules. The problem statement is as follows: given a set of routes for a fleet of Battery Electric Buses (BEB), generate an optimal charging schedule to minimize the consumption cost (amount of electricity used over a certain time) and the demand cost (rate at which electricity is being used) within the constraints that the buses must maintain sufficient charge to complete the working day and do not have any delays in their respective routes.

Simulated Annealing (SA) shall be introduced and utilized as a means of finding the global optimum. The SA algorithm shall be constrained by a set of Mixed Integer Linear Program (MILP) constraints derived from the Position Allocation Problem (PAP). These constraints are set in place to ensure validity of the proposed charging schedules. A set of objective functions describing consumption cost and demand cost, as stated above, shall be minimized to reduce power consumption and total cost of using the BEBs.

## 2 Problem Description

Given a set of bus arrivals to a charging station  $i \in \{1, \dots, I\} = \mathcal{I} \subset \mathcal{Z}$  with a set of chargers to be queued  $q \in \{1, \dots, Q\} = \mathcal{Q} \subset \mathcal{Z}$  where the bus is indicated by an identification number  $b \in \{1, \dots, B\} = \mathcal{B} \in \mathcal{Z}$ . Each bus arrival,  $i$ , can be represented by the tuple:  $(b_i, a_i, e_i, u_i, d_i, v_i, \eta_i, \xi_i)$ , in which the ordered elements denote the bus identification number arrival time to the station, departure time from the station, initial charging time, charge end time, the charger queue for the bus to be placed into, and the initial State Of Charge (SOC). The term "tuple" refers to an ordered set.

It is assumed that each visit occurs over the time horizon  $T$ . The set of all arrivals is represented by the set  $\mathbb{I} = \{(b_i, a_i, e_i, u_i, d_i, v_i, \eta_i, \xi_i) : b_i, \xi_i \in B, a_i, e_i, u_i, d_i \in T, v_i \in Q\}$ . The concept of "arrivals" is derived from the PAP [7]. This idea of arrivals is useful in the sense that it is easy to describe the state of any arbitrary arrival; however, it also assumes that each arrival is unique (i.e. no two bus arrives twice) therefore a system must be put in place to track each bus over each arrival. That is why a bus identifier is placed in the tuple, and in that way each bus can be tracked over each arrival.

For each arrival a bus must be placed in a singular queue,  $v_i \in Q$ . The charger  $q$  is assumed to be either a fast or slow charger. The bus is not allowed to change queues mid-charge. The amount of time the bus is allowed to charge is dictated by the scheduled arrival time and required departure time,  $[a_i, e_i]$ . Although an arrival must be placed in a queue, if a bus does not require much charge, or none at all, partial charges, or no charging, is allowed. It is not allowed for the bus to charge over its battery capacity limit. The battery charging rate is modeled as linear, which remains accurate up to about an SOC of 80% charge [5].

Each bus arrival, with the exception of the last arrival for each bus, has a paired "route" that the bus must perform. This route, as one would expect, causes the bus to discharge by some certain amount. This paper assumes an average discharge over a period of time allowing an estimation to be calculated for each route,  $\Delta_i$ . The charge supplied while at the station is assumed to supply enough charge for each route (battery charge does not deplete to zero) with an additional battery capacity percentage,  $m$ , acting as a safety factor.

The scheduler's task shall be to schedule the set of arrivals  $\mathbb{I}$  to fulfill the minimum charge requirements over the time horizon  $T$  as well as minimize the demand cost as well as minimize over the consumption cost. The objective function and constraints are discussed in further detail in section Subsection 3.2.

## 3 Optimization Problem

This sections introduces the problem in the form of the objective function as well MILP constraints. The objective function is required to allow comparisons between candidate solutions. In the context of this formulation, the objective function is broken down into two major components as alluded in the introduction: consumption cost and demand cost. The constraints ensure that candidate solutions are in the feasible region. They are composed of a series of equations defined by decision variables which are unknown variables that are manipulated in the attempt to optimize the objective functions and input variables predefined input variables that are assumed to be known. Furthermore, the decision variables have components that are directly and indirectly manipulated. This will be further discussed in Subsubsection 3.1.2.

### 3.1 Parameter Definitions

This section defines the input variables and decision variables used by the system. The input variables are the parameters that are assumed to be known prior to optimizing the system. The decision variables are the values that the SA algorithm has the freedom to manipulate. The values produced by the SA algorithm will be interpreted as a candidate charging solution. This is further described in Section 4.

Table 1: Table of variables used in the paper.

Variable	Description
Input constants	
$C$	Penalty method gain factor
$B$	Number of buses in use
$I$	Number of total visits
$J(v, u, d)$	Objective function
$K$	Local search iteration amount
$Q$	Number of chargers
$\mathcal{T}$	Time horizon
$T$	Temperature
Input variables	
$\Delta_i$	Discharge of visit over route $i$
$\alpha_b$	Initial charge percentage time for bus $b$
$\beta_i$	Final charge percentage for bus $i$ at the end of the time horizon
$\delta_i$	Discharge rate for vehicle $i$ per mile
$\epsilon_q$	Cost of using charger $q$
$\kappa_b$	Battery capacity for bus $b$
$\rho_i$	Route distance after visit $i$
$\xi_i$	Value indicating the next index visit $i$ will arrive
$a_i$	Arrival time of visit $i$
$b_i$	ID for bus visit $i$
$e_i$	Time visit $i$ must exit the station
$k$	Local search iteration $k$
$m$	Minimum charge percentage allowed for each visit
$r_q$	Charge rate of charger $q$
Decision Variables	
$\eta_i$	Initial charge for visit $i$
$\phi_i$	Binary term to enable/disable charge penalty for visit $i$
$\psi_{ij}$	Tracks spatial overlap for visit pair $(i, j)$
$\sigma_{ij}$	Tracks temporal overlap for visit pair $(i, j)$
$d_i$	Detach time from charger for visit $i$
$p_{dem}(t)$	Demand cost
$s_i$	Amount of time spent on charger for visit $i$ (service time)
$u_i$	Initial charge time of visit $i$
$v_i$	Assigned queue for visit $i$

### 3.1.1 Input Variables

The input values of any MILP system are defined prior to the solving of the system. They define initial conditions, known state properties, etc. Roughly following the order in Table 1, each variable will be introduced.

$\Delta_i$  is the amount power required to complete the bus route after visit  $i$ . Because there is no route after the last visit,  $\Delta_I = 0$ . The discharge for visit  $i$  is defined by equation Equation 1.

$$\Delta_i = \delta_i * \rho_i \quad (1)$$

Where  $\delta_i$  is the amount of energy consumed by the bus per mile and  $\rho_i$  is the route mileage after visit  $i$ . As discussed before, since there is no route after the last visit  $\rho_I = 0$ .  $\alpha_b$  is the initial SOC percentage of bus  $b$  at the beginning of the working day. The initial SOC for bus  $b$  can be represented as

$$\eta_{i_0}^b = \alpha_b * \kappa_b. \quad (2)$$

Where  $\kappa_b$  is the battery capacity for bus  $b$ ,  $\eta_{i_0}^b$  is special notation that will temporarily be used to indicate the initial charge for bus  $b$ .  $\eta_i$  will be further discussed in Subsubsection 3.1.2.  $\epsilon_q$  is the cost for assigning a charger to queue  $q$ . This parameter is utilized by the objective function and is further discussed in Subsection 3.2.  $\xi_i$  represents the next arrival index for bus  $b_i$ . In other words, given a set of bus visit IDs  $b = \{1, 2, 3, 1\}$ . Using a starting index of 1,  $\xi_1 = 4$ .  $a_i$  and  $e_i$  are the arrival and departure times of bus visit  $i$  to the station, respectively.  $k$  represents the local iteration search for the SA algorithm. This is further discussed in Section 4. Lastly,  $r_q$  represents the rate of charge for the charger in queue  $q$ . As will be discussed in Subsection 3.2, fast chargers and slow chargers relate to high and low costs,  $\epsilon_q$ , respectively.

### 3.1.2 Decision Variables

Decision variables are the defined by the optimizer and are therefore unknown prior to running the optimization algorithm. In this case the optimizer is SA. Once SA has been ran and each of the decision variables have been specified and the fitness of the solution is defined by the objection functions outlined in Subsection 3.2 are determined. The variables will be broken into two sections: direct and indirect decision variables. Decision variables that are direct are values that the system has direct control over and indirect variables are those that are influenced by the direct.

**Direct Decision Variables** Decision variables that are direct are variables that can be immediately chosen by SA. The first two variables are  $u_i$  and  $d_i$ . They represent the initial and final charging times. These values must remain within range of the arrival time and departure time for visit  $i$ ,  $[a_i, e_i]$ . The last direct decision variable is the queue that bus visit  $i$  can be placed in to charge,  $v_i \in q$ .

**Indirect Decision Variables** Indirect decision variables are variables that are dependent on direct decision variables. For example  $\eta_i$  is the initial charge for visit  $i$ . These variables are chained together per bus by using the bus identifier,  $b$ , and next index,  $\xi_i$ . The initial charges must be chained so that the battery charge can be calculated per bus as it is charged and discharged over each visit,  $[u_i, d_i]$ .  $\phi_i$  is a boolean decision variable,  $\phi_i \in \{0, 1\}$ , that either enables or disables the charge penalty defined in Subsection 3.2.  $\sigma_{ij}$  and  $\psi_{ij}$  are used to indicate whether a visit pair  $(i, j)$  overlap the same space as show in Figure 1. These variables will be further elaborated on in Subsection 3.3.  $p_{dem}$  is the demand cost of the overall charging schedule. It is calculated at after all the decision variables have been assigned. This is further described in Subsection 3.2.

### 3.2 Objective Function

The objective function is used to compare the fitness of different candidate solutions against one another. This objective function takes in a set input variables and decision variables to calculate some value of measure. The calculated objective function value can either be maximized or minimized. The desired option is dependent on the problem to be solved as well as the formulation of said objective function. Let  $J$  represent the objective function. The objective function for this problem has four main considerations: charger assignment, consumption cost, demand cost, and sufficient charge.

Suppose the objective function is of the form  $\min J = AC(u_i, d_i, v_i, \eta_i) + PC(u_i, d_i, v_i)$ .  $AC(u_i, d_i, v_i, \eta_i)$  is the assignment cost, and  $PC(u_i, d_i, v_i)$  is the power usage cost. The assignment cost represents the costs of assigning a bus to a particular queue as well as the chosen charging period,  $[u_i, d_i]$  as shown in Equation 3.

$$AC(u_i, d_i, v_i, \eta_i) = \sum_{i=1}^I \epsilon_{v_i} (d_i - u_i) + \frac{1}{2} C \phi_i (\eta_i - m \kappa_i)^2 \quad (3)$$

Where  $v_i \in q$  is the charger index,  $u_i$  is the initial charge time,  $d_i$  is the detach time for visit  $i$ ,  $\psi_i$  is a binary decision variable,  $m$  is the minimum charge percentage allowed,  $\kappa_i$  is the battery capacity for visit  $i$ , and  $\eta_i$  is the initial charge for visit  $i$ . The first term in the summation represents the calculation of the cost for assigning a bus to queue  $q$  (i.e. cost of using the charger multiplied by the usage time). The second term is the penalty function that is either enabled or disabled by  $\phi_i$  which is discussed in Subsection 3.3. This form is the most common form that penalty methods are found in [6]. Note that the variables  $\psi_i$  and  $\eta_i$  are both decision variables that are being multiplied together. This is called a bilinear term. Using a traditional MILP solver, this would require linearization [9]; however, because SA handles nonlinearities easily these bilinear terms will be ignored [8].

The power cost contains the demand cost and the consumption cost. It can be divided into two components: demand cost and the consumption cost. The demand cost quantifies the amount of power being used over a given period and adjusts the cost accordingly. The consumption cost calculates the total amount of power being consumed by the chargers. The power cost is shown in Equation 4. Note that the demand cost is written as a function. This is because it is calculated post generation of the candidate solution with no obvious MILP representation.

$$PC(u_i, d_i, v_i) = DemandCost(schedule) + \sum_{i=1}^I r[v_i] (d_i - u_i) \quad (4)$$

As stated before, the demand cost is calculated based on 15 minute increments (0.25 hours). This cost is also referred to as the peak 15. The peak 15 is represented by Equation 5.

$$p_{15}(t) = 0.25 \int_{t-15}^t p(\tau) d\tau \quad (5)$$

Which represents the energy used over the last 15 minutes. Because worst case must be assumed to always ensure enough power is supplied, the maximum value found is retained as represented in Equation 6.

$$p_{max}(t) = \max_{\tau \in [0, t]} p_{15}(\tau) \quad (6)$$

Because the cost has a minimum threshold, a fixed minimum cost is introduced. In a similar manner as  $p_{max}$ , the maximum value is retained.

$$p_{dem}(t) = \max(p_{fix}, p_{max}(t))s_r \quad (7)$$

Where  $s_r$  is the demand rate. Equation 7, again, retains the largest  $p_{15}$  value with a starting fixed value of  $p_{fix}$ . To calculate this numerically, an integration algorithm is required to iteratively calculate the  $p_{15}(t)$ . In turn,  $p_{dem}(T)$  can be defined. This process is defined in Algorithm 1.

**Algorithm:** DemandCost

**Input:** Candidate solution: (schedule)

**Output:** Demand cost: (p-dem)

```

1 begin
2   p15  $\leftarrow \emptyset$ ;
3   for  $dt \leftarrow 0$  to  $T$  do
4     | Union( $p_{15}$ , Integrate( $schedule, (dt, dt+0.25)$ ))
5   end
6   p-old  $\leftarrow$  p-new  $\leftarrow$  p-dem  $\leftarrow$  p-fix;
7   foreach element  $p$  in  $p_{15}$  do
8     | p-old  $\leftarrow$  p-new;
9     | p-new  $\leftarrow$   $p$ ;
10    | if  $p_{new} > p_{old}$  then
11      | | p-dem  $\leftarrow$  p-new;
12      | | p-old  $\leftarrow$  p-new;
13    | end
14  end
15  return  $p_{dem}$ 
16 end

```

**Algorithm 1:** Algorithm to calculate the demand cost.

Where **schedule** is the set  $\mathbb{I} = \{(b_i, a_i, e_i, u_i, d_i, v_i, \eta_i) : b_i \in B, a_i, e_i, u_i, d_i \in T, v_i \in Q\}$  and **p-fix** is the initial, fixed cost.

### 3.3 Constraints

Now that a method of calculating the fitness of a schedule has been established, a method for determining the feasibility of a schedule must be established. Feasible schedules require that the schedule maintain a certain list of properties. These properties are enforced by a set of constraints derived from the MILP PAP. The constraints must ensure no overlap temporally or spatially, receives enough charge to complete route after each visit  $i$ , bus visit  $i$  cannot be over charged, each visit,  $i$ , departs on time. The aforementioned constraints are shown in Equation 8.

$$u_i - d_j - (\sigma_{ij} - 1)T \geq 0 \quad (8a)$$

$$v_i - v_j - (\psi_{ij} - 1)Q \geq 0 \quad (8b)$$

$$\sigma_{ij} + \sigma_{ji} \leq 1 \quad (8c)$$

$$\psi_{ij} + \psi_{ji} \leq 1 \quad (8d)$$

$$\sigma_{ij} + \sigma_{ji} + \psi_{ij} + \psi_{ji} \geq 1 \quad (8e)$$

$$\Delta_i = \delta_i(a_{\xi_i} - d_i) \quad (8f)$$

$$\eta_{\xi_i} = \eta_i + r_{v_i}(d_i - u_i) - \Delta_i \quad (8g)$$

$$\kappa_i \geq \eta_i + r_{v_i}(d_i - u_i) \quad (8h)$$

$$\eta_i - m_{k_i} \leq T(1 - \phi_i) \quad (8i)$$

$$\eta_i - m_{k_i} < T\phi_i \quad (8j)$$

$$a_i \leq u_i \leq d_i \leq e_i \leq T \quad (8k)$$

Where the valid queue Equation 8a - Equation 8e define the spatial and temporal constraints of the system. These constraint enforce that the buses are placed in such a way that only one bus is allowed at a charger at any given time. Particularly Equation 8a determines if the initial charge time of visit  $i$  is after the final charge time of visit  $j$ . Similarly, Equation 8b determines if visit  $i$  or visit  $j$  are scheduled to be on the same queue. Equation 8c describes whether one of the visits come after the other temporally while Equation 8d describes if the chargers are placed in different queues. Equation 8e pulls all the previous constraints together and verifies that at least one of the conditions are true for each visit pair  $(i, j)$ . The concept of the temporal and spatial constraints can be visualized by Figure 1. The y-axis represents the possible queues for a bus visit to be placed into and the x-axis represents the time that can be reserved for each visit. The shaded rectangles represent time that has been scheduled for each bus visit. The set of constraints Equation 8a - Equation 8e aim to ensure that these shaded rectangles never overlap. Equation 8f calculates the discharge for the route after visit  $i$ . Equation 8g calculates the initial charge for the next visit for bus  $b_i$ . Equation 8h ensures that the bus is not being over charged. Equation 8i and Equation 8j are used to enable and disable the penalty method in Equation 3. This is done by checking if the initial charge for visit  $i$  is greater than or equal to the minimum allowed charge. Equation 8k ensures the continuity of the times (i.e. the arrival time is less than the initial charge which is less than the detach time which is less than the time the bus exits the station and all must be less than the time horizon).

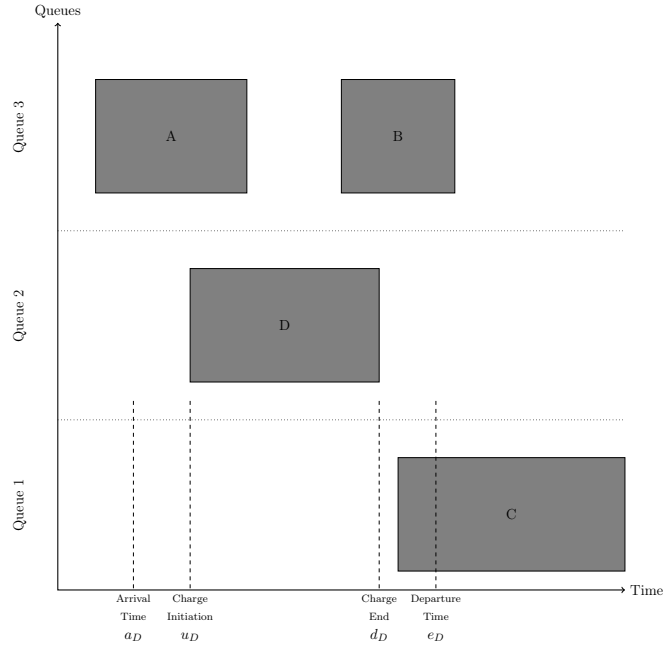


Figure 1: Visualization of the temporal and spatial aspect of the aspect of the schedule.

## 4 Simulated Annealing

SA is a local search (exploitation oriented) single-solution based (as compared to population based) meta-heuristic approach in which its main advantage is its simplicity both theoretically and in its implementation as well its inherit ability to overcome nonlinearities [1, 8]. This model is named after its analogized process where a crystalline solid is heated then allowed to cool very slowly until it achieves its most regular possible crystal lattice configuration [3]. There are five key components to SA: initial temperature, cooling

schedule (temperature function), generation mechanism, acceptance criteria, local search iteration count (temperature change counter) [4].

#### 4.1 Cooling Equation (Experimental)

The initial temperature and cooling schedule are used to regulate the speed at which the solution attempts to converge to the best known solution. When the temperature is high, SA encourages exploration. As it cools down (in accordance to the cooling schedule), it begins to encourage local exploitation of the solution [10, 3]. There are three basic types of cooling equations as shown in Figure 2 [4]. The different types merely dictate the rate at which we begin disallowing exploration. A linear cooling schedule is defined by Equation 9.

$$T[n] = T[n - 1] - \Delta_0 \quad (9)$$

with  $T[0] = T_0$  and  $\Delta_0 = 1/2 \text{ } ^\circ\text{C}$  in Figure 2. A geometric cooling schedule is mostly used in practice [4]. It is defined by Equation 10.

$$T[n] = \alpha T[n - 1] \quad (10)$$

where  $\alpha = 0.995$  in Figure 2. An Exponential cooling schedule is defined by the difference equation is define as Equation 11.

$$T[n] = e^\beta T[n - 1] \quad (11)$$

where  $\beta = 0.01$  in Figure 2. The initial temperature,  $T_0$ , in the case of Figure 2, is set to  $500^\circ\text{C}$  and each schedule's final temperature is  $1^\circ\text{C}$ .

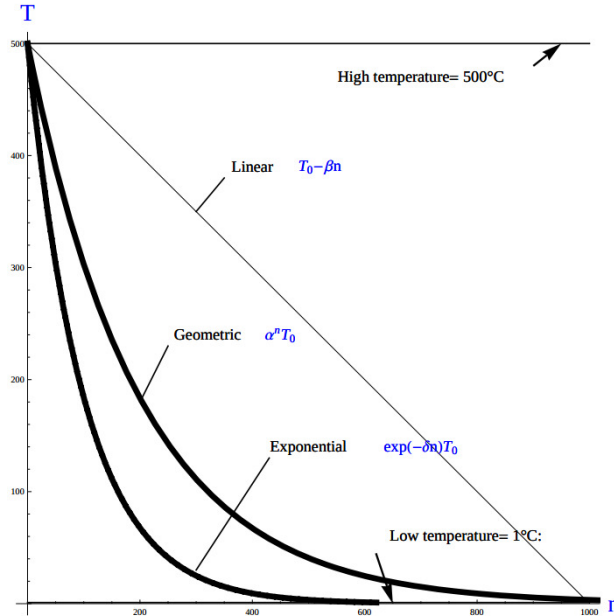


Figure 2: Cooling equations



## 4.2 Acceptance Criteria

Acceptance criteria describes the method to accept or reject a given candidate solution. In SA, if a new candidate solution is more fit than the currently stored solution it is always accepted as the new solution. However, within SA, worse candidate solutions may be accepted as the new solution. The probability of accepting the candidate solution is described by the function  $\exp(\frac{J(x)-J(x')}{T})$  where  $J(\cdot)$  is the objective functions described in Subsection 3.2. The probability of acceptance is a function of the cooling equation just described and difference of the current solution and a new candidate solution. Let  $\Delta E \equiv J(x) - J(x')$  where  $x$  is the current solution and  $x'$  is the new candidate solution. The probability of acceptance of  $x'$  is defined by Equation 12 [4].

$$f(x, x', T) = \begin{cases} 1 & \Delta E > 0 \\ e^{-\frac{\Delta E}{T}} & \text{otherwise} \end{cases} \quad (12)$$

## 4.3 Generation Mechanisms

Generation mechanisms in SA are used to generate random solutions to propose to the optimizer, these are known as candidate solutions. In the case of the problem statement made in Section 2, five generation mechanism shall be used: new visit, slide visit, new charger, remove, new window. The purpose of each of these generators is to assign new visits to a charger, adjust a bus visits initial and final charge time within the same time frame/queue, remove a bus from a charger, and place a bus visit into a new time slot/queue. Each generator will be discussed in more detail in Subsubsection 4.3.2.

These generator mechanisms will in turn be utilized by three wrapper functions. The purpose of the route generation to create a set of bus route data to feed to the SA algorithm. Although, strictly speaking, is not a part of the SA algorithm. It is vital in specifying the initial conditions and "setting the stage" for the SA algorithm to solve. The schedule generation is to used create candidate solutions for SA to compare with other solutions, and the perturb schedule generator is used to take a candidate solution and alter it slightly in an attempt to fall into a global/local minimum.

### 4.3.1 Generator Input/Output

This section discusses in detail the expected inputs and output of each generator. It is important to discuss these parameters in order to have an understanding of the generating algorithms derived. The input consists of the bus visit index of interest, information about the current state of arrivals,  $\mathbb{I}$ , and the current state of the chargers' availability,  $\mathbb{C}$ . The output of each generator affects the tuple of decision variables  $(v_i, u_i, d_i) \subset \mathbb{I}$ .

**Generator Input** Each generator has the tuple input of  $(i, \mathbb{I}, \mathbb{C})$  where  $i$  is the visit index,  $\mathbb{I}_i$  is the tuple  $(b_i, a_i, e_i, u_i, d_i, v_i, \eta_i, \xi_i)$  (Section 2), that describes the set of visits generated by the route generation algorithm (Section 4.3.3), and  $\mathbb{C}$  is the set that describes the availability for all chargers  $q \in \mathcal{Q}$ . In other words,  $\mathbb{C}$  defines the set of times when the chargers are not being utilized or are "inactive".

To derive  $\mathbb{C}$ , consider its inverse,  $\mathbb{C}'$ , which is the set of "active" time periods for each charger,  $\mathbb{C}' = \bigcup \{\mathbb{C}'_q : q \in \mathcal{Q}\}$  where  $\mathbb{C}'_q \subset \mathbb{C}'$  describes the active times for charger  $q$ . Focusing on an individual charger, consider  $\mathbb{C}'_q$  before a schedule has been imposed upon it,  $\mathbb{C}'_q \in \emptyset$ . In other words, no buses have been assigned to be charged over the time period of  $[u_i, d_i]$ . After the scheduling process is complete,  $\mathbb{C}'_q$  will have a set of active periods of the form  $\mathbb{C}'_q \in \{[u_j, d_j] : j \in \mathcal{J}\}$  where  $\mathcal{J} \subset \mathcal{I}$ . For  $\mathbb{C}'_q$  to be of value, its compliment is to be found,  $\mathbb{C}_q$ .

To determine the inverse of  $\mathbb{C}'_q$ , begin by noting  $\mathbb{C}'_q \cap \{[u_j, d_j] : j \in \mathcal{J}\} = \emptyset$ , in other words is said to be disjoint [2]. The inverse of a disjoint set can be found by the De Morgan Law as shown in Equation 13. Using Equation 13, the set of inactive periods can be written as  $\mathbb{C}_q \equiv \bigcup \{[u_j, d_j]' : j \in \mathcal{J}\}$ .

$$(A \cap B)' = A' \cup B' \quad (13)$$

**Generator Output** The output,  $x'_i \equiv (v_i, u_i, d_i) \in \mathbb{I}_i$  defines tuple of the chosen queue, initial charge time, and detach time from the generator,  $(v_i, u_i, d_i)$ . The nature of SA implies that the generators have a sense of randomness. Because of that, some of the generators may have multiple choices for what  $x'_i$  may be. Let the set of candidates for the output be defined as  $x'_i \in X'_i$ .

### 4.3.2 Generators

This section describes and outlines the algorithm pool for the different generator types that are utilized in the wrapper functions. Note that to satisfy constraints,  $B$  extra idle queues that provide no power to the bus. Because of this, the set of queues is fully defined as  $q \in \{1, \dots, Q, Q+1, \dots, Q+b\}$  where  $Q$  is the total amount of chargers and  $b$  is the bus ID. The use case for this is for when a bus is not to be placed on a charger, it will be placed in the queue,  $v_i \in \{Q+1, \dots, Q+b\}$ , which will satisfy the constraints above while allowing the bus to be "set aside" while others charge.

**New visit** The new visit generator describes the process of moving bus  $b$  from the idle queue,  $v_i \in \{Q+1, \dots, Q+b\}$  to a valid charging queue,  $v_i \in \{1, \dots, Q\}$ . Line 2 initializes the set of solutions to the empty set. Line 3 loops through each charger availability set and line 4 loops thorough each of the available ranges, denoted as  $L$  and  $U$  for lower and upper free time. Line 5 checks if the range  $[a_i, e_i] \subset [L, U]$ , and line 6 adds it to the set of candidates. Line 10 chooses picks a candidate solution,  $x'_i \in X'_i$ , with a discrete uniform distribution which is denoted by  $\mathcal{U}_{\{\cdot\}}$ .

**Algorithm: New Visit**

**Input:** ( $\mathbb{S}$ )

**Output:**  $x'_i$

```

1 begin
2    $i \leftarrow \{i : i \in \mathbb{S}\}$  ;                                     /* The index of the visit  $i$  */
3    $a_i \leftarrow \{a_i \in \mathbb{I}_i : \mathbb{I}_i \in \mathbb{I} \subset \mathbb{S}\}$  ;           /* Get the arrival time for visit  $i$  */
4    $e_i \leftarrow \{e_i \in \mathbb{I}_i : \mathbb{I}_i \in \mathbb{I} \subset \mathbb{S}\}$  ;         /* Get the departure time for visit  $i$  */
5    $X'_i \leftarrow \emptyset$  ;                                     /* Begin with the empty set */
6   foreach  $\mathbb{C}_q \in \mathbb{C}$  do                                       /* For set of available times for charger  $q$  */
7     foreach  $C \in \mathbb{C}_q$  do                                     /* For each inactive region in  $\mathbb{C}_q$  */
8       if findFreeTime( $C, (a_i, e_i)$ )  $\notin \emptyset$  then      /* If there is time available in  $C$  */
9          $x'_i \rightarrow \text{findFreeTime}(C, (a_i, e_i))$  ;      /* Assign  $x'_i$  to the found region */
10         $X'_i \cup x'_i$  ;                                       /* Add  $x'_i$  to the set of candidates */
11      end
12    end
13  end
14  return  $\mathcal{U}_{X'_i}$                                            /* Return a random candidate */
15 end

```

**Algorithm 2:** New visit algorithm

The algorithm to find free time is defined in Algorithm 3.  $L$  and  $U$  are the lower and upper bound of the time between scheduled times. The possible use cases are depicted in Figure 3.

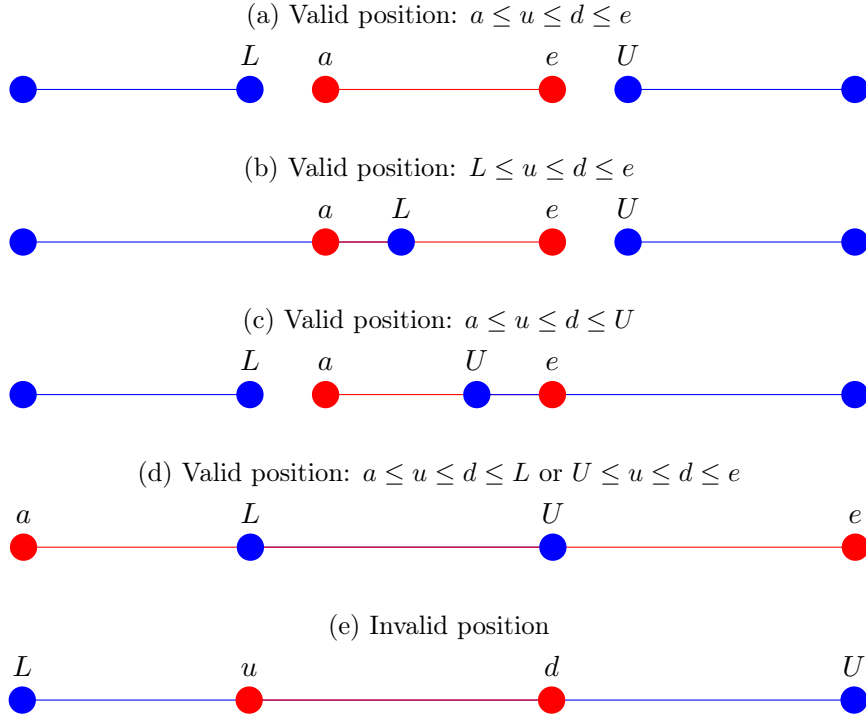


Figure 3: Outlines the different cases that requested time and charger allocated time can overlap

**Algorithm:** Find Free Time

**Input:**  $(L, U, a, e)$

**Output:**  $(u, d)$

```

1 begin
2   if  $L \leq a$  and  $U \geq e$  then                                     /* If  $L < a < e < U$ ] (Figure 3a) */
3     |  $u \leftarrow \mathcal{U}_{[a,e]}$ ;
4     |  $d \leftarrow \mathcal{U}_{[u,e]}$ ;
5   end
6   else if  $L > a$  and  $U \geq e$  then                                   /* Else if  $a < L < e < U$  (Figure 3b) */
7     |  $u \leftarrow \mathcal{U}_{[L,e]}$ ;
8     |  $d \leftarrow \mathcal{U}_{[u,e]}$ ;
9   end
10  else if  $L \leq a$  and  $U < e$  then                                   /* Else if  $L < a < U < e$  (Figure 3c) */
11    |  $u \leftarrow \mathcal{U}_{[a,U]}$ ;
12    |  $d \leftarrow \mathcal{U}_{[u,U]}$ ;
13  end
14  else if  $L > a$  and  $U < e$  then                                   /* Else if  $a \leq u \leq d \leq L$  or  $U \leq a \leq d \leq e$  (Figure 3d) */
15    |  $u \leftarrow \mathcal{U}_{[a,L],[U,e]}$ ;
16    |  $d \leftarrow \mathcal{U}_{[u,L],[u,e]}$ ;
17  end
18  else                                                             /* Otherwise the bus cannot be scheduled in this time frame (Figure 3e) */
19    |  $u \leftarrow \emptyset$ ;
20    |  $d \leftarrow \emptyset$ ;
21  end
22  return  $(u, d)$ 
23 end

```

**Algorithm 3:** Find free time algorithm searches and returns the available time frames

**Slide visit** Slide visit is used for buses that have already been scheduled. Because of the constraint Equation 8k there may be some room to move  $u_i$  and  $d_i$  within the window  $[a_i, e_i]$ . Two new values,  $u_i$  and  $d_i$  are selected with a uniform distribution to satisfy the constraint  $a_i \leq u_i \leq d_i \leq e_i$ . Lines 1 and 2 get the arrival and departure times, and line 3 gets the current queue that visit  $i$  is assigned to.

**Algorithm:** Slide Visit

**Input:**  $\mathbb{S}$

**Output:**  $x'_i$

```

1 begin
2    $X'_i \in \{[u_i, d_i]\}$  ;                               /* Initialize candidates with the current charge time */
3   foreach  $C \in \mathbb{C}_q$  do                                  /* For each inactive region in  $\mathbb{C}_q$  where  $q = v_i$  */
4     if findFreeTime( $C, (a_i, e_i)$ )  $\notin \emptyset$  then    /* If there is time available in  $C$  */
5        $x'_i \rightarrow$  findFreeTime( $C, (a_i, e_i)$ ) ;      /* Assign  $x'_i$  to the found reigion */
6        $X'_i \cup x'_i$  ;                                    /* Add  $x'_i$  to the set of candidates */
7     end
8   end
9   return  $x'_i$ 
10 end

```

**Algorithm 4:** Slide Visit Algorithm

**New charger** The new charger generator takes a visit  $\mathbb{I}_i$  and changes the charger it is on while maintianing the same charge time,  $[u_i, d_i]$ . Similarly to 2, the new candidate,  $x'_i$ , must be checked before being added to the set  $X'_i$ .

**Algorithm:** New Charger

**Input:** Visit index, route data, Charger data:  $(i, \text{route-data}, \text{charger-data})$

**Output:**  $x'_i$ :  $(v, u, d)$

```

1 begin
2    $X'_i \leftarrow \emptyset$  ;                               /* Begin with the empty set */
3   foreach  $\mathbb{C}_q \in \mathbb{C}$  do                                  /* For set of availabile times for charger  $q$  */
4     foreach  $C \in \mathbb{C}_q$  do                                  /* For each inactive region in  $\mathbb{C}_q$  */
5       /* If  $L < a < e < U$  or  $a < L < e < U$  or  $L < a < U < e$  or  $L > a$  or  $U < e$  */
6       if  $L \leq a \ \& \ U \geq e$  or  $L > a \ \& \ U \geq e$  or  $L \leq a \ \& \ U < e$  or
7          $a \leq u \leq d \leq L$  or  $U \leq a \leq d \leq e$  then
8          $X'_i \cup (q, u_i, d_i)$ 
9       end
10    end
11  end
12  return  $\mathcal{U}_{X'_i}$ 
13 end

```

**Algorithm 5:** New Charger Algorithm

**Remove** The remove generator simply removes a bus from a charger queue and places it in its idle queue,  $v_i \in \{Q, \dots, Q + B\}$ .

**Algorithm:** Remove

**Input:** Visit index, route data, Charger data:  $(i, \text{route-data}, \text{charger-data})$

**Output:**  $x'_i: (v, u, d)$

```

1 begin
2    $v \leftarrow Q + b$  ;
3    $u \leftarrow \text{route-data}[i].u$ ;
4    $d \leftarrow \text{route-data}[i].d$ ;
5   return  $(v, u, d)$ 
6 end

```

**Algorithm 6:** Remove algorithm

**New Window** New window is a combination of the remove and then new visit generators (Section 11 and Section 6). By this it is meant that current scheduled tuple  $(v_i, u_i, d_i)$  is removed and added back in as if it were a new visit.

**Algorithm:** New Window

**Input:** Visit index, route data, Charger data:  $(i, \text{route-data}, \text{charger-data})$

**Output:**  $x'_i: (v, u, d)$

```

1 begin
2    $v \leftarrow \text{route-data}[i].v$ ;
3    $u \leftarrow \text{route-data}[i].u$ ;
4    $d \leftarrow \text{route-data}[i].d$ ;
5    $(v, u, d) = \text{Remove}(v, u, d)$ ;
6    $(v, u, d) = \text{NewVisit}(v, u, d)$ ;
7   return  $(v, u, d)$ 
8 end

```

**Algorithm 7:** New window algorithm

#### 4.3.3 Generator Wrappers

This section covers the algorithms utilized to select and execute different generation processes for the SA process. The generator wrappers are the method immediately called by SA. Each wrapper utilizes the generators previously described and returns either metadata about the bus routes or a new valid charger schedule.

**Route Generation** The objective of route generation is to create a set of metadata about bus routes given the information in Figure 5. Specifically, the objective is to generate  $I$  routes for  $B$  buses. Each visit will have an initial charge (specified for first visit only), arrival time, departure time, final charge (minimum allowed charge specified for final visit only).

This is created by following the "GenerateSchedule" state in the state diagram found in Figure 4. In essence the logic is as follows: Generate  $B$  random numbers that add up to  $I$  visits (with a minimum amount of visits set for each bus). For each bus and for each visit, set a departure time that is between the range  $[\text{min}_{\text{rest}}, \text{max}_{\text{rest}}]$  (Figure 5), set the next arrival time to be  $j \cdot \frac{T}{J}$  where  $j$  is the  $j^{\text{th}}$  visit for bus  $b$  and  $J$  is the total number of visits for bus  $b$ . Finally, calculate the amount of discharge from previous arrival to the departure time.

**Algorithm:** RouteGeneration

**Input:** Route YAML metadata: (mdata)

**Output:** Array of route events: (route-data)

```

1 begin
2   while not schedule-created do
3     arrival-new  $\leftarrow$  0.0;
4     arrival-old  $\leftarrow$  0.0;
5     departure-time  $\leftarrow$  0.0;
6     schedule-created  $\leftarrow$  false;
7     foreach  $b \in B$  do
8       foreach  $n \in J_b$  do
9         arrival-old  $\leftarrow$  arrival-new;
10        if  $j = J_b$  then
11          final-visit = true;
12        end
13        else
14          final-visit = false;
15        end
16        departure-time  $\leftarrow$  DepartureTime(arrival-old, final-visit);
17        arrival-new  $\leftarrow$  current-visit  $\ast \frac{T}{J_b}$ ;
18        discharge  $\leftarrow$  discharge-rate*(next-arrival, depart-time) ;
19        Union(route-data, (arrival-old, departure-time, discharge));
20      end
21    end
22    schedule-created  $\leftarrow$  Feasible(route-data);
23    SortByArrival(route-data);
24  end
25 end

```

**Algorithm 8:** Route generation algorithm

Where `discharge-rate` is read from YAML data shown in Figure 5, the `Departure` algorithm is shown in Algorithm 9, and the `Feasible` method is used to determine if the generated schedule is valid (conditions covered in Subsection 3.3). This is done by attempting to generate a schedule that is in the solution space. This is further elaborated on later.

**Algorithm:** DepartureTime

**Input:** Previous arrival and final visit flag: (arrival-old and final-visit)

**Output:** Next departure time: (depart)

```

1 begin
2   if final-visit then
3     depart  $\leftarrow$  T;
4   end
5   else
6     depart  $\leftarrow$  arrival-old +  $\mathcal{U}_{[\text{min-rest}, \text{max-rest}]}$ ;
7   end
8   return depart
9 end

```

**Algorithm 9:** Departure time algorithm

**Schedule Generation** The objective of this generator is to generate a candidate solution to the given schedule. To generate a candidate solution the generator is given the route schedule data that was previous generated. A bus is picked at random,  $b \in B$ , then a random route is picked for bus  $b$ . The new arrival

generator is then utilized. This process is repeated for each visit. The state diagram is depicted in the state diagram in Figure 6 and outlined in Algorithm 10.

**Algorithm:** ScheduleGeneration

**Input:** Route data: (route-data)

**Output:** Candidate charging schedule: (schedule)

```

1 begin
2   schedule  $\leftarrow \emptyset$ ;
3   for  $i$  in  $I$  do
4     bus  $\leftarrow \mathcal{U}_{[0,B]}$ ;
5     visit  $\leftarrow \mathcal{U}_{[0,I]}$ ;
6     Union(schedule, NewVisit( $(visit.a, visit.e)$ ));
7   end
8   return schedule
9 end

```

**Algorithm 10:** Schedule generation algorithm

Where **schedule** is  $\mathbb{I} = \{(b_i, a_i, e_i, u_i, d_i, v_i, \eta_i) : b_i \in B, a_i, e_i, u_i, d_i \in T, v_i \in Q\}$ .

**Perturb Schedule** As described in SA, local searches are also employed to try and exploit a given solution [8]. The method that will be employed to exploit the given solution is as follows: pick a bus, pick a visit, pick a generator. This state diagram is depicted in Figure 7 and outlined in Algorithm 11.

**Algorithm:** PerturbSchedule

**Input:** Schedule candidate solution: (schedule)

**Output:** Perturbed schedule: (schedule)

```

1 begin
2   for  $i$  in  $I$  do
3     visit  $\leftarrow \mathcal{U}_{[0,I]}$ ;
4     generator  $\leftarrow \mathcal{U}_{[0,generator-count]}$ ;
5     schedule  $\leftarrow$  GeneratorCallback [generator](visit, route-data, charger-data);
6   end
7   return schedule
8 end

```

**Algorithm 11:** Perturb schedule algorithm

## 5 Optimization Algorithm

This final section combines the generation algorithms and the optimization problem into a single algorithm. The objective is to outline the SA process from start to finish. Algorithm 8 generates a set of bus routes utilizing the route metadata in Figure 5. The initial temperature and cooling schedule will be selected prior to execution and passed into the SA optimization algorithm. A new candidate solution will be generated. The candidate solution will be checked if it is feasible by using the equations from Subsection 3.3. For each step in the cooling schedule will have  $K$  iterations to attempt to find a local maxima. Each perturbation to the system is then compared to the current candidate solution. If the new candidate solution is better it is kept; however, if the candidate solution is worse, the solution may still be kept with a calculated probability as described in Subsection 4.2. This process is summarized in Algorithm 12.

**Algorithm: SA PAP****Input:** Bus route metadata: (file-path)**Output:** Optimal charging schedule: (schedule)

```

1 begin
2    $T_0 \leftarrow \text{InitTemp}()$ ;
3    $T_{\text{schedule}} \leftarrow \text{GetCoolingEquation}()$ ;
4   route-metadata  $\leftarrow \text{LoadYaml}(\text{file-path})$ ;
5   routes  $\leftarrow \text{RouteGeneration}(\text{route-metadata})$ ;
6   best-solution  $\leftarrow v \in \text{ScheduleGeneration}(\text{routes})$ ;
7   foreach  $T \in T_{\text{schedule}}(T_0)$  do
8     candidate-solution  $\leftarrow \text{ScheduleGeneration}(\text{routes})$ ;
9     if InSolutionSpace(candidate-solution) then
10      foreach  $k \in K$  do
11        del-sol  $\leftarrow J(\text{candidate-solution}) - J(\text{best-solution})$ ;
12        if del-sol  $\leq 0$  then
13          | best-solution  $\leftarrow$  candidate-solution;
14        end
15        else if del-sol  $\geq 0$  then
16          | best-solution  $\leftarrow$  candidate-solution with probability  $\exp(\text{del-sol}\tau_k)$ ;
17        end
18        schedule  $\leftarrow \text{PerturbSchedule}(\text{schedule})$ ;
19      end
20    end
21  end
22 end

```

**Algorithm 12:** Simulated annealing approach to the position allocation problem

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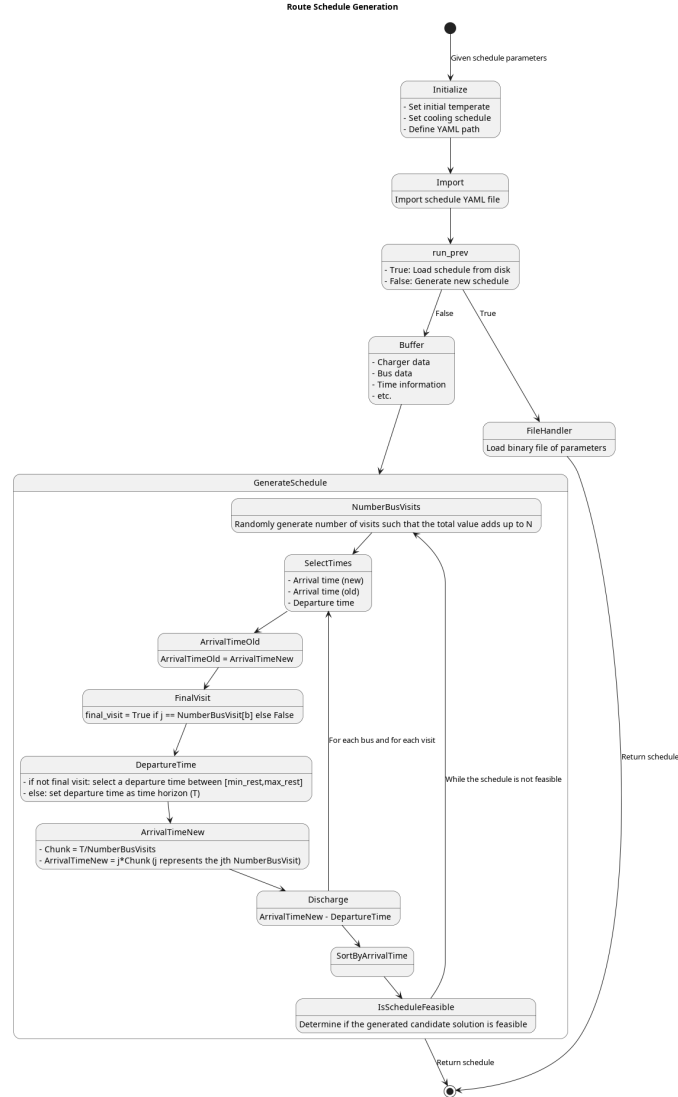


Figure 4: Route generation state diagram

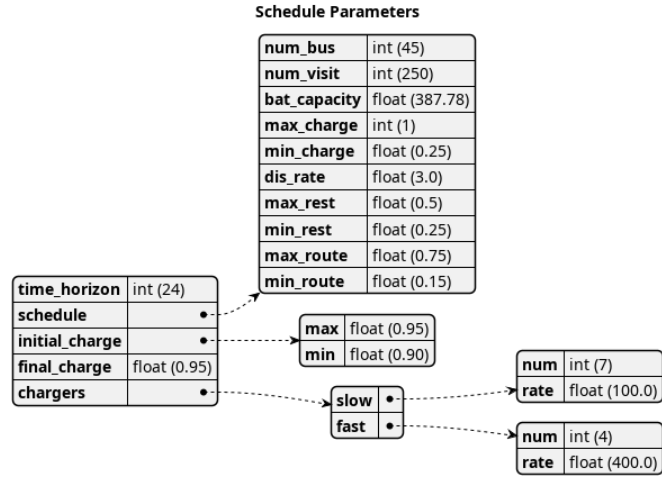


Figure 5: Route YAML file with example data

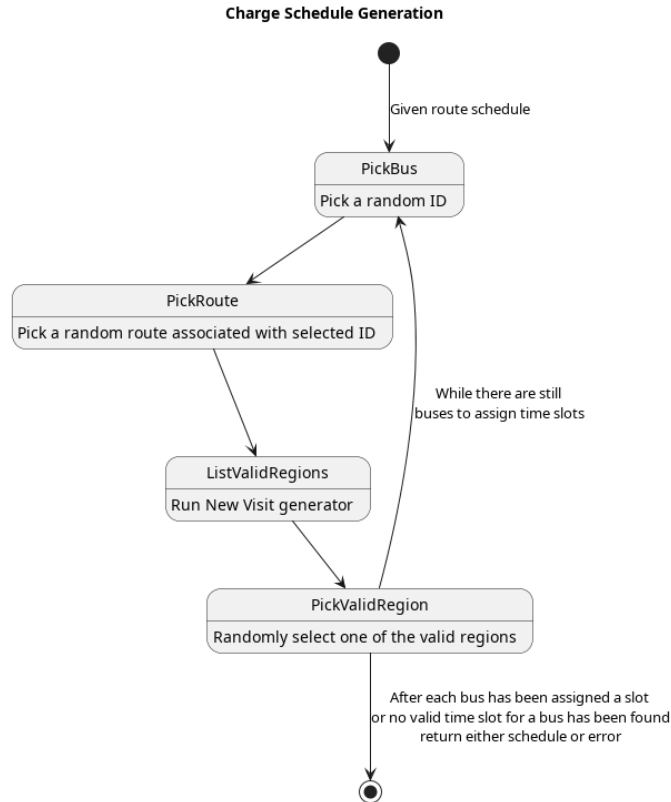


Figure 6: Charge solution state diagram

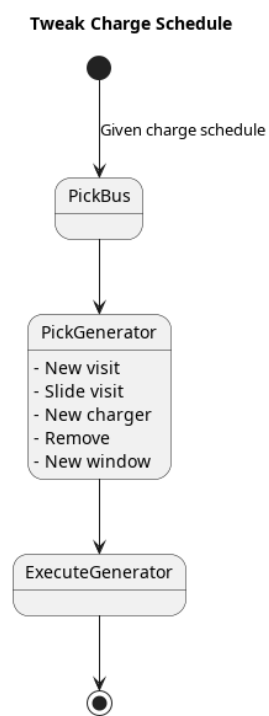


Figure 7: Solution perturb state diagram