1)

Given

$$\dot{W} = -V_g e_{py} \sin\left(\chi^{\infty} \tfrac{2}{\pi} \tan^{-1}(k_{path} e_{py})\right)$$

Find:

a) Find $\dot{W}$ given $W = \frac{1}{2} e_{py}^2$

b) Show $\dot{W} \leq 0$ for e_{py}

Solution:

$$W(e_{py}) = \frac{1}{2} e_{py}^2$$

$$\dot{W}(e_{py}) = \frac{1}{2} \cdot 2 e_{py} \dot{e}_{py} = \underline{e_{py} \dot{e}_{py}}$$

We know

$$\underline{\dot{e} = V_g \sin(\chi - \chi_q)}$$

and

$$\chi = \chi_q + \chi^d(e_{py})$$

$$\Rightarrow \quad \underline{\chi^d(e_{py}) = \chi - \chi_q = -\chi^{\infty} \frac{2}{\pi} \tan^{-1}(k_{path} e_{py})}$$

$$\Rightarrow \quad \dot{W} = e_{py}\left(V_g \sin\left(-\chi^{\infty} \frac{2}{\pi} \tan^{-1}(k_{path} e_{py})\right)\right)$$

$$= \underline{\underline{-V_g e_{py} \sin\left(\chi^{\infty} \frac{2}{\pi} \tan^{-1}(k_{path} e_{py})\right)}}$$

Theorem

Let $V: D \to \mathbb{R}$ be a continuously differentiable function such that

1) $V(0) = 0$

2) $\forall x \in D \setminus \{0\}, \; V(x) > 0$

S) If $\forall x \in D \; \dot{V}(x) \leq 0$ then $x = 0$ is stable

AS) If $\forall x \in D \; \dot{V}(x) \leq 0$

$\forall x \in D \setminus \{0\} \; \dot{V}(x) < 0$

then $x = 0$ is asymptotically stable

If $e_{py} < 0 \Rightarrow \tan^{-1}(\cdot) < 0 \Rightarrow \dot{W}(\cdot) < 0$

If $e_{py} > 0 \Rightarrow \tan^{-1}(\cdot) > 0 \Rightarrow \dot{W}(\cdot) < 0$

The only way $\dot{W}(\cdot) = 0$ is if $e_{py} = 0$

$\therefore$ the system is A.S.

2)

Given:

$$\chi_q = \operatorname{atan2}(q_e, q_n) + 2\pi m.$$

Find:

a) Describe in your own words why $+2\pi m$ is necessary

b) Give two examples on how it helps UAV turn in correct direction.

Solution:

The course angle, as measured from north, is given by

$$\chi_q := \operatorname{atan2}(q_e, q_n) \qquad (1)$$

(1) can cause undesireable behavior because atan2(·) returns a value between $\pm\pi$. The $2\pi m$ is to ensure that

$$-\pi \leq \chi_q - \chi \leq \pi$$

$$-\pi \leq \chi^d(e_{py}) \leq \pi$$

This bound is to ensure that the UAV turns "the right way"

As described in the book:

If χ_q is slightly smaller than π, the MAV will turn right

If χ_q is slightly smaller than $-\pi$ the MAV will turn left

↳ In other words, take the long way around.

1/

3)

Given:

$w = 1/2\,(d-\rho)$

Find:

$\dot{w}$

Solution:

$w = 1/2\,(d-\rho)^2$ $\quad$ ρ is constant

$= \frac{1}{2}(d-\rho)(d-\rho) = \frac{1}{2}(d^2 - 2d\rho - \rho^2)$

$\Rightarrow \quad \dot{w} = \frac{1}{2}(2d\dot{d} - 2\dot{d}\rho - 0) = \underline{(d-\rho)\dot{d}}$

$\underline{\dot{d} = V_g \cos(\chi - \phi)}$

$\chi = \chi^d(d-\rho, \lambda) = \chi^o + \lambda \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)$

$= \underline{\phi + \lambda\frac{\pi}{2} + \lambda \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)}$

$\Rightarrow \quad \dot{w} = V_g \cos\left(\cancel{\phi} + \lambda\frac{\pi}{2} + \lambda\tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right) \cancel{-\phi}\right)$

And assume $\lambda = -1$ (CCW)

$\dot{w} = -V_g \cos\left(\pi/2 + \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)\right)$

$\sin(\theta + \pi/2) = \cos(\theta)$

$\Rightarrow \quad \underline{\dot{w} = -V_g \sin\left(\tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)\right)}$

4)

Given:

(10.13) & (10.14) in book

Find:

Show how they are related

Solution:

$$x^c(t) = \phi + \lambda\left[\frac{\pi}{2} + \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)\right]$$

$$x_\lambda(d-\rho, \lambda) = x^0 + \lambda \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)$$

We know $x^0 = \phi + \lambda\frac{\pi}{2}$, if we plug it into $x_\lambda(\cdot)$

$$x_\lambda(d-\rho, \lambda) = \phi + \lambda\frac{\pi}{2} + \lambda \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)$$

$$= \phi + \lambda\left[\frac{\pi}{2} + \tan^{-1}\left(k_{orbit}\left(\frac{d-\rho}{\rho}\right)\right)\right]$$

$$= x^c(t)$$