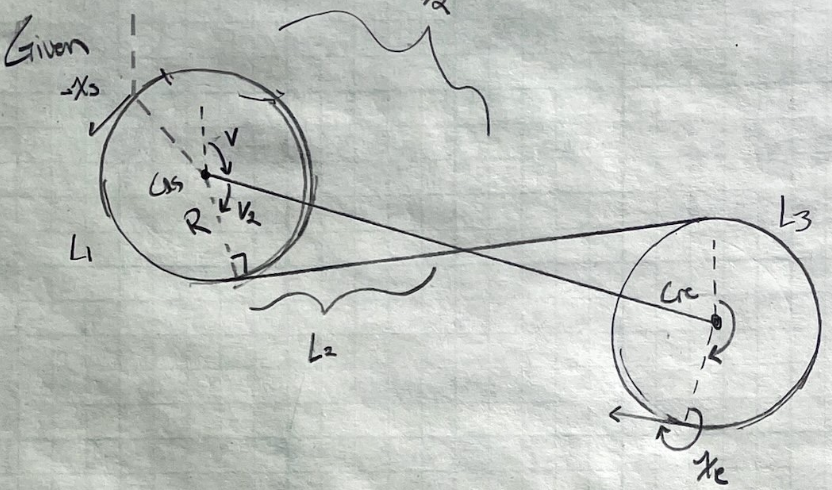
1)

Given



$$L_3 = \sqrt{\lambda^2 + 4R^2} + R(2\pi + (\chi_s + \pi/2) - (V + V_2)) +$$
$$R(2\pi + (\chi_e - \pi/2) - (V + V_2 - \pi))$$

Find: Derive L_3

Solution:

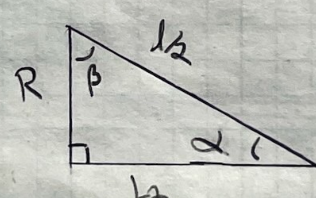
The total length is devided into three parts

1) Arc length about C_{rs}
2) Straight line
3) Arc length about C_{re}

Let $L = L_1 + L_2 + L_3$

L_2 is the easiest so lets start there

$L_2 = \sqrt{\lambda^2 - 4R^2}$ is found based on the figure to the right



Lets now derive L_1. Consider the figure given on page (1).

To find γ, we begin by recalling how to find the angle between two points:

$$\theta = \text{atan2}^{-1}(y, x) \quad \text{where} \quad y = y_2 - y_1, \quad x = x_2 - x_1$$

$$\Rightarrow \gamma = \text{atan2}^{-1}(C_{rey} - C_{lsy}, C_{rex} - C_{lsx})$$

Note that γ is measured from $\pi/2$

$$\Rightarrow \gamma = \text{atan2}^{-1}(C_{rey} - C_{lsy}, C_{rex}\ C_{lsx}) + \pi/2$$

To find γ_1 consider the figure of the triangle on page (1)

$\gamma_1 := \beta$

$$\Rightarrow \cos\beta = \frac{2R}{\ell} \quad \Rightarrow \beta = \gamma_2 = \cos^{-1}\left(\frac{2R}{L}\right)$$

Note that arc length $\lambda = R\theta$. Therefore the angle traveled is

$$(2\pi + (x_s + \pi/2) - (\gamma + \gamma_2))$$

one rev + start ang − end ang

$$\Rightarrow L_1 = R(2\pi + (x_s + \pi/2) - (\gamma + \gamma_2))$$

Similarly for L_3, begin by defining the angles

To get exit angle: $(x_e - \pi/2)$

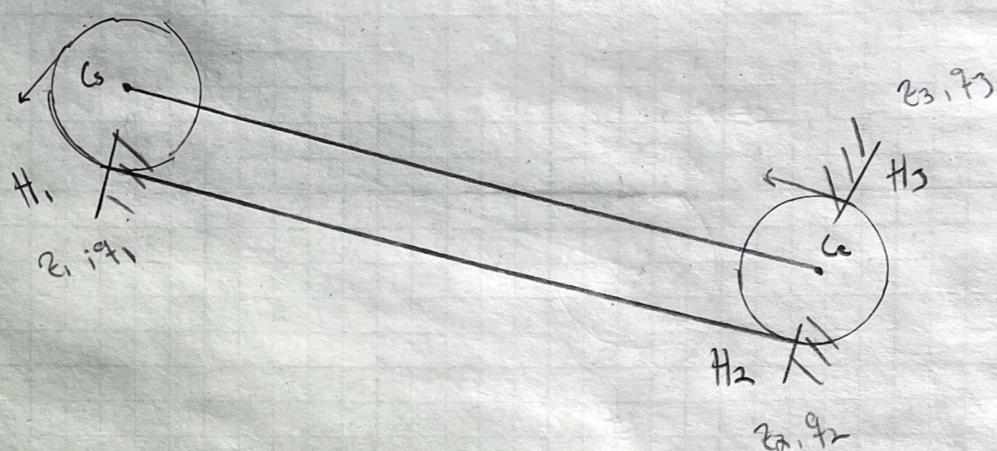
To get entry angle: Note the angles are the same, but inverted

$(\gamma + \gamma_2 - \gamma_1)$ gives use the "opposite side".

$$\Rightarrow L_3 = R(2\pi + (x_e - \pi/2) - (\gamma + \gamma_2 - \gamma_1))$$

2)

Given:



Find: Derive and explain the half spaces for switching in the Left - Straight - Left case.

Solution:

To find the point on the circle that gives us a point on the half plane as we can be found using

$z_i = C_x + R\theta$

$\Rightarrow$ $z_1 = C_s + R(R_z(\pi/2)\, q_1)$

$z_2 = C_e + R(R_z(\pi/2)\, q_1)$

$R_z(\pi/2)$ is chosen to get the vector normal to the trajectory

$z_3 = P_e$

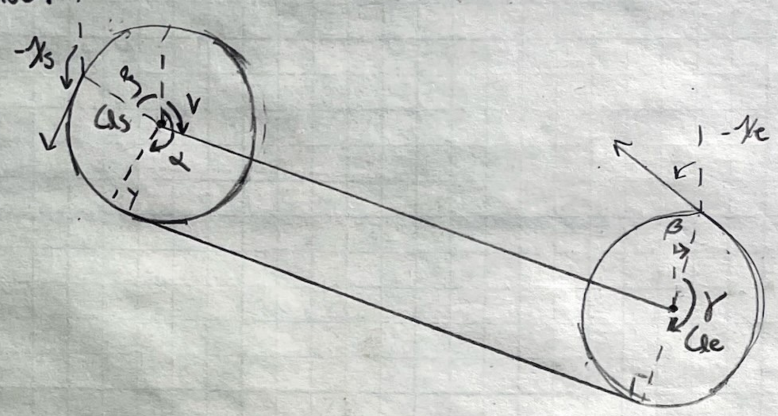
$q_1 = q_2 = \dfrac{C_e - C_s}{\|C_e - C_s\|}$ $\qquad$ $q_3 = R_z(\chi_e)\,[1\ 0\ 0]^T$

Rotate unit vector to get new orientation base on χ_e

The half spaces are defined at the point of transition from line/circle or vice versa.

3)

Given:



$$L = \| C_{rs} - C_{re} \| + R\langle 2\pi + \langle \chi_s + \pi/2 \rangle - \langle \nu + \pi/2 \rangle \rangle +$$
$$R\langle 2\pi + \langle \nu + \pi/2 \rangle - \langle \chi_e + \pi/2 \rangle \rangle$$

Find: Derive L

Solution:

Similar to problem (1) let

$L = L_1 + L_2 + L_3$ define the same components as before. Lets also begin with the straight line.

Note that the path trajectory is parallel to the line between the origins of each circle. Furthermore, the distances are also equivalent.

$\Rightarrow$ $L_2 = \| C_{rs} - C_{re} \|$

To compute L_1

$\nu = \text{atan2}^{-1}(C_{rey} - C_{rsy}, C_{rex} - C_{rsx}) + \pi/2$

$\alpha = \nu + \pi/2$ (the lines drawn for trajectory and connected origins is a rectangle)

$\xi = \chi_s + \pi/2$

$\Rightarrow$ $L_1 = R\langle 2\pi + \langle \chi_s + \pi/2 \rangle - \langle \nu + \pi/2 \rangle \rangle$

For L_3

$\beta = \gamma_e + \pi/2$ (entry is perpendicular to entry trajectory)

$\gamma = \nu + \pi$ (same but opposite)

$\Rightarrow$ $L_3 = R\langle 2\pi + \langle \nu + \pi/2 \rangle - \langle \gamma_e + \pi/2 \rangle \rangle$