4.54.2)

Given:

$$\dot{x} = -(1+u)x^3 - x^5$$

Find:

Investigate Input-to-State stability

Solution

Suppose $V(x) = \frac{x^2}{2} = \alpha_1(x) = \alpha_2(x)$

$$\begin{aligned}\dot{V}(x) = x\dot{x} &= x(-(1+u)x^3 - x^5) \\ &= -x^4 - ux^4 - x^6\end{aligned}$$

If $u > -\frac{x^6}{x^4} = -x^2$

$$\sqrt{u} < |x|$$

Then $\dot{V}(x)$ will be negative definite and if $-W_3(x) = -x^4$

$$\dot{V}(x) < -W_3(x)$$

Therefore the system is input-to-stable by (1)