4.32.4)

Given

$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_1 - x_2 - x_3 - x_1 x_3 \dot{x}_3 = (x_1 + 1)x_2$$

Find

Investigate whether the origin is stable, asympotically stable, or unstable.

Solution

$$J = \frac{\partial f}{\partial x} = \left.\begin{bmatrix} -1 & 0 & 0 \\ -x_3 - 1 & -1 & -x_1 - 1 \\ x_2 & x_1 + 1 & 0 \end{bmatrix}\right|_{x=[0]} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Finding the eigenvalues by using `eivals(*)` in Maxima

```
/* Define equation */
xd1: -x1;
xd2: -x1 - x2 - x3 - x1*x3;
xd3: (x1 + 1)*x2;

/* Solve for roots */
solve([xd1=0, xd2=0, xd3=0])

/* Calculate jacobian */
J: jacobian([xd1, xd1, xd3], [x1, x2, x3]);

/* Evaluate jacobian at roots */
float(eivals(psubst([x1=0, x2=0, x3=0], J)))
```

Theorem: Let $x = 0$ be an equilibrium point for

$$\dot{x} = f(x)$$

where $f : D \to \mathbb{R}^n$ is continuously differentiable and D is in a neighborhood of the origin. Let

$$A = \frac{\partial f}{\partial x}(0)$$

Let λ_i denote an eigenvalue of A

1. If $\forall \lambda_i Re(\lambda_i) < 0$, then the origin is asympotically stable
2. If $\exists \lambda_i$ such that $Re(\lambda_i) > 0$ then the origin is not stable

Therefore, by the previous stated theorem, the system is asympotically stable.