(x)$ such that $||\psi(x)|| < k \forall x$ where k is a positive constant.

Solution:

Begin by defining the linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Let $V(x) = x^T P x$, therefore

$$
\begin{aligned}
\dot{V}(x) \quad &= x^T(P + P^T)\dot{x} \\
&= x^T(PA + P^T A)x + x^T(PB + P^T B)u \\
&= x^T(PA + A^T P)x + x^T(PB + B^T P)u \\
&= x^T(PA + A^T P)x + x^T(PB)u = -B^T P x u
\end{aligned}
$$

Let $C = B^T P x$, therefore

$$\dot{V} = x^T(PA + A^T P)x + x^T(PB)u = yx$$

Which means signifies that the system is passive. Furthermore, setting $u = 0$ and stating $x = 0 \implies y = 0$ which means that the system is zero state observable. Therefore we choose the control to by $u = -ky$ where $k > 0$.