14.42

Given:

$$\begin{aligned}&\dot{x} = Ax + Bu\\ &PA + A^T P \leq 0\\ &P > 0\end{aligned}$$

Find:

A globally stabilizing feedback law $u = -\psi(x)$ such that $||\psi(x)|| < k \forall x$ where k is a positive constant.

Solution:

Begin by defining the linear system

$$\begin{aligned}&\dot{x} = Ax + Bu\\ &y = Cx\end{aligned}$$

Let $V(x) = x^T P x$, therefore

$$\begin{aligned}\dot{V}(x) &= x^T(P + P^T)\dot{x}\\ &= x^T(PA + P^T A)x + x^T(PB + P^T B)u\\ &= x^T(PA + A^T P)x + x^T(PB + B^T P)u\\ &= x^T(PA + A^T P)x + x^T(PB)u = -B^T P x u\end{aligned}$$

Let $C = B^T P x$, therefore

$$\dot{V} = x^T(PA + A^T P)x + x^T(PB)u = yx$$

Which means signifies that the system is passive. Furthermore, setting $u = 0$ and stating $x = 0 \implies y = 0$ which means that the system is zero state observable. Therefore we choose the control to by $u = -ky$ where $k > 0$.
