4.3.3

Given

$$\dot{x}_1 = x_2(1 - x_1^2)$$
$$\dot{x}_2 = -(x_1 + x_2)(1 - x_2^2)$$

Find

See (4.3.1)

Solution

Suppose $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$

$$\dot{V}(x) = x_1\dot{x}_1 + x_2\dot{x}_2 = x_2^2x_1^2 - x_2^2$$

For values $<< 1$ (much smaller than 1) $x_2^2x_1^2 << x_2^2$. Therefore, the origin is asymptotically stable for points near the origin.