1)

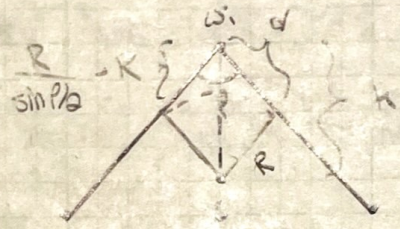
Given:

$$d = \frac{R}{\tan\frac{\rho}{2}}$$

Find:

Prove this

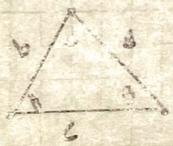
Solution:



Note; Line is tangent to the circle.

Let $h = w - c := R + \frac{R}{\sin\rho/2} - R = \frac{R}{\sin\rho/2}$

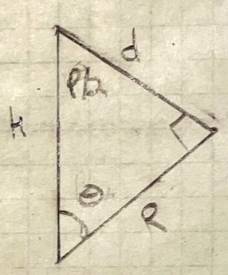
Law of sines:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (1)$$

The triangle we are looking at is

$$180° = \pi = \theta + \frac{\pi}{2} + \frac{\rho}{2}$$



$$\Rightarrow \frac{\pi}{2} = \theta + \frac{\rho}{2} \rightarrow \theta = \frac{\rho}{2} + \frac{\pi}{2}$$

$$\frac{d}{\sin\theta} = \frac{R}{\sin\rho/2} \Rightarrow d = \frac{R\sin\theta}{\sin\rho/2} = \frac{R\sin(\rho/2 + \pi/2)}{\sin(\rho/2)}$$

$$\Rightarrow d = R\frac{\cos(\rho/2)}{\sin(\rho/2)} = \frac{R}{\tan(\rho/2)}$$

2)

Given:

$$k = \frac{R}{\sin\frac{\beta}{2}}$$

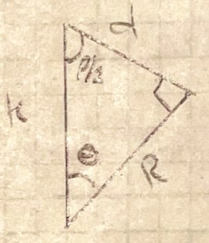
Find:

Prove this

Solution:

Using the image from problem (1) and Theorem (1).

⇒ The main objective is to find the distance from W_i to the circle. Lets begin with the triangle



$$l := k - R \Rightarrow k = l + R$$

$$\theta := \beta/2 + \pi/2$$

$$\Rightarrow \frac{k}{\sin\pi} = k = \frac{R}{\sin\beta/2} = l + R$$

$$\Rightarrow l = \frac{R}{\sin\beta/2} - R$$

3)

Given:

A singularity happens in a straight-line switching half plane.

Find:

- When does this happen?
- Use math to justify.

Solution:

We know that the half plane is defined by

The unit vector pointing in the direction of $\overline{W_i W_{i+1}}$

$$q = \frac{W_{i+1} - W_i}{\|W_{i+1} - W_i\|}$$

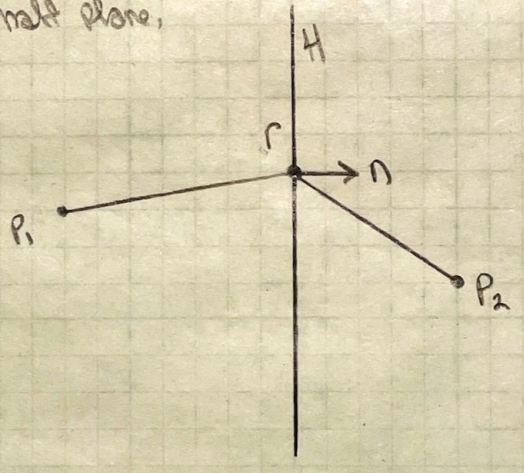
and the normal vector that separates $W_{i-1}W_i$ from $\overline{W_i W_{i+1}}$

$$n_i = \frac{q_{i-1} - q_i}{\|q_{i-1} + q_i\|}$$

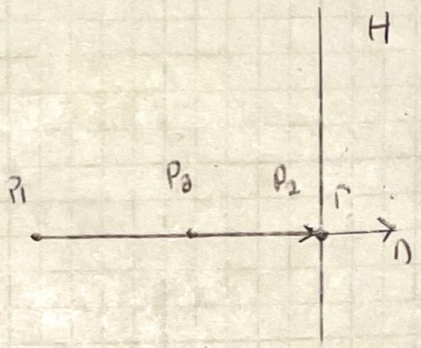
and finally the half plane

$$H(r, n) := \{ p \in \mathbb{R}^3 : (p - r)^T n \geq 0 \}$$

Which states that same vector defined be p and r in $\mathbb{R}^3$ and normal vector $n \in \mathbb{R}^3$ can be used to determine if p has past the half plane.



Now suppose the scenario:



$$n_3 = \frac{P_2 + P_3}{\|P_2 + P_3\|}$$

Note:

$$P_2 = \frac{W_{iH} - W_i}{\|W_{iH} - W_i\|}$$ which is a unit vector

$$n_3 = \frac{-P_3 + P_3}{\|-P_3 + P_3\|}$$ ⇒ singularity

This will happen every time one wants to backtrace over the previous ray path traveled.

4).

Given:

$$C = W_i - \left(\frac{R}{\sin\frac{\rho}{2}}\right)\frac{q_{i-1} - q_i}{\|q_{i-1} - q_i\|}$$

Find:

- When singularites occur
- Defend with math

Solution:

C defines the center of the fillet.

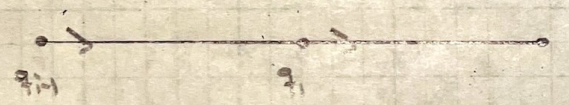
$\frac{R}{\sin\frac{\rho}{2}}$ defines the distance from W_i to C

$\frac{q_{i-1} - q_i}{\|q_{i-1} - q_i\|}$ defines the average direction normal.

If $\sin\frac{\rho}{2} = 0$, meaning that the next waypoint goes back to original direction

↳ $\rho = 0, 2\pi, \ldots, 2\pi n$

The second singularity occurs when $q_{i-1} = q_i$. In other words, the two waypoints create a straight line.



Because q_i is a unit vector pointing in the direction of travel, the magnitude does not matter.

5)

Given:

$$|W_f| = |W| + \sum_{i=2}^{N} \left(R(\pi - \rho_i) - \frac{2R}{\tan\frac{\rho_i}{2}} \right)$$

Find:

Derive this function

Given:

We know

$$|W| := \sum_{i=2}^{N} \|w_i - w_{i-1}\|$$ which defines the length of the waypoint path.

Now to calculate the length of the fillet traversal.

$L = r \cdot \theta$ & $r := R$ $\theta := \pi - \rho$ (derived in problem 1)

$\Rightarrow$ $L = R(\pi - \rho)$

Furthermore, the straight line that is removed by the fillet d (derived in (1)) is

$$d = \frac{R}{\tan\frac{\rho}{2}}$$ Note,

$\Rightarrow$ $$|W_f|' = |W| + \sum_{i=2}^{N} \left(R(\pi - \rho_i) - \frac{2R}{\tan\frac{\rho_i}{2}} \right)$$

Straight line

Plus arc lengths

Subtract out linear portions that are replaced.

ch11a_line_and_fillet_path_manager

April 1, 2022

0.1 Problem 0: Path manager implementation

Implement the following and ensure the unit tests pass. * `chap11/path_manager_utilities.py`: * `inHalfSpace(...)` * `chap11/line_manager.py`: * `construct_line(...)` * `line_manager(...)` * `chap11/fillet_manager.py`: * `construct_fillet_line(...)` * `construct_fillet_circle(...)` * `fillet_manager(...)`

0.1.1 Hints on implementation

- `np.linalg.norm(...)` will be very useful for computing the norm of a vector
- Make sure to set the airspeed on your paths. The airspeed can be found using `path.airspeed = get_airspeed(waypoints, ptr)`
- Problems 1 and 2 help guide you to accounting for singularities
- The path manager does not clear after use, so you will see the previous paths on the display if you run all of them sequentially.

0.1.2 Note on the unit tests

There will be no unit tests for the `line_manager(...)` and `fillet_manager(...)` functions. Furthermore, there are several scenarios where dividing by zero becomes an issue. As these are singularities that can be handled in various ways, there is not a correct answer. Thus, these cases may not all arise in the unit tests, but they will be tested in the problems 1 and 2.

[1]:
```
import mav_sim.parameters.planner_parameters as PLAN
import numpy as np
from mav_sim.chap3.mav_dynamics import DynamicState
from mav_sim.chap11.run_sim import run_sim
from mav_sim.message_types.msg_sim_params import MsgSimParams
from mav_sim.message_types.msg_waypoints import MsgWaypoints

from mav_sim.chap11.waypoint_viewer import WaypointViewer
from mav_sim.chap3.data_viewer import DataViewer
from mav_sim.tools.display_figures import display_data_view, display_mav_view

# The viewers need to be initialized once due to restart issues with qtgraph
if 'path_view' not in globals():
    print("Initializing waypoint viewer")
    global waypoint_view
    waypoint_view = WaypointViewer()
```

```
if 'data_view' not in globals():
    print("Initializing data_view")
    global data_view
    data_view = DataViewer()

# Initialize the simulation parameters
sim_params_default = MsgSimParams(end_time=130., video_name="cha11.avi") # Sim␣
 ↪ending in 10 seconds
state = DynamicState()

# Function for running simulation and displaying results
def run_sim_and_display(waypoints: MsgWaypoints, sim_params: MsgSimParams =␣
 ↪sim_params_default):
    global waypoint_view
    global data_view
    data_view.reset(sim_params.start_time)
    (waypoint_view, data_view) = run_sim(sim=sim_params, waypoints=waypoints,␣
 ↪init_state=state, waypoint_view=waypoint_view, data_view=data_view)
    display_data_view(data_view)
    display_mav_view(waypoint_view)
```

```
136.62760706610337
Initializing waypoint viewer
Initializing data_view
```

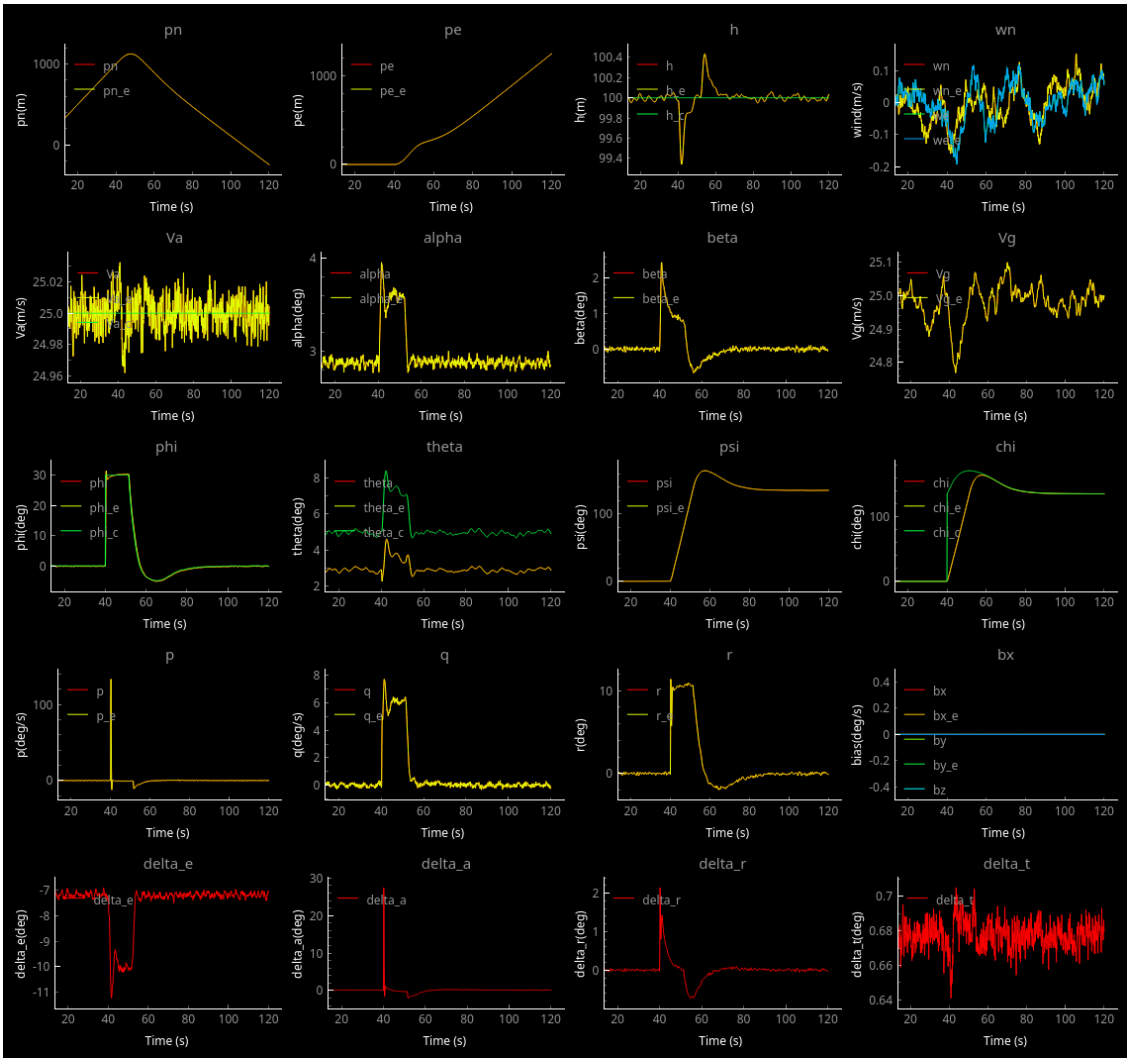
0.2 Problem 1 - Straight line fillet paths

There are two scenarios where the fillet path equations have a singularity. The first is the most common: the waypoints cannot form a straight line. This is very problematic because many paths have three waypoints that are colinear. In fact, the default motion for the UAV once it has reached the final waypoint is to proceed in a straight line.

Modify `construct_fillet_line` and/or `construct_fillet_circle` so that the fillet path manager will work for the code below. Make sure that the unit tests still pass.

[2]:
```
# Waypoint definition
waypoints = MsgWaypoints()
waypoints.type = 'straight_line'
Va = PLAN.Va0
waypoints.add(np.array([[0, 0, -100]]).T, Va, np.radians(0), np.inf, 0, 0)
waypoints.add(np.array([[1000, 0, -100]]).T, Va, np.radians(45), np.inf, 0, 0)
waypoints.add(np.array([[0, 1000, -100]]).T, Va, np.radians(45), np.inf, 0, 0)

# Run the simulation
run_sim_and_display(waypoints=waypoints)
```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Va_e
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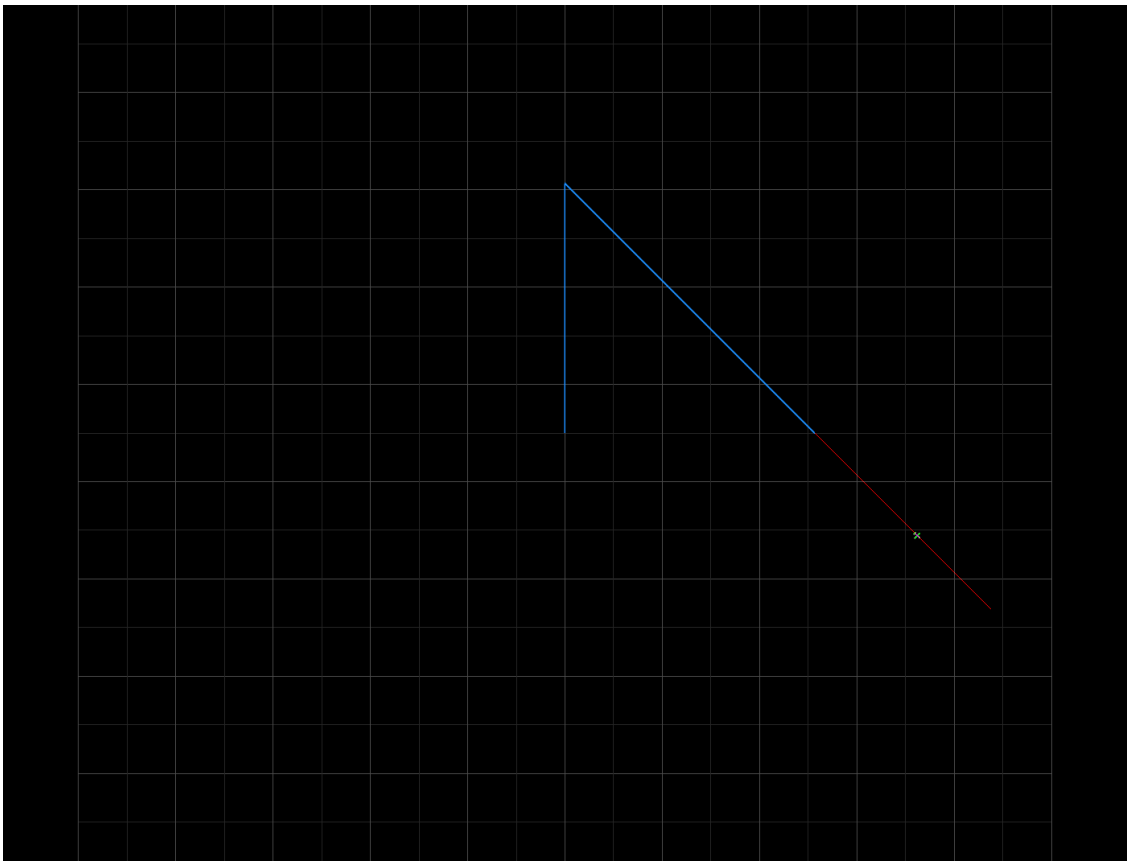
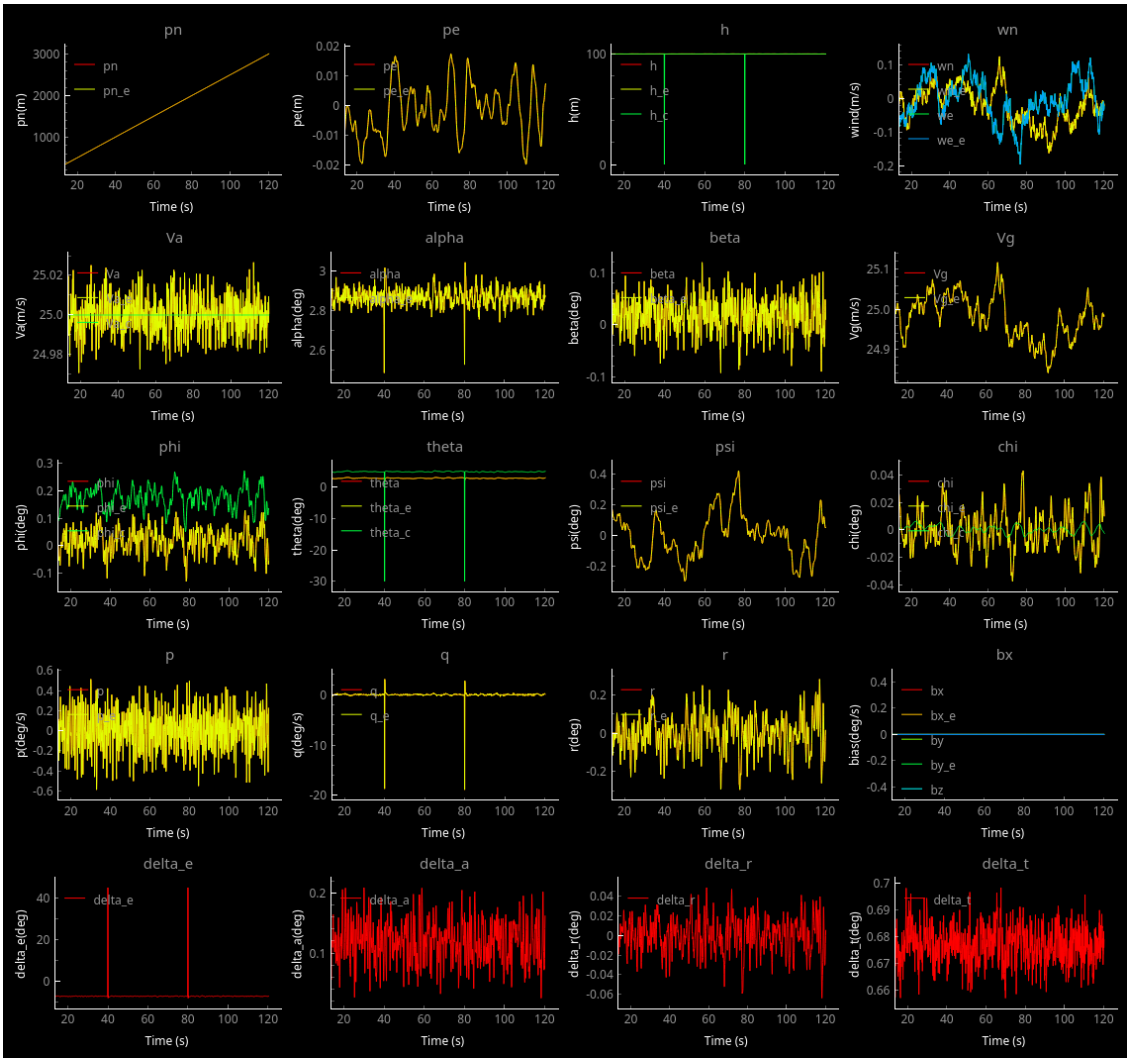


[3]:
```
# Waypoint definition
waypoints = MsgWaypoints()
waypoints.type = 'fillet'
Va = PLAN.Va0
waypoints.add(np.array([[0, 0, -100]]).T, Va, np.radians(0), np.inf, 0, 0)
waypoints.add(np.array([[1000, 0, -100]]).T, Va, np.radians(45), np.inf, 0, 0)
waypoints.add(np.array([[2000, 0, -100]]).T, Va, np.radians(45), np.inf, 0, 0)

# Run the simulation
run_sim_and_display(waypoints=waypoints)
```


pn
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h_e
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Time (s)
wn
wind(m/s)
wn
wn_e
we
we_e
Time (s)
Va
Va(m/s)
Va
Va_e
Time (s)
alpha
alpha(deg)
alpha
alpha_e
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beta
beta(deg)
beta
beta_e
Time (s)
Vg
Vg(m/s)
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Vg_e
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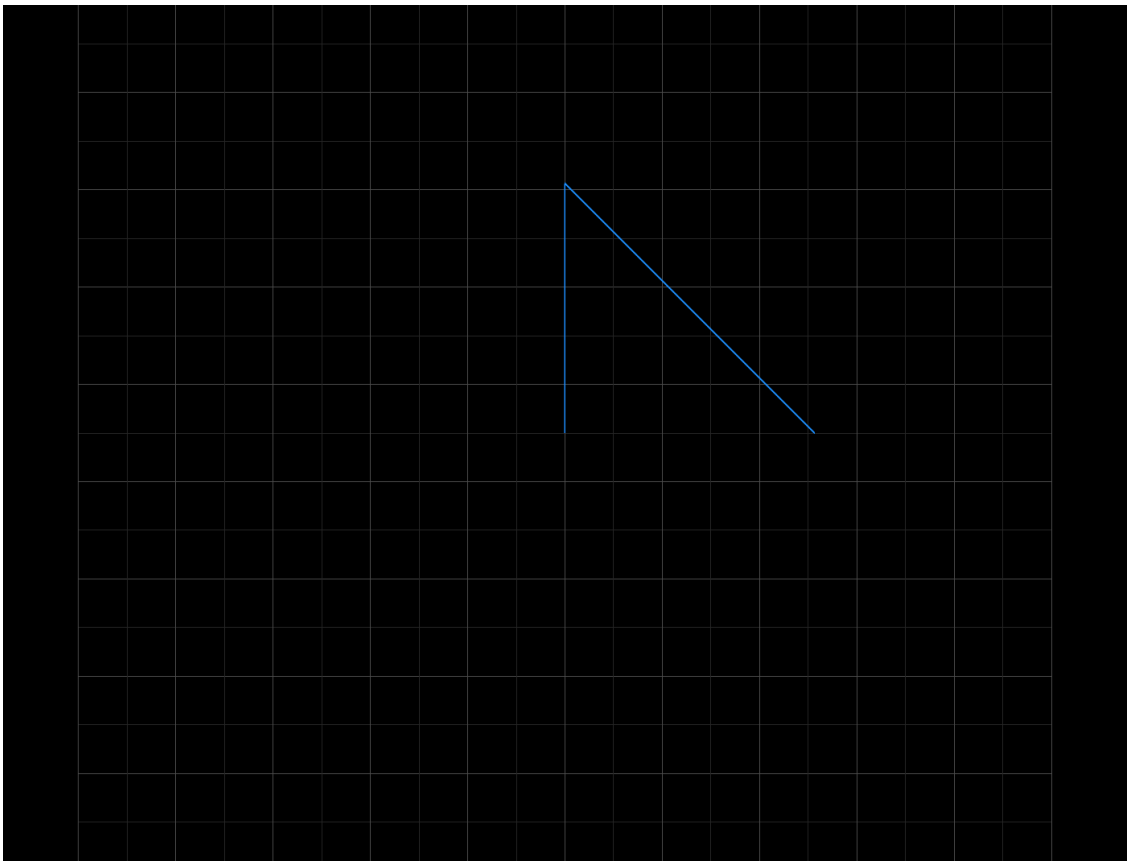


0.3 Problem 2 - Paths that fold back

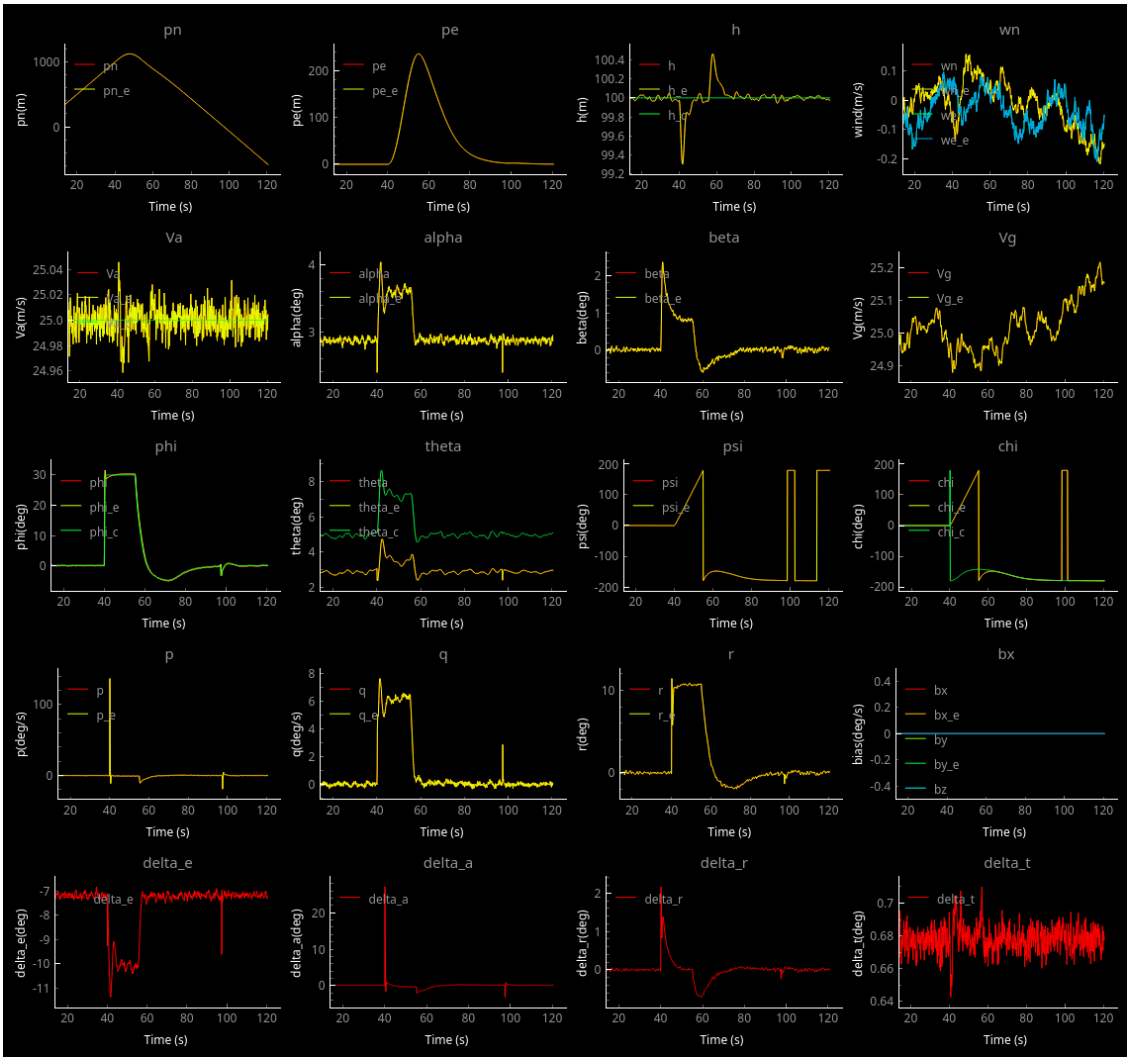
The second scenario with singularities actually occurs in both straight-line and fillet paths. The case occurs when the path folds back onto itself. While not as common of an occurance, it must be dealt with regardless.

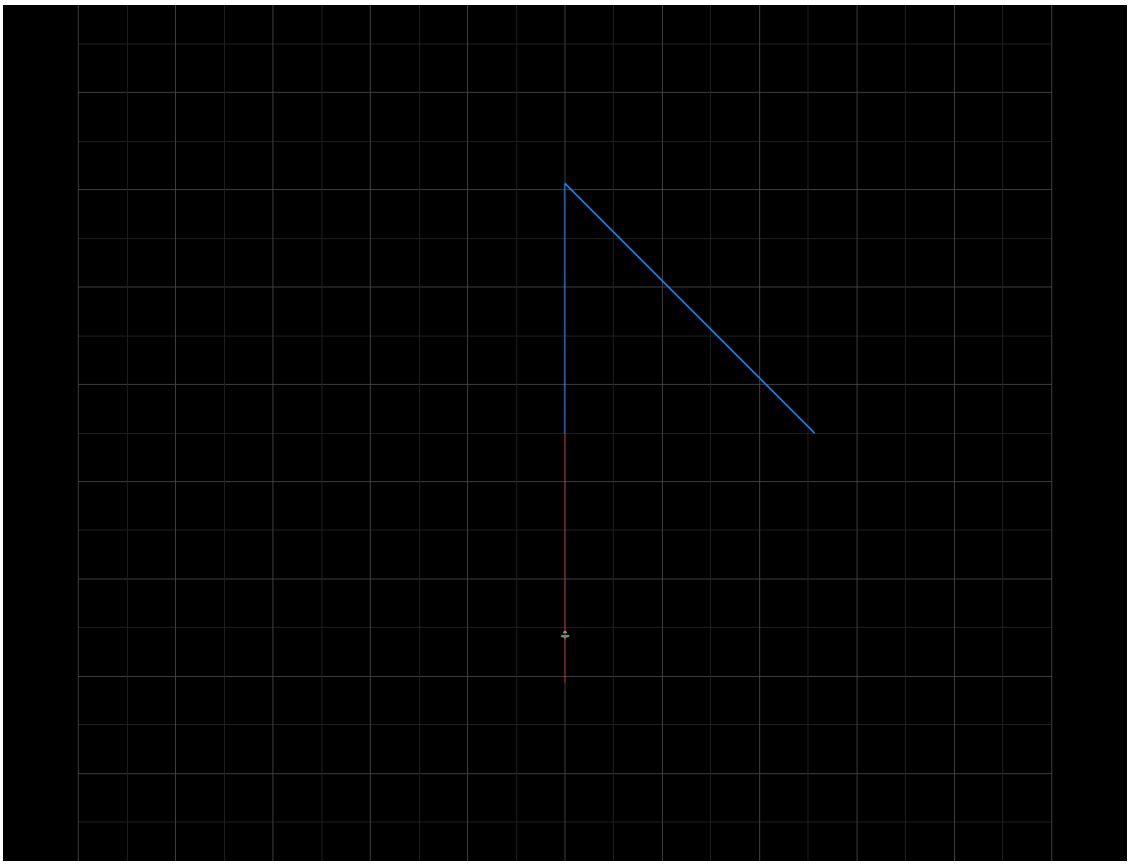
Modify `construct_fillet_line`, `construct_fillet_circle`, and `construct_line` so that the path managers will work for the code below. The switching behaviors are ill-defined in this case. Redesign them so that if a path will fold back onto itself, the UAV will make it all the way to the waypoint before starting back. Make sure that the unit tests still pass.

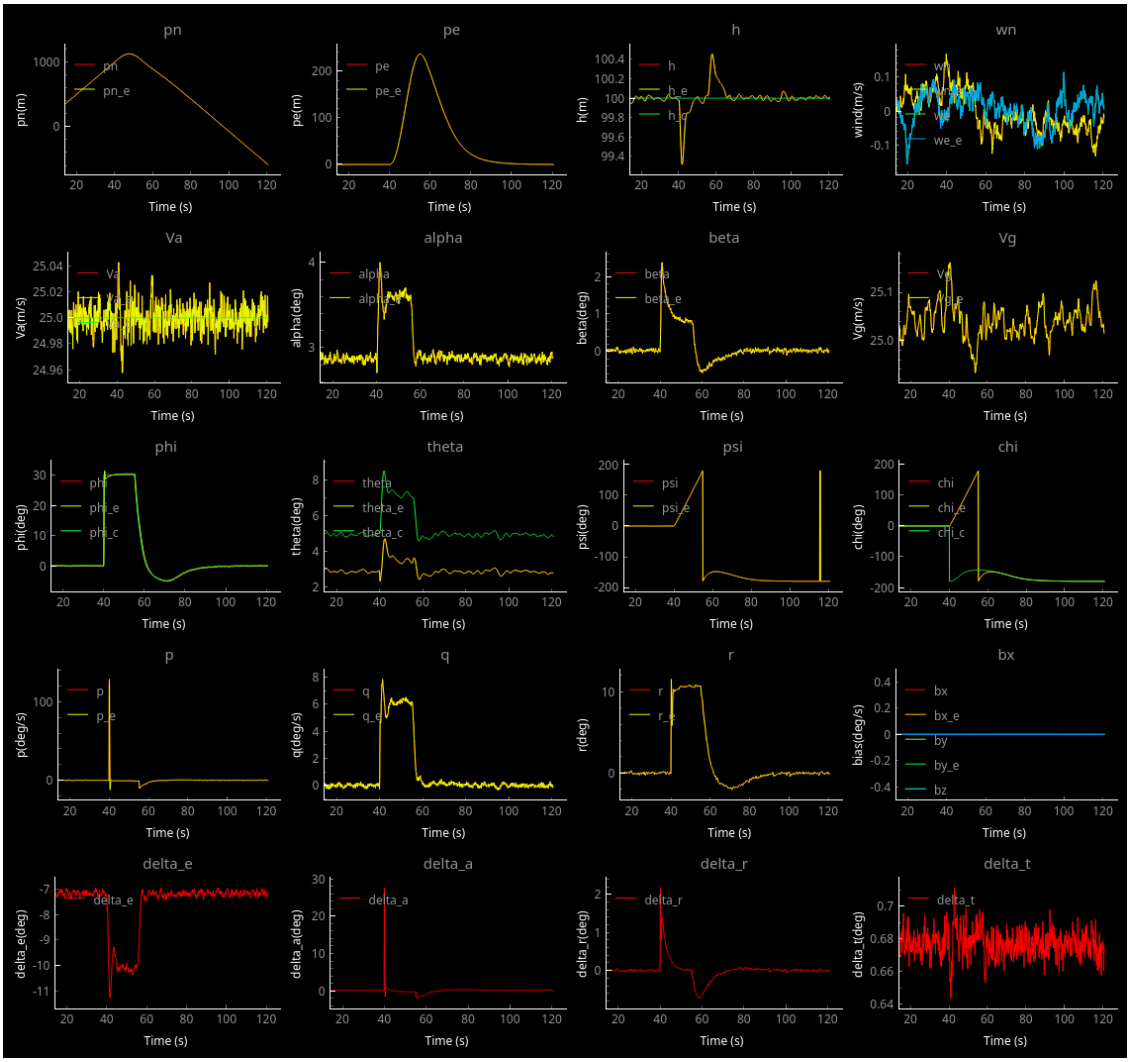
[4]:
```
# Waypoint definition
waypoints = MsgWaypoints()
waypoints.type = 'fillet'
Va = PLAN.Va0
waypoints.add(np.array([[0, 0, -100]]).T, Va, np.radians(0), np.inf, 0, 0)
waypoints.add(np.array([[1000, 0, -100]]).T, Va, np.radians(45), np.inf, 0, 0)
waypoints.add(np.array([[0, 0, -100]]).T, Va, np.radians(45), np.inf, 0, 0)

# Run the simulation for the fillet path manager
run_sim_and_display(waypoints=waypoints)
```

```
# Run the simulation for the straight line path manager
waypoints.type = 'straight_line'
run_sim_and_display(waypoints=waypoints)
```






pn
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h(m)
Time (s)
wn
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we
we_e
wind(m/s)
Time (s)
Va
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Va(m/s)
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alpha
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Vg
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chi_c
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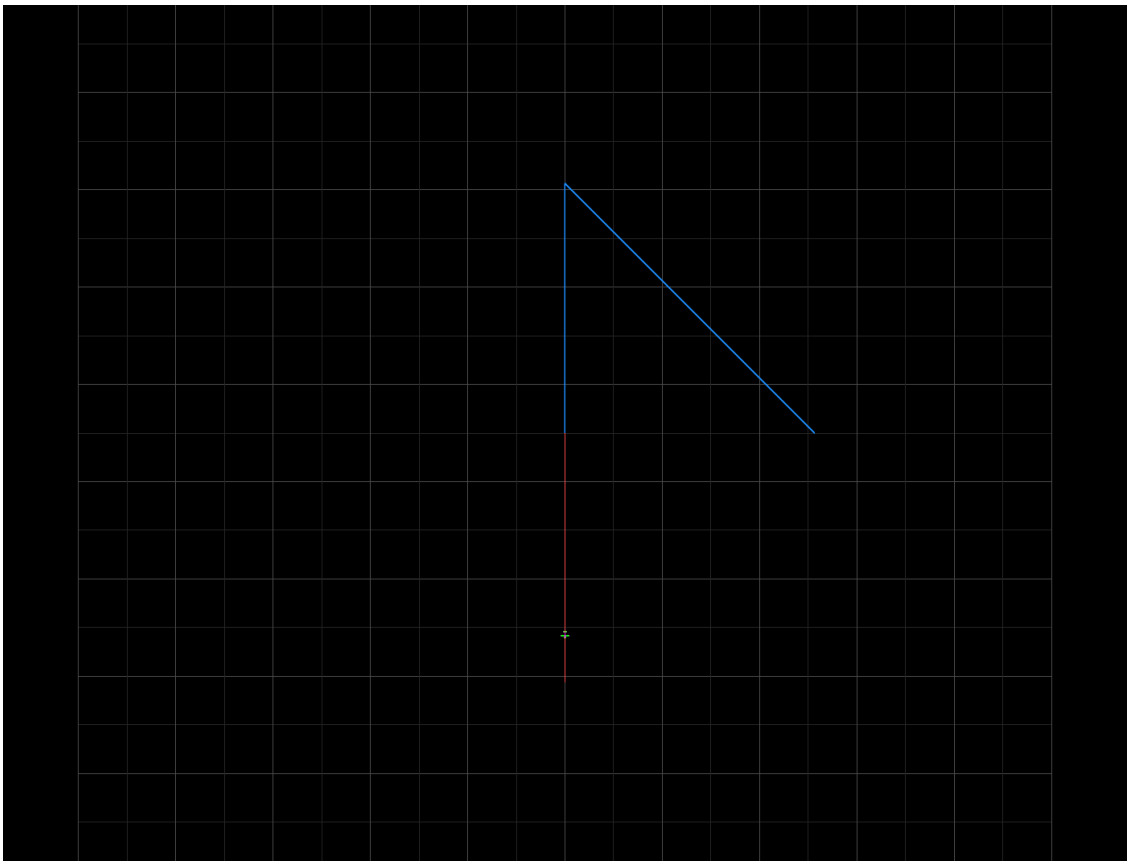


0.4 Problem 3 - Compare paths

Create five waypoints that are not colinear with the first and last waypoints being $w = \begin{bmatrix} 0 \\ 0 \\ -100 \end{bmatrix}$.

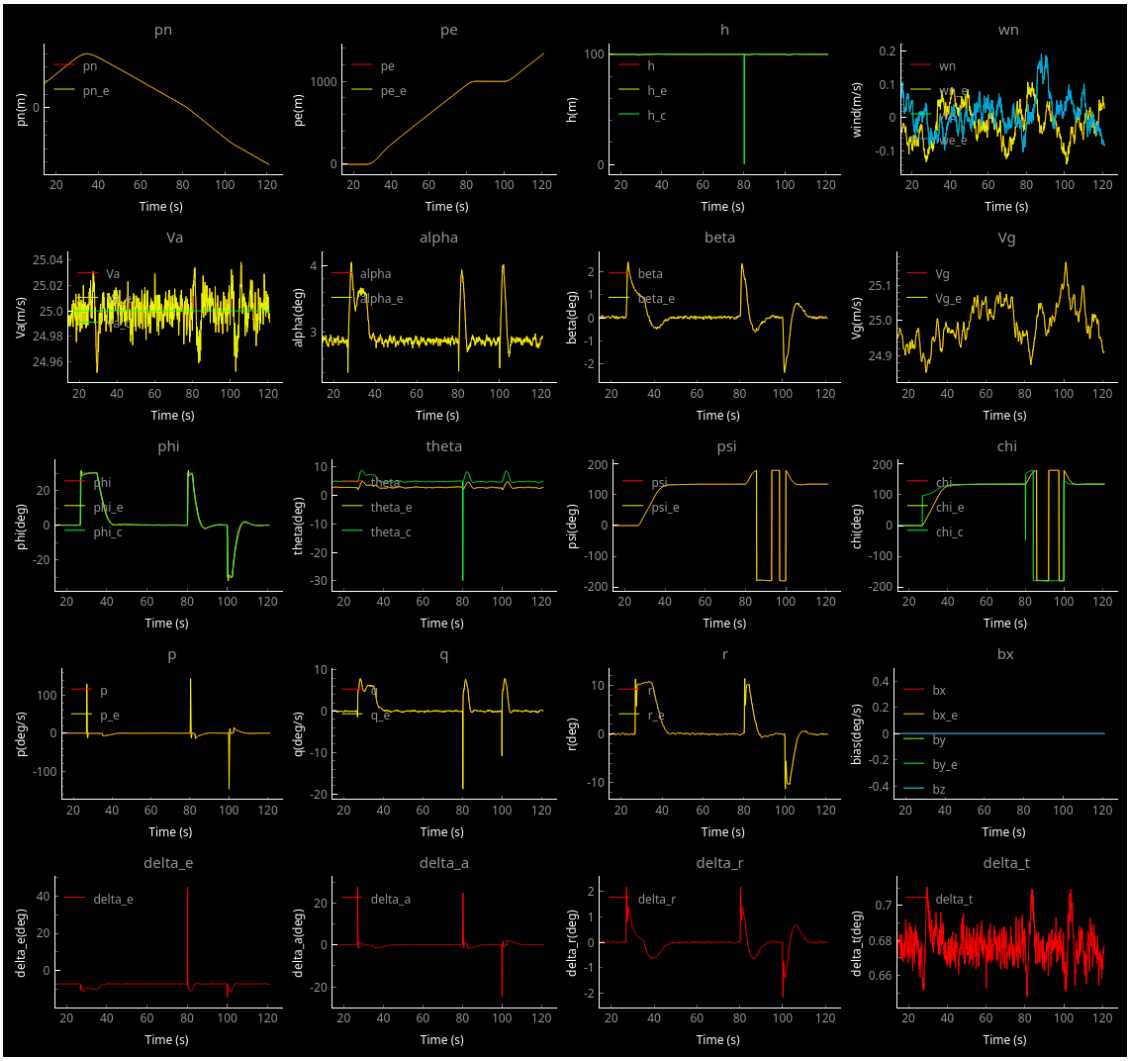
Ensure that there is sufficient spacing between waypoints such that the fillet circles do not overlap. Run it for both the `straight-line` and `fillet` approaches. Ensure that the simulation time is sufficient to run passed the final waypoint.

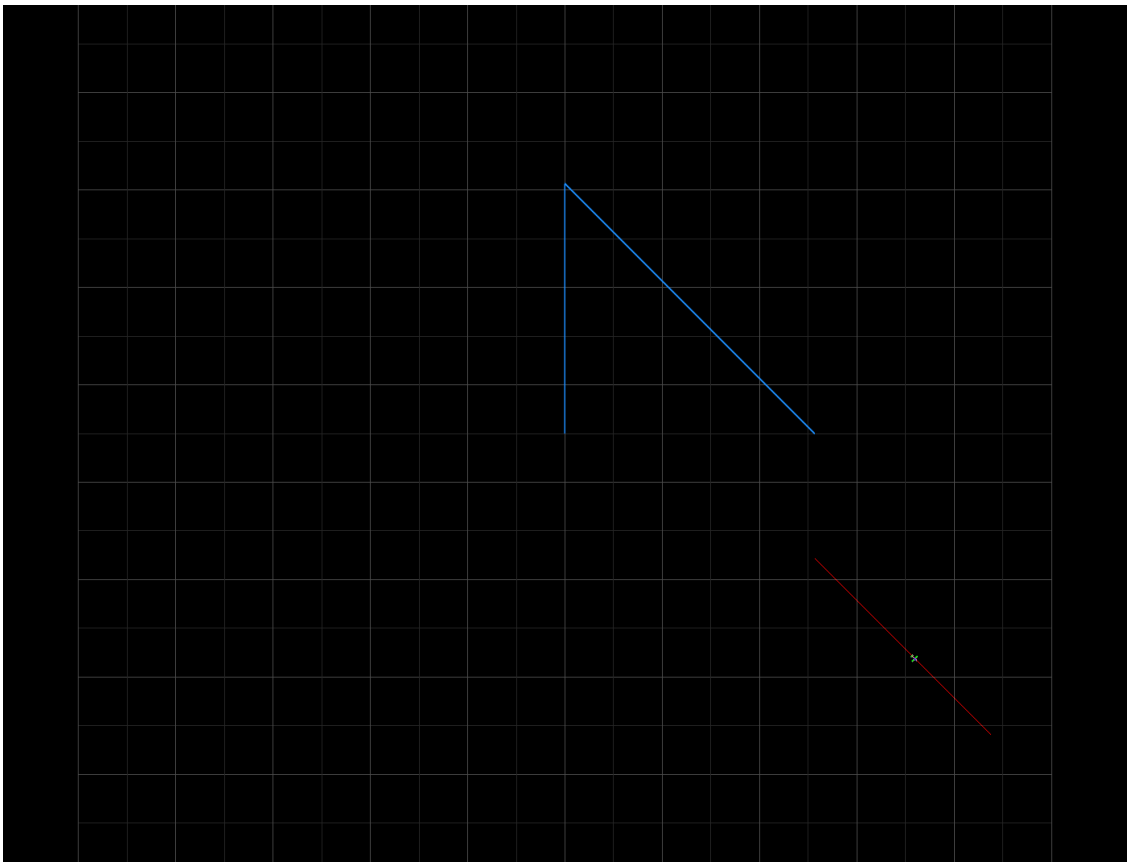
0.4.1 Question: what is the difference between the performance of the straight_line path and the fillet path?

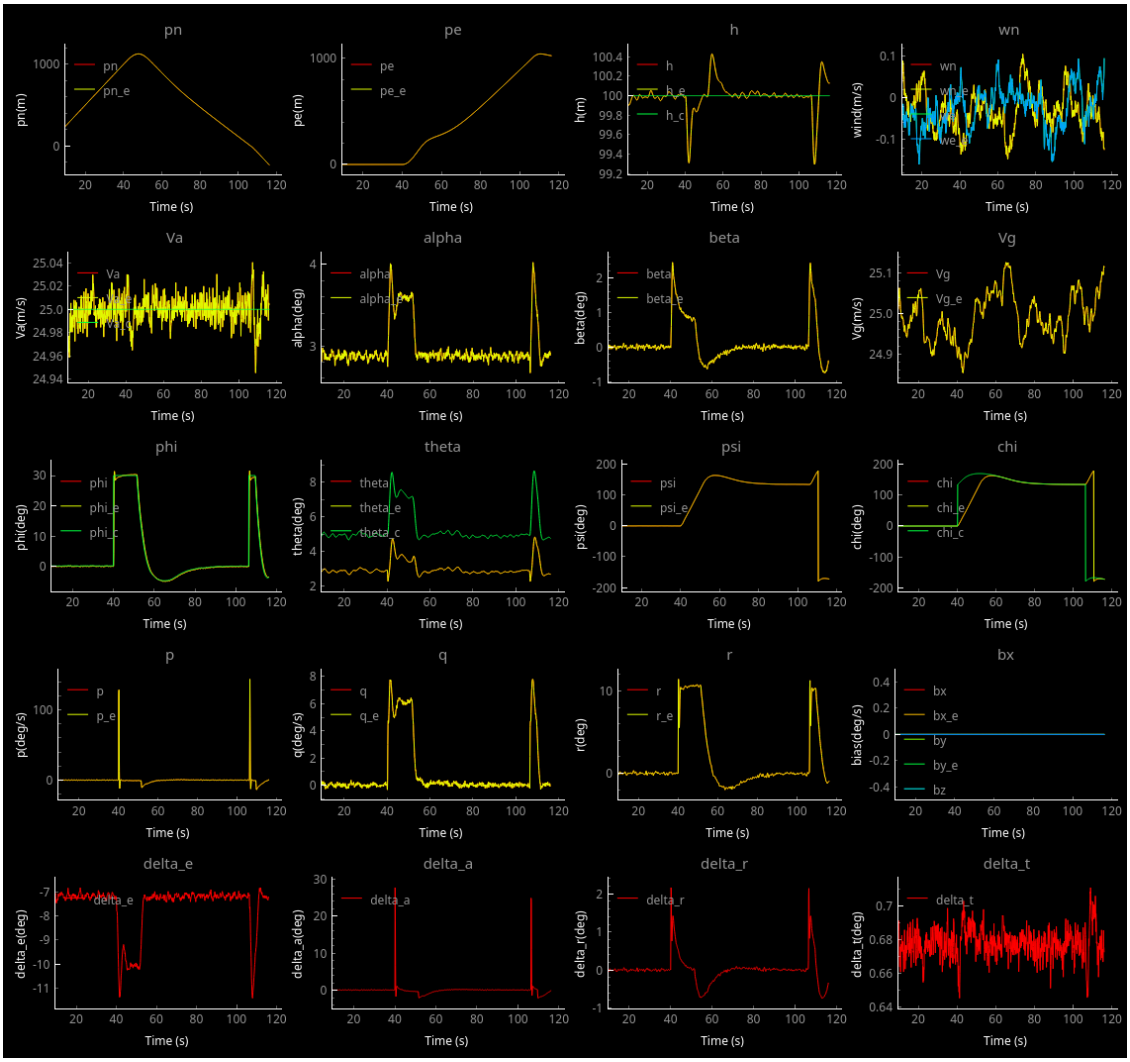
- The filet path is much more efficient than the straight line, however it does not reach the waypoints.

[5]:
```
# Waypoint definition
waypoints = MsgWaypoints()
waypoints.type = 'fillet'
Va = PLAN.Va0
waypoints.add(np.array([[0, 0, -100]]).T, Va, np.radians(0), np.inf, 0, 0)
waypoints.add(np.array([[1000, 0, -100]]).T, Va, np.radians(45), np.inf, 0, 0)
```

```
waypoints.add(np.array([[0, 1000, -100]]).T, Va, np.radians(45), np.inf, 0, 0)
waypoints.add(np.array([[-500, 1000, -100]]).T, Va, np.radians(0), np.inf, 0, 0)
waypoints.add(np.array([[-1000, 1500, -100]]).T, Va, np.radians(45), np.inf, 0,␣
↪0)

# Initialize the simulation parameters
sim_params_default = MsgSimParams(end_time=125., video_name="cha11.avi") # Sim␣
↪ending in 10 seconds

# Run the simulation for the fillet path manager
run_sim_and_display(waypoints=waypoints, sim_params=sim_params_default)

# Run the simulation for the straight line path manager
waypoints.type = 'straight_line'
run_sim_and_display(waypoints=waypoints, sim_params=sim_params_default)
```



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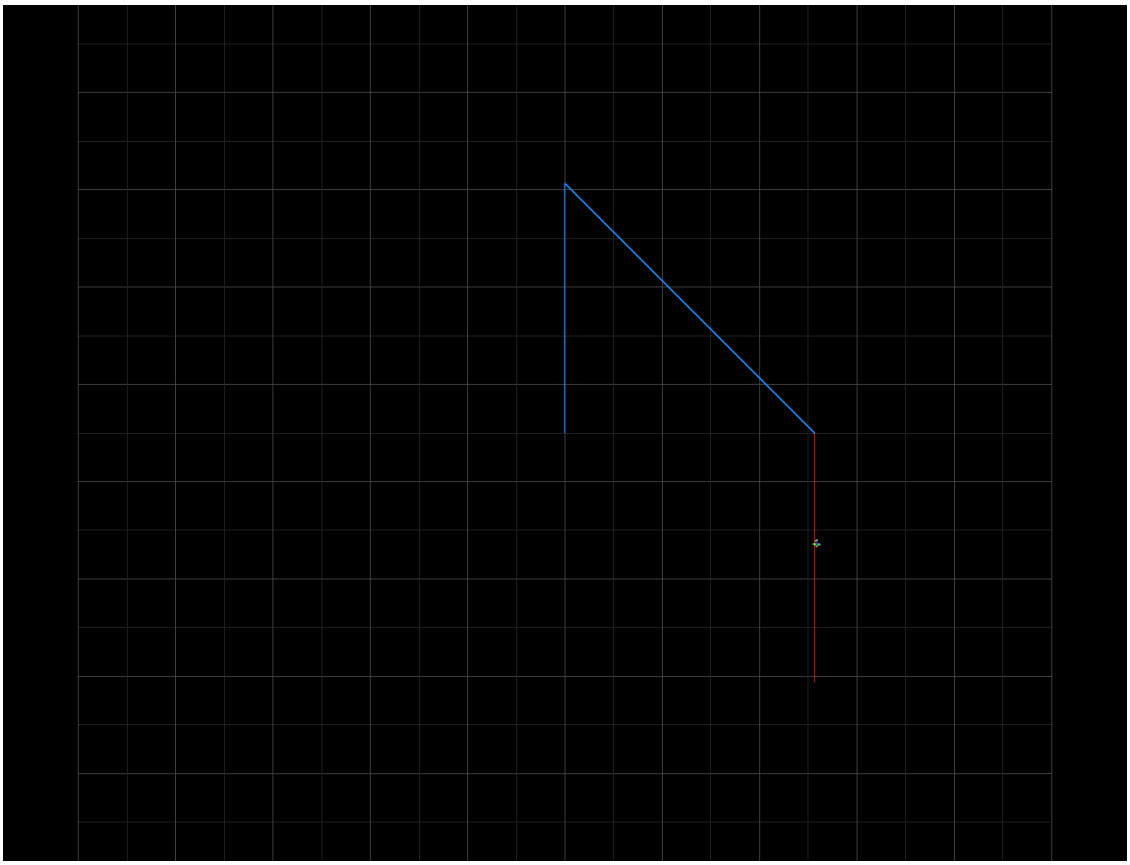
Vg_e
Vg(m/s)
Time (s)
phi
phi
phi_e
phi_c
phi(deg)
Time (s)
theta
theta
theta_e
theta_c
theta(deg)
Time (s)
psi
psi
psi_e
psi(deg)
Time (s)
chi
chi
chi_e
chi_c
chi(deg)
Time (s)
p
p
p_e
p(deg/s)
Time (s)
q
q
q_e
q(deg/s)
Time (s)
r
r
r_e
r(deg)
Time (s)
bx
bx
bx_e
by
by_e
bz
bias(deg/s)
Time (s)
delta_e
delta_e
delta_e(deg)
Time (s)
delta_a
delta_a
delta_a(deg)
Time (s)
delta_r
delta_r
delta_r(deg)
Time (s)
delta_t
delta_t
delta_t(deg)
Time (s)



0.5 Simple code checking

The following code does not need to change. It should just be used as a sanity check so that you know the code is implemented properly. The output should not have any lines reading `Failed test!`

[6]:
```
from mav_sim.unit_tests.ch11_line_and_fillet_path_manager_test import␣
  ↪run_all_tests
run_all_tests()
```

```
Starting inHalfSpace test
Failed test!
Calculated output:
True
Expected output:
False
Failed on test id: 377
End of test

Starting construct_line test
Failed test!
```

```
Calculated output:

type= line
plot_updated= False
airspeed= 10.0
line_origin= [[-100.]
 [-100.]
 [-100.]]
line_direction= [[-0.57735027]
 [-0.57735027]
 [-0.57735027]]
orbit_center= [[0.]
 [0.]
 [0.]]
orbit_radius= 50
orbit_direction= CW

normal= [[-0.64055498]
 [-0.64055498]
 [-0.42353116]]
point= [[-108.]
 [-108.]
 [-108.]]
Expected output:

type= line
plot_updated= False
airspeed= 11.0
line_origin= [[-100.]
 [-100.]
 [-100.]]
line_direction= [[-0.57735027]
 [-0.57735027]
 [-0.57735027]]
orbit_center= [[0.]
 [0.]
 [0.]]
orbit_radius= 50
orbit_direction= CW

normal= [[-0.64055498]
 [-0.64055498]
 [-0.42353116]]
point= [[-108.]
 [-108.]
 [-108.]]
Failed on test id: 0 list index: 0
End of test
```

```
Starting construct_fillet_line test
Failed test!
Calculated output:

type= line
plot_updated= False
airspeed= 25
line_origin= [[-100.]
 [-100.]
 [-100.]]
line_direction= [[-0.57735027]
 [-0.57735027]
 [-0.57735027]]
orbit_center= [[0.]
 [0.]
 [0.]]
orbit_radius= 50
orbit_direction= CW

normal= [[-0.57735027]
 [-0.57735027]
 [-0.57735027]]
point= [[-106.96048968]
 [-106.96048968]
 [-106.96048968]]
Expected output:

type= line
plot_updated= False
airspeed= 11.0
line_origin= [[-100.]
 [-100.]
 [-100.]]
line_direction= [[-0.57735027]
 [-0.57735027]
 [-0.57735027]]
orbit_center= [[0.]
 [0.]
 [0.]]
orbit_radius= 50
orbit_direction= CW

normal= [[-0.57735027]
 [-0.57735027]
 [-0.57735027]]
point= [[-106.96048968]
 [-106.96048968]
```

```
 [-106.96048968]]
Failed on test id: 0 list index: 0
End of test

Starting construct_fillet_circle test
Failed test!
Calculated output:

type= line
plot_updated= False
airspeed= 25
line_origin= [[0.]
 [0.]
 [0.]]
line_direction= [[1.]
 [0.]
 [0.]]
orbit_center= [[0.]
 [0.]
 [0.]]
orbit_radius= 50
orbit_direction= CW

normal= [[-0.68348613]
 [-0.68348613]
 [-0.2563073 ]]
point= [[-106.29212334]
 [-106.29212334]
 [-106.29212334]]
Expected output:

type= orbit
plot_updated= False
airspeed= 11.0
line_origin= [[0.]
 [0.]
 [0.]]
line_direction= [[1.]
 [0.]
 [0.]]
orbit_center= [[-111.04297259]
 [-111.04297259]
 [ -98.79552387]]
orbit_radius= 10
orbit_direction= CCW

normal= [[-0.68348613]
 [-0.68348613]
```

```
 [-0.2563073 ]]
point= [[-109.23060631]
 [-109.23060631]
 [-108.46147736]]
Failed on test id: 0 list index: 0
End of test
```