1)

Given

$\dot{x}_1 = -x_1^3 + x_2$
$\dot{x}_2 = x_1 - x_2^3$

Find

1. Find all equilibrium points
2. Determine the type of each isolated equilibrium
3. Draw vector field plot

Solution

Find equilibrium points

>>>

```
/* Define equations */
xd1: -x1^3 + x2;
xd2: x1 - x2^3;

/* Find roots */
solve([xd1=0, xd2=0]);
```

```
x2-x1^3

x1-x2^3

[[x2 = -(-1)^(1/4),x1 = sqrt(-%i)],[x2 = (-1)^(3/4),x1 = (-1)^(1/4)],
        [x2 = %i,x1 = -%i],[x2 = -%i,x1 = %i],[x2 = -1,x1 = -1],
        [x2 = 1,x1 = 1],[x2 = 0,x1 = 0]]
```

Determine equilibrium point types

>>>

```
/* Calculate Jacobian */
J:jacobian([xd1, xd2], [x1, x2]);

/* Calculate eigenvalue for each equilibrium point  */
/* The eigenvalue output is of the following format */
/* [[eigenvalues], [multiplicity]]                  */

float(eivals(psubst([x1=1, x2=1], J)));
float(eivals(psubst([x1=-1, x2=-1], J)));
float(eivals(psubst([x1=0, x2=0], J)));
```

```
matrix([-3*x1^2,1],[1,-3*x2^2])

[[-2.0,-4.0],[1.0,1.0]]

[[-2.0,-4.0],[1.0,1.0]]

[[-1.0,1.0],[1.0,1.0]]
```

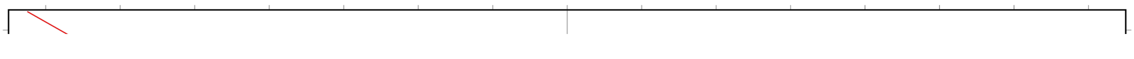
Based on the eigevalues found from the Jacobians found at each of the equilibrium points, we can determine that they are:

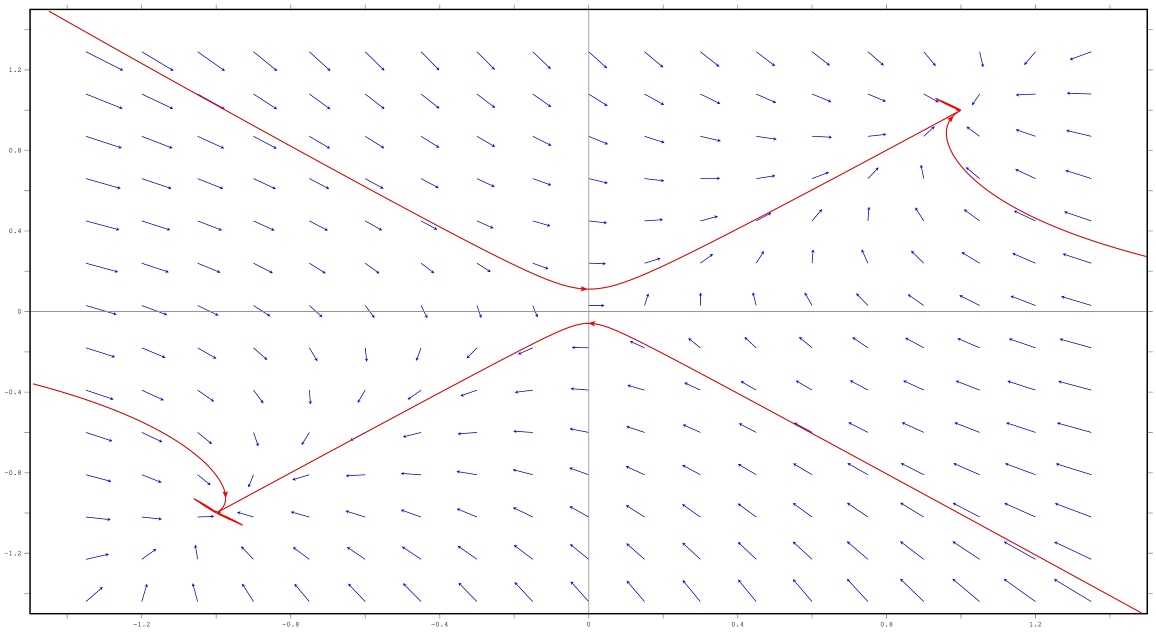
- (1,1): Stable node
- (-1,-1): Stable Node
- (0,0): Saddle

Draw vector field

>>>

```
plotdf([xd1, xd2], [x1, x2], [x1, -1.5, 1.5], [x2, -1.5, 1.5])$
```





3)

Given

$$\dot{x}_1 = x_1^3 - x_2$$
$$\dot{x}_2 = x_1 - x_2$$

Find

1. Find all equilibrium points
2. Determine the type of each isolated equilibrium
3. Draw vector field plot

Solution

Find equilibrium points

>>>

```
/* Define equations */
xd1: x1^3 - x2;
xd2: x1 - x2;

/* Find roots */
solve([xd1=0, xd2=0]);
```

```
x1^3-x2
```

```
x1-x2
```

```
[[x2 = -1,x1 = -1],[x2 = 1,x1 = 1],[x2 = 0,x1 = 0]]
```

Determine equilibrium point types

```
>>> /* Calculate Jacobian */
J:jacobian([xd1, xd2], [x1, x2]);

/* Calculate eigenvalue for each equilibrium point  */
/* The eigenvalue output is of the following format */
/* [[eigenvalues], [multiplicity]]                  */

float(eivals(psubst([x1=1, x2=1], J)));
float(eivals(psubst([x1=-1, x2=-1], J)));
float(eivals(psubst([x1=0, x2=0], J)));
```

```
matrix([3*x1^2,-1],[1,-1])

[[-0.7320508075688772,2.732050807568877],[1.0,1.0]]

[[-0.7320508075688772,2.732050807568877],[1.0,1.0]]

[[-0.5*(1.732050807568877*%i+1.0),0.5*(1.732050807568877*%i-1.0)],[1.0,
                                                                    1.0]]
```

Based on the eigevalues found from the Jacobians found at each of the equilibrium points, we can determine that they are:

- (1,1): Saddle
- (-1,-1): Saddle
- (0,0): Saddle

Draw vector field

```
>>> plotdf([xd1, xd2], [x1, x2], [x1, -1.5, 1.5], [x2, -1.5, 1.5])$
```