4.22)

Given:

$$PA + A^T P = -C^T C \quad \text{where } (A, C) \text{ is observable}$$

Find:

- Show A is Hurwitz iff $P = P^T > 0$ that satisfies the equation
- Show that if A is Hurwitz, the Lyapunov function will have a unique solution.

Solution:

Hint: Apply LaSalle's Theorem and recall that for an observable pair (A, C), the vector $C\exp(At)x \equiv 0 \ \forall t$ iff $x = 0$.

LaSalle's Theorem: Let $\Omega \subset D$ be a compact set that is positively invariant with respect to (1)

$$\dot{x} = f(x)$$

Let $V: D \to R$ be a continuously differentiable function s.t. $\dot{V}(x) \leq 0$ in Ω. Let E be the set of all points in Ω where $\dot{V}(x) = 0$. Let M be the largest invariant set in E. Then every solution starting in Ω approaches M as $t \to \infty$.

Where: Positively Invariant: $x(0) \in M \Rightarrow x(t) \in M \quad \forall t \geq 0$

Invariant Set: $x(0) \in M \Rightarrow x(t) \in M \quad \forall t \in R$

Theorem: A matrix is Hurwitz (2)

i) $(\forall \lambda_i, \ Re(\lambda_i) < 0)$

iff

ii) $\forall Q = Q^T > 0 \ \exists P = P^T > 0$ s.t.

$$PA + A^T P = -Q$$

(3)

Corrollary 4.2: (Invariance Principle) Let $x = 0$ be an equilibrium point for

$$\dot{x} = f(x)$$

Let $V: \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable, radially unbounded, positive definite function s.t. $\dot{V}(x) \leq 0 \ \forall \ x \in \mathbb{R}^n$. Let $S = \{x \in \mathbb{R}^n \mid \dot{V}(x) = 0\}$ and suppose that no solution can stay identically on S, other than the trivial solution $x(t) \equiv 0$. Then the origin is A.S.

Lets use a similar approach used to prove Hurwitz in class (pg 80)

i) Suppose $V(x) = x^T P x$

$\Rightarrow$ $\dot{V}(x) = x^T P \dot{x} + \dot{x}^T P x = x^T (PA + A^T P) x$

$$\dot{x} = Ax$$

Now substitute $PA + A^T P = -C^T C$

$\Rightarrow$ $\dot{V}(x) = x^T(-C^T C)x = -x^T C^T C x$

By invariance principle: if $C^T x \equiv 0 \Rightarrow \dot{V}(x) = 0$

ii) Now suppose A is Hurwitz (all eigs < 0)

Define

$$P = \int_0^\infty e^{A^T t} Q e^{At} \, dt$$

Now to show P is positive definite, suppose that it is not

$$\exists x \in \mathbb{R}^n \setminus \{0\} \text{ s.t. } x^T P x = 0$$

$\Rightarrow$ $\int_0^\infty e^{A^T t} Q e^{At} x \, dt = 0$

$\Rightarrow$ $e^{At} x = 0 \Rightarrow x = 0$ Which contradicts what was stated above. $\therefore$ P is positive definite.

This can be said because (A, C) is observable.

Now to show that it is a solution of the Lyapanov Equation we use direct substitution

$$PA + A^T P = \int_0^\infty e^{A^T t} C^T C e^{At} A \, dt + \int_0^\infty A e^{A^T t} C^T C e^{At} \, dt$$

$$= \int_0^\infty \frac{d}{dt}\left(e^{A^T t} C^T C e^{At}\right) dt$$

$$= e^{A^T t} C^T C e^{At} \Big|_0^\infty = \underline{\underline{-C^T C}}$$

Which is a unique solution

4.27)

Given:

$$\dot{x}_1 = -x_2x_3 + 1$$

$$\dot{x}_2 = x_1x_3 - x_2$$

$$\dot{x}_3 = x_3^2(1 - x_3)$$

Find:

a) Show that the system has a unique equilibrium point

b) Using linearization, show that that the equilibrium point is asymptotically stable. Is it G.A.S.?

Solution:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_2x_3 + 1 \\ x_1x_3 - x_2 \\ x_3^2(1 - x_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3^2(1 - x_3) \Rightarrow x = 0, 1$

$\underline{x_3 = 0}$:

$-x_2(0) + 1 = 0$ $\qquad\qquad$ $x_1(0) - x_2 = 0$

$\underline{1 \neq 0}$

$\underline{x_3 = 1}$:

$-x_2(1) + 1 = 0$ $\qquad\qquad$ $x_1(1) - x_2 = 0$

$-x_2 = -1$ $\qquad\qquad$ $\underline{x_1 = x_2 = 1}$

$\underline{x_2 = 1}$

The only equilibrium point is $x^{eq} = [1 \;\; 1 \;\; 1]^T$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & -x_3 & -x_2 \\ x_3 & -1 & x_1 \\ 0 & 0 & 2x_3 - 3x_3^3 \end{bmatrix} \Bigg|_{x = x^*}$$

$$= \begin{bmatrix} 0 & -1 & -1 \\ 1 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} := A$$

$$\text{eig}(A) = \frac{-1 \pm i\sqrt{3}}{2} \quad \text{or} \quad -1$$

Because $Re(\lambda_i) < 0$ then we can say it is A.S.
However for $x_3 = 0$, x_1 grows unbounded. ∴ the system is not G.A.S.

4.32.1)

Given:

$\dot{x}_1 = -x_1 + x_1^2 \qquad \dot{x}_2 = -x_2 + x_3^2 \qquad \dot{x}_3 = x_3 - x_1^2$

Find:

Investigate whether the origin is stable, asymptotically stable, or unstable

Solution:

To determine stability of the origin we can try to linearize about it to see if any information can be extracted.

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \\ \vdots & \frac{\partial f_2}{\partial x_2} & \\ \vdots & & \ddots \end{bmatrix} = \begin{bmatrix} 2x_1 - 1 & 0 & 0 \\ 0 & -1 & 2x_3 \\ -2x_1 & 0 & 1 \end{bmatrix}\Bigg|_{x=[0]}$$

$$\Rightarrow \quad J = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{eig}(J) = -1, -1, \underline{1}$$

The origin is unstable because:

Theorem: Let $x=0$ be an equilibrium point for (4)

$$\dot{x} = f(x)$$

where $f: D \to \mathbb{R}^n$ is continuously differentiable and D is a neighborhood of the origin. Let

$$A = \frac{\partial f}{\partial x}(0)$$

Let λ_i denote an eigenvalue of A

i) If $\forall \lambda_i$ $Re(\lambda_i) < 0$, then the origin is A.S.

ii) If $\exists \lambda_i$ s.t. $Re(\lambda_i) > 0$ then the origin is not stable.

4.32.2)

Given:

$\dot{x}_1 = x_2 \quad \dot{x}_2 = -\sin x_3 + x_1[-2x_3 - sat(y)]^2 \quad \dot{x}_3 = -2x_3 - sat(y)$

where $y = -2x_1 - 5x_2 + 2x_3$

Find:

See (4.32.1)

Solution:

If we assume $y(0) = 0$ we get

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ -2x_3^2 & 0 & -\cos x_3 - 4x_3 x_1 \\ 0 & 0 & -2 \end{bmatrix} \Bigg|_{x=0}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & +2 \end{bmatrix} \Rightarrow eig(J) = \text{No good}$$

Lets try assuming that y is not saturated near the origin.

$sat(y) = y = -2x_1 - 5x_2 + 2x_3$

$\Rightarrow \dot{x}_1 = x_2$

$\dot{x}_2 = -\sin x_3 + x_1[-2x_3 - (-2x_1 - 5x_2 + 2x_3)]^2$

$\dot{x}_3 = -2x_3 - (-2x_1 - 5x_2 + 2x_3)$

$$J = \begin{bmatrix} 0 & 1 & 0 \\ (2x_1 + 5x_2 - 4x_3)^2 + x_1(8x_1 + 20x_2 - 16x_3) & x_1(20x_1 + 50x_2 - 40x_3) & -\cos x_3 - x_1(16x_1 - 40x_2 - 32x_3) \\ 2 & 5 & -4 \end{bmatrix}$$

$$J\big|_{x=[0]} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 2 & 5 & -4 \end{bmatrix} \Rightarrow eig(J) = \begin{array}{l} -0.3146 \pm 0i \\ \underline{2.1573 \pm 1.205i} \end{array}$$

Which implies origin is unstable

4.32.3)

Given:

$$\dot{x}_1 = -2x_1 + x_1^3 \qquad \dot{x}_2 = -x_2 + x_1^2 \qquad \dot{x}_3 = -x_3$$

Find:

See (4.3.2.1)

Solution:

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 3x_1^2 - 2 & 0 & 0 \\ 2x_1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\Bigg|_{x=[0]}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \text{eig}(J) = -2, \underline{-1, -1}$$

By (4), the system is A.S.

4.32.4)

Given:

$\dot{x}_1 = -x_1 \qquad \dot{x}_2 = -x_1 - x_2 - x_3 - x_1 x_3 \qquad \dot{x}_3 = (x_1 + 1) x_2$

Find:

See (4.32.1)

Solution:

$$J = \frac{\partial f}{\partial x} = \left. \begin{bmatrix} -1 & 0 & 0 \\ -x_3 - 1 & -1 & -x_1 - 1 \\ x_2 & x_1 + 1 & 0 \end{bmatrix} \right|_{x=[0]}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \operatorname{eig}(J) = \begin{matrix} -0.5 \pm 0.866i \\ -1 \end{matrix}$$

By (4) the system is A.S.

2.1.1)

Given:

$$\dot{x}_1 = -x_1 + 2x_1^3 + x_2 \qquad \dot{x}_2 = -x_1 - x_2$$

Find:

- Determine type of isolated equilibrium
- Plot vector field

Solution:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + 2x_1^3 + x_2 \\ -x_1 - x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\underline{x_1 = -x_2}$

$$-x_1 + 2x_1^3 + x_2 = 0$$

$$-(-x_2) + 2(-x_2)^3 + x_2 = 0$$

$$x_2 - 2x_2^3 + x_2 = 0$$

$$\cancel{2}x_2 - \cancel{2}x_2^3 = 0$$

$$x_2(1 - x_2^2) = 0$$

$$\underline{x_2 = 0, \pm 1}$$

$$x^{eq} = [0, 0] \; ; \; [1, -1] \; ; \; [-1, 1]$$

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 6x_1^2 - 1 & 1 \\ -1 & -1 \end{bmatrix}$$

i) [0,0]

$$J\big|_{[0,0]} = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \Rightarrow \text{eig}(J) = \underline{-1 \pm 1i}$$

Stable Focus

ii) [1,-1]

$$J\big|_{[1,-1]} = \begin{bmatrix} 6-1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & -1 \end{bmatrix} \Rightarrow \text{eig}(J) = \begin{matrix} 4.82 \\ -0.82 \end{matrix}$$

Saddle ($\pm$ eigen value)

Same as before

iii) [-1,1]

$$J\big|_{[-1,1]} = \begin{bmatrix} 6-1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & -1 \end{bmatrix} \Rightarrow \text{eig}(J) = \begin{matrix} 4.82 \\ -0.82 \end{matrix}$$

Saddle

2.1.2)

Given:

$\dot{x}_1 = x_1 + x_1 x_2$ $\dot{x}_2 = -x_2 + x_2^2 + x_1 x_2 - x_1^3$

Find:

See (2.1.1)

Solution:

$$\dot{x} = \begin{bmatrix} x_1 + x_1 x_2 \\ -x_2 + x_2^2 + x_1 x_2 - x_1^3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(x_1 + x_1 x_2) = x_1(1 + x_2) = 0 \Rightarrow \underline{x_1 = 0 \text{ or } x_2 = -1}$

$x_1 = 0$

$-x_2 + x_2^2 + (0)x_2 - 0^3 = 0$

$-x_2 + x_2^2 = 0$

$x_2(-1 + x_2) = 0$

$\underline{x_2 = 0, 1}$

$x_2 = -1$

$-(-1) + (-1)^2 - x_1 - x_1^3 = 0$

$2 - x_1 - x_1^3 = 0$

$\underline{x_1 = 1}$

$\Rightarrow x_{eq} = [0, 0]; [0, 1]; [1, -1]$

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} 1 + x_2 & x_1 \\ x_2 - 3x_1^2 & -1 + 2x_2 + x_1 \end{bmatrix}$$

i) $[0,0]$

$$J|_{[0,0]} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow eig(J) = \pm 1$$

Saddle

ii) $[0,1]$

$$J|_{[0,1]} = \begin{bmatrix} 1+1 & 0 \\ 1-0 & -1+2+0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$\Rightarrow eig(J) = 1, 2$ Unstable node

iii) $[1,-1]$

$$J|_{[1,-1]} = \begin{bmatrix} 1+(-1) & 1 \\ -1-3(1) & -1+2(-1)+1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$\Rightarrow eig(A) = -1 \pm 1.7321i$ Stable focus

Given:

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -0.8x_1 - 10x_1^2 x_2 + u$$

Find:

Develop an approximate, higher order linearization of the following system.

Solution:

Lets attempt a simple redefinition of

$$\tilde{x}_1 = x_2 \; ; \quad \tilde{x}_2 = -0.8x_1 \qquad \tilde{x}_3 = -10x_1^2 x_2$$

Differentiating gives

$$\dot{\tilde{x}}_1 = \dot{x}_2 = -0.8x_1 - 10x_1^2 x_2 + u = \tilde{x}_2 + \tilde{x}_3 + u$$

$$\dot{\tilde{x}}_2 = \underline{-0.8\dot{x}_1} = -0.8x_2 = \underline{-0.8\tilde{x}_1}$$

$$\dot{\tilde{x}}_3 = -10\,2x_1\dot{x}_1 x_2 - 10x_1^2 \dot{x}_2$$

$$= \underline{\frac{20\tilde{x}_2 x_1^2}{0.8} - \frac{10x_2^2 x_1}{0.8}}$$

Continue in PDF

2.1.1)

Given

$$\dot{x}_1 = -x_1 + 2x_2^3 + x_2$$
$$\dot{x}_2 = -x_1 - x_2$$

Find

1. Find all equilibrium points
2. Determine the type of each isolated equilibrium
3. Draw vector field plot

Solution

Find equilibrium points

>>>

```
/* Define equations */
xd1: -x1 + 2*x1^3 + x2;
xd2: -x1 - x2;

/* Find roots */
solve([xd1=0, xd2=0]);
```

$$\mathrm{x2} + 2\mathrm{x1}^3 - \mathrm{x1}$$

$$[[\mathrm{x2} = 1, \mathrm{x1} = -1], [\mathrm{x2} = -1, \mathrm{x1} = 1], [\mathrm{x2} = 0, \mathrm{x1} = 0]]$$

$$-\mathrm{x2} - \mathrm{x1}$$

Determine equilibrium point types

>>>

```
/* Calculate Jacobian */
J:jacobian([xd1, xd2], [x1, x2]);

/* Calculate eigenvalue for each equilibrium point  */
/* The eigenvalue output is of the following format */
/* [[eigenvalues], [multiplicity]]                  */

float(eivals(psubst([x1=-1, x2=1], J)));
float(eivals(psubst([x1=1, x2=-1], J)));
float(eivals(psubst([x1=0, x2=0], J)));
```

$$[[-0.8284271247461907, 4.828427124746191], [1.0, 1.0]]$$

$$[[-0.8284271247461907, 4.828427124746191], [1.0, 1.0]]$$

$$[[-1.0i - 1.0, i - 1.0], [1.0, 1.0]]$$

$$\begin{pmatrix} 6\mathrm{x1}^2 - 1 & 1 \\ -1 & -1 \end{pmatrix}$$

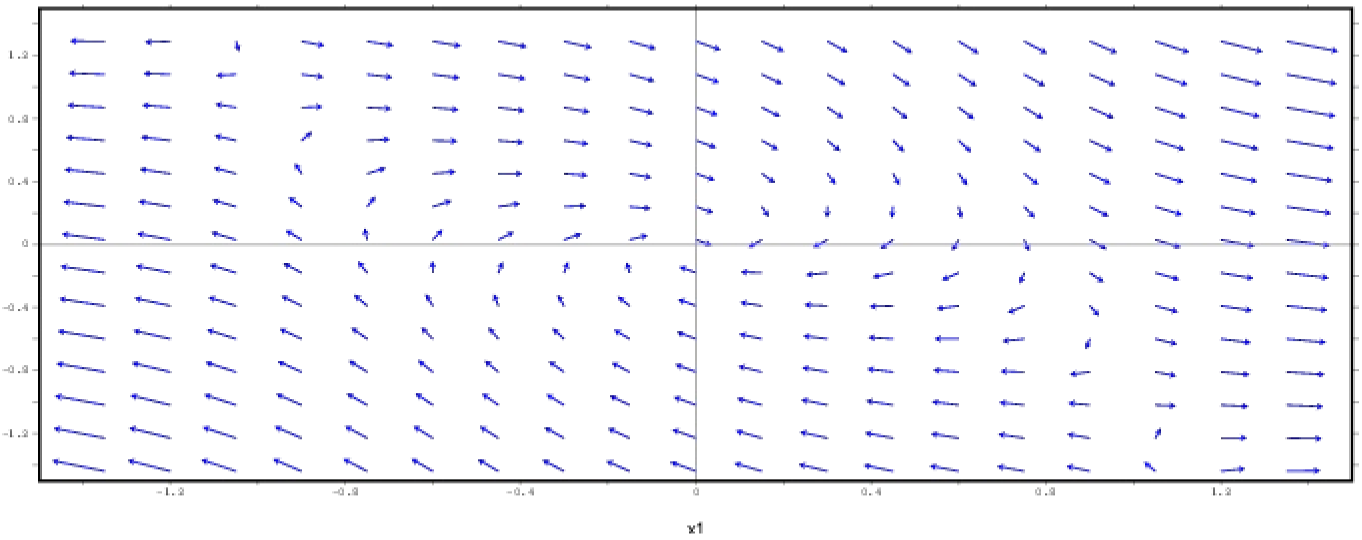
Based on the eigenvalues found from the Jacobians found at each of the equilibrium points, we can determine that they are:

- (-1, 1): Saddle
- (0,0): Stable focus
- (1, -1): Saddle

Draw vector field

>>>

```
plotdf([xd1, xd2], [x1, x2], [x1, -1.5, 1.5], [x2, -1.5, 1.5])$
```



2.1.2)

Given

$$\dot{x}_1 = x_1 + x_1 x_2$$
$$\dot{x}_2 = -x_2 + x_2^2 + x_1 x_2 - x_1^3$$

Find

1. Find all equilibrium points
2. Determine the type of each isolated equilibrium
3. Draw vector field plot

Solution

Find equilibrium points

>>>
```
/* Define equations */
xd1: x1 + x1*x2;
xd2:-x2 + x2^2 + x1*x2 - x1^3;

/* Find roots */
solve([xd1=0, xd2=0]);
```

$$\left[[\mathrm{x2}=1,\mathrm{x1}=0],[\mathrm{x2}=0,\mathrm{x1}=0],[\mathrm{x2}=-1,\mathrm{x1}=1],\left[\mathrm{x2}=-1,\mathrm{x1}=-\frac{\sqrt{7}i+1}{2}\right],\left[\mathrm{x2}=-1,\mathrm{x1}=\frac{\sqrt{7}i-1}{2}\right]\right]$$

$$\mathrm{x1x2}+\mathrm{x1}$$

$$\mathrm{x2}^2+\mathrm{x1x2}-\mathrm{x2}-\mathrm{x1}^3$$

Determine equilibrium point types

>>>

```
/* Calculate Jacobian */
J:jacobian([xd1, xd2], [x1, x2]);

/* Calculate eigenvalue for each equilibrium point  */
/* The eigenvalue output is of the following format */
/* [[eigenvalues], [multiplicity]]                  */

float(eivals(psubst([x1=0, x2=1], J)));
float(eivals(psubst([x1=0, x2=0], J)));
float(eivals(psubst([x1=1, x2=-1], J)));
```

$[[-1.0, 1.0], [1.0, 1.0]]$

$$\begin{pmatrix} x2+1 & x1 \\ x2-3x1^2 & 2x2+x1-1 \end{pmatrix}$$

$[[-1.732050807568877i - 1.0, 1.732050807568877i - 1.0], [1.0, 1.0]]$

$[[1.0, 2.0], [1.0, 1.0]]$

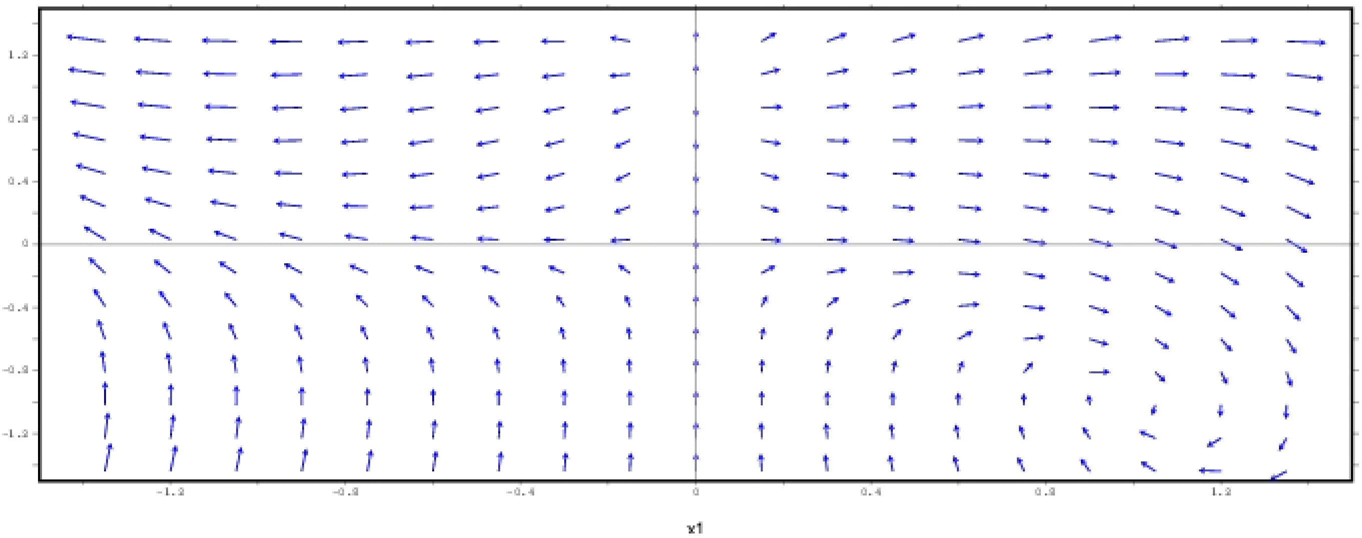
Based on the eigenvalues found from the Jacobians found at each of the equilibrium points, we can determine that they are:

- (0,1): Unstable node
- (0,0): Saddle
- (1,-1): Stable focus

Draw vector field

>>>

```
plotdf([xd1, xd2], [x1, x2], [x1, -1.2, 1.5], [x2, -1.5, 3.5])$
```



Canvas Problem)

Given

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.8x_1 - 10x_1^2 x_2 + u$$

Find

1. Develop an approximate, higher order linearization
2. Plot results

Solution

```
>>> /* Define original state */
xd1: x2$
xd2:-0.8*x1 - 10*x1^2*x2$

/* Find roots */
solve([xd1=0, xd2=0]);

/* Calculate Jacobian */
J:jacobian([xd1, xd2], [x1, x2])$

/* Calculate eigenvalue for each equilibrium point  */
/* The eigenvalue output is of the following format */
/* [[eigenvalues], [multiplicity]]                  */
float(eivals(psubst([x1=0, x2=0], J)));

/* Draw vector field */
plotdf([xd1, xd2], [x1, x2], [x1, -1.2, 1.5], [x2, -1.5, 3.5])$
```

```
rat: replaced -0.8 by -4/5 = -0.8
```

$$[[\mathrm{x1} = 0, \mathrm{x2} = 0]]$$

$$[[-0.8944271909999159i, 0.8944271909999159i], [1.0, 1.0]]$$

Define new linearized states

```
>>> /* Define state */
xt1: x2(t)$
xt2: -0.8*x1(t)$
xt3: -10*x1(t)^2*x2(t)$

/* Differentiate to get forms of xt# */
diff(xt1);
diff(xt2);
diff(xt3);

/* Define first derivative of new state */
xdt1: x2 + x3$
xdt2: -0.8*x1$
xdt3: (20*x2*x1^2)/(0.8) - (10*x2^2*x1)/(0.8)$

/* Calculate Jacobian */
J:jacobian([xdt1, xdt2, xdt3], [x1, x2, x3])$

/* Evalulate Jacobian at equilibrium point */
float(eivals(psubst([x1=0, x2=0], J)));
```

$$-0.8\left(\frac{d}{dt}\mathrm{x1}\left(t\right)\right)dt$$

$$\left(-10\mathrm{x1}\left(t\right)^2\left(\frac{d}{dt}\mathrm{x2}\left(t\right)\right) - 20\mathrm{x1}\left(t\right)\mathrm{x2}\left(t\right)\left(\frac{d}{dt}\mathrm{x1}\left(t\right)\right)\right)dt$$

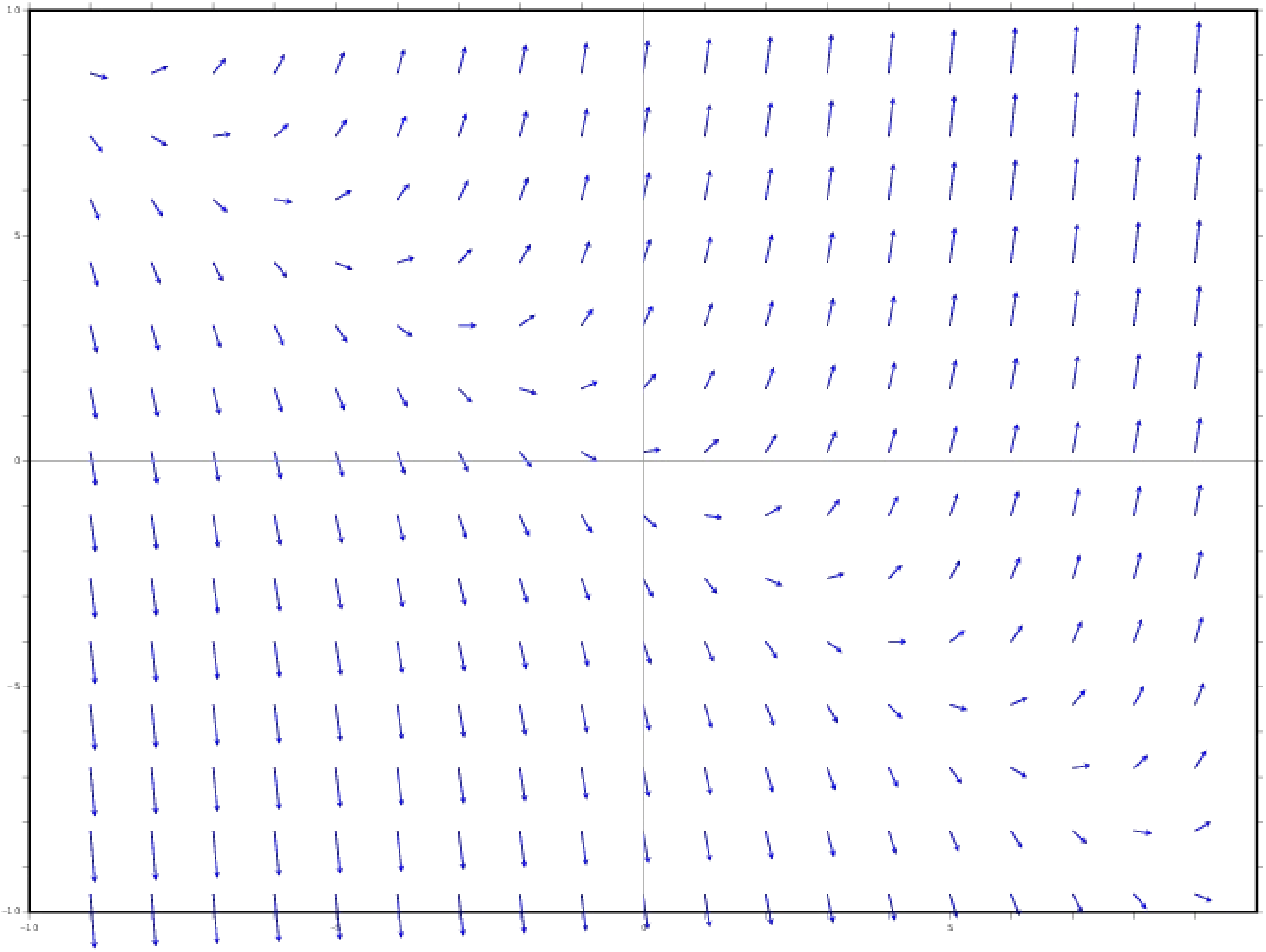
$$[[-0.8944271909999159i, 0.8944271909999159i, 0.0], [1.0, 1.0, 1.0]]$$

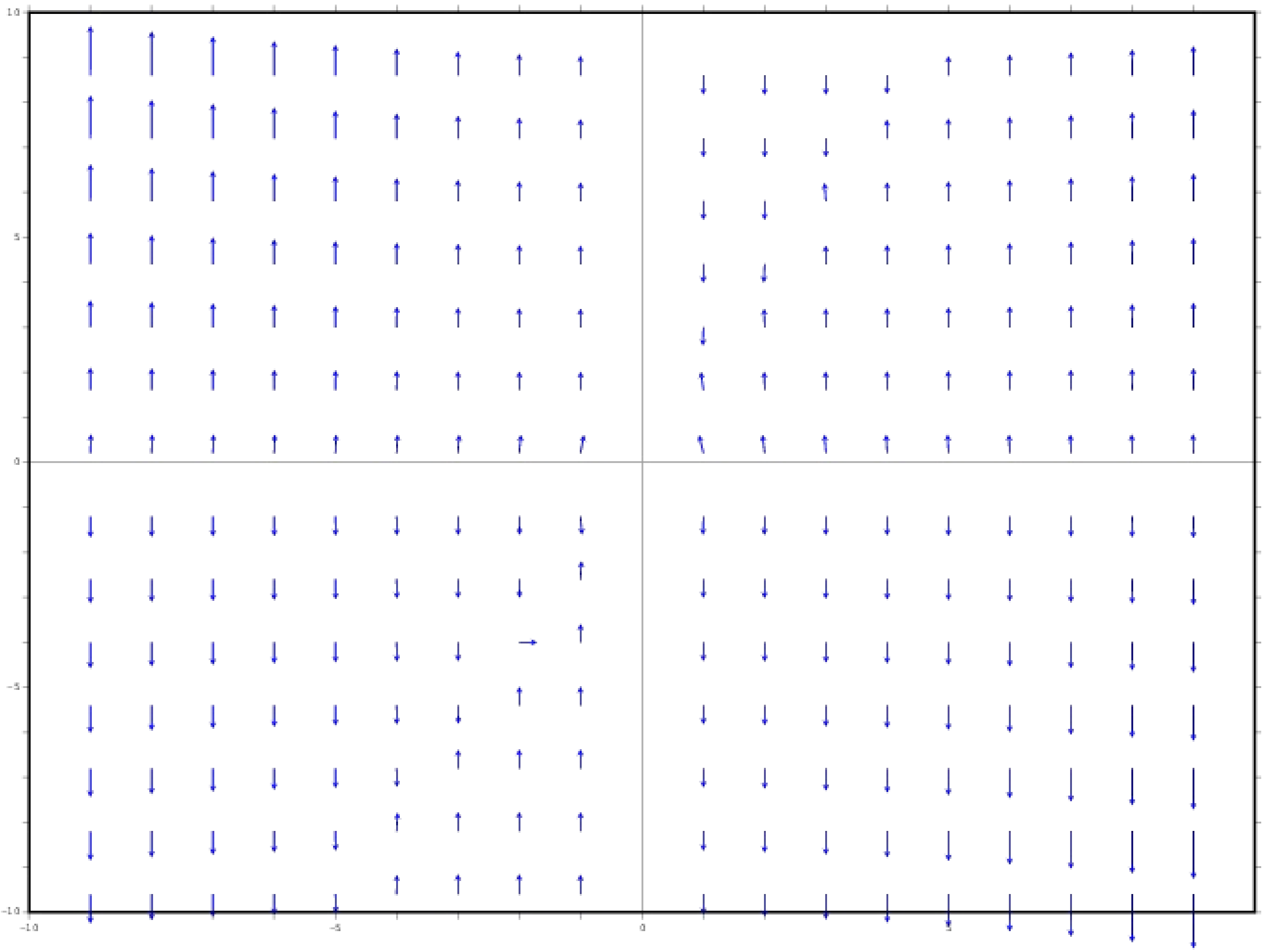
$$\frac{d}{dt}\mathrm{x2}\left(t\right)dt$$

From this we can see that we have retained the same eigenvalues (showing that the higher order state is another relization). Below are some slices of the contour plots.

```
>>> /* Draw vector field slices */
plotdf([xdt1], [x2, x3])$
```

```
>>>   plotdf([xdt2, xdt3], [x1, x2])$
```





4.32.4)

Given

$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_1 - x_2 - x_3 - x_1 x_3 \dot{x}_3 = (x_1 + 1)x_2$$

Find

Investigate whether the origin is stable, asympotically stable, or unstable.

Solution

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} -1 & 0 & 0 \\ -x_3 - 1 & -1 & -x_1 - 1 \\ x_2 & x_1 + 1 & 0 \end{bmatrix}\Bigg|_{x=[0]} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Finding the eigenvalues by using `eivals(*)` in Maxima

```
/* Define equation */
xd1: -x1;
xd2: -x1 - x2 - x3 - x1*x3;
xd3: (x1 + 1)*x2;

/* Solve for roots */
solve([xd1=0, xd2=0, xd3=0])

/* Calculate jacobian */
J: jacobian([xd1, xd1, xd3], [x1, x2, x3]);

/* Evaluate jacobian at roots */
float(eivals(psubst([x1=0, x2=0, x3=0], J)))
```

Theorem: Let $x = 0$ be an equilibrium point for

$$\dot{x} = f(x)$$

where $f : D \to \mathbb{R}^n$ is continuously differentiable and D is in a neighborhood of the origin. Let

$$A = \frac{\partial f}{\partial x}(0)$$

Let λ_i denote an eigenvalue of A

1. If $\forall \lambda_i Re(\lambda_i) < 0$, then the origin is asympotically stable
2. If $\exists \lambda_i$ such that $Re(\lambda_i) > 0$ then the origin is not stable

Therefore, by the previous stated theorem, the system is asympotically stable.