Canvas Problem

Given

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\phi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos(\phi+\theta) \\ sin(\phi+\theta) \\ sin(\theta) \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

And the Lie Brackets $q_1,\ q_2,\ [q_1, q_2],\ [q_1, [q_1, q_2]]$ spans $\mathbb{R}^4$ making the system STLC from any x

Find

Show that the system is STLC

Solution

```
>>> /* Create grad function */
grad(f,a,b,c,d) := [diff(f,a), diff(f,b), diff(f,c), diff(f,d)]$

/* Create Lie Bracket function */
LB(g1,g2) := block([dg1: grad(g1,x1,x2,phi,theta), dg2: grad(g2,x1,x2,phi,theta)], return(dg2.g1 -
dg1.g2))$

/* Define variables */
g1: matrix([cos(phi+theta)], [sin(phi+theta)], [sin(theta)], [0])$
g2: matrix([0],[0],[0],[1])$

/* Calculate Lie Brackets */
lb1: LB(g1,g2)$
lb2: LB(g1,lb1)$

M: mat_fullunblocker(matrix([g1,g2,lb1,lb2]))$
columnspace(M);
```

$$\mathrm{span}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \cos(\vartheta+\varphi) \\ \sin(\vartheta+\varphi) \\ \sin\vartheta \\ 0 \end{pmatrix}, \begin{pmatrix} \sin(\vartheta+\varphi) \\ -\cos(\vartheta+\varphi) \\ -\cos\vartheta \\ 0 \end{pmatrix}, \begin{pmatrix} \sin\vartheta\cos(\vartheta+\varphi) - \cos\vartheta\sin(\vartheta+\varphi) \\ \sin\vartheta\sin(\vartheta+\varphi) + \cos\vartheta\cos(\vartheta+\varphi) \\ 0 \\ 0 \end{pmatrix} \right)$$

$$\mathrm{notequal}(\sin\vartheta, 0) \wedge \mathrm{notequal}(\sin\vartheta, 0) \wedge \mathrm{notequal}(\sin\vartheta\sin(\vartheta+\varphi) + \cos\vartheta\cos(\vartheta+\varphi), 0) \wedge \mathrm{notequal}$$

It can easily be seen that the span is R^4 .

```
>>> /* Sanity check, solve the unicycle problem done in class */

/* Create grad function */
grad(f,a,b,c) := [diff(f,a), diff(f,b), diff(f,c)]$

/* Create Lie Bracket function */
LB(g1,g2) := block([dg1: grad(g1,x1,x2,phi), dg2: grad(g2,x1,x2,phi)], return(dg2.g1 - dg1.g2))$

/* Define variables */
g1: matrix([cos(phi)], [sin(phi)], [0])$
g2: matrix([0],[0],[1])$

/* Calculate Lie Brackets */
lb1: LB(g1,g2)$


M: mat_fullunblocker(matrix([g1,g2,lb1]));
columnspace(M);
```

$$\begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ \sin\varphi & 0 & -\cos\varphi \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathrm{notequal}\left(\cos\varphi, 0\right) \wedge \mathrm{notequal}\left(\cos\varphi, 0\right) \wedge \mathrm{notequal}\left(-\sin^2\varphi - \cos^2\varphi, 0\right)$$

$$\mathrm{span}\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix}, \begin{pmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{pmatrix}\right)$$

13.13)

Given:

$$\dot{x}_1 = tan(x_3)$$
$$\dot{x}_2 = -\frac{tan(x_2)}{acos(x_3)} + \frac{tan(u)}{bcos(x_2)cos(x_3)}$$
$$\dot{x}_3 = \frac{tan(x_2)}{acos(x3)}$$

Find:

1. Show system is feedback linearizeable
2. Find the domain of validity of the exact linear model
3. With $a = b = 1$ design a state feedback controller to stabilize the origin and simulate

13.13)

Given:

$$\dot{x}_1 = tan(x_3)$$
$$\dot{x}_2 = -\frac{tan(x_2)}{acos(x_3)} + \frac{tan(u)}{bcos(x_2)cos(x_3)}$$
$$\dot{x}_3 = \frac{tan(x_2)}{acos(x3)}$$

Find:

1. Show system is feedback linearizeable
2. Find the domain of validity of the exact linear model
3. With $a = b = 1$ design a state feedback controller to stabilize the origin and simulate

---

Solution

Note: This second just covers part (3)

>>>
```
/* Define system */
xd1: tan(x3)$
xd2: -tan(x2)/(a*cos(x3)) + tan(u)/(b*cos(x2)*cos(x3))$
xd3: tan(x2)/(a*cos(x3))$
```

Choose $z1 = h(x) = sin(x3)$

>>>
```
/* Define and calcualte derivatives */
/* z1  = sin(x3)                    */
/* z2  = dz1/dt                     */
/* zd2 = dz2/dt                     */

z1: sin(x3);
z2: trigsimp(xd3*cos(x3));
zd2: trigsimp(sec(x2)^2/a*xd2);
```

$$\sin \mathrm{x3}$$

$$-\frac{b\cos u\sin \mathrm{x2} - a\sin u}{a^2 b\cos u\cos^3 \mathrm{x2}\cos \mathrm{x3}}$$

$$\frac{\sin \mathrm{x2}}{a\cos \mathrm{x2}}$$

>>>