Utah State University
ECE 6030
Homework # 3
Due Friday, Feb 10, 2023

- Reading: Read sections 2.1 through 2.6
- Reading: Read the brief summary of random (stochastic) processes in Appendix D. We have mentioned in class about autocorrelation functions (section D.1), power spectral density functions (D.3), and linear systems with stochastic inputs (D.4). As a summary, this appendix does not provide derivations. This is intended to familiarize you with notation and concepts.
- Homework:

  1. (20 pts) Using MATLAB, plot the set $S_p = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_p = 1\}$, for $p \in \{4, 3, 2, 1, 0.8, 0.5, 0.2\}$. (That is, the "spheres" according to the p-norm.) Describe what happens as p gets large: Why is
  $$S_\infty = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_\infty = 1\}$$
  (the sphere using the d_∞ norm) a natural generalization? Also, describe what happens in the limit as $p \to 0$.
  2. (10 pts) Problem A.5-17. To clarify a couple of things, it is re-written here:
  Let (X, d_2) be a metric space of functions defined on $\mathbb{R}$ with the Euclidean metric
  $$d_2^2(f, g) = \int_{-\infty}^{\infty} (f(t) - g(t))^2 \, dt.$$
  Define the mapping $\Phi_\phi : X \to \mathbb{R}$ by
  $$\Phi_\phi(x) = \int_{-\infty}^{\infty} x(t)\phi(t) \, dt.$$
  Show that if $\phi(t)$ is square integrable, that is,
  $$\int_{-\infty}^{\infty} \phi(t)^2 \, dt < \infty,$$
  then Φ_ϕ is continuous mapping.

Reminder: **Definition**: A function $g : \mathbb{R} \to \mathbb{R}$ is *continuous* if for every $\epsilon > 0$ there exists a $\delta > 0$ such that

$$|f(x) - f(y)| < \epsilon \text{ whenever } |x - y| < \delta.$$

Conceptually: If x and y are close, the $f(x)$ and $f(y)$ can be close. Extend this basic definition from calculus to this metric space. What is analogous to $|x - y|$? What is analogous to $|f(x) - f(y)|$?

Hint: The Cauchy-Schwartz inequality may be helpful:

$$\left| \int_{-\infty}^{\infty} a(t)b(t)\, dt \right| \leq \left(\int_{-\infty}^{\infty} a^2(t)\, dt \right)^{1/2} \left(\int_{-\infty}^{\infty} b^2(t)\, dt \right)^{1/2}.$$

3. Problem 2.1-1 (5 pts) , 2.1-4 (a:5 b: 5), 2.1-5 (5 pts), 2.1-6 (5 pts), 2.1-9 (5 pts)
4. Problems 2.1-14 (5 pts) 2.1-23 (5 pts)