Thus

$$\det(B_n) = 1 \;//$$

$$\|A\|_\infty = \max_i \sum_j |a_{ij}| \quad ; \quad K(A) = \frac{M}{m}$$

$$\Rightarrow K_\infty(B_n) = \frac{\max_i \sum_j |a_{ij}|}{\min_i \sum_j |a_{ij}|} = \frac{n}{1} \;//$$

b) The theorem provided above applies to diagonal matrices as well, thus

$$\det(D_n) = \prod_{k=1}^{n} 10^{-1} = 10^{-n} \;//$$

The values on the diagonal of a diagonal matrix are the eigenvalues. Thus

$$K(D_n) = \frac{\sqrt{\lambda_{max}}}{\sqrt{\lambda_{min}}} = 1 \;//$$

//

4.11-47) Given:

$$B^{-1} = (A + xy^H)^{-1} = A^{-1} - \frac{A^{-1}xy^H A^{-1}}{1 + y^H A^{-1} x} \qquad (1)$$

Find:

Show (1) is true

Solution:

If $B = (A + xy^H)$ and $B^{-1} = A^{-1} - \frac{A^{-1}xy^H A^{-1}}{1+y^H A^{-1}x}$

then $BB^{-1} = B^{-1}B = I$

$$(A + xy^H)\left(A^{-1} - \frac{A^{-1}xy^H A^{-1}}{1+y^H A^{-1}x}\right)$$

$$= AA^{-1} + xy^H A^{-1} - \frac{AA^{-1}xy^H A^{-1} + xy^H A^{-1}xy^H A^{-1}}{1 + y^H A^{-1} x}$$

$$= I + xy^H A^{-1} - \frac{xy^H A^{-1} + xy^H A^{-1} xy^H A^{-1}}{1 + y^{-1} A^{-1} x}$$

$$= I + xy^H A^{-1} - \frac{x(1 + y^H A^{-1} x) y^H A^{-1}}{1 + y^{-1} A^{-1} x}$$

$$= I + xy^H A^{-1} - xy^H A^{-1} = I \ //$$

//

4.11-51) Given:

$$E[t] = \sum_i \lambda^{t-i} |e[i]|^2 \qquad \lambda < 1$$

$$R[t] = \sum_{i=1}^{t} \lambda^{t-i} q[i] q^H[i]$$

$$p[t] = \sum_{i=1}^{t} \lambda^{t-i} q^H[i] d[t]$$

Find:

Show that under this weighting that

$$k[t] = \frac{\lambda^{-1} P[t-1] q[t]}{1 + \lambda^{-1} q^H[t] P[t-1] q[t]}$$

$$P[t] = \lambda^{-1} P[t-1] - \lambda^{-1} k[t] q^H[t] P[t-1]$$

$$\epsilon[t] = d[t] - q^H[t] h[t-1]$$

$$h[t] = h[t-1] + k[t] \epsilon[t]$$

Solution

we know

$$h = R^{-1} A^H d = R^{-1} p$$

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$$R^{-1}[t] = R^{-1}[t-1] - \frac{R^{-1}[t-1]\, q[t]\, q^H[t]\, R^{-1}[t-1]}{1 + q^H[t]\, R^{-1}[t-1]\, q[t]}$$

by Sherman-Morrison formula. Note that the λ is in $R^{-1}[\cdot]$.

If we let

$$K[t] = \frac{R^{-1}[t-1]\; q[t]}{1 + q^H[t]\, R^{-1}[t-1]\, q[t]}$$

$$= \frac{\lambda^{-1}\, R^{-1}[t-1]\, q[t]}{1 + \lambda^{-1} q^H[t]\, R^{-1}[t-1]\, q[t]}$$

Let $R^{-1}(\cdot) = P(\cdot)$

$$K[t] = \frac{\lambda^{-1} P[t-1]\; q[t]}{1 + \lambda^{-1} q^H[t]\, P[t-1]\, q[t]}$$

$$P[t] = R^{-1}[t-1] - K[t]\, q^H[t]\, R^{-1}[t-1]$$

$$= \lambda^{-1} P[t-1] - \lambda^{-1}[t]\, q^H[t]\, P[t-1]$$

$$h[t] = P[t]p[t] = P[t](p[t-1] + q[t]d[t]) \quad (4.39)$$

where

$$P[t]P^{-1}[t-1] = h[t-1] - k[t]q^H[t]h[t-1]$$

$$\Rightarrow h[t] = h[t-1] - k[t]q^H[t]h[t-1] + P[t]q[t]d[t]$$
$$= h[t-1] + k[t]\underbrace{(d[t] - q^H[t]h[t-1]}_{\epsilon[t]}$$

$$\Rightarrow h[t] = h[t-1] + k[t]\epsilon[t]$$

$$\epsilon[t] = d[t] - q^H[t]h[t-1] \quad //$$

4.11-53) Given: Consider a sequence of vectors

$s_{11}, s_{12}, \ldots, s_{1N}$
$s_{21}, s_{22}, \ldots, s_{2N}$

Find:

a) Determine a transformation T s.t.

$$J = \sum_{i=1}^{N} \| T s_{1i} - s_{2i} \|^2$$

is minimized. Hint: Use the fact that scalar J, $J = tr[J]$, Use gradient formulas in Appendix E.

b) Take this solution and make it recursive, Determine initial conditions for recursive algorithm

c) Code and test in MATLAB

Solution:

a)

$$J = \sum_{i=1}^{N} \| T s_{1i} - s_{2i} \|^2$$

$$tr(J) = J = tr\left(\sum_i \| T s_{1i} - s_{2i} \|^2\right)$$

Note

$$\| T s_{1i} - s_{2i} \| \, \| T s_{1i} - s_{2i} \| = T s_{1i}^2 + 2 T s_{1i}^T s_{2i} + s_{2i}^2$$

$$\frac{\partial J}{\partial S} = \frac{\partial}{\partial S} \sum_i tr\left(T_{S1i}^2 - 2T_{S1i}^T S_{2i} + S_{2i}^2 \right)$$

$$= \sum_i tr\left(\begin{bmatrix} 2T_{S1i} - 2T_{S2i} & \cdots \\ 2T_{S1i} - 2S_{2i} & \cdots \end{bmatrix} \right)$$

$$= \sum_i 2T_{S1i} - 2T_{S2i} = 0$$

$$= \sum_i S_{1i} = \sum_i T^{*} T_{S2i}$$ //

Let $T : \mathbb{R}^n \to R(S_{2i})$ //

See P53.m