

80

Truth Tables

Problem 1

Compute the truth table for the following expressions and their subexpressions.

a) $(p \rightarrow q) \equiv (\text{NOT } p \text{ OR } q)$

p	q	$p \rightarrow q$	NOT p	NOT p OR q	$(p \rightarrow q) = (\text{NOT } p \text{ OR } q)$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

b) $p \rightarrow (q \rightarrow (r \text{ OR NOT } p))$

p	q	r	NOT p	r OR NOT p	$q \rightarrow (r \text{ OR NOT } p)$	$p \rightarrow (q \rightarrow (r \text{ OR NOT } p))$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	1	1
1	0	1	0	1	1	1
1	1	0	0	0	0	0
1	1	1	0	1	1	1

c) $(p \text{ OR } q) \rightarrow (p \text{ AND } q)$

p	q	p AND q	p OR q	$(p \text{ AND } q) \rightarrow (p \text{ OR } q)$
0	0	0	0	1
0	1	0	1	0
1	0	0	1	0
1	1	1	1	1

-2

Problem 2

The binary exclusive-or function, \otimes , is defined to have value TRUE if and only if exactly one of its arguments are TRUE.

- a) Draw the truth table for \otimes .

p	q	$p \otimes q$
0	0	0
0	1	1
1	0	1
1	1	0

- b) Is \otimes commutative? Is it associative?

I know \otimes is identity, because in encryption we can use a randomly generated stream of binary bits and \otimes it with plain text to get ciphertext which is reversible to decrypt.

I'd say it's commutative since " $a \otimes b = b \otimes a$ ", I have no idea how to prove it though. I'd say associative as well since I'm 99% sure " $a \otimes (b \otimes c) = (a \otimes b) \otimes c$ "

From Boolean Functions to Logical Expressions

Problem 3

The truth table below defines two Boolean functions, a and b, in terms of variables p, q, and r. Write sum-of-products expressions for each of these functions.

p	q	r	a	b
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	0	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

a: $pqr + \overline{p}qr + p\overline{q}r + pqr + \overline{p}q\overline{r} + pqr + pqr + pqr$

b: $\overline{p}qr + \overline{p}qr + \overline{p}qr + pqr + pqr + pqr + pqr + pqr$

only the ones with a or b = 1

Problem 4

Write product-of-sums expressions for

- a) Function a of the previous table.
 - i) $(p + q + r)(p + q + r)(p + !q + r)(p + !q + !r)(p + q + r)(p + q + r)(p + q + r)(p + q + r)$
- b) Function b of the previous table.
 - i) $(!p + !q + !r)(p + q + r)(p + q + r)(p + !q + !r)(!p + q + r)(!p + q + r)(!p + !q + r)(p + q + r)$
- c) Function z of the table below.
 - i) $(x + y + c)(x + y + c)(x + y + c)(x + !y + !c)(x + y + c)(!x + y + !c)(!x + !y + c)(x + q + y)$

only when the row is \emptyset

Designing Logical Expressions by Karnaugh Maps

Problem 5

Draw the Karnaugh maps for the following functions of variables p, q, r, and s.

- a) The function that is TRUE if one, two, or three of p, q, r, and s are TRUE, but not if zero or all four are TRUE.

		pq			
rs		00	01	11	10
	00	0	1	1	1
	01	1	1	1	1
	11	1	1	0	1
	10	1	1	1	1

- b) The function that is TRUE if up to two of p, q, r, and s are TRUE, but not if three or four are TRUE.

		rs			
pq		00	01	11	10
	00	1	1	1	1
	01	1	1	0	1
	11	1	0	0	0
	10	1	1	0	1

- c) The function that is TRUE if one, three, or four of p, q, r, and s are TRUE, but not if zero or two are TRUE.

		pq			
rs		00	01	11	10
	00	0	1	0	1
	01	1	0	1	0
	11	0	1	1	1
	10	1	0	1	0

- d) The function represented by the logical expression $pqr \rightarrow s$.

		pq			
rs		00	01	11	10
	00	0	1	0	1
	01	1	0	1	0
	11	0	1	1	1
	10	1	0	0	0

- e) The function that is TRUE if pqrs, regarded as a binary number, has value less than ten.

		pq			
rs		00	01	11	10
	00	1	1	0	1
	01	1	1	0	1
	11	1	1	0	0
	10	1	1	0	0

Problem 6

Find the implicants - other than the minterms - for each of your Karnaugh maps from Problem 5. Which of them are prime implicants? For each function, find a sum of prime implicants that covers all the 1's of the map. Do you need to use all the prime implicants?

- a) $\overline{p}r + p\overline{r} + \overline{r}s + \overline{p}q + r\overline{s} + p\overline{q}$
b) $\overline{p}r + \overline{p}q + \overline{r}s + \overline{q}r + \overline{p}s + \overline{q}s$
i) Conjunctive

- 1) $(!q + !r + !s)$
 - 2) $(!p + !q + !s)$
 - 3) $(!p + !q + !r)$
 - 4) $(!p + !r + !s)$
- c) $pqs + pqr + prs + qrs$
- d) $pqs + pqr + prs + qrs$
- e) $!p + !r$

Tautologies

Problem 7

Which of the following expressions are tautologies?

- a) $pqr \rightarrow p + q$ **YES**

p	q	r	pqr	p + q	$pqr \rightarrow p + q$
0	0	0	0	0	1
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	1	1

- b) $((p \rightarrow q)(q \rightarrow r)) \rightarrow (p \rightarrow r)$ **YES**

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r)(q \rightarrow r)$	$((p \rightarrow r)(q \rightarrow r)) \rightarrow (p \rightarrow r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	1	1	1	1
1	1	0	0	0	0	1
1	1	1	1	1	1	1

c) $(p \rightarrow q) \rightarrow p$ **NO**



p	q	$p \rightarrow q$	$(p \rightarrow q) \rightarrow p$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

d) $(p \equiv (q + r)) \rightarrow (q \rightarrow pr)$ **YES**

X -2

p	q	r	$q + r$	$p = (q + r)$	pr	$q \rightarrow pr$	$(p \equiv (q + r)) \rightarrow (q \rightarrow pr)$
0	0	0	0	1	0	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	0	0	1	1
1	0	1	1	1	1	1	1
1	1	0	1	1	0	0	0
1	1	1	1	1	1	1	1

Problem 8

Apply DeMorgan's laws to turn the following expressions into expressions where the only NOT's are applied to propositional variables (i.e., the NOT's appear in literals only).

1. $\text{NOT}(pq + pr)$
 - a. $\text{NOT}((pq)(\text{NOT } pr))$
 - b. $\text{NOT } pq + \text{NOT } pr$
 - c. $\text{NOT } p + \text{NOT } q + \text{NOT NOT } p + \text{NOT NOT } r$
 - d. **$\text{NOT } p + \text{NOT } q + p + r$**
2. $\text{NOT}(\text{NOT } p + q(\text{NOT}(r + s)))$
 - a. $\text{NOT}((\text{NOT } p) q(\text{NOT}(r + \text{NOT } s)))$
 - b. $\text{NOT NOT } p + \text{NOT } q + \text{NOT}(\text{NOT}(r + \text{NOT } s))$
 - c. $p + \text{NOT } q + \text{NOT}(r \text{ NOT } s)$
 - d. $p + \text{NOT } q + \text{NOT } r + \text{NOT NOT } s$



5

x

e. $p + \text{NOT } q + \text{NOT } r + s$