Fault-Tree Analysis by Fuzzy Probability

Hideo Tanaka, Member IEEE
University of Osaka Prefecture, Osaka
L. T. Fan
Kansas State University, Manhattan
F. S. Lai
US Grain Marketing Research Laboratory, Manhattan
K. Toguchi
Mitsubishi Kakoki Kaisha Ltd, Kawasaki

Key Words—**Fault-tree analysis, Fuzzy-set theory, Extension principle, Possibility of failure, Probability of failure.**

Reader Aids—
Purpose: Tutorial
Special math needed for explanations: Elementary statistics
Special math needed to use results: Same
Results useful to: Fault-tree analysts, reliability analysts

Abstract—**In conventional fault-tree analysis, the failure probabilities of components of a system are treated as exact values in estimating the failure probability of the top event. For many systems, it is often difficult to evaluate the failure probabilities of components from past occurrences because the environments of the systems change. Furthermore, it might be necessary to consider possible failure of components even if they have never failed before. We, therefore, propose to employ the *possibility* of failure, viz. a fuzzy set defined in probability space. The notion of the *possibility* of failure is more predictive than that of the probability of failure; the latter is a limiting case of the former.**

In the present approach based on a fuzzy fault-tree model, the maximum *possibility* of system failure is determined from the *possibility* of failure of each component within the system according to the *extension principle*. In calculating the *possibility* of system failure, some approximation is made for simplicity.

1. INTRODUCTION

Fault-tree analysis (FTA) is a logical and diagrammatic method to evaluate the probability of an accident resulting from sequences and combinations of faults and failure events. A fault tree describes an accident-model and interprets the relations between malfunctions of the components and observed symptoms. Thus, the fault tree is useful for understanding logically the mode of occurrence of an accident. Furthermore, given the failure probabilities of system components, the probability of the top event can be calculated.

In conventional fault-tree analysis, the failure probabilities of system components are treated as exact values. For many systems, however, it is often difficult to evaluate the failure probabilities of components from past occurrences, because the environments of the systems change. Moreover, we often need to consider failure of components which have never failed before.

Instead of the probability of failure, we propose the *possibility* of failure, viz. a fuzzy set [1] defined in probability space. By resorting to this concept, we can allocate a degree of uncertainty to each value of the probability of failure; in this manner, different aspects of uncertainty, probability, and *possibility* can be simultaneously treated. For example, if information that "the probability of failure is between 0.01 and 0.1, and is perhaps around 0.07" is given, it can be represented as a fuzzy set, ie. *possibility* of failure. This *possibility* of failure includes the probability of failure as a limiting case, and thus, the present approach might be more predictive and useful than the conventional uncertainty analysis.

In the present approach based on fuzzy fault-tree model [2], the *possibility* of failure of the top event is calculated from the *possibilities* of failure of its components according to the *extension principle* [3, 4]. In this paper, the *possibilities* of failure are limited to the trapezoid shape for simplicity; this assumption leads to a reasonable approximation to the mode by which we assess the *possibilities* of failure.

2. NOTATION

X, Y, X_1, X_2	subsets in real space.
x, y, x_1, x_2	element in each subset.
P, p, q	probabilities.
X_i	event i.
α, ℓ, r	superscript for representing a scale of the trapezoid.
$\tilde{A}$	fuzzy set of A.
$\tilde{<}$	fuzzy inequality.
$\tilde{A} \subset \tilde{B}$	fuzzy inclusion (see theorem 4).
$\odot$	approximate product of fuzzy sets.

3. PRELIMINARIES ON FUZZY SETS

Fuzzy set theory has been developed to deal with fuzzy phenomena [1, 4]. Fuzziness of a phenomenon stems from the lack of clearly defined boundaries, and such a phenomenon can be defined by a fuzzy set. A fuzzy set $\tilde{A}$ of X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which is associated with a number in the interval [0, 1], representing the degree of x belonging to X.

Definition 1. (Extension principle)

Let $y = f(x_1, x_2)$

where $y \in Y$, $x_1 \in X_1$, $x_2 \in X_2$. Given fuzzy sets $\tilde{A}_1$ and $\tilde{A}_2$ in X_1 and X_2, respectively, the fuzzy set —

$$\tilde{Y} = f(\tilde{A}_1, \tilde{A}_2).$$

is defined by the membership function:

$$\mu_{\tilde{Y}}(y) \equiv \begin{cases} \max\{\mu_{\tilde{A}_1}(x_1) \wedge \mu_{\tilde{A}_2}(x_2)\} & ; [x_1, x_2 | y = f(x_1, x_2)] \neq \phi \\ [x_1, x_2 | y = f(x_1, x_2)] & \\ 0 & ; \text{otherwise} \end{cases}$$

where $\wedge$ stands for the minimum. The concept underlying this definition is called the *extension principle* [3, 4]. Using definition 1, we can define the maximum or minimum of two fuzzy sets [5, 6].

Definition 2. The maximum of two fuzzy sets:

$$\tilde{A}_M \equiv \max\{\tilde{A}_1, \tilde{A}_2\}$$

is defined by:

$$\mu_{\tilde{A}_M}(x) \equiv \max_{[x_1, x_2 | x = \max\{x_1, x_2\}]} \{\mu_{\tilde{A}_1}(x_1) \wedge \mu_{\tilde{A}_2}(x_2)\}.$$

Similarly, the minimum of two fuzzy sets:

$$\tilde{A}_m = \min(\tilde{A}_1, \tilde{A}_2)$$

is defined by:

$$\mu_{\tilde{A}_m}(x) \equiv \max_{[x_1, x_2 | x = \min\{x_1, x_2\}]} \{\mu_{\tilde{A}_1}(x_1) \wedge \mu_{\tilde{A}_2}(x_2)\}.$$

This definition, constructed by means of the *extension principle,* is useful for ranking fuzzy sets on the probability space. For simplicity, suppose that fuzzy sets are normal and convex. A fuzzy set $\tilde{A}$ is normal and convex if and only if there exists x^* such that $\max_x\{\mu_{\tilde{A}}(x)\} = \mu_{\tilde{A}}(x^*) = 1$, and $A_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}$ is a convex set for α.

Definition 3.

Let $\max(\tilde{A}_1, \tilde{A}_2) = \tilde{A}_1$ or $\min(\tilde{A}_1, \tilde{A}_2) = \tilde{A}_2$.

Then, the order relation is defined as:

$\tilde{A}_1 \tilde{>} \tilde{A}_2$.

If $\tilde{A}_1 \supset \tilde{A}_2$ holds, then:

$\tilde{A}_1 \tilde{>} \tilde{A}_2$.

Otherwise, $\tilde{A}_1$ and $\tilde{A}_2$ are deemed equivalent:

$\tilde{A}_1 \approx \tilde{A}_2$.

Definition 3 induces an order relation among fuzzy sets.

Example. Consider fuzzy probabilities shown in figure 1. According to definition 3, the order relation among them is:

$\tilde{A}_1 \tilde{>} \tilde{A}_2 \tilde{>} \tilde{A}_3 \approx \tilde{A}_4 \tilde{>} \tilde{A}_5 \tilde{>} \tilde{A}_6$

because —

$\max\{\tilde{A}_1, \tilde{A}_2\} = \tilde{A}_1$

$\max\{\tilde{A}_2, \tilde{A}_3\} = \tilde{A}_2$

$\max\{\tilde{A}_4, \tilde{A}_5\} = \tilde{A}_4$

$\tilde{A}_5 \supset \tilde{A}_6$,

and no order relation exists between $\tilde{A}_3$ and $\tilde{A}_4$.

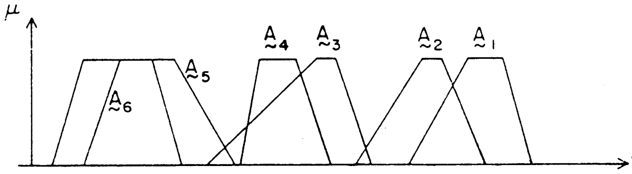

Fig. 1. Fuzzy sets ordered as $\tilde{A}_1 \tilde{>} \tilde{A}_2 \tilde{>} \tilde{A}_3 \approx \tilde{A}_4 \tilde{>} \tilde{A}_5 \tilde{>} \tilde{A}_6$.

4. FORMULATION OF FUZZY FAULT-TREE ANALYSIS

This section generalizes evaluation of a fault tree to fuzzy sets. Fault-tree analysis consists of two major parts: construction and evaluation. Here, we are mainly concerned with the fuzzy evaluation of failure probability of the top event of a fault-tree.

A fault-tree is a logic model that represents the combinations of events which lead to the top (undesirable) event. Figure 2 is an example that uses two types of event symbols and two types of gates. The rectangle defines an intermediate or top event that is the output of a logic gate. The circle indicates a fundamental event, viz, a primary failure of a system element. The symbol "+" stands for an OR gate and the symbol "•" for an AND gate.

In figure 2, the top event can be expanded as:

$$\begin{aligned} T &= A_1 \cup A_2 \\ &= (X_1 \cap X_2) \cup (X_3 \cup A_3) \\ &= (X_1 \cap X_2) \cup X_3 \cup (X_4 \cap X_5). \end{aligned}$$

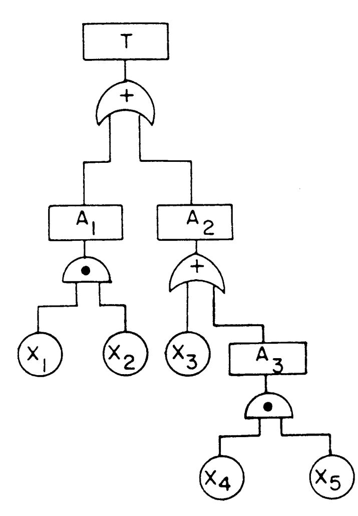

Fig. 2. An example of fault tree.

Given p_{X_i}, the failure probability of X_i, the failure probability of the top event T is:

$$P_T(p_{X_1}, p_{X_2}, \ldots, p_{X_5}) = 1 - (1 - p_{X_1} p_{X_2})(1 - p_{X_3})(1 - p_{X_4} p_{X_5}).$$

As described in the Introduction, it is often difficult to assign a unique numerical value between 0 and 1 to a failure probability. To circumvent this difficulty, the failure probability can be defined as a fuzzy set on [0, 1]. Specifically, the *possibility* of failure defined in a certain range on [0, 1] is used instead of a unique value of probability. Now the problem is to calculate the *possibility* of failure of the top event as a fuzzy set, given the *possibilities* of failure of fundamental events. Instead of a specific value of probability, we deal with a fuzzy number on [0, 1], viz. fuzzy-probability or *possibility* of failure. Mathematically speaking, it is equivalent to determining the following fuzzy set according to definition 1;

$$\tilde{P}_T(\tilde{P}_{X_1}, \tilde{P}_{X_2}, \ldots, \tilde{P}_{X_5}) = 1 - (1 - \tilde{P}_{X_1}\tilde{P}_{X_2})(1 - \tilde{P}_{X_3})(1 - \tilde{P}_{X_4}\tilde{P}_{X_5})$$

where $\tilde{P}_{X_i}$ is a fuzzy set defined on [0, 1]. More specifically, the following type of fuzzy sets is considered in analyzing a fault tree.

Definition 4. The *possibility* of failure probability of X_i is:

$$\tilde{P}_{X_i} \equiv (q_i^\ell, p_i^\ell, p_i^r, q_i^r)$$

which is defined by the membership function (see figure 3):

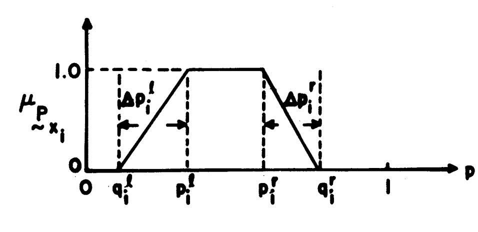

Fig. 3. Sketch of fuzzy probability $\tilde{P}_{X_i}$.

$$\mu_{\tilde{P}_{X_i}}(p) = \begin{cases} 0; & 0 \le p \le q_i^\ell \\ 1 - \theta_i^\ell; & q_i^\ell < p \le p_i^\ell \\ 1; & p_i^\ell < p \le p_i^r \\ 1 + \theta_i^r; & p_i^r < p \le q_i^r \\ 0; & q_i^r < p \le 1 \end{cases}$$

$$\Delta p_i^\alpha \equiv p_i^\alpha - q_i^\alpha, \ \alpha = \ell, r;$$

$$\theta_i^\alpha \equiv (p_i^\alpha - p)/\Delta p_i^\alpha.$$

The *extension principle* described in definition 1 results in:

Theorem 1. The multiplication of two fuzzy sets $\tilde{P}_{X_i}$ and $\tilde{P}_{X_j}$, ie. $\tilde{P}_{X_i} \cdot \tilde{P}_{X_j}$, gives rise to the following membership function;

$$\mu_{\tilde{P}_{X_i} \cdot \tilde{P}_{X_j}}(p) = \begin{cases} 0 & ; \ 0 \le p \le q_i^\ell q_j^\ell \\ 1 - \phi_{ij}^\ell & ; \ q_i^\ell q_j^\ell < p \le p_i^\ell p_j^\ell \\ 1 & ; \ p_i^\ell p_j^\ell < p \le p_i^r p_j^r \\ 1 + \phi_{ij}^r & ; \ p_i^r p_j^r < p \le q_i^r q_j^r \\ 0 & ; \ q_i^r q_j^r < p \le 1 \end{cases}$$

$$g_{ij}^\alpha \equiv \frac{p - p_i^\alpha p_j^\alpha}{\Delta p_i^\alpha \Delta p_j^\alpha}$$

$$h_{ij}^\alpha \equiv \frac{\Delta p_i^\alpha p_j^\alpha + \Delta p_j^\alpha p_i^\alpha}{2\Delta p_i^\alpha \Delta p_j^\alpha} \ ; \alpha = \ell, r$$

$$\phi_{ij}^\alpha \equiv h_{ij}^\alpha - [g_{ij}^\alpha + (h_{ij}^\alpha)^2]^{1/2}.$$

Theorem 1 is proved in the Supplement [7]. As described in theorem 1, the manipulation involved in the multiplication of two fuzzy sets is complicated. Thus, the following approximate product $\odot$ and the complement are used here:

$$P_{X_i} \odot P_{X_j} \equiv (q_i^\ell q_j^\ell, p_i^\ell p_j^\ell, p_i^r p_j^r, q_i^r q_j^r) \approx \tilde{P}_{X_i} \cdot \tilde{P}_{X_j} \ ;$$

$$1 - \tilde{P}_{X_i} \equiv (1 - q_i^r, 1 - p_i^r, 1 - p_i^\ell, 1 - q_i^\ell).$$

Since the approximate product of fuzzy sets is obtained through linearization, it is desirable that the relationship between the approximate product $\tilde{P}_{X_i} \odot \tilde{P}_{X_j}$ and the exact product $\tilde{P}_{X_i} \cdot \tilde{P}_{X_j}$ be given.

Theorem 2. Two equivalent relations are:

$$\tilde{P}_{X_i} \cdot \tilde{P}_{X_j} \tilde{<} \tilde{P}_{X_i} \odot \tilde{P}_{X_j}$$

$$\max\{\tilde{P}_{X_i} \cdot \tilde{P}_{X_j}, \tilde{P}_{X_i} \odot \tilde{P}_{X_j}\} = \tilde{P}_{X_i} \odot \tilde{P}_{X_j}.$$

Theorem 2 is proved in the Supplement [7]. The relation in theorem 2 is illustrated in figure 4. Theorem 2 implies that the approximate product $\odot$ over-estimates the possibility of failure, thereby resulting in a greater margin of safety. Theorem 3 facilitates comparison of the relative dominance of an event.

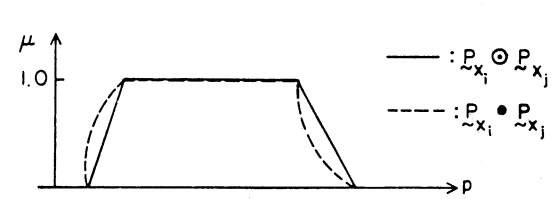

Fig. 4. Explanation of $\tilde{P}_{X_i} \odot \tilde{P}_{X_j} \tilde{>} \tilde{P}_{X_i} \cdot \tilde{P}_{X_j}$.

Theorem 3.

$$\tilde{P}_T(\tilde{P}_{X_1}, \ldots, \tilde{P}_{X_{i-1}}, 0, \tilde{P}_{X_{i+1}}, \ldots, \tilde{P}_{X_n}) \tilde{<} \tilde{P}_T(\tilde{P}_{X_1}, \ldots, \tilde{P}_{X_i}, \ldots, \tilde{P}_{X_n})$$

(The proof of theorem 3 is straightforward, since the probability of the top event, $P_T(p)$, is an increasing function with respect to the probability p.)

For simplicity, define:

$$\tilde{P}_T(\tilde{P}_{X_1}, \ldots, \tilde{P}_{X_i}, \ldots, \tilde{P}_{X_n}) \equiv \tilde{P}_T$$

$$\tilde{P}_T(\tilde{P}_{X_1}, \ldots, \tilde{P}_{X_{i-1}}, 0, \tilde{P}_{X_{i+1}}, \ldots, \tilde{P}_{X_n}) \equiv \tilde{P}_{T_i}.$$

The index, V, measures the difference between $\tilde{P}_T$ and $\tilde{P}_{T_i}$ and indicates the extent of improvement in eliminating the event X_i:

$$V(\tilde{P}_T, \tilde{P}_{T_i}) \equiv (q_T^\ell - q_{T_i}^\ell) + (p_T^\ell - p_{T_i}^\ell) + (p_T^r - p_{T_i}^r) + (q_T^r - q_{T_i}^r) > 0.$$

For example, if —

$$V(\tilde{P}_T, \tilde{P}_{T_i}) \geq V(\tilde{P}_T, \tilde{P}_{T_j}),$$

then eliminating X_i is more effective than removing X_j. Using this index, we can rank the basic events according to the degree of effectiveness in improving the system.

The logical equation of a fault tree can be expanded. With the definitions of the two operators, $\tilde{P}_{X_i} \odot \tilde{P}_{X_j}$ and $(1 - \tilde{P}_{X_i})$ in hand, we can evaluate the fault tree which is represented by minimal cut and path sets (see figure 5). In these examples of fault trees, the following equations correspond to the two structures:

$$\tilde{P}_T \equiv 1 - (1 - \tilde{P}_{X_1} \odot \tilde{P}_{X_2}) \odot, \ldots, \odot (1 - \tilde{P}_{X_{n-1}} \odot \tilde{P}_{X_n});$$

minimal cut sets

$$\tilde{P}_T \equiv \{1 - (1 - \tilde{P}_{X_1}) \odot (1 - \tilde{P}_{X_2})\} \odot, \ldots, \odot \{1 - (1 - \tilde{P}_{X_{n-1}}) \odot (1 - \tilde{P}_{X_n})\};$$ minimal path sets.

Neither of the above equations should be expanded under the approximate product $\odot$; such expansion gives rise to fuzzy sets different from the ones defined by these equations according to the *extension principle*; see theorem 4.

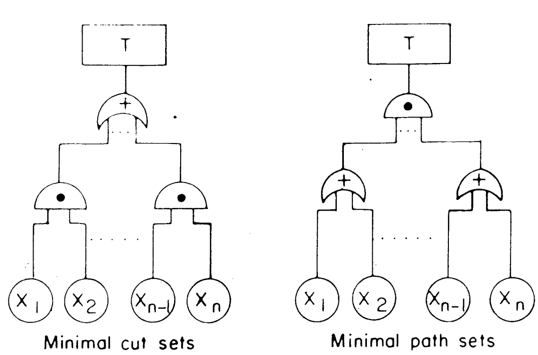

Fig. 5. Examples of fault trees represented by minimal cut sets and minimal path sets.

Theorem 4.

$$(1 - \tilde{P}_A) \odot (1 - \tilde{P}_B) \subset 1 - (\tilde{P}_A + \tilde{P}_B) + \tilde{P}_A \odot \tilde{P}_B$$

where $\tilde{P}_A \subset \tilde{P}_B$

implies, for all p, $\mu_{\tilde{A}}(p) \leq \mu_{\tilde{B}}(p)$,

$$(P_{\tilde{A}} + P_{\tilde{B}}) \equiv \{(q_A^\ell + q_B^\ell) \wedge 1, (p_A^\ell + p_B^\ell) \wedge 1, (p_A^r + p_B^r) \wedge 1, (q_A^r + q_B^r) \wedge 1\}.$$

The proof is in the Supplement [7].

Numerical Example. To explain the present approach, consider the numerical example in figure 2. Fuzzy probabilities are given in table I. Using the approximate operator $\odot$, we have:

$$\tilde{P}_T(\tilde{P}_{X_1}, \tilde{P}_{X_2}, \ldots, \tilde{P}_{X_5}) = 1 - (1 - \tilde{P}_{X_1} \odot \tilde{P}_{X_2}) \odot (1 - \tilde{P}_{X_3}) \odot (1 - \tilde{P}_{X_4} \odot \tilde{P}_{X_5}).$$

TABLE I
Fuzzy Probabilities of Fundamental Events

Fuzzy Probability	$(q^\ell, p^\ell, p^r, q^r)$
$\tilde{P}_{X_1}$	(0.1, 0.15, 0.2, 0.25)
$\tilde{P}_{X_2}$	(0.02, 0.03, 0.05, 0.08)
$\tilde{P}_{X_3}$	(0.01, 0.02, 0.03, 0.04)
$\tilde{P}_{X_4}$	(0.2, 0.25, 0.35, 0.5)
$\tilde{P}_{X_5}$	(0.006, 0.008, 0.01, 0.012)

This expression yields the fuzzy probability of the top event as follows:

$$\tilde{P}_T(\tilde{P}_{X_1}, \tilde{P}_{X_2}, \ldots, \tilde{P}_{X_5}) = (0.013, 0.026, 0.043, 0.065)$$

This is illustrated in figure 6. With prevention of the failure of fundamental events X_i, we obtain the corresponding fuzzy probability of top event, $\tilde{P}_{T_i}$ and $V(\tilde{P}_T, \tilde{P}_{T_i})$, as listed in table II. The values of $V(\tilde{P}_T, \tilde{P}_{T_i})$ indicate that preventing the failure of X_3 is most effective; this yields:

$$\tilde{P}_{T_3} = (0.003, 0.006, 0.013, 0.026).$$

If the failure of X_1 or X_2 is prevented, the fuzzy probability of the top event becomes:

$$\tilde{P}_{T_1} = \tilde{P}_{T_2} = (0.011, 0.022, 0.033, 0.046).$$

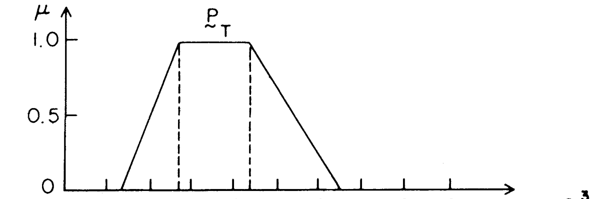

Fig. 6. Sketch of $\tilde{P}_T$ = (0.013, 0.026, 0.043, 0.065).

TABLE II
$\tilde{P}_{T_i}$ and $V(\tilde{P}_T, \tilde{P}_{T_i})$

	$\tilde{P}_{T_i} = (q^\ell, p^\ell, p^r, q^r)$	$V(\tilde{P}_T, \tilde{P}_{T_i})$
$\tilde{P}_{T_1} = \tilde{P}_{T_2}$	(0.011, 0.022, 0.033, 0.046)	0.035
$\tilde{P}_{T_3}$	(0.003, 0.006, 0.013, 0.026)	0.099
$\tilde{P}_{T_4} = \tilde{P}_{T_5}$	(0.012, 0.024, 0.040, 0.059)	0.012

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AUTHORS

Hideo Tanaka; Department of Industrial Engineering; University of Osaka Prefecture; Sakai, Osaka 591, JAPAN.

Dr. H. Tanaka (M'81) was born in 1938 and is Associate Professor of Industrial Engineering, University of Osaka Prefecture. He received BS from Kobe University, and MS and PhD in Control Engineering from Osaka City University. He is a member of IEEE, SICE (Japan), and Operation Research Soc. of Japan.

L. T. Fan; Department of Chemical Engineering; Kansas State University; Manhattan, Kansas 66506 USA (where all correspondence should be sent.)

Dr. L. T. Fan was born in 1929 and is Professor and Head of Chemical Engineering, Kansas State University. He received BS from National Taiwan University, MS from Kansas State University, PhD in Chemical Engineering from West Virginia University, and MS in Mathematics from West Virginia University. He is a member of American Institute of Chemical Engineers, American Chemical Society, and Society of Engineering Sciences.

F. S. Lai; Grain Marketing Research Laboratory; US Department of Agriculture; Manhattan, Kansas 66502 USA.

Dr. F. S. Lai was born in 1941 and is Research Leader of Engineering Research Unit at Grain Marketing Research Laboratory, USDA. He received BS from National Taiwan University, MS from University of Notre Dame, and PhD in Chemical Engineering from Kansas State University. He is a member of AIChE, ACS, and American Society of Agricultural Engineers.

Kenichi Toguchi; Mitsubishi Kakoki Kaisha Ltd., 2-1 Okawa, Kawasaki 210, JAPAN.

Mr. K. Toguchi was born in 1950 and received BS and MS in Chemical Engineering from University of Tokyo. He is a member of AIChE, Society of Chemical Engineers (Japan), and Japan Petroleum Institute.

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Probability of Component or Subsystem Failure Before System Failure

(continued from page 452)

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AUTHORS

Dr. John Buzacott; Dept. of Management Sciences; University of Waterloo; Waterloo, Ont. N2L 3G1, CANADA.

John Buzacott has a BE in Electrical Engineering from the University of Sydney, Australia, and MSc and PhD degrees from the University of Birmingham. Prior to moving to the University of Waterloo in January 1984 he had been in the Department of Industrial Engineering at the University of Toronto since 1967. His research interests are in production system modelling and reliability. He is President of the Canadian Operational Research Society.

Dr. George J. Anders; Ontario Hydro, 700 University Ave.; Toronto, Ont. M5G 1X6, CANADA.

George J. Anders (M'79) received a Masters Degree in Electrical Engineering from Technical University of Lodz in Poland in 1973, and an MSc Degree in Mathematics and PhD Degree in Power System Reliability from the University of Toronto in 1977 and 1980. Since 1975 he has been employed by Ontario Hydro, most recently in the Reliability Section of the Transmission System Design Department of System Planning Division.

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