Polytechnics School of ULB
First Master in Computer Engineering
Section: Computational Intelligence

Learning Dynamics

Assignment 3 : Nash Equilibrium

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Question 1

1. Two people have 10€ to divide between themselves. They use the following procedure. Each person names a certain amount (a non negative integer), at most equal to 10€. If the sum of the amounts that the persons name is at most 10€, then each person receives the amount of money she named (and the remainder is destroyed). If the sum of the amounts that they name exceeds 10€ and the amounts named are different, then the person who named the smaller amount receives that amount and the other person receives the remaining money. If the sum of the amounts that the players named exceeds 10€ and the amounts are the same, then each person receives 5 €.

Write down this game as the strategic game (players, actions, payoff)

Determine the Nash equilibria using best response functions

Answer

Legend:

- Best Player 1's actions for a given action of Player 2: green
- Best Player 2's actions for a given action of Player 1: red

According to the following table we can see that the best response function gives 4 Nash Equilibriums:

- (5,5)
- (5,6)
- (6,5)
- (6,6)

	1	2	3	4	5	6	7	8	9	10
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(1,9)	(1,9)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)	(2,8)	(2,8)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,7)	(3,7)	(3,7)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,6)	(4,6)	(4,6)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)
6	(6,1)	(6,2)	(6,3)	(6,4)	(5,5)	(5,5)	(6,4)	(6,4)	(6,4)	(6,4)
7	(7,1)	(7,2)	(7,3)	(6,4)	(5,5)	(4,6)	(5,5)	(7,3)	(7,3)	(7,3)
8	(8,1)	(8,2)	(7,3)	(6,4)	(5,5)	(4,6)	(3,7)	(5,5)	(8,2)	(8,2)
9	(9,1)	(8,2)	(7,3)	(6,4)	(5,5)	(4,6)	(3,7)	(2,8)	(5,5)	(9,1)
10	(9,1)	(8,2)	(7,3)	(6,4)	(5,5)	(4,6)	(3,7)	(2,8)	(1,9)	(5,5)

Question 2

2. Find the mixed strategy Nash equilibria in

	L	R
T	6,0	0,6
B	3,2	6,0

	L	R
T	0,1	0,2
B	2,2	0,1

Answer

	L	R
T	6,0	0,6
B	3,2	6,0

Player 1

We are looking for $U_L = f(\sigma)$

$$U_L = \sigma_T * 0 + (1 - \sigma_T) * 2$$

We are looking for $U_R = f(\sigma)$

$$U_R = \sigma_T * 6 + (1 - \sigma_T) * 0$$

So we have:

$$U_L = U_R$$

$$2 - 2 * \sigma_T = 6 * \sigma_T$$

$$\sigma_T = 1/4$$

Player 2

We are looking for $U_T = f(\sigma)$

$$U_T = \sigma_L * 6 + (1 - \sigma_L) * 0$$

We are looking for $U_B = f(\sigma)$

$$U_B = \sigma_L * 3 + (1 - \sigma_L) * 6$$

So we have:

$$U_T = U_B$$

$$\sigma_L * 6 = 3 * \sigma_L + (1 - \sigma_L) * 6$$

$$9 * \sigma_L = 6$$

$$\sigma_T = 2/3$$

Mixed Strategy Nash Equilibrium is

$$<\sigma_T = 1/4, \sigma_L = 2/3>$$

	L	R
T	0,1	0,2
B	2,2	0,1

Player 1

We are looking for $U_L = f(\sigma)$

$$U_L = \sigma_T * 1 + (1 - \sigma_T) * 2$$

We are looking for $U_R = f(\sigma)$

$$U_R = \sigma_T * 2 + (1 - \sigma_T) * 1$$

So we have:

$$U_L = U_R$$

$$\sigma_T * 1 + (1 - \sigma_T) * 2 = \sigma_T * 2 + (1 - \sigma_T) * 1$$

$$-2 * \sigma_T = -1$$

$$\sigma_T = 1/2$$

Player 2

We are looking for $U_T = f(\sigma)$

$$U_T = \sigma_L * 0 + (1 - \sigma_L) * 0$$

We are looking for $U_B = f(\sigma)$

$$U_B = \sigma_L * 2 + (1 - \sigma_L) * 0$$

So we have:

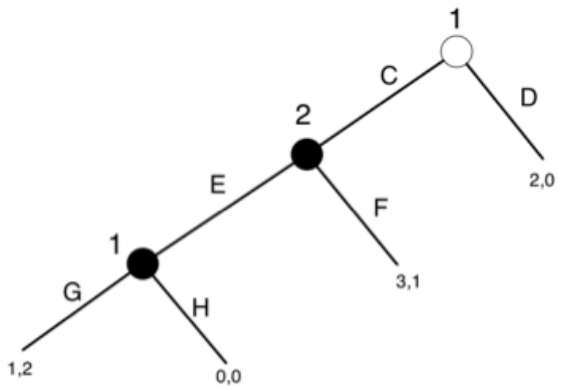
$$U_T = U_B$$
$$\sigma_L * 6 = 3 * \sigma_L + (1 - \sigma_L) * 6$$
$$9 * \sigma_L = 6$$
$$\sigma_L = 0$$

Mixed Strategy Nash Equilibrium is

$$<\sigma_T = 1/2, \sigma_L = 0>$$

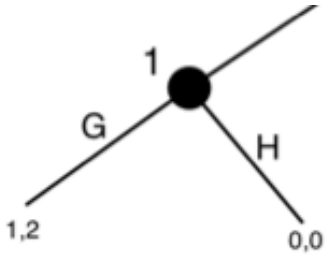
Question 3

2. Find the Nash equilibria and determine which of them is subgame perfect



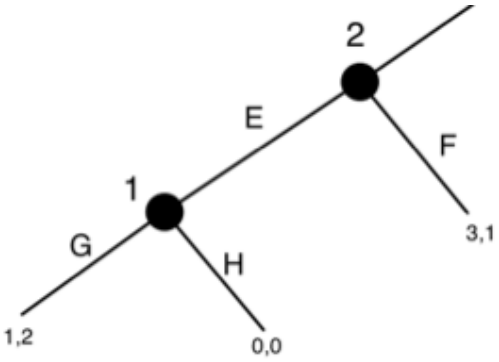
Subgames

- Subgame 1:



G	H
1,2	0,0

- Subgame 2:



E	F
1,2	3,1

→ Player 1 has the following strategies:

1. DG: D, but if C were chosen, then choose G.
2. DH: D, but if C were chosen, then choose H.
3. CG: C and G following 2's play.
4. CH: C and H following 2's play.

→ Player 2 has the following strategies:

1. E(C): E if 1 plays C.
2. E(D): E if 1 plays D.
3. F(C): F if 1 plays C.
4. F(D): F if 1 plays D.

	E(C)	E(D)	F(C)	F(D)
DG	-1,-1	2,0	2,0	2,0
DH	-1,-1	2,0	2,0	2,0
CG	1,2	-1,-1	3,1	-1,-1
CH	0,0	-1,-1	3,1	-1,-1

(The value -1 is chosen for impossible actions.)

There are 2 Nash Equilibriums:

- (CG, F(C)) is not a subgame perfect because F(C) is not played in the subgame beginning after C.
- (CH, F(C)) is not a subgame perfect because F(C) is not played in the subgame beginning after C.

We can see graphically that there is no subgame perfect in this game → no line go straight from the top to the bottom of the tree.