Contents

1 GITHUB COMMANDS

shows all the configurations of github $gitconfig - -list$

link for Endnote and Latex `https://www.rhizobia.co.nz/latex/convert`

1.1 Fix the Personal Access Token

Ref:
`https://stackoverflow.com/questions/68775869/support-for-password-authentication-was-removed-please-use-a-personal-access-to`

From August 13, 2021, GitHub is no longer accepting account passwords when authenticating Git operations. You need to add a PAT (Personal Access Token) instead, and you can follow the below method to add a PAT on your system.
Create Personal Access Token on GitHub:

- From your GitHub account, go to Settings.
- Developer Settings
- Personal Access Token
- Generate New Token (Give your password)
- Fillup the form => click Generate token
- Copy the generated Token, it will be something like $ghp\_sFhFsSHhTzMDreGRLjmks4Tzuzgthdvfsrt$

Now follow below method based on your machine:

- For Windows OS:
  - Go to Credential Manager from Control Panel
  - Windows Credentials
  - find git:https://github.com
  - Edit, on Password replace with with your GitHub Personal Access Token
  - You are Done
  - If you dont find `git:https://github.com`
    - Click on Add a generic credential
    - Internet address will be `git:https://github.com`
    - Fill username, password is your GitHub Personal Access Token
    - Click Ok and you are done.

2 LINEAR ALGEBRA - GILBERT STRANG

2.1 Video 1 - 4

operation matrix on the left is for row, on the right is for column [1]
A = LU = LDU, with LDU, the diaganol in the L,U is ones.
A = LU helps the reduce the computational efforts a lot.

2.2 Video 5 (Nov 2, 2021)

Symmetric matrix A' = A
R' x R is always a symmetric matrix. Prove: (R'.R)' = R'.R" = R'.R
Chapter 3: Vector spaces
Every vector spaces has a vector zeros, R2 = [2 3], [pi e], R3 = [2 3 4], [3 2 0] 8 rules in the book. Every subspace must have the zero vector because we must be allowed to do the math operation with zero. subspace of R2: whole r2, 2 > any lines go through the origin, 3 > zero vector [0 0]. how to find the subspace: matrix A -> take columns of A, find all the combinations of the column then you have the column subspaces of the matrix A

2.3 Video 6 (Nov 2, 2021)

Read chapter 3 of the book.
Vector spaces and subspaces
Column space of A: solving Ax = b
Nullspace of A: solving Ax = 0
Vector space requirements: v + w and cv are in the space, all the combinations cv + dw are in the space.
We can solve the A.x = b exactly if b is in the column space of A, noted as C(A). Because in the same subspace, any linear combination of the columns in the A with x is B, and B will belongs to that subspace.
Pivot column, the column that is independent (27m)
NULL space of A, noted as N(A), is all the combination of x that make A.x = 0, another ways is that with b = 0, solve A.x = b.
Check that the solution of A.x = 0 always give a subspace. What do we have to check? If Av = 0 and Aw = 0, how about A(v + w) must be = 0
Ax = b, do the solutions form the subspace -> NO, check if the zero-vector is not a solution or not. Although there are many solutions but there is no zero-vector solution (origin). So the solution is the line and the plan which does not go through the origin.

2.4 Video 7 (Nov 3, 2021)

Solving Ax = 0, break A -> U -> R, matlab function is rref (reduced row echelon form).
If the matrix has m (let's say m = 3) columns, and the rank (aka number of

pivots point of that matrix) = 2, then the matrix has 3 - 2 = 1 free columns.
x = c*[-F , I], [-F , I] is the nullspace matrix, called N. Nullspace is the matrix, whose column is the special solution
Null space N = [-Free, I], nullspace stores all the solution.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 6 & 8 \\ 2 & 8 & 10 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = U \tag{1}$$

Rank of the matrix is 2 because there are 2 pivots point, number 1 in the first row, and the first number 2 in the second row.
There are 2 pivot columns:

$$Pivot\_columns = \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

There are 1 free column:

$$Free\_column = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

From U to R

$$U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{pmatrix} = R = \tag{4}$$

There are 2 main matrices we need to focus on, one is the Identity matrix at the top left corner of the matrix R, the other is the free (F) part next to the Identity matrix at the top right corner of the matrix R. Identity matrix is:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

Free matrix is

$$F = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{6}$$

The nullspace, also known as the solution, called N, is the matrix:

$$x = N = c * \begin{pmatrix} -F \\ I \end{pmatrix} \tag{7}$$

2.5 Video 8 (Nov 11, 2021)

Summarize the lecture so far at the end of the videos.
The rank of the matrix tell you about the number of solution for the equation Ax = b
There are full ROW rank, full COL rank.
The solvability of the equation Ax = b.
The form of the solution of Ax = b is X = Xparticular + Xnullspace.
X-nullspace always for Ax = 0.

2.6 Video 9 (Nov 16, 2021)

Independence, Basis and dimension
Definition of independence vectors.
18m: Vector span a space
24m: basics
nho viet lai theo dang item cho cai note nay
35m: given a space, every basis for the space has the same number of vectors, that number of vectors is called dimension, this is the definition of the dimension of the spaces
rank of matrix A = number of pivot columns = dimension of space of A, written as C(A)

2.7 Video 10 (Nov 16, 2021)

Four fundamental subspace is the matrix
Fix the error in the video 9, the error is the 2 rows is the same so the rank in row is only 2, so it is not invertible, because of the transposition A'.
4m: the 4 subspaces
10m: the picture of 4 spaces

3 Control Bootcamp - Steve Bruton

3.1 Vid 01 - Overview

We could classify the control into these groups:

- Passive Control
- Active Control
  - Open-loop Control
  - Closed-loop Control

Why feedbacks?

- Uncertainty

- Intability
- Disturbances Rejection
- Efficient Control
  - You have to continuously move the inverted pendulum up and down to keep it stable (openloop)
  - But if you have a good fdback controller, you barely have to control the cart, and the stick is still stable
  - You only need to control whenever you need it, not all the time

3.2 Vid 02 - Linear System

3.2.1 Linear System $\dot{X} = A.X,\ with X \in \mathbb{R}^n$

The solution for the equation is

$$X(t) = e^{At}.X(0) \tag{8}$$

with X is a vector, A is a matrix.
Use Taylor series to expand e^{At}

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \ldots + \frac{A^nt^n}{n!} \tag{9}$$

But if we have A is a 5×5 matrix, calculating e^{At} is not practical to compute, so we will use eignenvalue and eigenvector of A
to get a coordinate transformation from X coordinates to eigenvector's coordinates.
so that it is easier for us to understand the dynamics themselves $\dot{X} = A.X$ and easier to write down e^{At}

With

$$\dot{X} = A.X \tag{10}$$

Use MATLAB to find T, D matrices

$$[T, D] = eig(A) \tag{11}$$

Matrix D is a diaganol matrix

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \tag{12}$$

From D we calculate the e^{Dt}

$$e^{Dt} = \begin{bmatrix} e^{\lambda_1 t} & & & \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ & & & e^{\lambda_n t} \end{bmatrix} \tag{13}$$

3.2.2 Eignevalue and Eigenvector

with $A = TDT^{-1}$

$$\begin{aligned} e^{At} &= e^{TDT^{-1}t} \\ &= I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \ldots + \frac{A^nt^n}{n!} \\ &= TIT^{-1} + TDT^{-1}t + \frac{TDT^{-1}.TDT^{-1}t^2}{2!} + \ldots + \frac{(TDT^{-1})^nt^n}{n!} \\ &= T.[I + Dt + \frac{D^2t^2}{2!}] + \frac{D^3t^3}{3!} + \ldots + \frac{D^nt^n}{n!}].T^{-1} \\ &= T.e^{Dt}.T^{-1} \end{aligned} \tag{14}$$

Try to write the system in this form AT = TD (8:19 timestamp)

$$\begin{aligned} A.T &= T.D \\ T^{-1}.A.T &= T^{-1}.T.D \\ T^{-1}.A.T &= D \end{aligned} \tag{15}$$

We have

$$\begin{aligned} X &= TZ \\ \dot{X} &= T\dot{Z} = AX \\ T\dot{Z} &= AX \\ T\dot{Z} &= ATZ \\ T^{-1}T\dot{Z} &= T^{-1}ATZ \\ \dot{Z} &= T^{-1}ATZ \end{aligned} \tag{16}$$

with $T^{-1}.A.T = D$

$$\dot{Z} = DZ \tag{17}$$

Why do we really need D to be diaganol?
Because D now is a diaganol matrix so that all the component of Z is fully decoupled to each other. In another words, Z_1 only effects $\dot{Z}_1$, Z_2 only effects $\dot{Z}_2$

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \vdots \\ \dot{Z}_n \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_n \end{bmatrix} \tag{19}$$

The solution for $\dot{Z} = DZ$ is $Z(t) = e^{Dt}.Z(0)$ (13:46 tstamp) We think immediately in our mind when we see the $\dot{X} = AX$ is think about it in eigenvector $X = TZ$ because it is simple for the solution $Z(t)$

3.2.3 Eigenvector Relationship

Start with this Taylor series to expand e^{At}

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \ldots + \frac{A^nt^n}{n!} \tag{20}$$

The transformation from $e^{At} = Te^{Dt}T^{-1}$ (20:43 tstamp), we do this because it is very easy to compute the e^{Dt} due to the diaganol characteristic of matrix D

IMPORTANT: The main idea (concept) is $X(t) = Te^{Dt}T^{-1}.X(0)$, with $X = TZ$

- $Z(0) = T^{-1}.X(0)$: mapping the IC from x coordinates to eigenvector coordinates
- Then multiply with e^{Dt} to form $Z(t) = e^{Dt}.T^{-1}.X(0)$: it is intuitive because the dynamics is decoupled already
- Then multiply with T to map it back to X coordinates, $X(t) = Te^{Dt}T^{-1}.X(0)$ which is required in the physical world - what we care is x, not z.

3.2.4 Next step

- Analyze the stability using eigenvectors of matrix A
- Build discrete model instead of using continuous model $\dot{X} = A.X$ by discretize the system with λT
- Add the control Bu part to start controlling the dynamics $\dot{X} = A.X + B.u$

3.3 Vid 03 - Stability and Eigenvalues

$$\dot{X} = A.X \tag{21}$$

Use MATLAB to find T, D matrices

$$[T, D] = eig(A) \tag{22}$$

Matrix D is a diaganol matrix

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \tag{23}$$

$$e^{Dt} = \begin{bmatrix} e^{\lambda_1 t} & & & \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ & & & e^{\lambda_n t} \end{bmatrix} \tag{24}$$

3.3.1 Stability of $\dot{X} = A.X$

With
$X(t) = Te^{Dt}T^{-1}.X(0)$
$\lambda = a \pm bi$
$e^{\lambda t} = e^{at}[cos(bt) \pm isin(bt)]$

- if ALL of the REAL parts of all the eigenvalues has $a < 0$ then the system is STABLE
- if any of the REAL parts of all the eigenvalues has $a > 0$ then the system is unstable
- If any real parts of these lambdas are positive then the e^{Dt} blows up and the $X(t) = Te^{Dt}T^{-1}.X(0)$ will blow up too.

3.3.2 Discrete time model, tstamp 09:14

$X_{k+1} = \tilde{A}.X_k$
$\tilde{A} = e^{A\Delta t}$

We have $X_0, \Delta t$, and matrix A

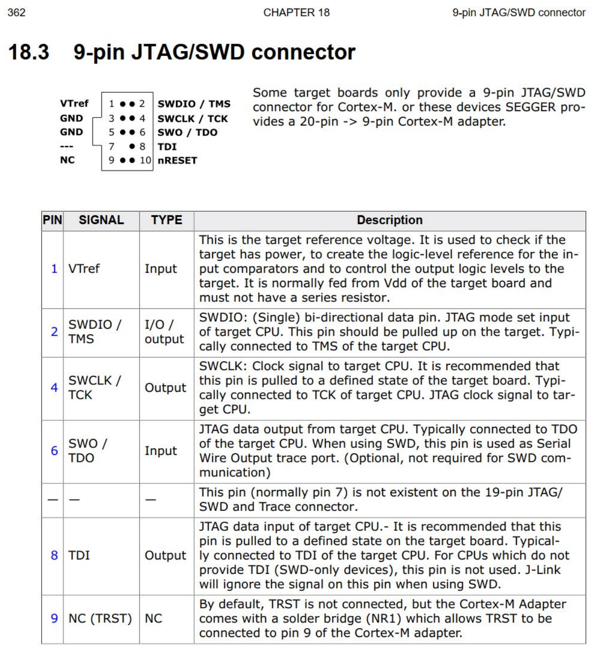
$$\begin{aligned} X_1 &= \tilde{A}^1 X_0, \lambda = 1 \\ X_2 &= \tilde{A}^2 X_0, \lambda = 2 \\ X_3 &= \tilde{A}^3 X_0, \lambda = 3 \\ &\dots \\ X_N &= \tilde{A}^N X_0, \lambda = N \end{aligned} \tag{25}$$

Using polar coordinate to describe the

$$\begin{aligned} \lambda &= R.e^{i\theta} \\ \lambda^N &= R^N e^{iN\theta} \end{aligned} \tag{26}$$

We see that λ^N has the same direction with λ, the radius R is powered by N (getting bigger or smaller), and the angle θ is rotating around
So if the radius R of ALL the $\lambda = R.e^{i\theta}$

- $R < 1$ the system is STABLE because $R^N \to 0$ when $t \to \infty$
- $R > 1$ the system is unstable because $R^N \to \infty$ when $t \to \infty$



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18.3 9-pin JTAG/SWD connector

Some target boards only provide a 9-pin JTAG/SWD connector for Cortex-M. or these devices SEGGER provides a 20-pin -> 9-pin Cortex-M adapter.

PIN	SIGNAL	TYPE	Description
1	VTref	Input	This is the target reference voltage. It is used to check if the target has power, to create the logic-level reference for the input comparators and to control the output logic levels to the target. It is normally fed from Vdd of the target board and must not have a series resistor.
2	SWDIO / TMS	I/O / output	SWDIO: (Single) bi-directional data pin. JTAG mode set input of target CPU. This pin should be pulled up on the target. Typically connected to TMS of the target CPU.
4	SWCLK / TCK	Output	SWCLK: Clock signal to target CPU. It is recommended that this pin is pulled to a defined state of the target board. Typically connected to TCK of target CPU. JTAG clock signal to target CPU.
6	SWO / TDO	Input	JTAG data output from target CPU. Typically connected to TDO of the target CPU. When using SWD, this pin is used as Serial Wire Output trace port. (Optional, not required for SWD communication)
—	—	—	This pin (normally pin 7) is not existent on the 19-pin JTAG/ SWD and Trace connector.
8	TDI	Output	JTAG data input of target CPU.- It is recommended that this pin is pulled to a defined state on the target board. Typically connected to TDI of the target CPU. For CPUs which do not provide TDI (SWD-only devices), this pin is not used. J-Link will ignore the signal on this pin when using SWD.
9	NC (TRST)	NC	By default, TRST is not connected, but the Cortex-M Adapter comes with a solder bridge (NR1) which allows TRST to be connected to pin 9 of the Cortex-M adapter.

Figure 1: Pinout of the Jlink segger edu mini

So if all the lambda (also described as vectors) inside the unit circle, then the system is stable

From matrix $\tilde{A}$ - discrete time, we calculate
$[\tilde{T}, \tilde{D}] = eig(\tilde{A})$

Always start with
$\tilde{A}\tilde{T} = \tilde{T}\tilde{D}$
$\tilde{A}\tilde{T}\tilde{T}^{-1} = \tilde{T}\tilde{D}\tilde{T}^{-1}$
$\tilde{A} = \tilde{T}\tilde{D}\tilde{T}^{-1}$

Plug $\tilde{A} = \tilde{T}\tilde{D}\tilde{T}^{-1}$ into these equations:

$$
\begin{aligned}
X_1 &= \tilde{A}^1 X_0, \lambda = 1, \tilde{D}^1 \\
X_2 &= \tilde{A}^2 X_0, \lambda = 2, \tilde{D}^2 \\
X_3 &= \tilde{A}^3 X_0, \lambda = 3, \tilde{D}^3 \\
&\dots \\
X_N &= \tilde{A}^N X_0, \lambda = N, \tilde{D}^N
\end{aligned} \tag{27}
$$

We see that the only thing could blow up the system is if the REAL part of

ANY lambda in the matrix D is > 0

$$\tilde{D} = \begin{bmatrix} \tilde{\lambda}_1 & & & \\ & \tilde{\lambda}_2 & & \\ & & \ddots & \\ & & & \tilde{\lambda}_N \end{bmatrix} \tag{28}$$

The stability of the system is fully depend on the eigenvalues of the matrix A

- continuos time, real part must be negative, in the left half plane
- discrete time, the eigenvalues must be in the unit circle

3.4 Vid 04 - Linearizing System around fixed point $\dot{X} = f(x) \rightarrow \dot{X} = AX,\ with\ X \in \mathbb{R}^n$

- Find the fixed point $\bar{x}$ st $\dot{x} = f(\bar{x}) = 0$
  - inverted pendulum, $\bar{x}$ is the top point, without disturbances, the pendulum will stay there forever
  - pendulum, $\bar{x}$ is the bottom point.
  - between the Earth and the Sun, $\bar{x}$ the point where the gravitational force of the Sun and the Earth are the same.
- Linearize about the fixed point $\bar{x}$
  - Take partial derivatives
  - Hartman–Grobman THEOREM,
    https://en.wikipedia.org/wiki/Hartman
    We could only linearize the system if the eigenvalues has NON-ZERO REAL PARTS,
    IF NOT, the linearization cannot show the dynamics of the system.

$$\frac{Df}{Dx}|_{\bar{x}} = \frac{\partial f_i}{\partial x_j} \tag{29}$$

  - examples

$$\begin{aligned} \dot{X}_1 &= f_1(x_1, x_2) = x_1 x_2 \\ \dot{X}_2 &= f_2(x_1, x_2) = {x_1}^2 + {x_2}^2 \\ \frac{Df}{Dx} &= \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} x_2 & x_1 \\ 2x_1 & 2x_2 \end{bmatrix} \end{aligned} \tag{30}$$

Examples: The pendulum.

- The Equation of Motion

$$\ddot{\theta} = -\frac{g}{L}.sin(\theta) - \delta\dot{\theta} \tag{31}$$

- Find the fixed point
  - Define the state of the system.

$$\begin{aligned} X &= \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \\ \dot{X} &= \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -sin(\theta) - \delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_2 \\ -sin(X_1) - \delta X_2 \end{bmatrix} \end{aligned} \tag{32}$$

  - Fixed point $\bar{X}$ so that $\dot{X} = f(\bar{X}) = 0$

$$\dot{X} = \begin{bmatrix} X_2 \\ -sin(X_1) - \delta X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} X_2 \\ -sin(X_1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{34}$$

$$\begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 + k\pi \end{bmatrix} \tag{35}$$

  In the physical world, there are 2 roots, which are the up position and down position of the pendulum.
  There are 2 fixed points:

$$\bar{X}_{down} = \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \; and \;\; \bar{X}_{up} = \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \pi \end{bmatrix} \tag{36}$$

- Linearize the system around the fixed points

$$\dot{X} = \begin{bmatrix} f_1(X_1, X_2) \\ f_2(X_1, X_2) \end{bmatrix} = \begin{bmatrix} X_2 \\ -sin(X_1) - \delta X_2 \end{bmatrix} \tag{37}$$

$$\frac{Df}{Dx} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -cos(x_1) & -\delta \end{bmatrix} \tag{38}$$

  Subsitute the value of 2 fixed point X into the Jacobian Matrix to find the matrix A for both cases

$$\bar{X}_{down} = \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad A_{down} = \begin{bmatrix} 0 & 1 \\ -1 & -\delta \end{bmatrix} \tag{39}$$

$$\bar{X}_{up} = \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \pi \end{bmatrix}, \quad A_{up} = \begin{bmatrix} 0 & 1 \\ 1 & -\delta \end{bmatrix} \tag{40}$$

- Find the eigenvalues of the 2 matrices A_up and A_down, with $\delta = 0.1$
  Using Matlab function eig(A)

$$A_{down} = \begin{bmatrix} 0 & 1 \\ -1 & -0.1 \end{bmatrix}, eigenvalues \;\; = \begin{bmatrix} -0.05 + 0.9987i \\ -0.05 - 0.9987i \end{bmatrix} \tag{41}$$

$$A_{up} = \begin{bmatrix} 0 & 1 \\ 1 & -0.1 \end{bmatrix}, eigenvalues \;\; = \begin{bmatrix} -1.0512 \\ 0.9512 \end{bmatrix} \tag{42}$$

- Conclusions
  - Pendulum in down position is stable, because the real part of 2 eigenvalues are negative
  - Pendulum in up position is unstable, because there are 1 eigenvalues is positve
  - The result is as same as we predict before doing this examples

3.5 Vid 5 - Controllability

Insert the right notations, Xdot, y, u = -kx, xdot = (a-bk)x
In real world, people give a matrix A, B, you cannot change it, you could only design the control law to make the system perform better
Column Rank and Row Rank
Controllability function in Matlab C = ctrb(A,B) only tells you YES or NO, not tell how controllable. If the rank (C) equals to the number of the states, then the system is controllable. So we need another way to know how well the system could be controlled
Fullstate feedback means you could measure all the elements in the state X
IF THE SYSTEM IS CONTROLLABLE, then you could drive the system to any state that you want. and place the pole any where you want. Link this with the Linear Algebra in Gilbert Strang class of MIT courseware
SVD - Single Value Decomposition of a CONTROLLABILITY MATRIX - C, the first part will show you which states of the System in the order from the easiest to control to the hardest to control. In other words, which states is easy to reach, which states are not.

3.5.1 Controllability

Example of uncontrollable system - tstamp 13:14 How do I modify B to make uncontrollable system to controllable.

3.6 Vid 6 - Controllability, Reachability, and Eigenvalue Placement

System is controllable or not?
if yes, we could use Pole placement, with any arbitary eigenvalues
Reachability, if the column is not depend, assume we 2 vectors in R2, then it is independent, it could represent the whole plane, whole state of the systems, watch gilbert strang linear algebra.
But the limit in reality is the limit in u of physical world. and the linear system disadvantage is it cannot represent the non-linear factor in the real life.
And in linear system equation, we could apply infinity value of U to get the X-dot as big as we want.

3.7 Vid 7 - Controllability, and the Discrete time Impulse Response

3.8 Vid 8 - Degrees of Controllability and Gramians

3.9 Vid 9 - Controllability and the Popov-Belevitch-Hautus (PBH) test

3.9.1 remind

$\dot{x} = Ax + Bu,\ with\ x \in R^n$
$C = [B\ AB\ A^2B\ \ldots A^{n-1}B]$
$>> rank(ctrb(A, B))$

3.9.2 The PBH Test

The PBH test is (A, B) is controllable if and only iff (iff) $rank[(A - \lambda I)\ B] = n$ with every λ in the complex plan $\mathbb{C}$

The PBH test

$$rank[(A - \lambda I)\ B] = n, \quad \forall \lambda \in \mathbb{C} \tag{43}$$

Good question: When will $(A - \lambda I)$ has rank n?
More specificly, when will $(A - \lambda I)$ NOT has rank n(2:43)?
$(A - \lambda I)$ is deefficient, when the determinent of the $(A - \lambda I)$ equals to 0. It is the determient equation

$$|(A - \lambda I)| = 0 \tag{44}$$

This equation satisfied with at most λ is the eigenvalues of matrix A.
If λ is not the eigenvalues of matrix A, then the $(A - \lambda I)$ alone has the rank n, and we DO NOT really need the matrix B in the $[(A - \lambda I)\ B]$ to have $rank[(A - \lambda I)\ B] =$ n.

3.9.3 The PBH Test - Important Conclusions

- $rank(A - \lambda I) = n$ except for the eigenvalues λ.
  It means we only need to test at eigenvalues, because $(A - \lambda I)$ has rank n if λ is not the eigenvalues of matrix A (3:50)
  So we reduce from testing for the whole complex plane $\mathbb{C}$ down to only testing at the eigenvalues of matrix A.

- B needs to have some components in each eigenvector (of matrix A) direction.
  If we pick λ as a eigenvalues of matrix A and plug it into the $(A - \lambda I)$, we see that the $(A - \lambda I)$ is rank deefficient, and in which direction that

$(A - \lambda I)$ is rank deefficient?
The answer is, we look at the Nullspace of $(A - \lambda I)$, which is the eigenvector of $(A - \lambda I)$. The $(A - \lambda I)$ is rank deefficient in EXACTLY the direction of the eigenvector. And the eigenvector is the reason that makes $(A - \lambda I)$ equals to 0.
We see that in order to have $rank(A - \lambda I) = n$, when $(A - \lambda I) = 0$ in some eigenvector directions of A, then the matrix B must complement for matrix A, or matrix B must at least has some component in all of the eigenvector directions to make $rank[(A - \lambda I)\ B] = n$.

- (ADVANCED) if B is a random vector, in another word, if B = randn(n,1), then (A,B) will be controllable with high probability.

  B cannot be aligned with ONLY ONE eigenvector of matrix A because $rank[(A - \lambda I)\ B]$ will be$= n$ for that specific λ, and for the other λ, the matrix B will not make $rank[(A - \lambda I)\ B] = n$ So that B must at least has some component in EVERY EIGENVECTOR of matrix A in order to make $rank[(A - \lambda I)\ B] = n$ with all the λ in the complex plan $\mathbb{C}$ $(\forall \lambda \in \mathbb{C})$

  A random vector B $\in R^n$, with HIGH PROBABILITY, it has a little bit in in ALL of those eigenvector direction of MATRIX A. And it is VERY EXTREMELY UNLUCKY for B to be aligned with ONLY one, two or a few eigenvector direction. So that it is a high probability that a random B will make , $rank[(A - \lambda I)\ B] = n$ with all the λ.

  Even for the very high dimension of matrix A (milion by milion dimensional system), if I pull out a random vector B from R^n, then with the HIGH PROBABILITY, it is going to be able to controll all the states x of the system $\dot{x} = Ax + Bu,\ with\ x \in R^n$

- The PBH test tell me that, with a GIVEN MATRIX A, what is the minimal number of columns B, minimum of control actuators that we need in order to make the system $\dot{x} = Ax + Bu$ CONTROLLABLE.

  If there are 2 eigenvector direction in the null space of this operator $(A - \lambda I)$, then we need matrix B has 2 columns in order to fill in the null space and have a rank n. That is also mean you need 2 CONTROL INPUTS for your system.

- WE USE THIS TEST TO THINK ABOUT HOW CONTROLLABLE IS THE SYSTEM?
  In case the system in the gray area, What if you have 2 different eigenvalues that is really close, or 2 eigenvectors but it is nearly the same (parallel) but it is not.
  The »rank(ctrb(A,B)) from MATLAB tell us the system is controllable (binary result), but in fact, the system is approximately DEGENERATED,

and be BARELY CONTROLLABLE.

SOLUTION: we want to have MULTIPLE COLUMNS of matrix B to boost the control authority of the system.

random vector la gi, nullspace, subspace

3.10 Vid 10 - Cayley-Hamilton Theorem

$\dot{x} = Ax + Bu, \ with \ x \in R^n$

3.11 Vid 12 - Inverted Pendulum on a Cart

We have 2 states but due to Newton 2nd law, we have 4 (double) coupled ODEs (2:15)

Because the rank of the Controllability matrix is 4, I can span all of my state space with this Controllability subspace and that means I can actually develop a controller to control the system.

Set up the problem, identify matrix A, B, and the initial condition. Find the fixed points of the system. Linearize the system at the fixed point. Analyze the stability of the system with eig(A) and the Controllability with rank(ctrb(A,B))

3.12 Vid 13 - Pole Placement for the Inverted Pendulum on a Cart

We cannot move the poles (eigenvalues) of the feedback system to the left half plane forever because

- the control power of the actuator u is limit, not unlimit to meet up with the control demand from the controller
- when the response is fast, the dynamics of the system is out of the range when we linearize the non linear system. So that the controller we built for the linearized system failed.

3.13 Vid 13 - Linear Quadratic Regulator (LQR) Control for the Inverted Pendulum on a Cart

What is the math behind pole placement and the LQR?

4 MATLAB SIMULINK

4.1 Dec 10, 21

We could make different color for the time legend, according to the video in the link: `https://www.youtube.com/watch?v=8jBOD7CD5Z8`

How to make cascaded simulation in simulink?

How to use git for simulink, matlab?

5 STM32

5.1 Nov 04, 2021

CubeIDE vs CubeMX differences?
add path trong environments, edit file path, link to the folder store the make (cygwin), git, etc. so we could use it in the terminal of the visual code.

5.2 Nov 05, 2021

Equivalences at tstamp 7:15
reference nhung noi de thay doi file makefile tuy theo loai chip trong video 02 - toanchung

5.3 Nov 29, 2021

Compare JTAG vs SWD The other good thing about SWD is you can use the serial wire viewer for your printf statements for debugging.

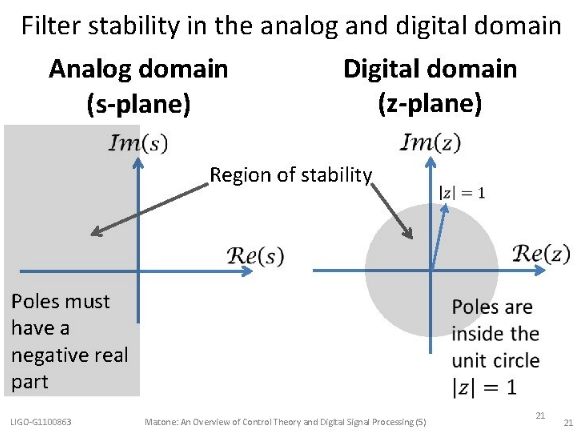

Figure 2: Mapping stable region between s-continuos and z-discrete domain

6 C - PROGRAMMING

Pointers
array[ ] different ? is it a 2 dimension, 3 dimensions array?

int * const array, use array name, so it is 1 dimension (single subscripted array?)
what is the principle of least privilege? opreands la gi?
Portability Tip 7.3
Most computers today have 2-byte or 4-byte integers. Some of the newer machines use 8- byte integers. Because the results of pointer arithmetic depend on the size of the objects a pointer points to, pointer arithmetic is machine dependent.
Pointers are valid operands in arithmetic expressions, assignment expressions and comparison expressions.
Arithmetic operations: cac phep tinh toan hoc, basics: + - x /
how to know the system is 2, 4 or 8 byte systems?
Pointers can be compared using equality and relational operators, but such comparisons are meaningless unless the pointers point to elements of the same array
Array element b[3] can alternatively be referenced with the pointer expression:
*(bPtr + 3)
The parentheses are necessary because the precedence of * is higher than the precedence of + .
string vs character array?
What is the end char of the string? null? "'slash 0'?" how to see it?
const int * xPtr
int * const xPtr
const char * const s2

DESIGN STEP
first think about the flow of the codes
which variable is fixed, should not be changed by mistakes, then use the qualifier const
should only pass the variable by value and return ONLY ONE value in each function for the principle of least of privilege
should not change the value of the variable inside the function if you dont really need to do that

7 WHAT'S NEXT

7.1 Dec 10, 21

- Watch 47 videos control Bootcamp - Bruton
- Answer Questions
- Fix GG Calendar - Done
- Watch (8-12 / 47) videos control Bootcamp - Bruton

7.2 Dec 06, 21

- Change ticket time - Done

- Sign Offer - Done
- Confirm the info of Gusto - Done
- Watch (1-8 / 47) videos control Bootcamp - Bruton

7.3 Nov 25, 21

- xem het video cua Embedded Linux, 1 ngay 2 videos, 5,6
- xem video Linear Algebra 2 videos 1 ngay, 8 9 10
- Github for simulink va matlab code
- setup cho stm32 toan chung