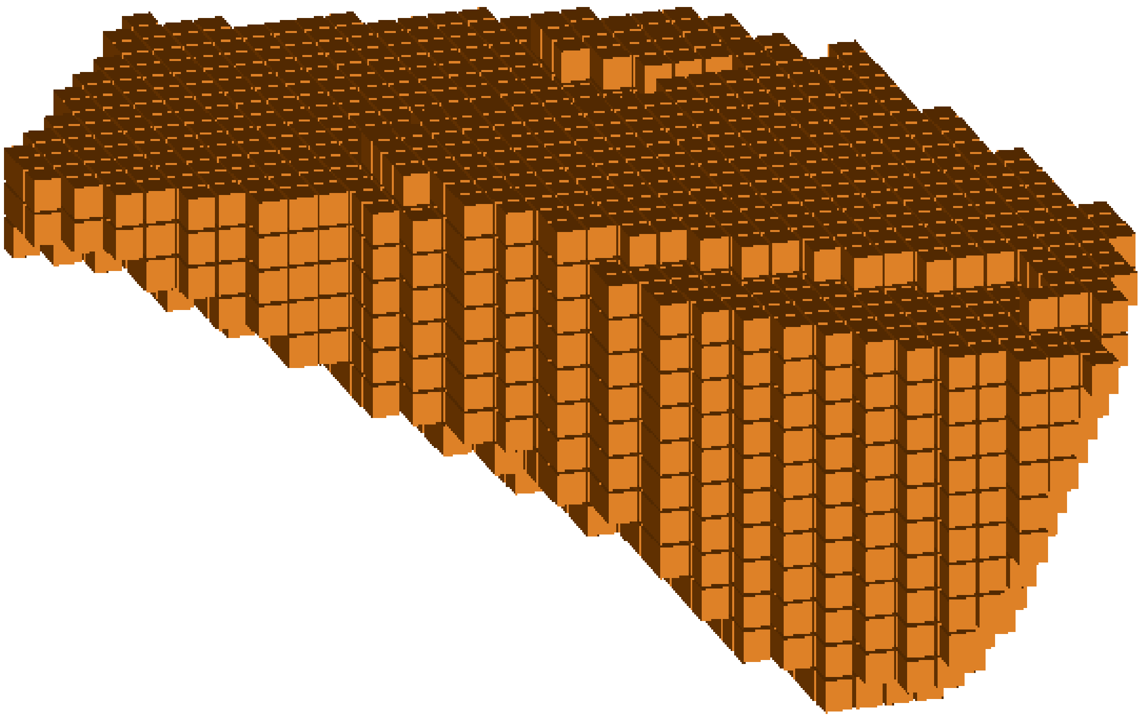
MiningMath Associates

Dense Cholesky CUDA implementation

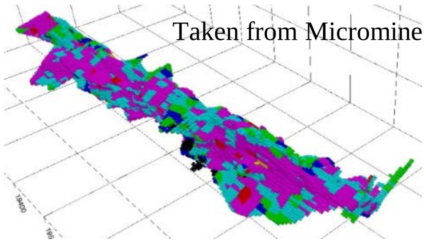
SimSched

Fabrício Ceolin e Alex Teixeira

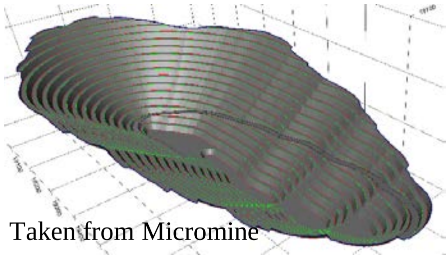
Block Model



The current state of the mine planning technology


Taken from Micromine

Pre-assigned destinations
Single block values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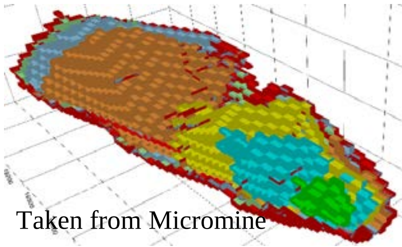
Lerchs-Grossman
Max. Undisc. Cash Flow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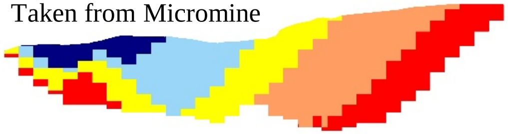
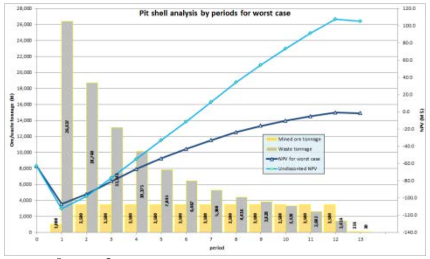


Period-by-period scheduling with blending

Stockpiling and Cutoff policy



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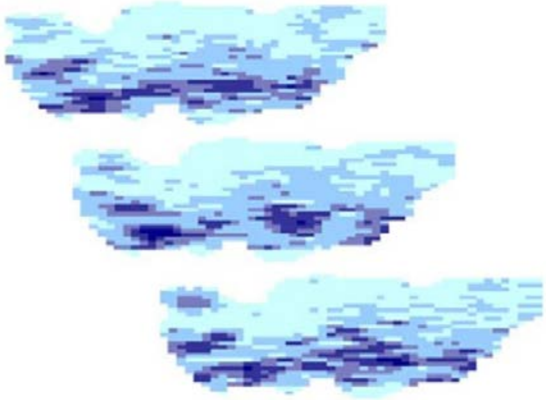
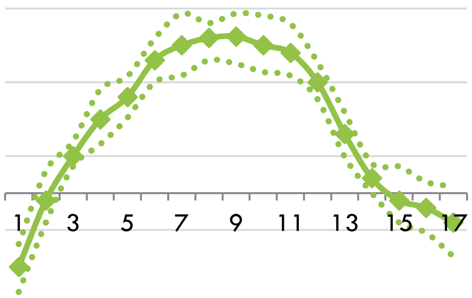


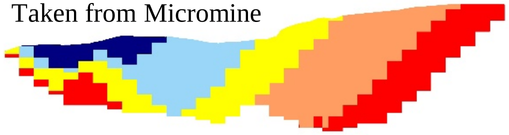
Leite and Dimitrakopoulos (2009)

Block values for each model and possible destination



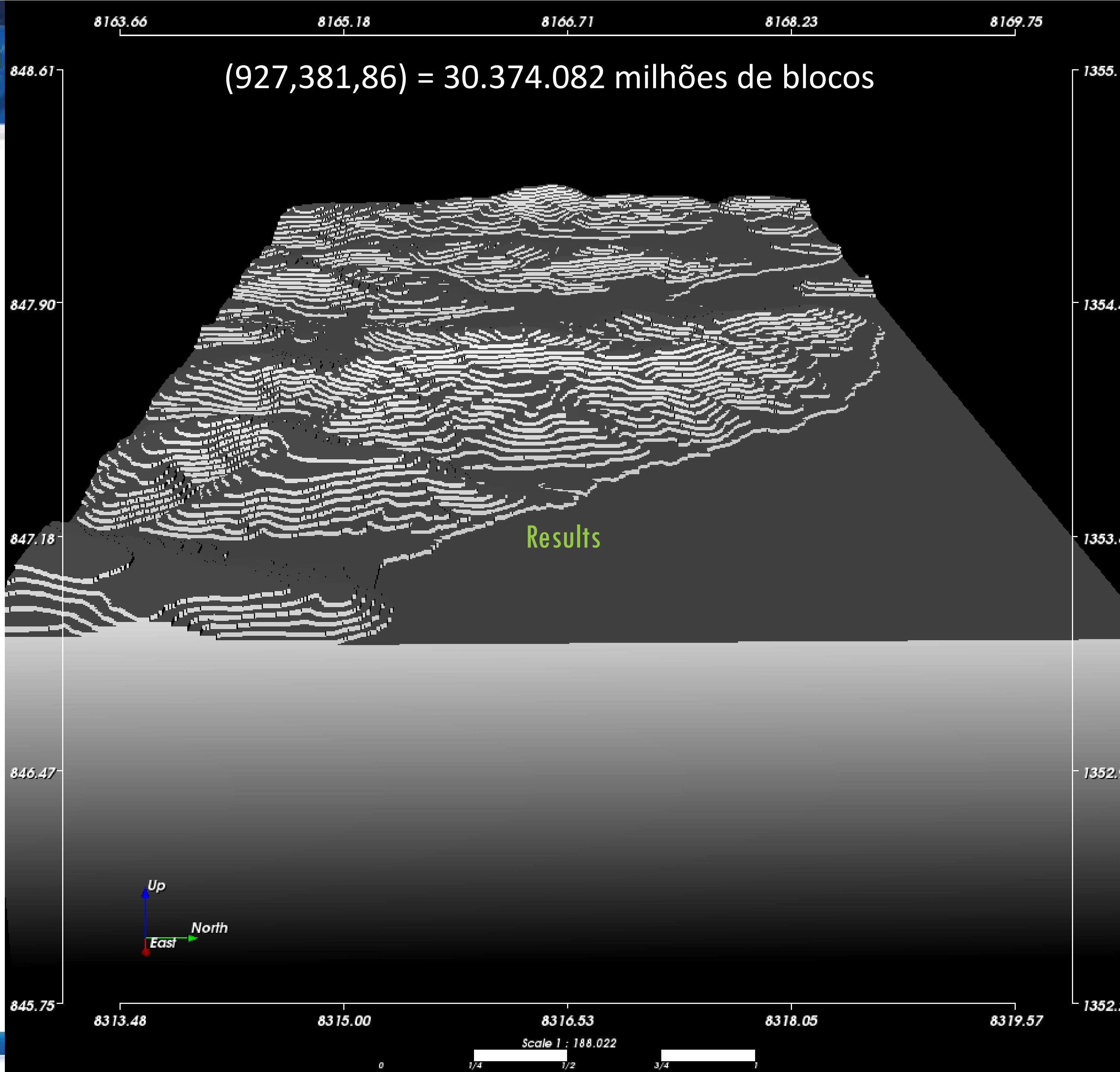
Indicators with risk profiles

Taken from Micromine



Multi-period scheduling under uncertainty with blending and stockpiling

Results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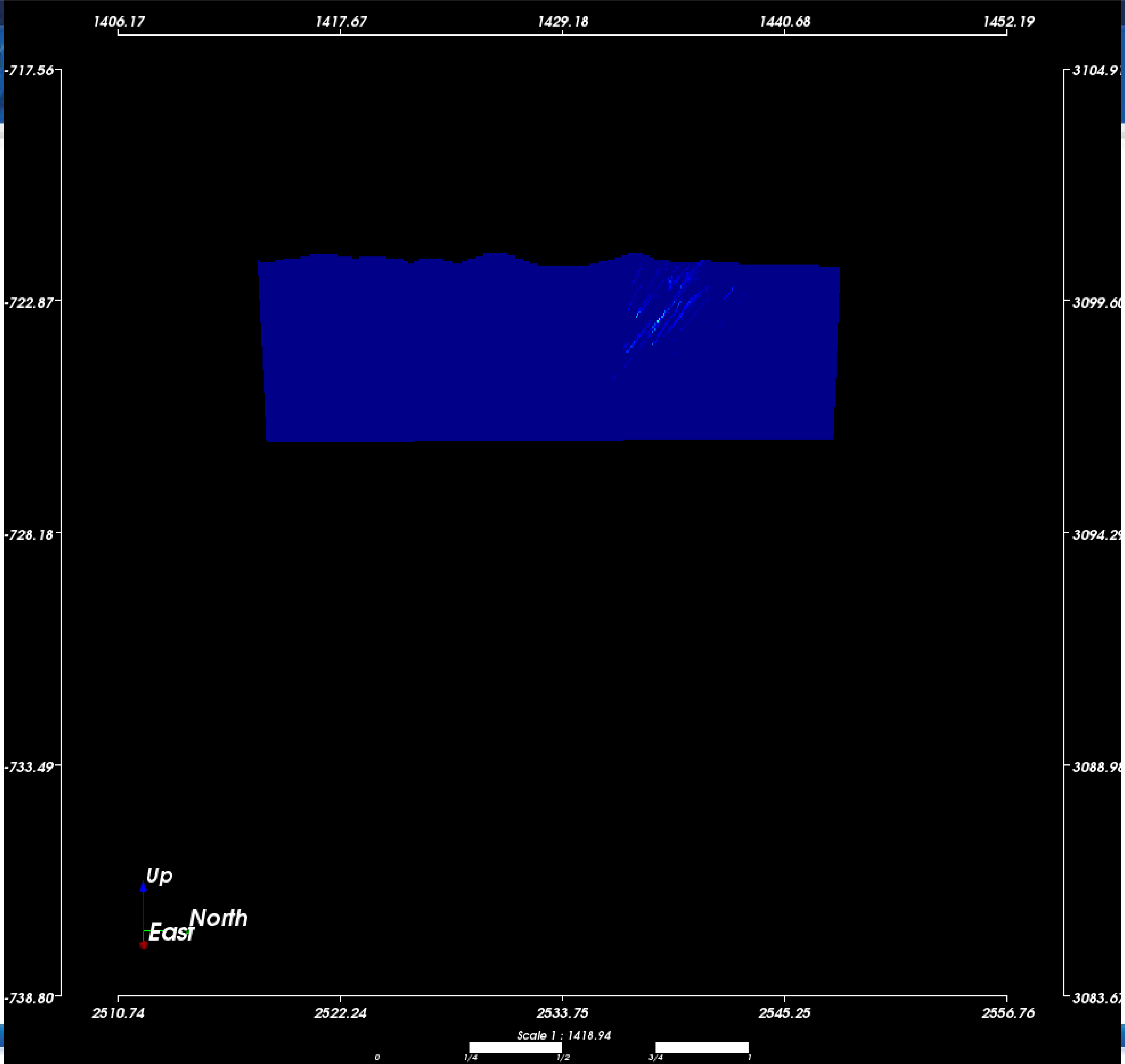
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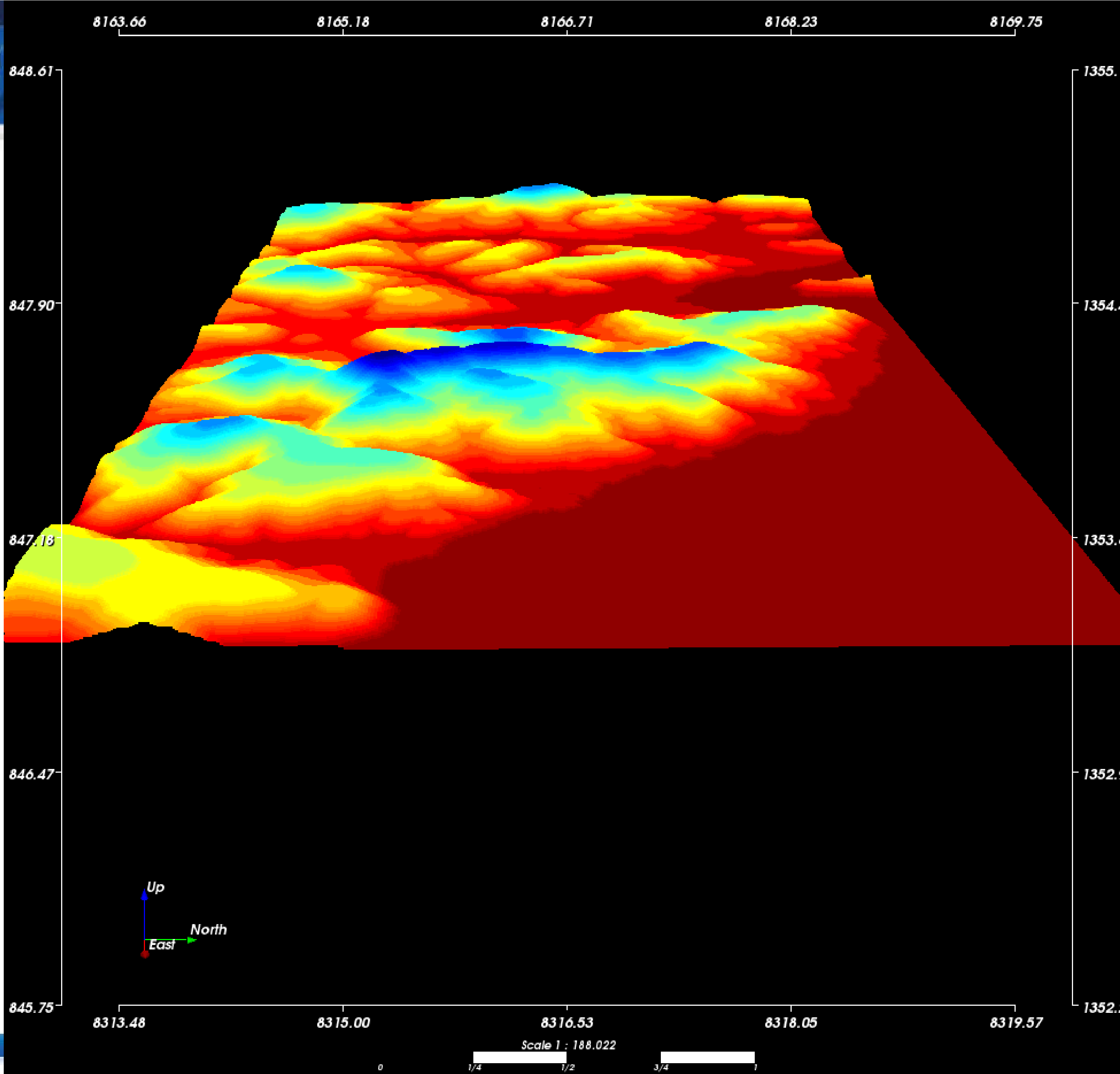
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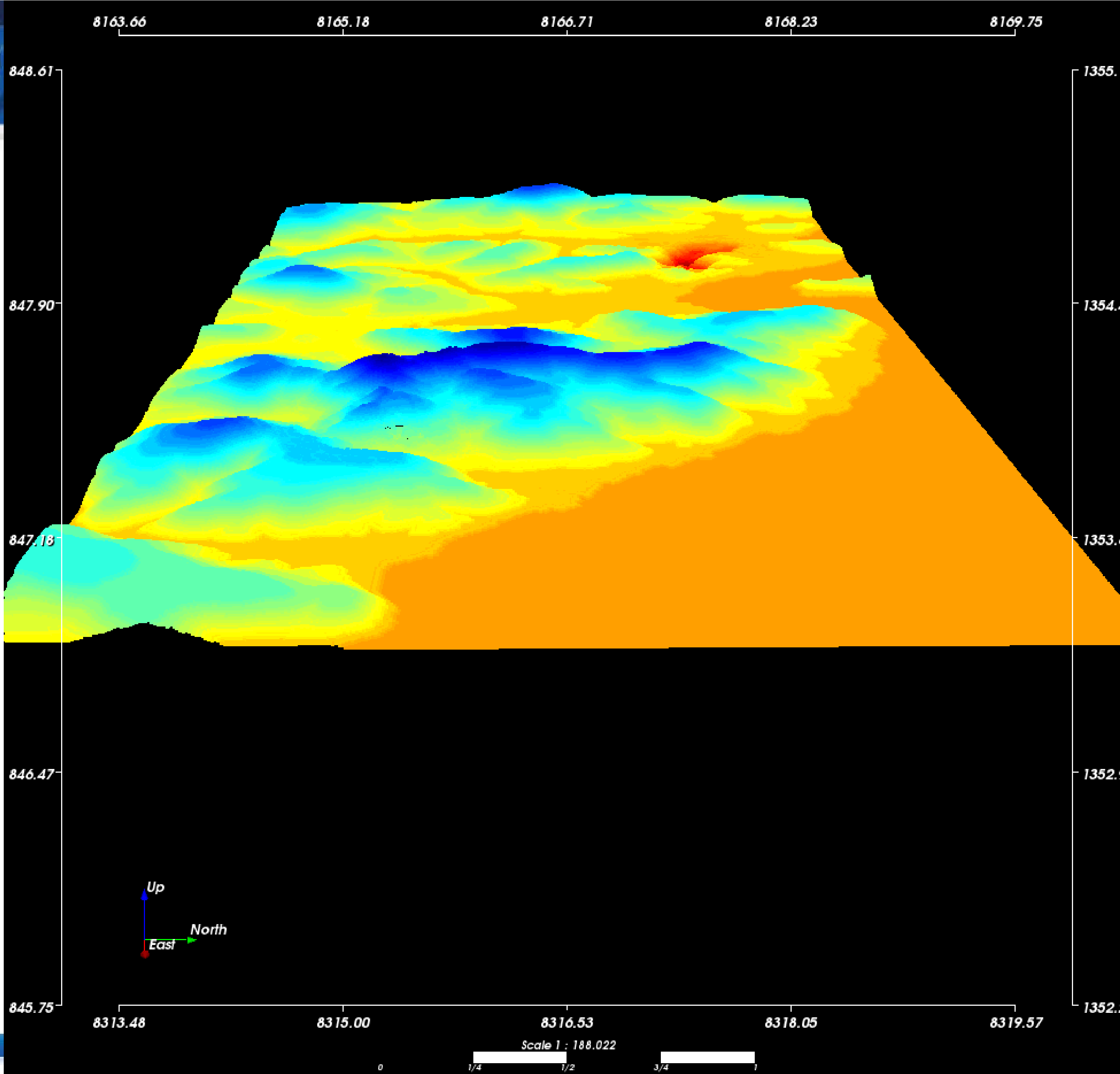
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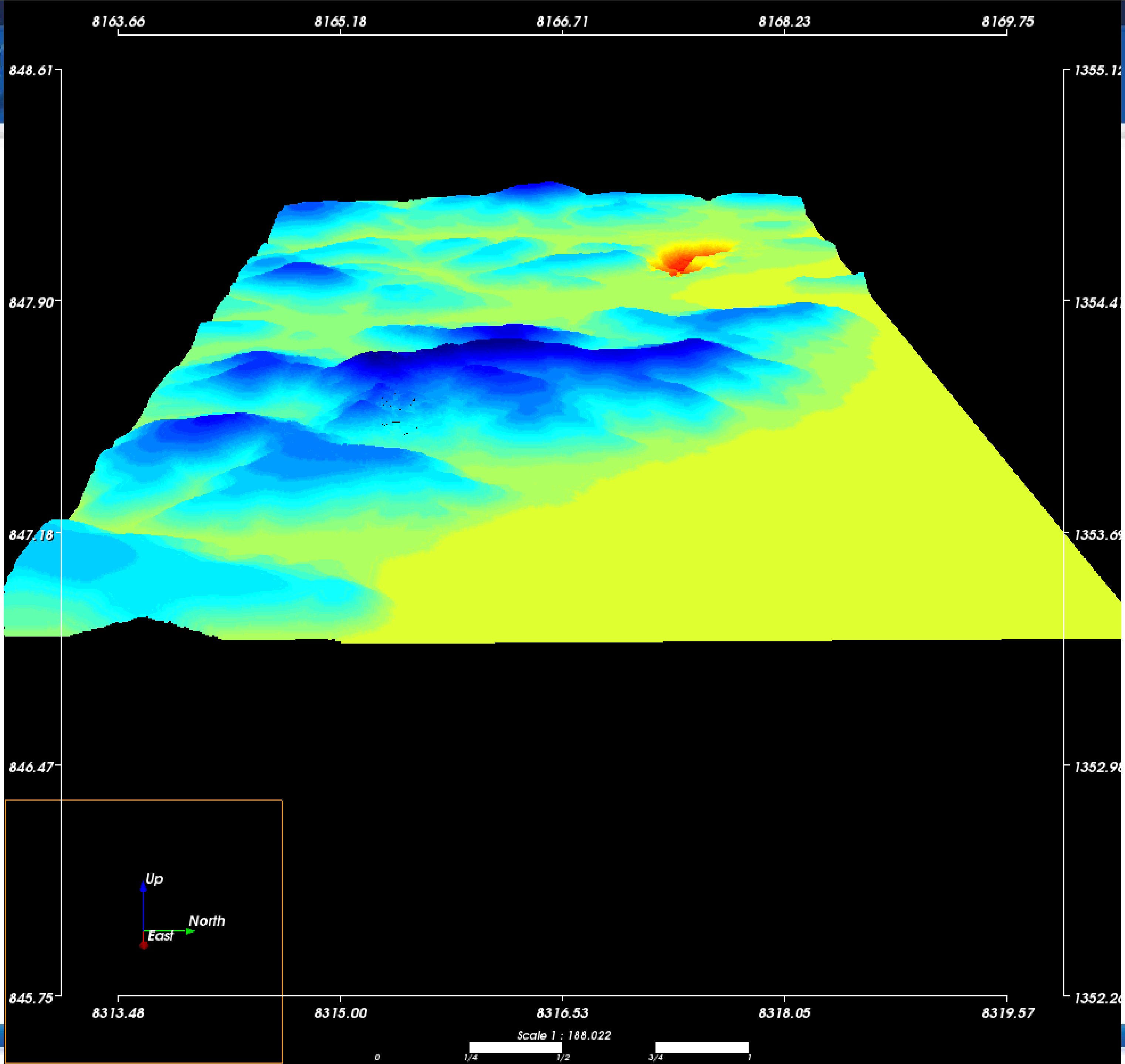
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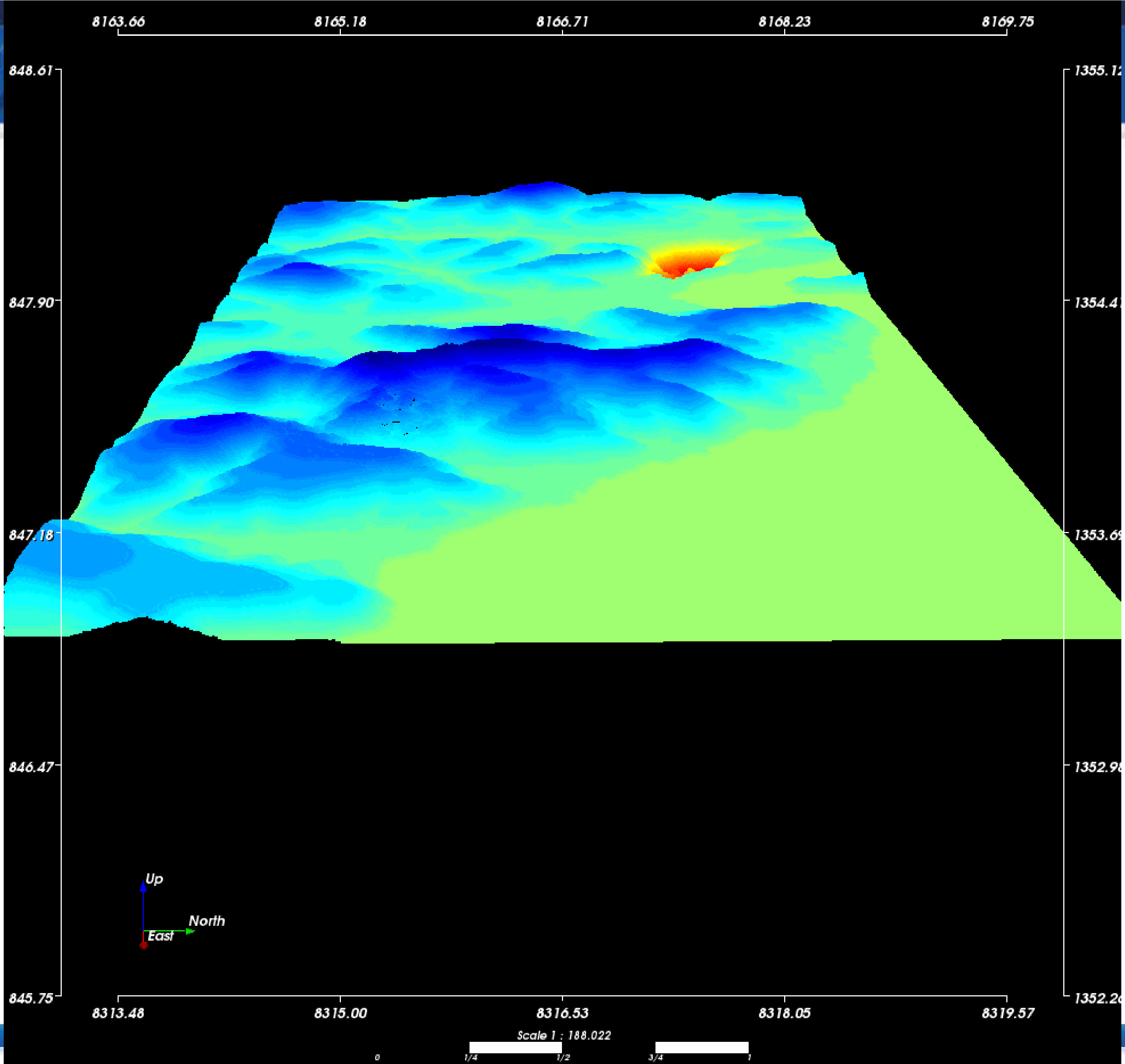
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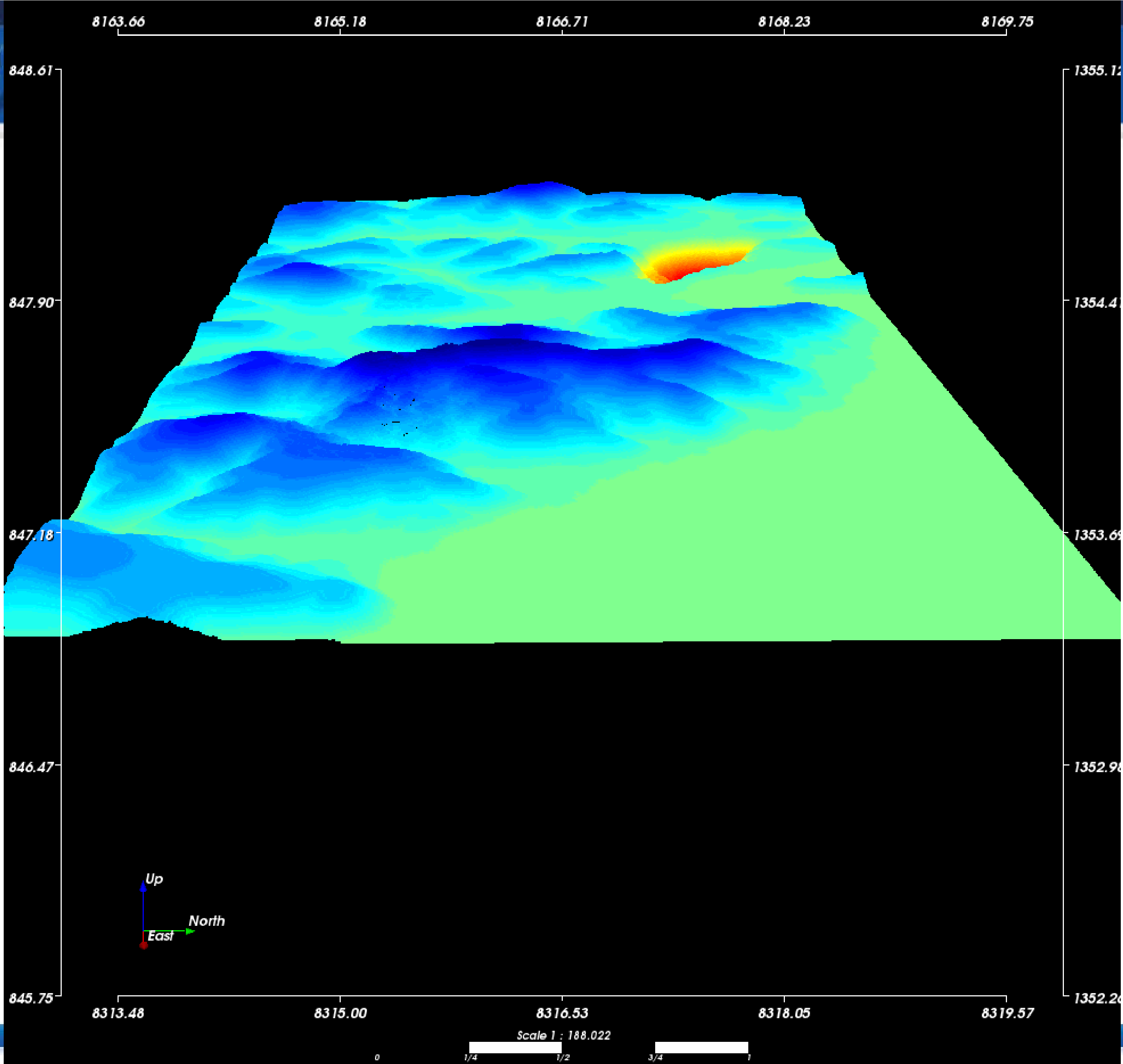
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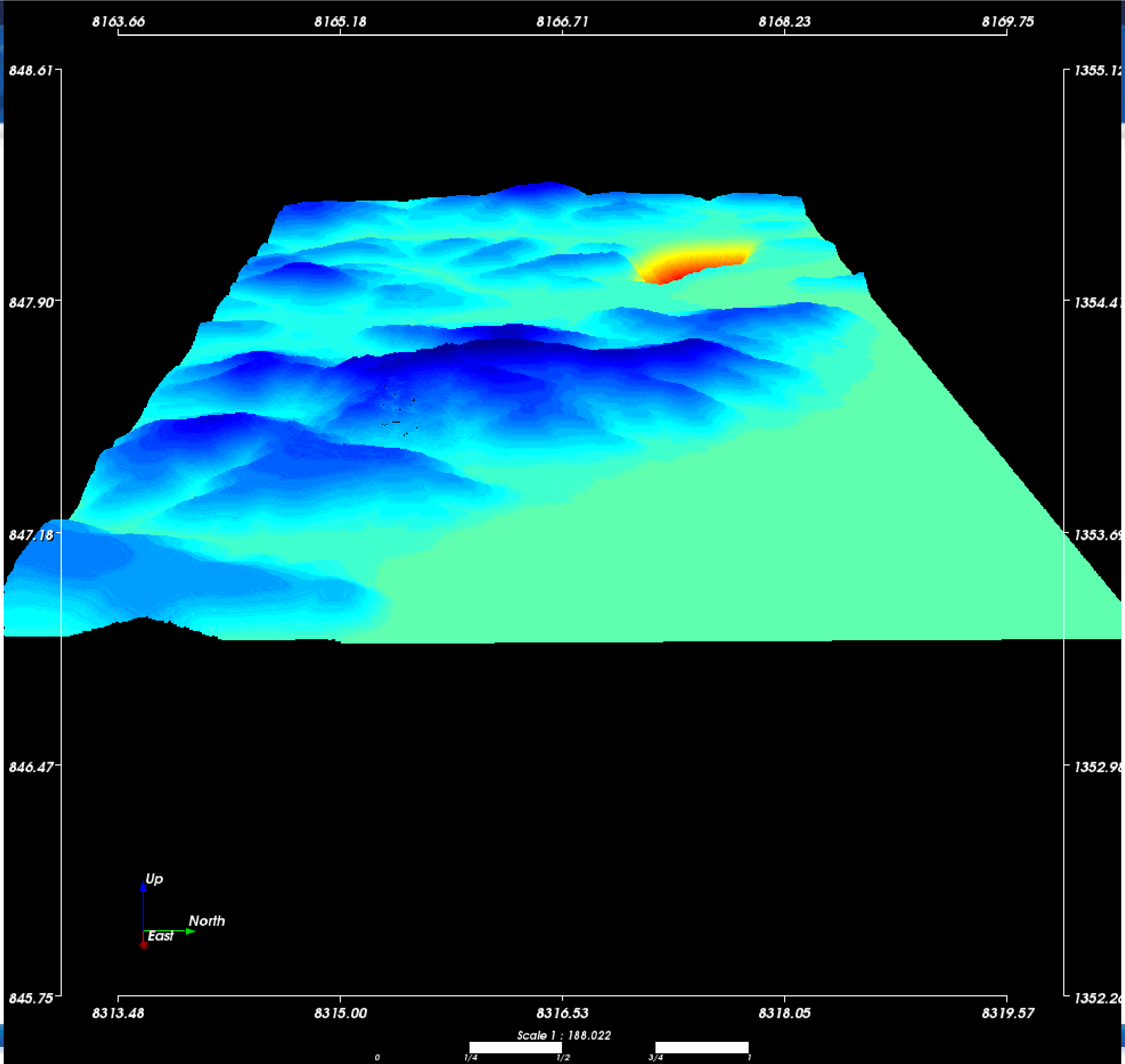
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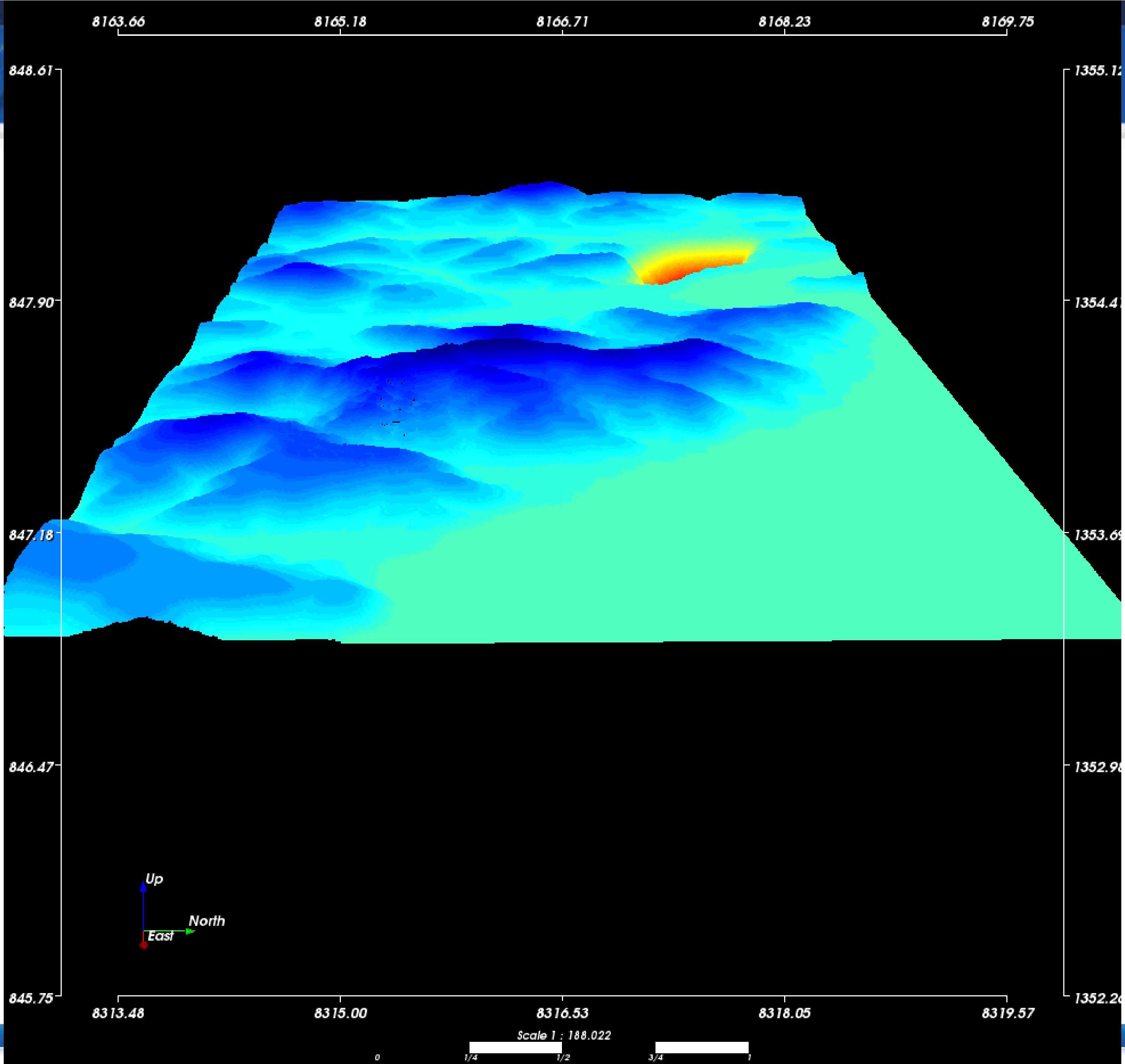
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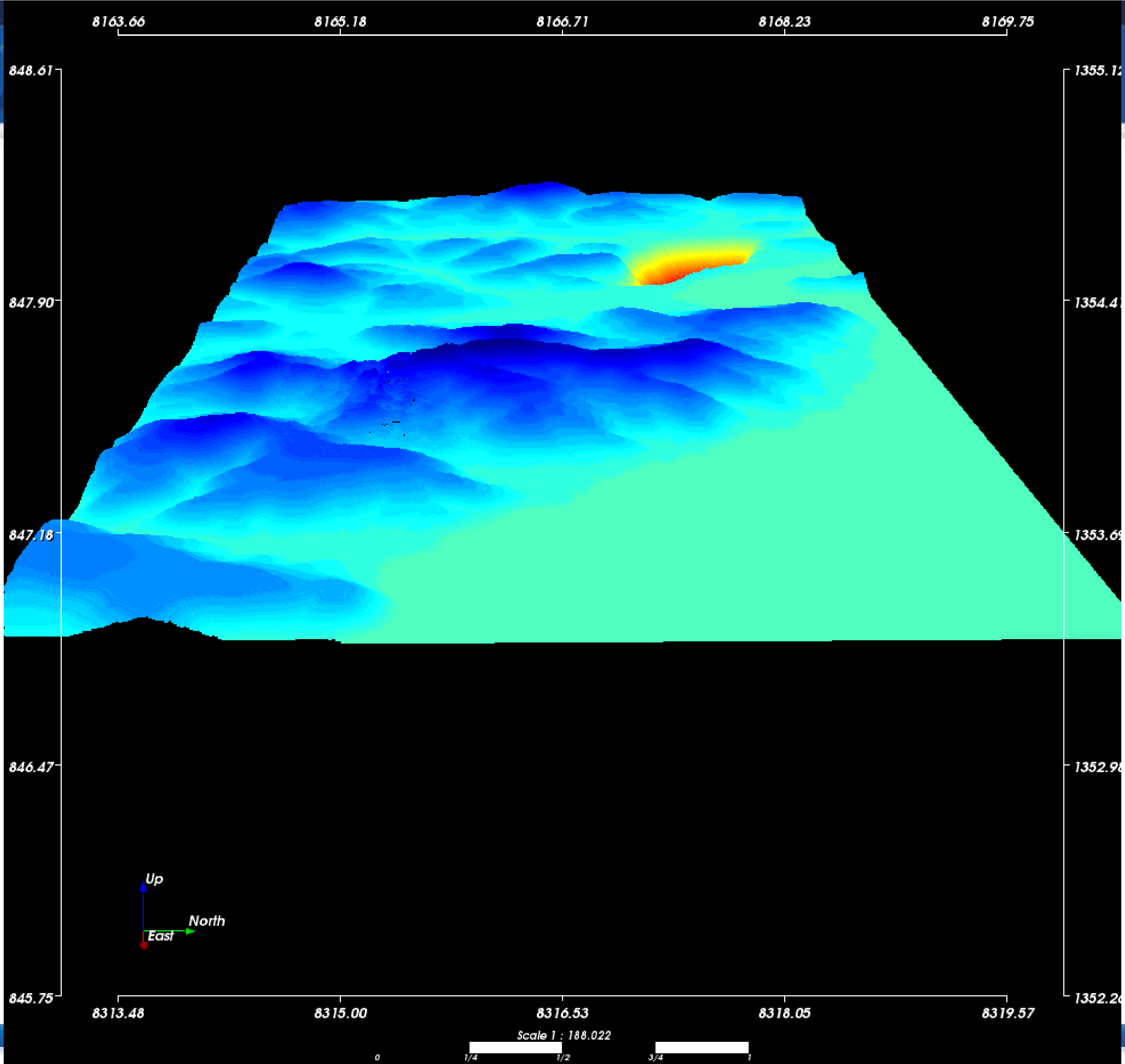

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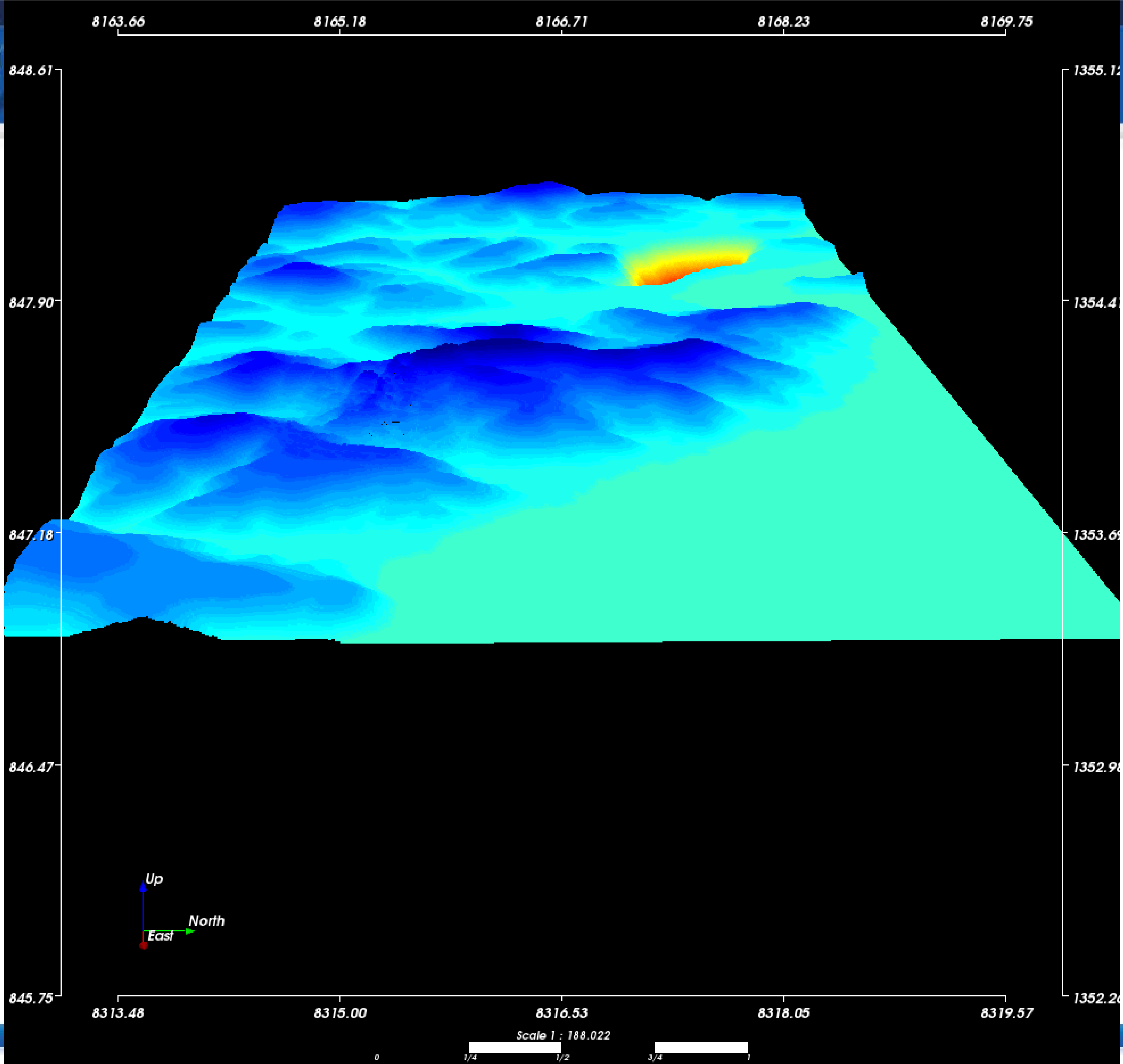

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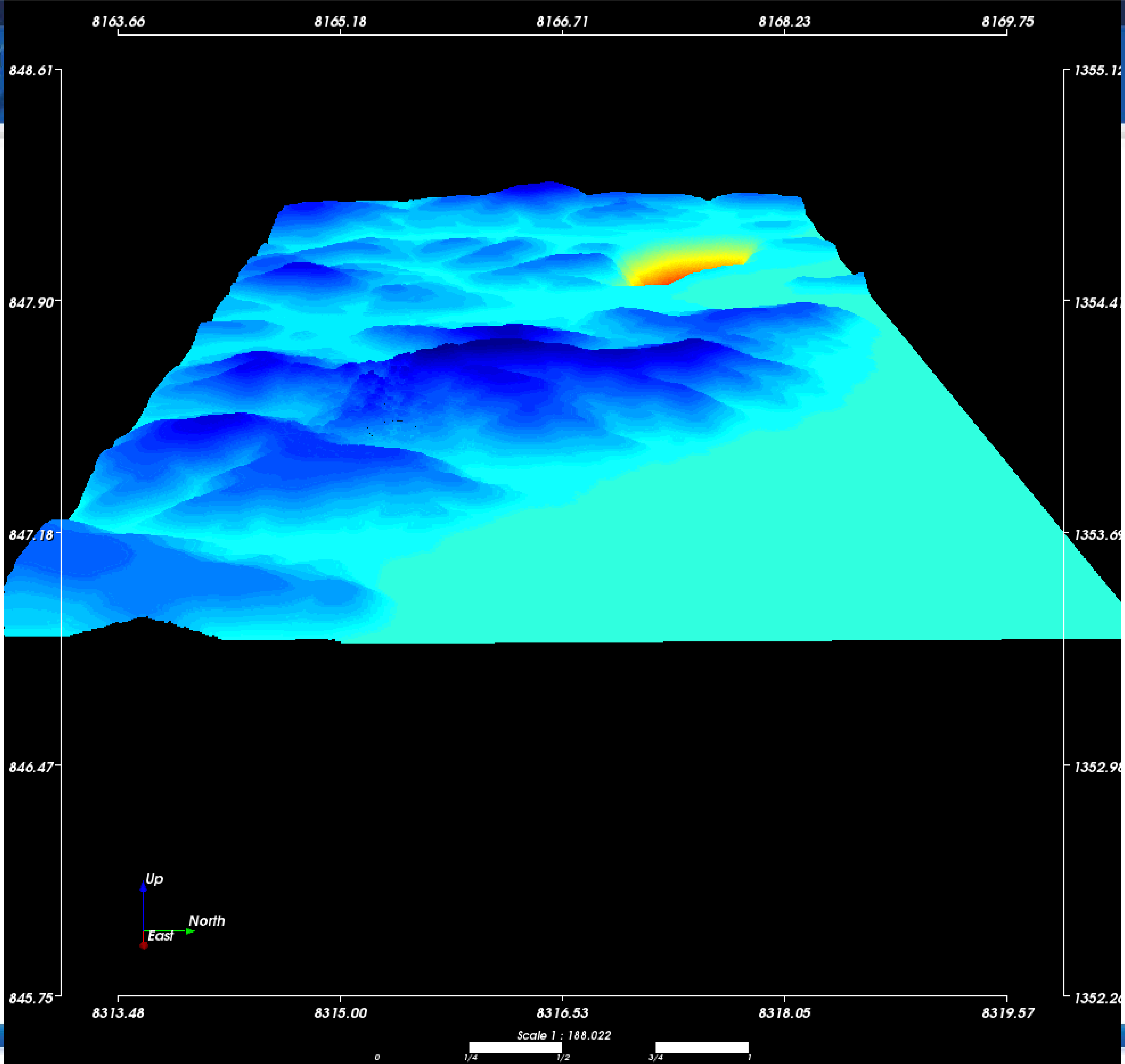

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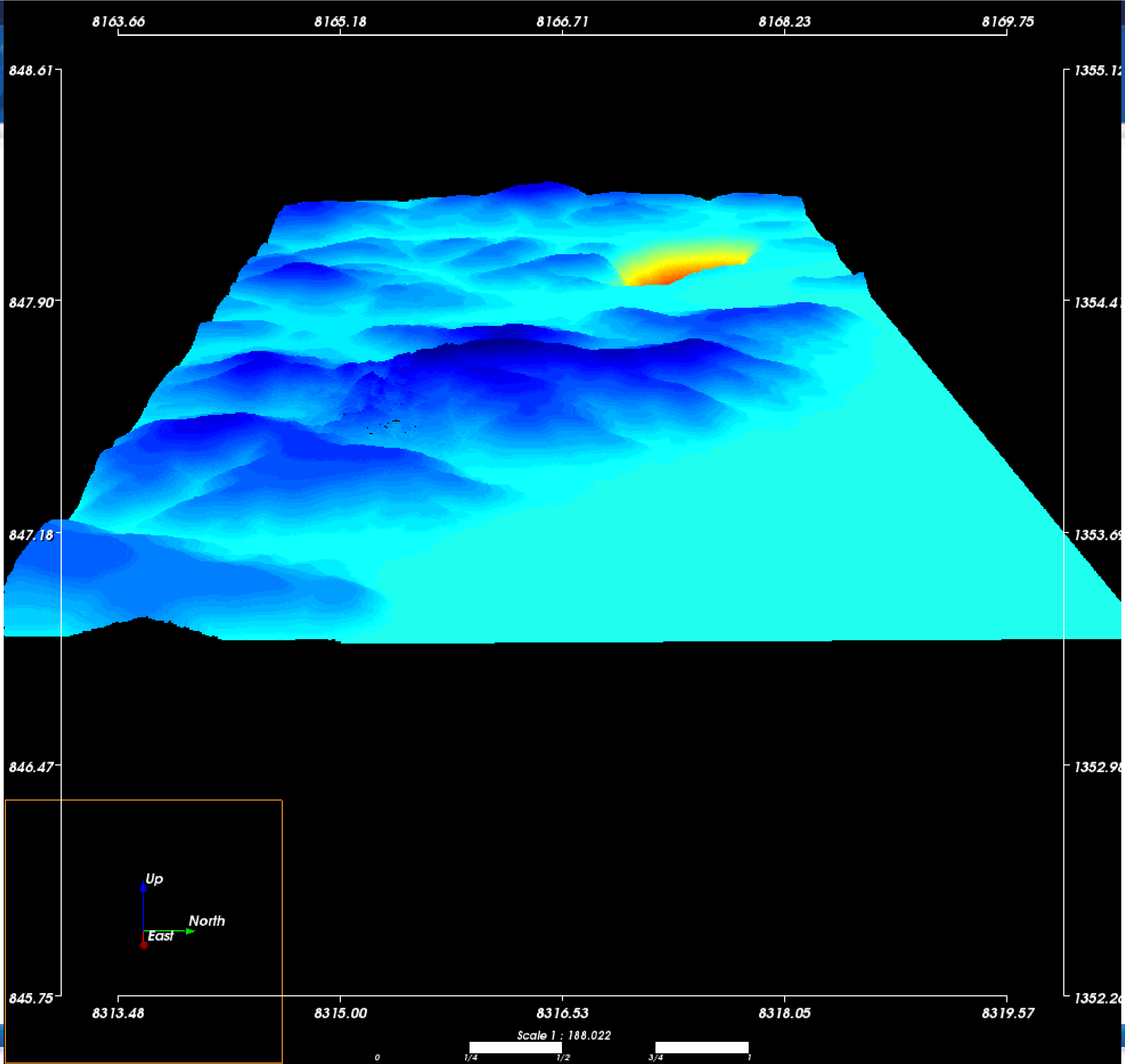

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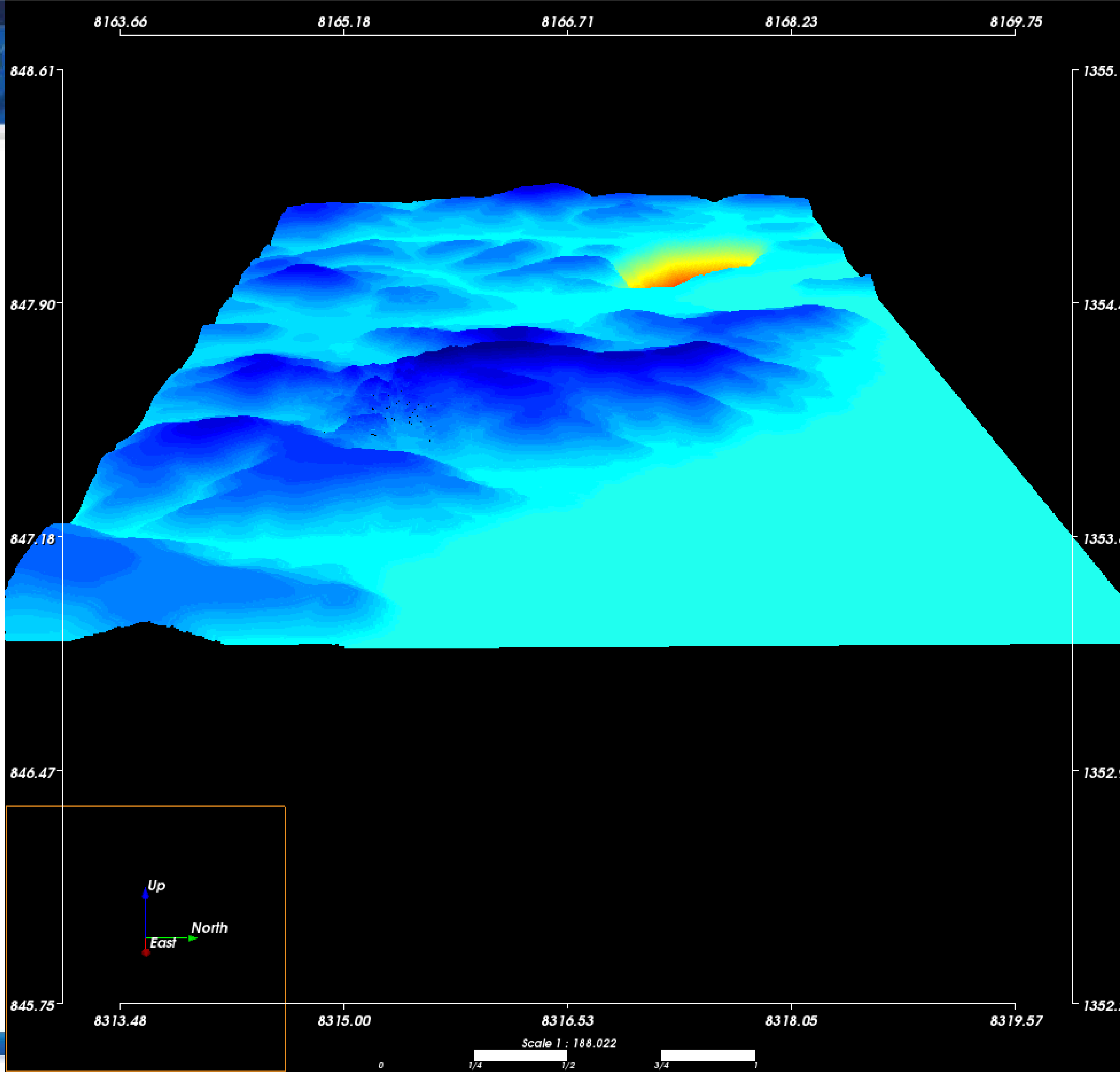
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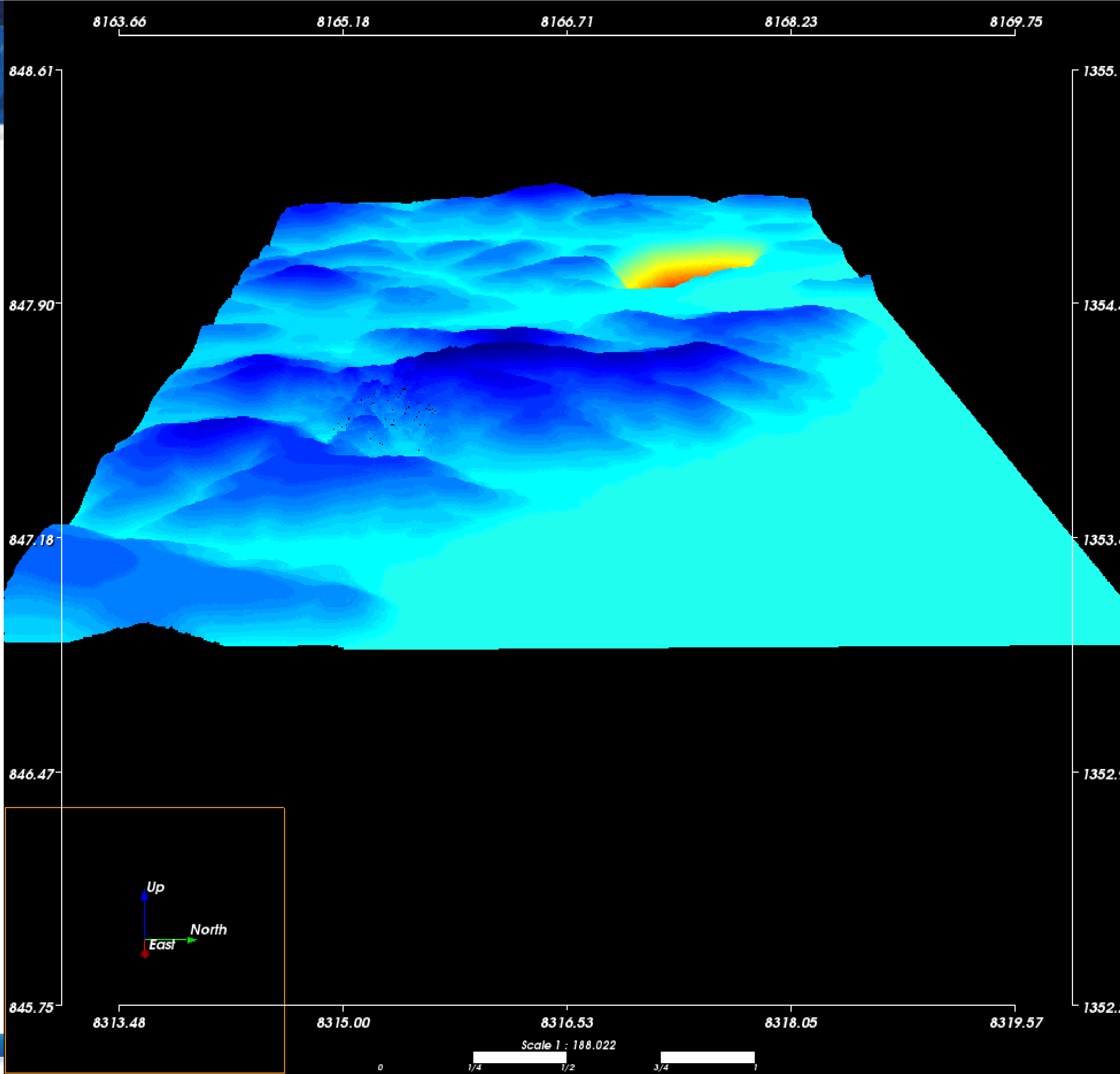
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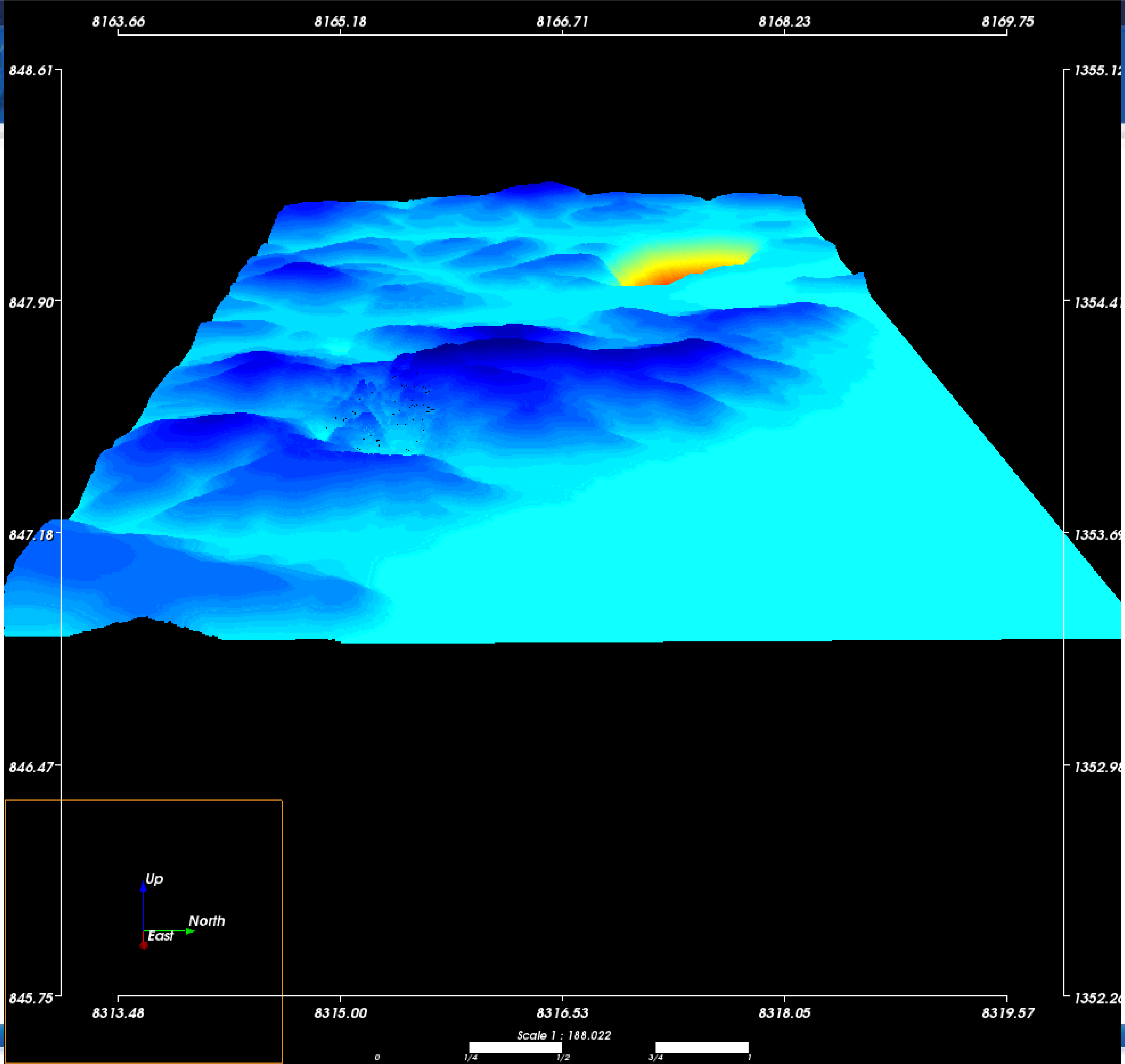
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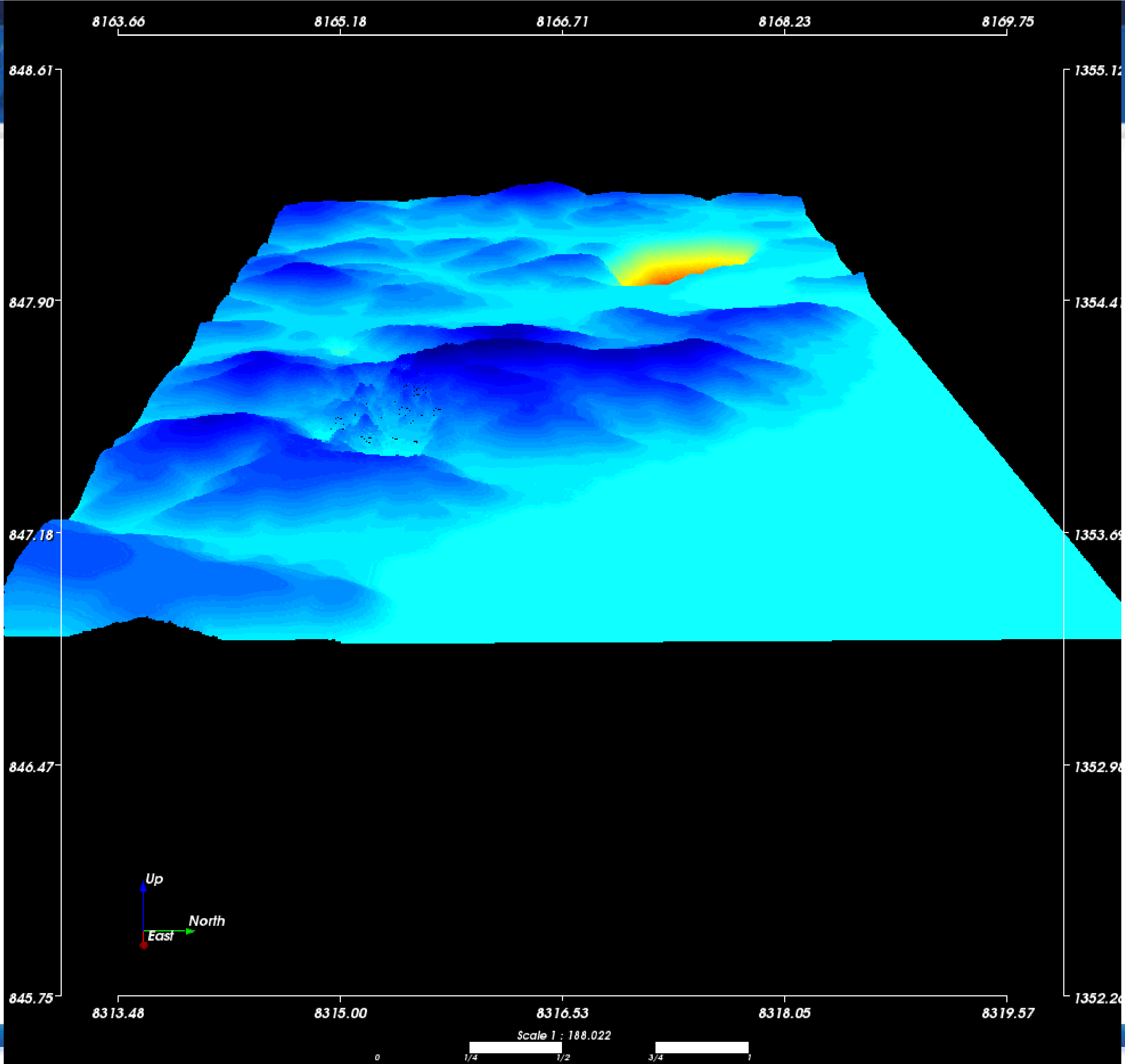
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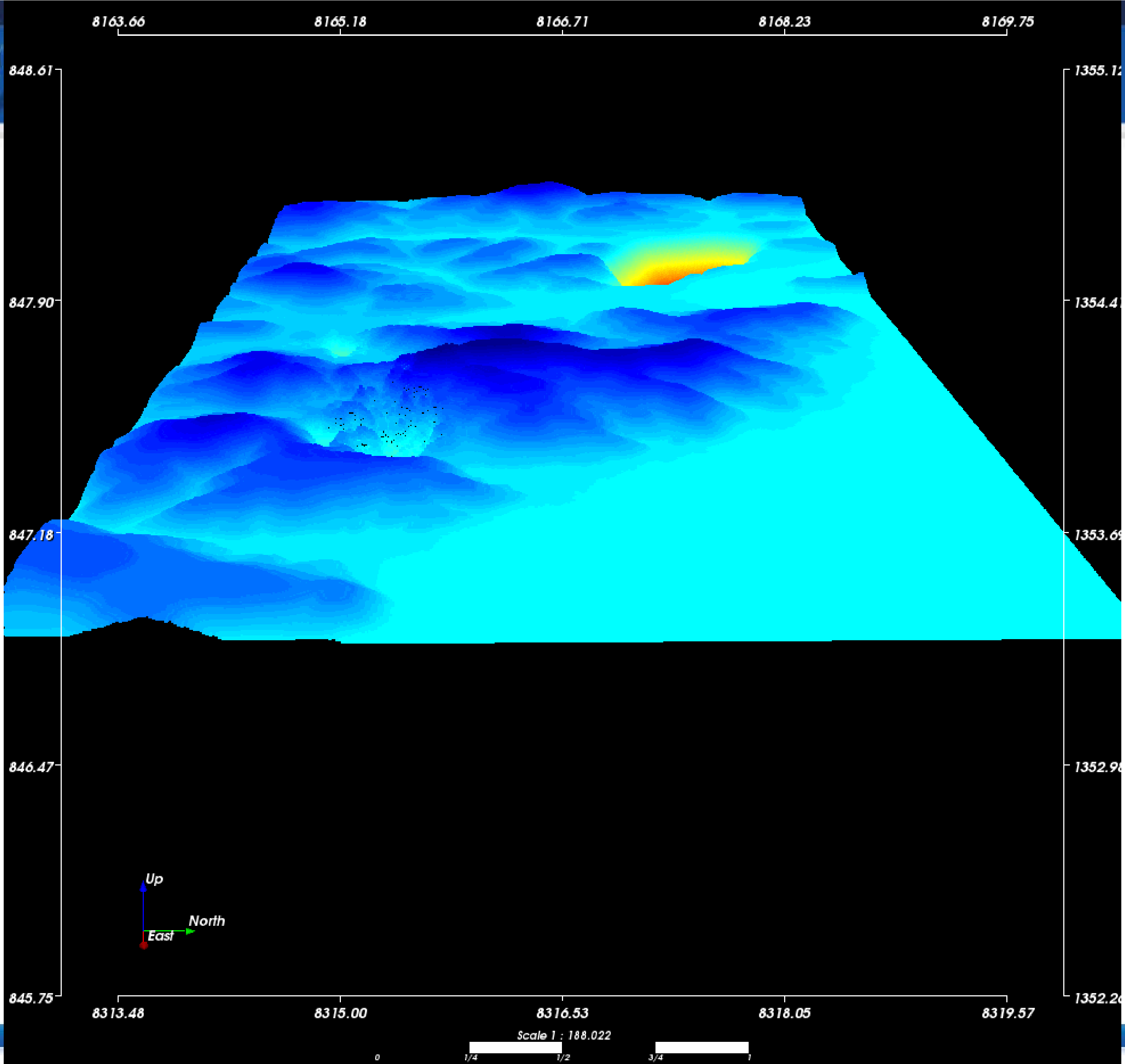

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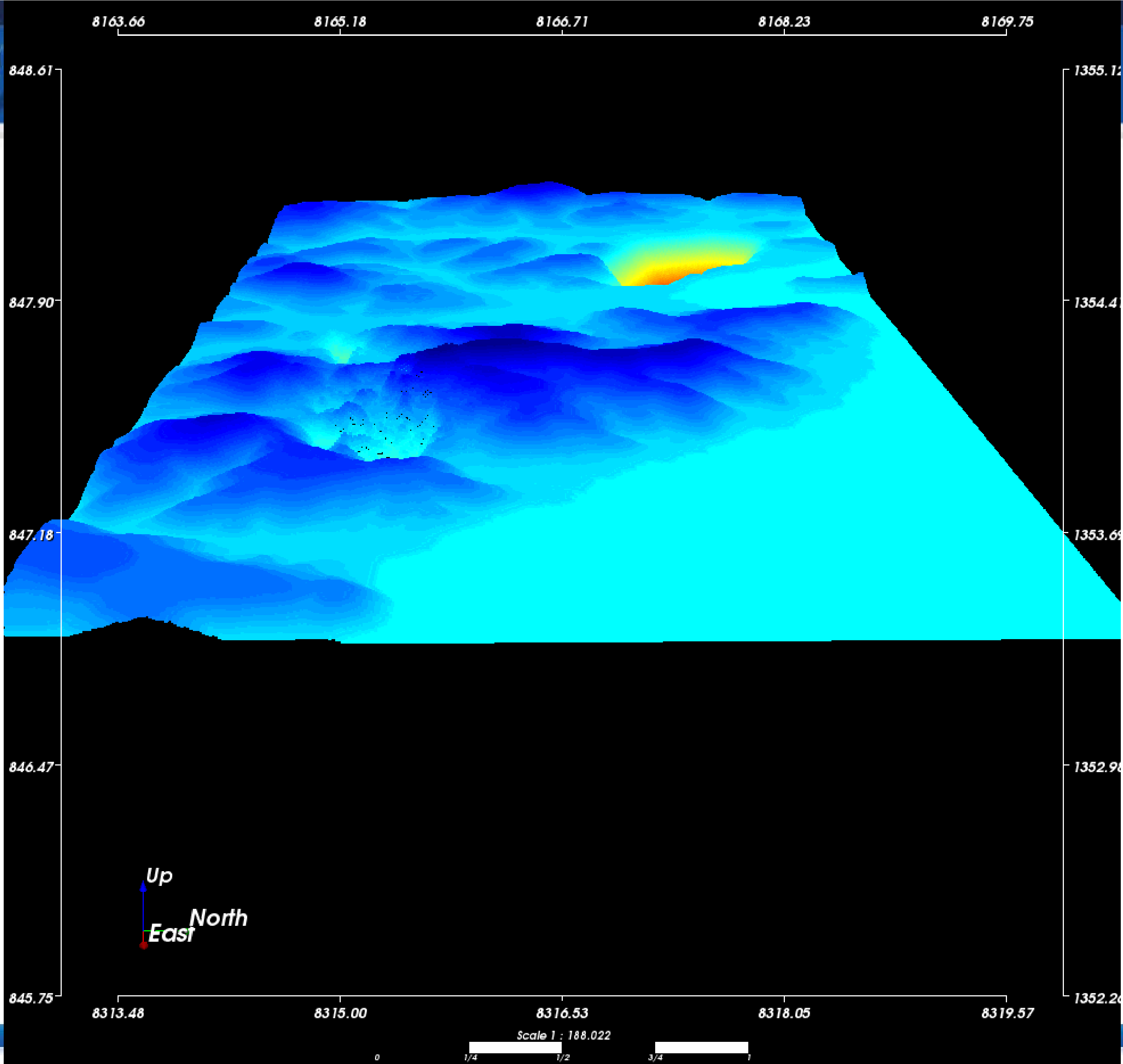

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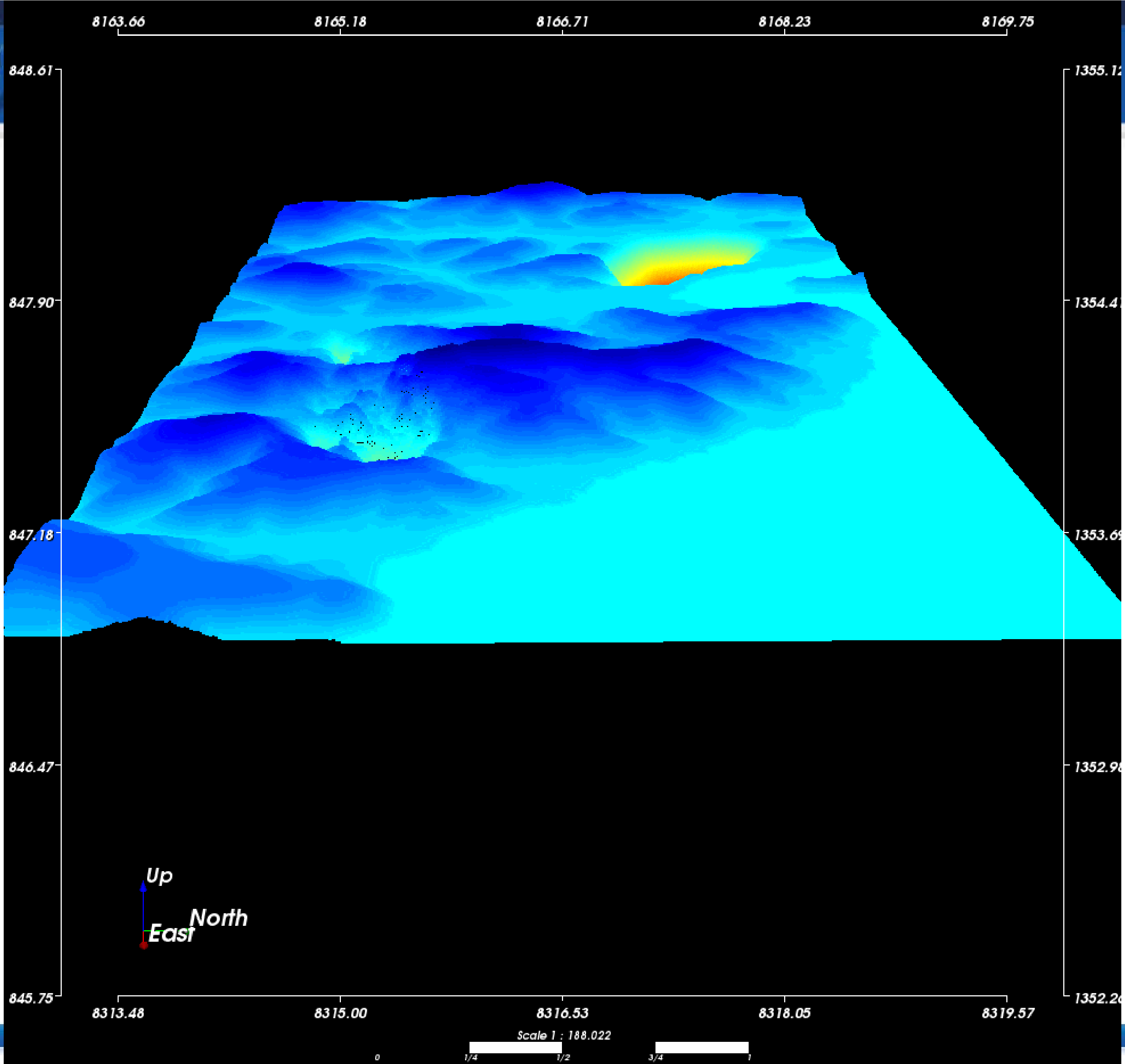
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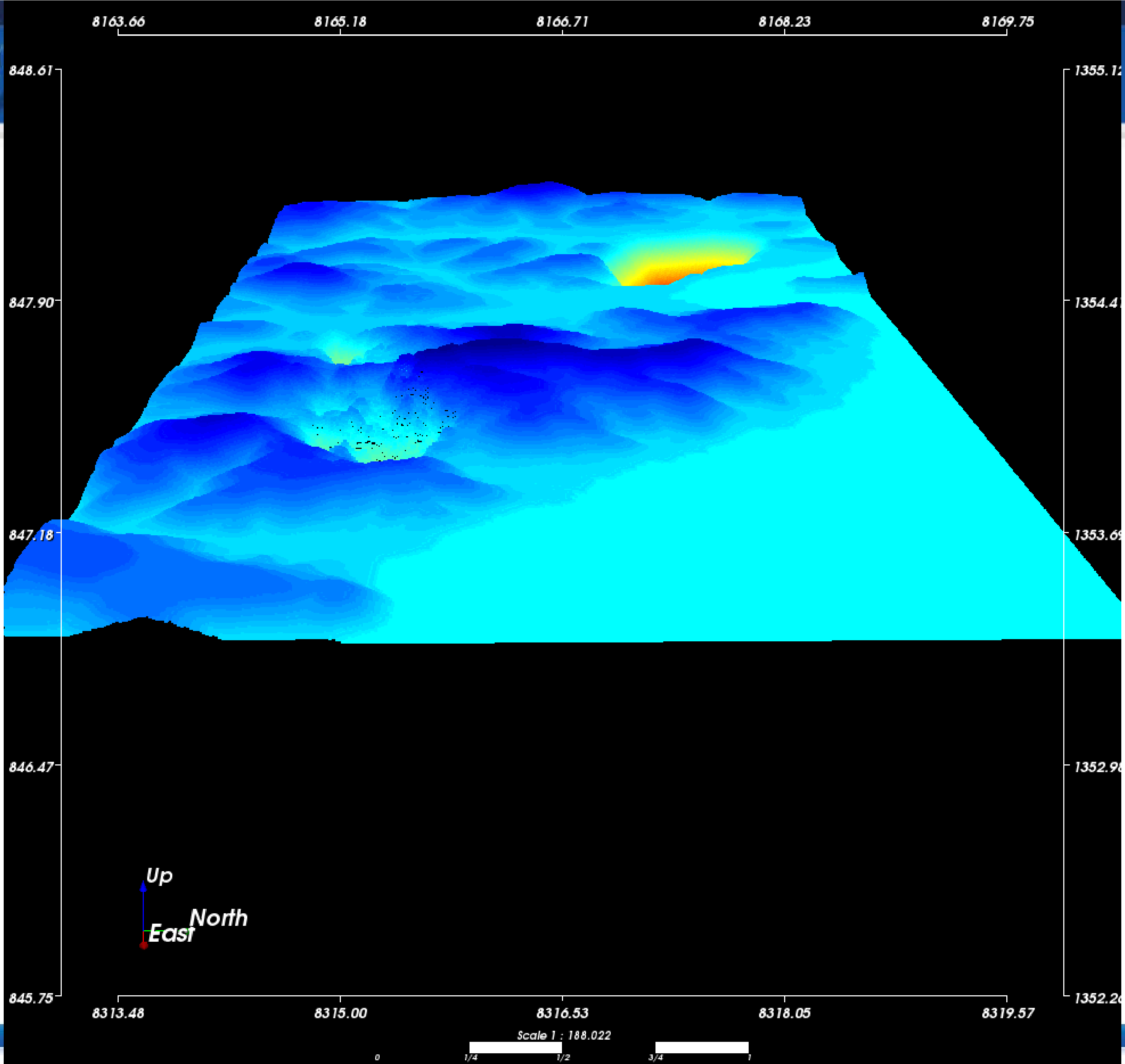
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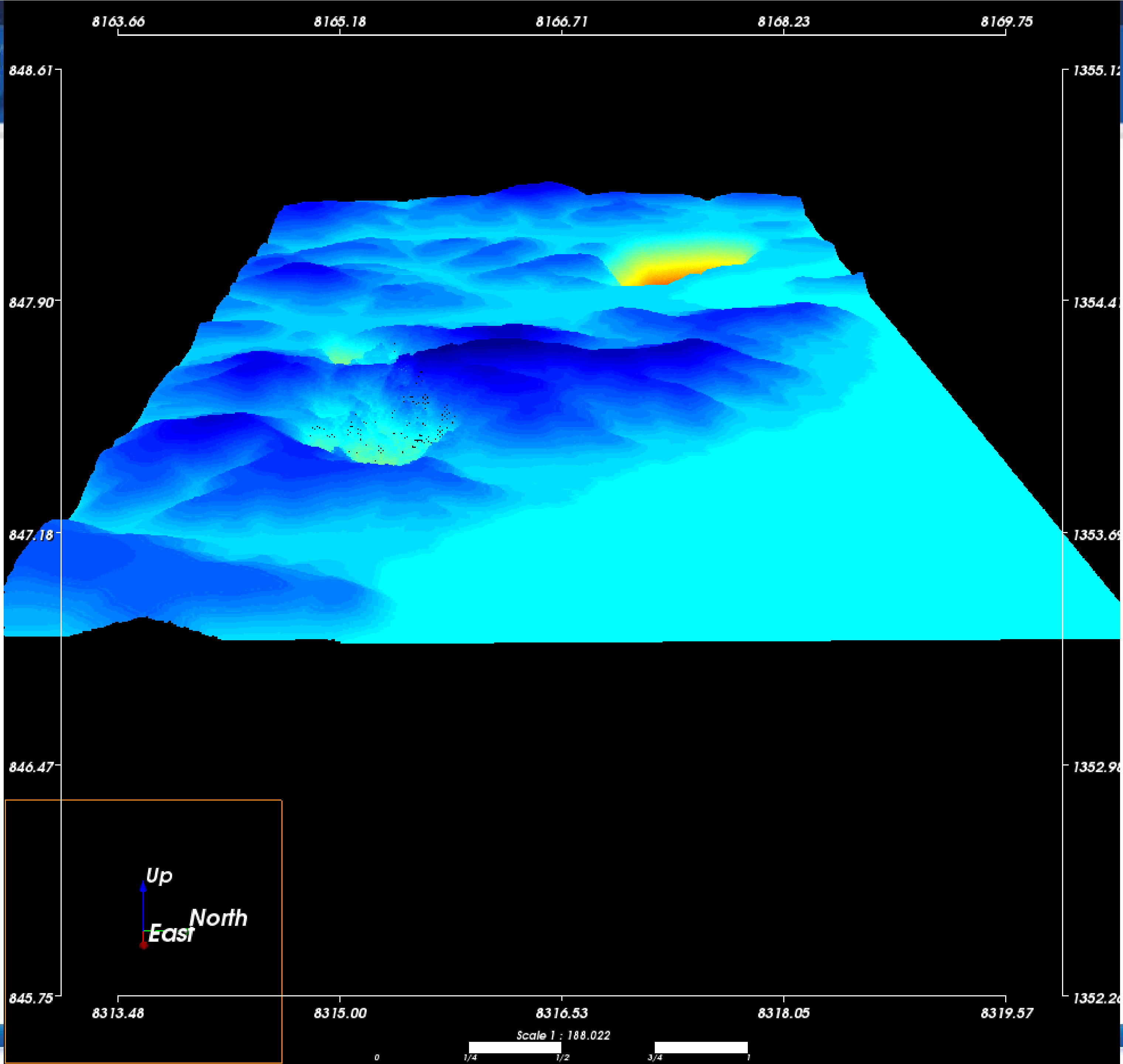
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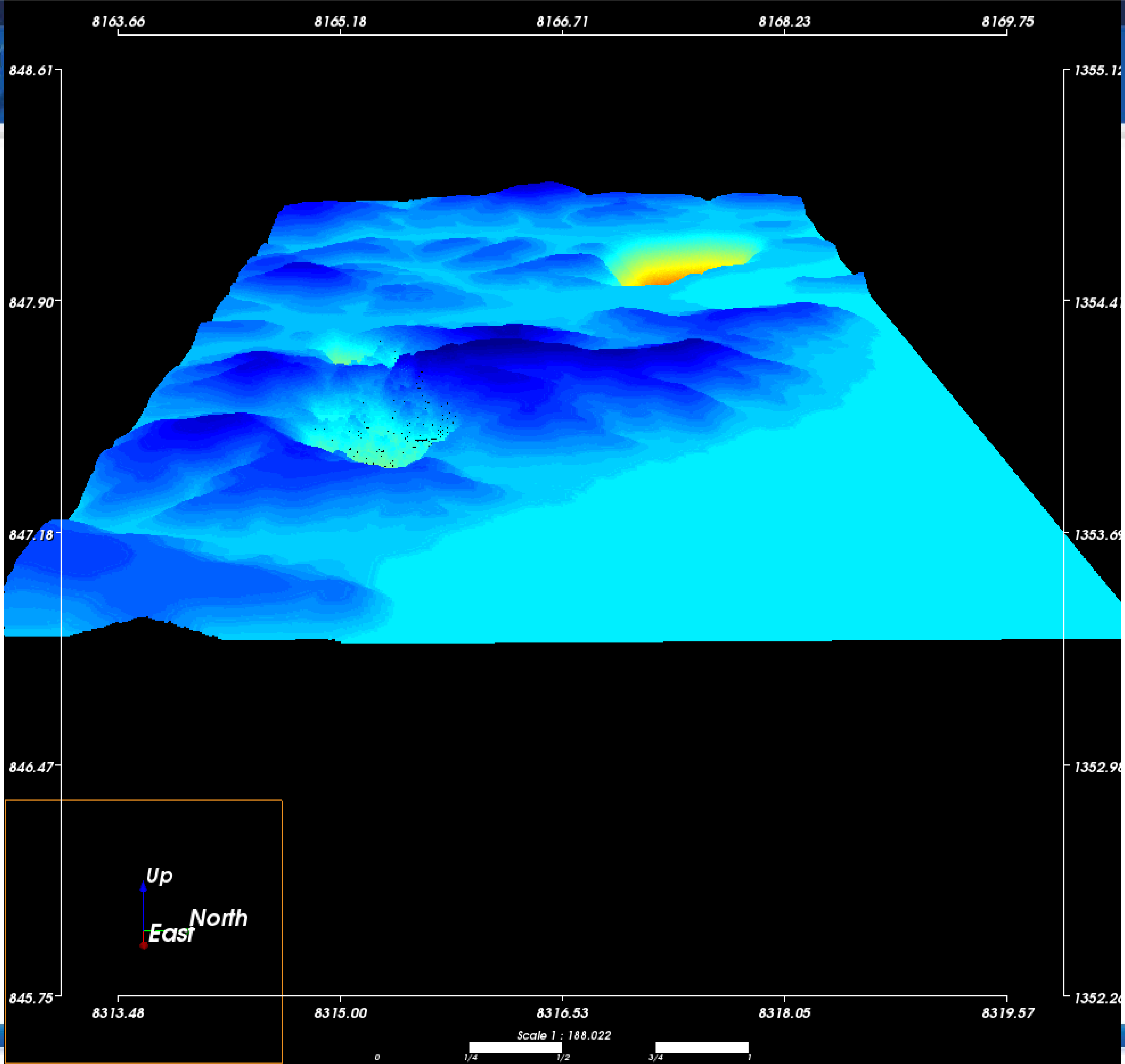
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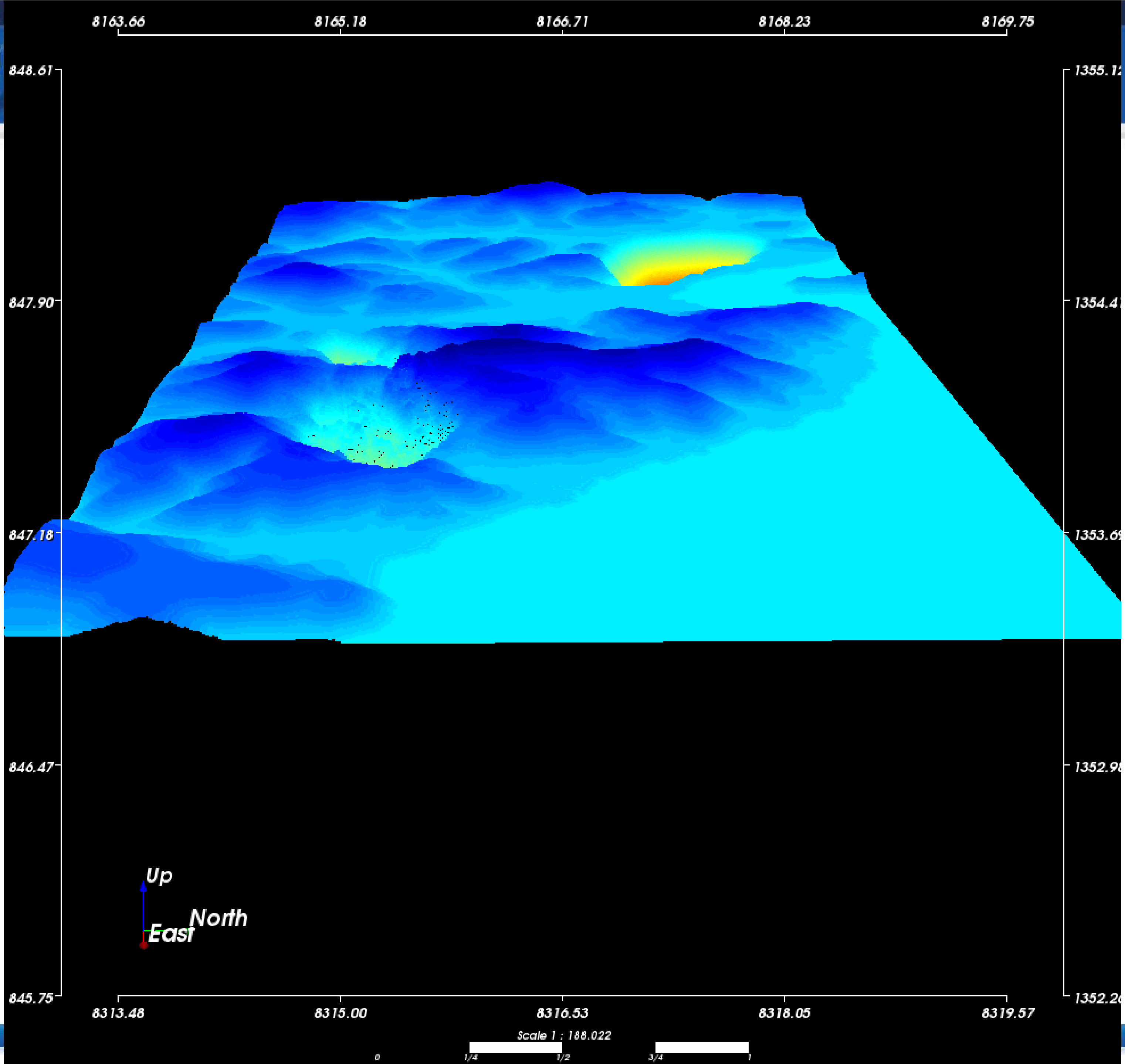

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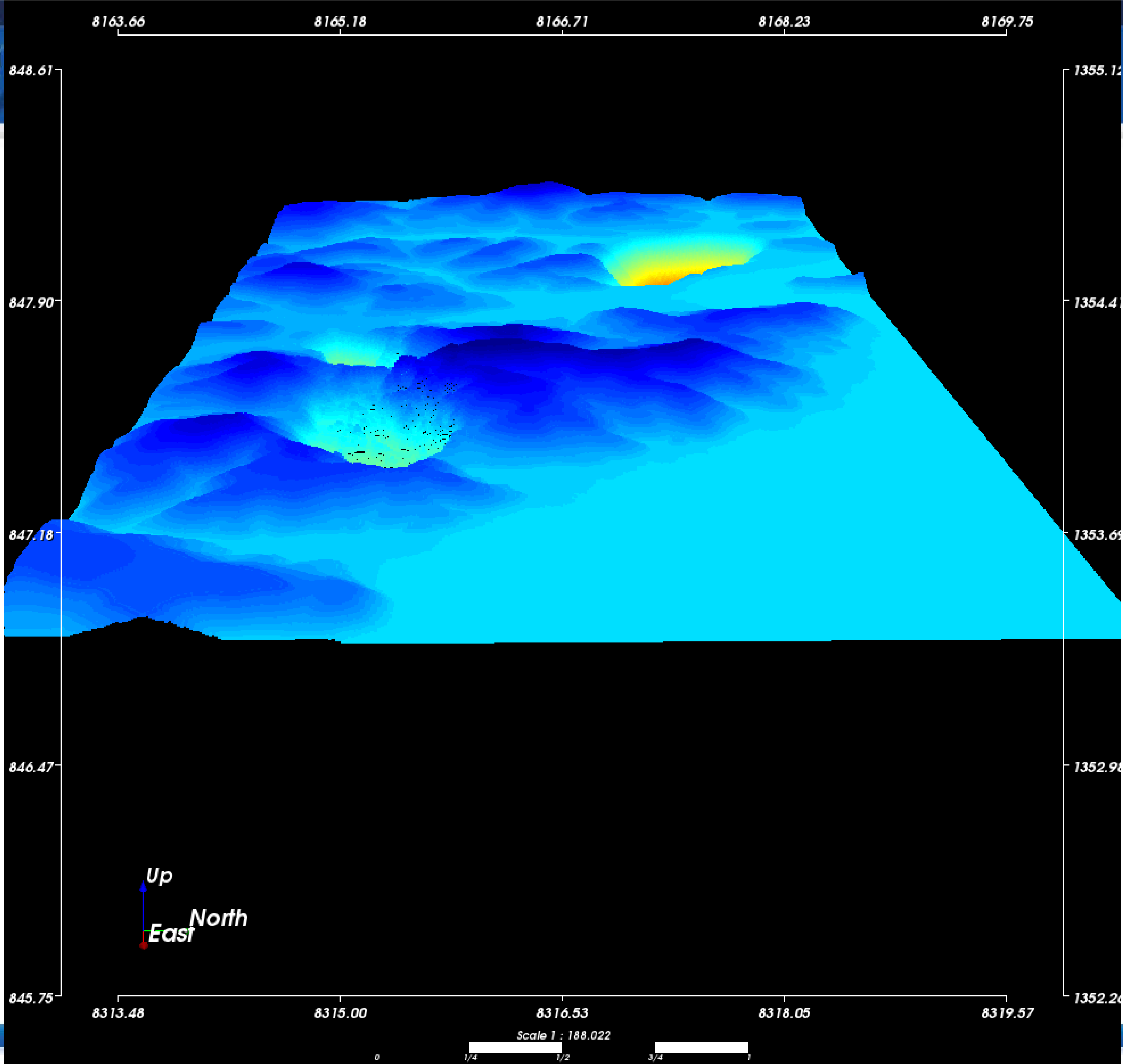

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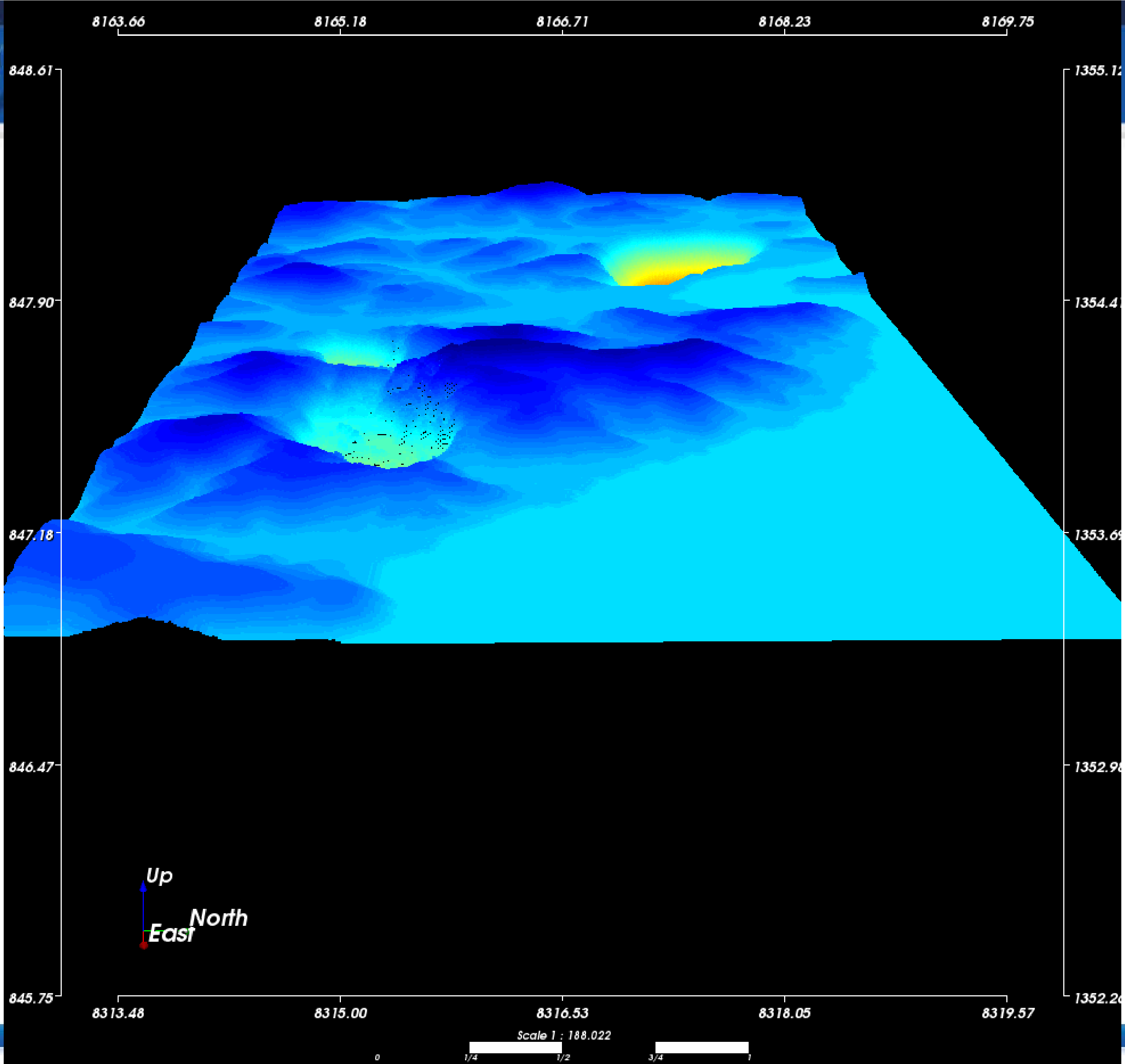
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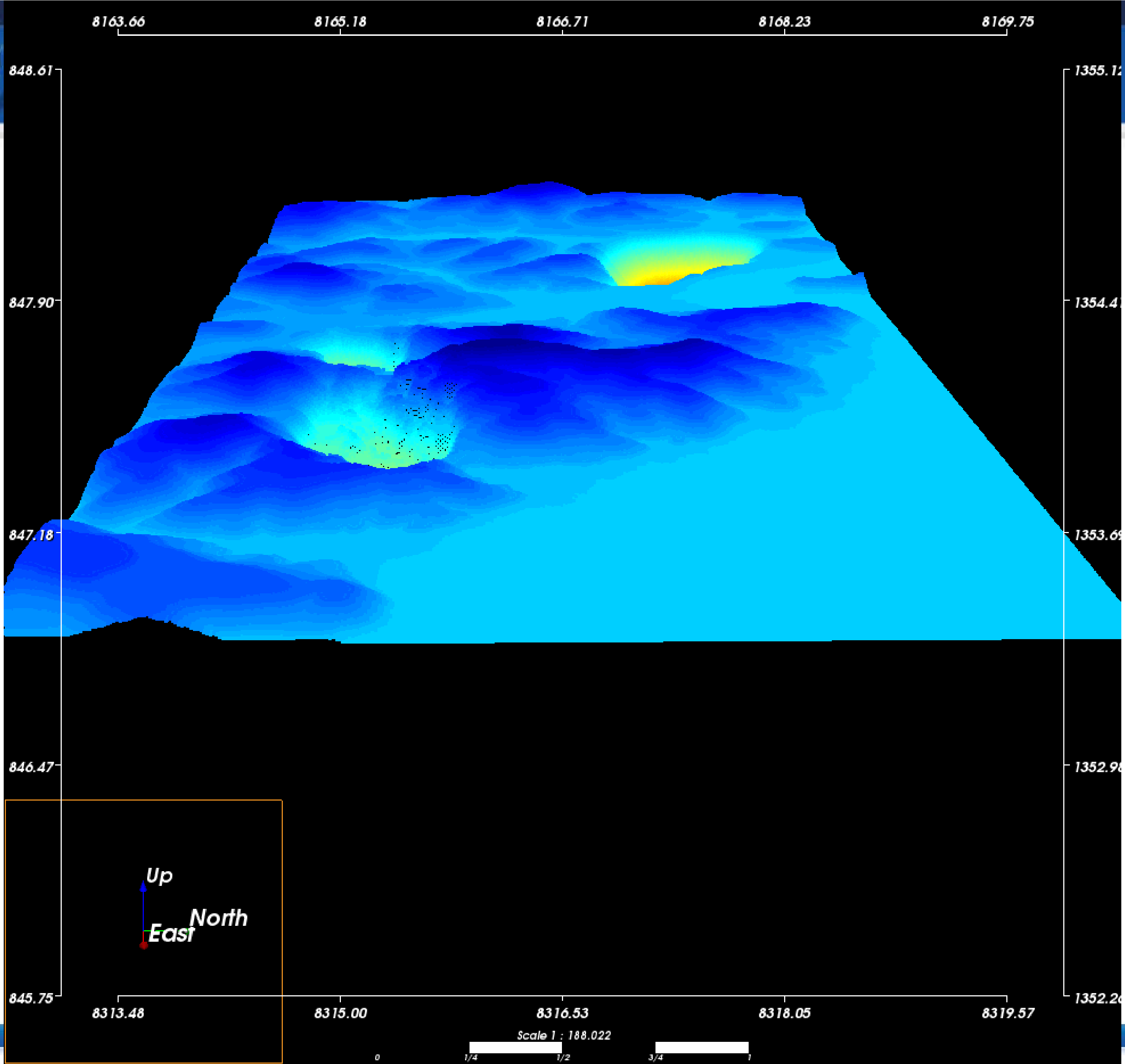

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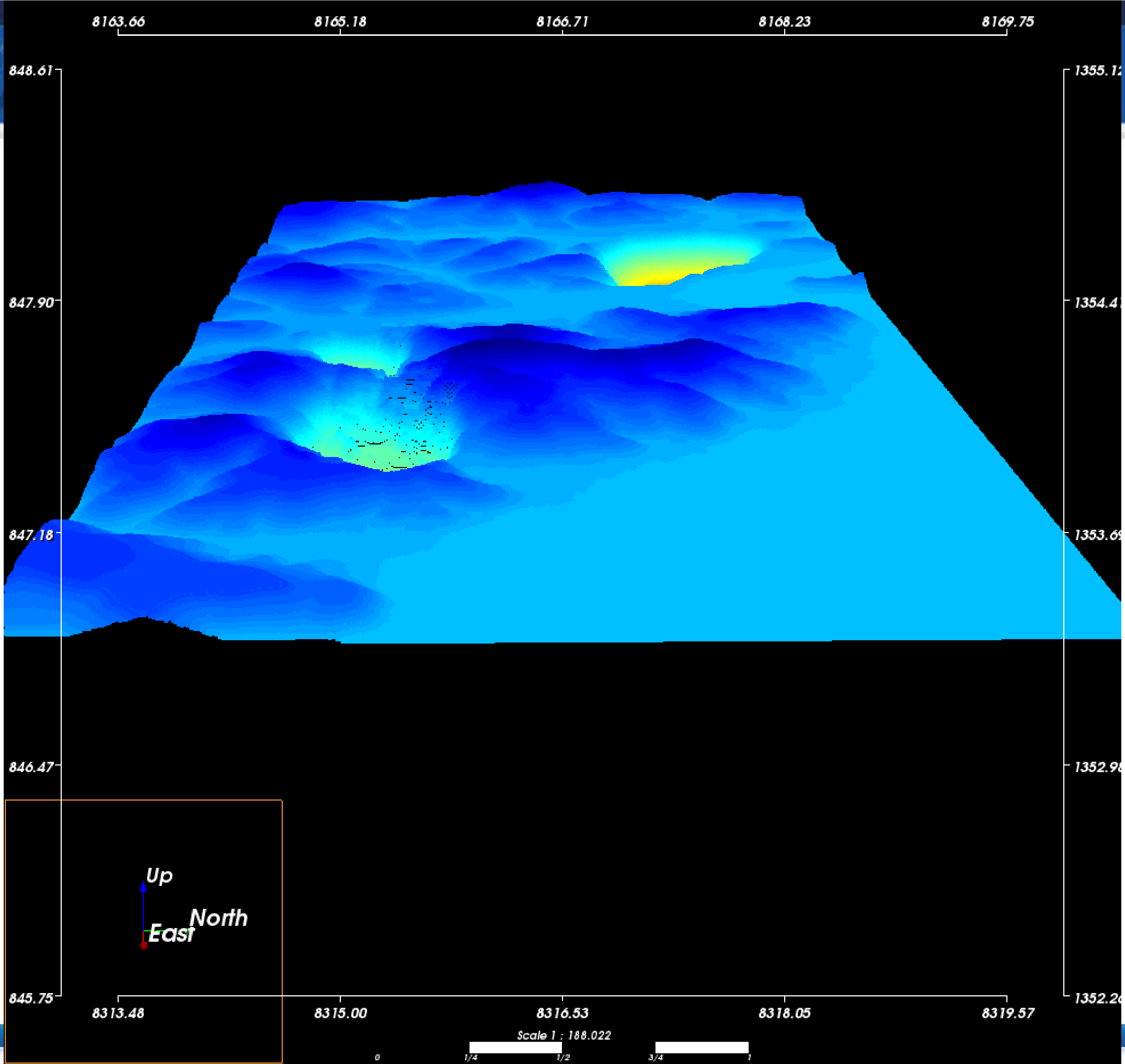

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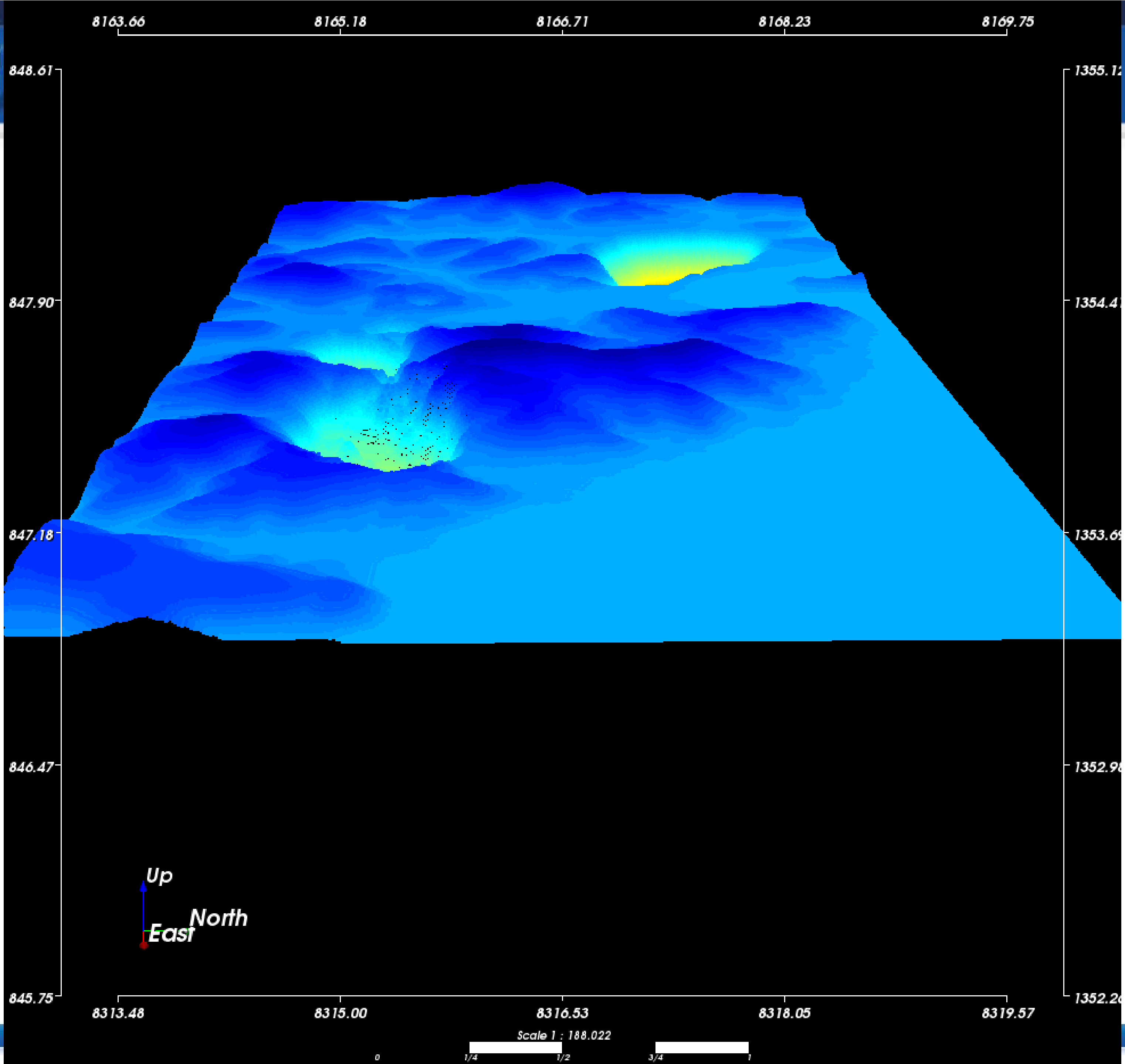
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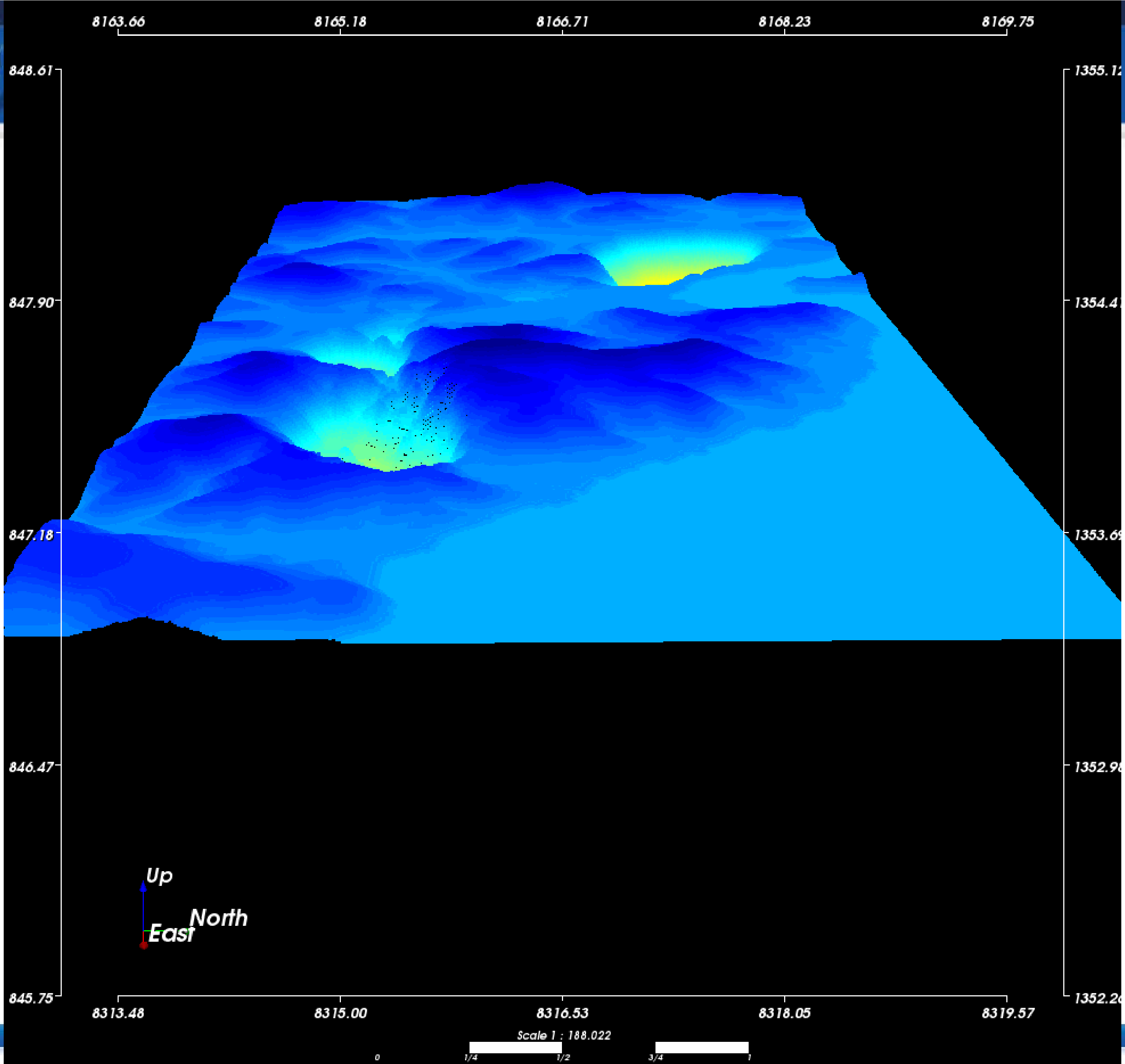
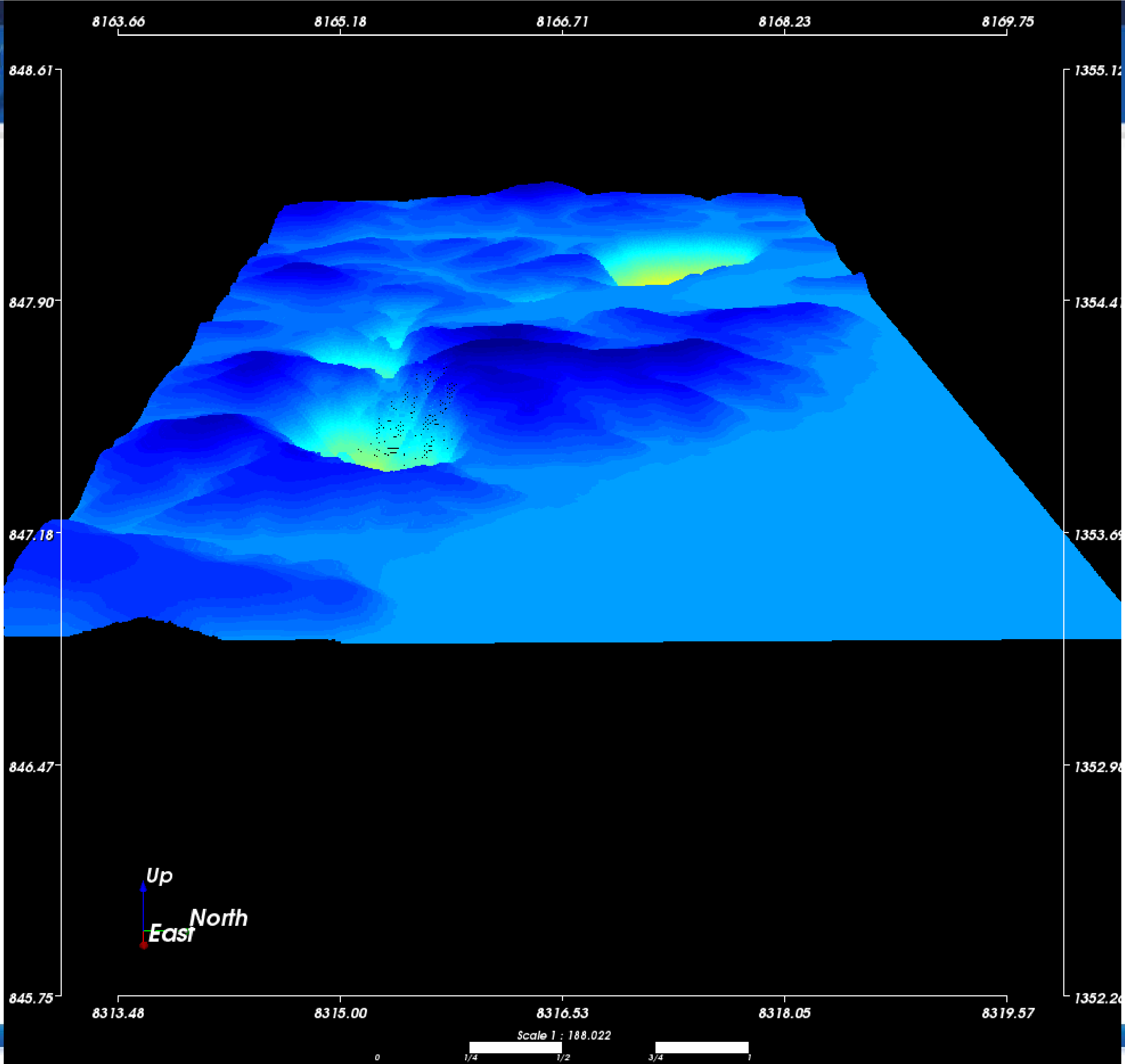
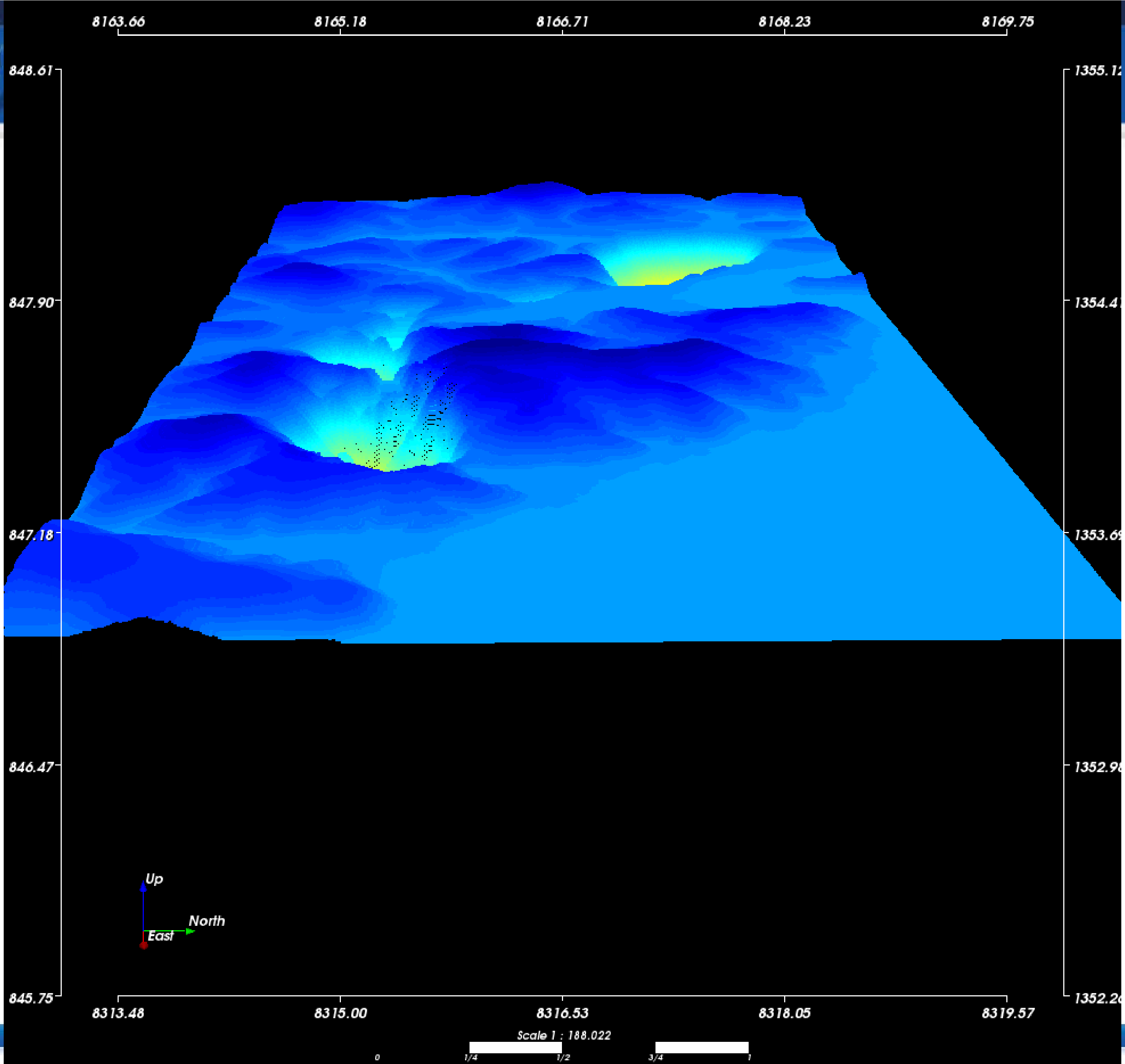
East
8313.48
8315.00
8316.53
8318.05
8319.57
Scale 1 : 188.022

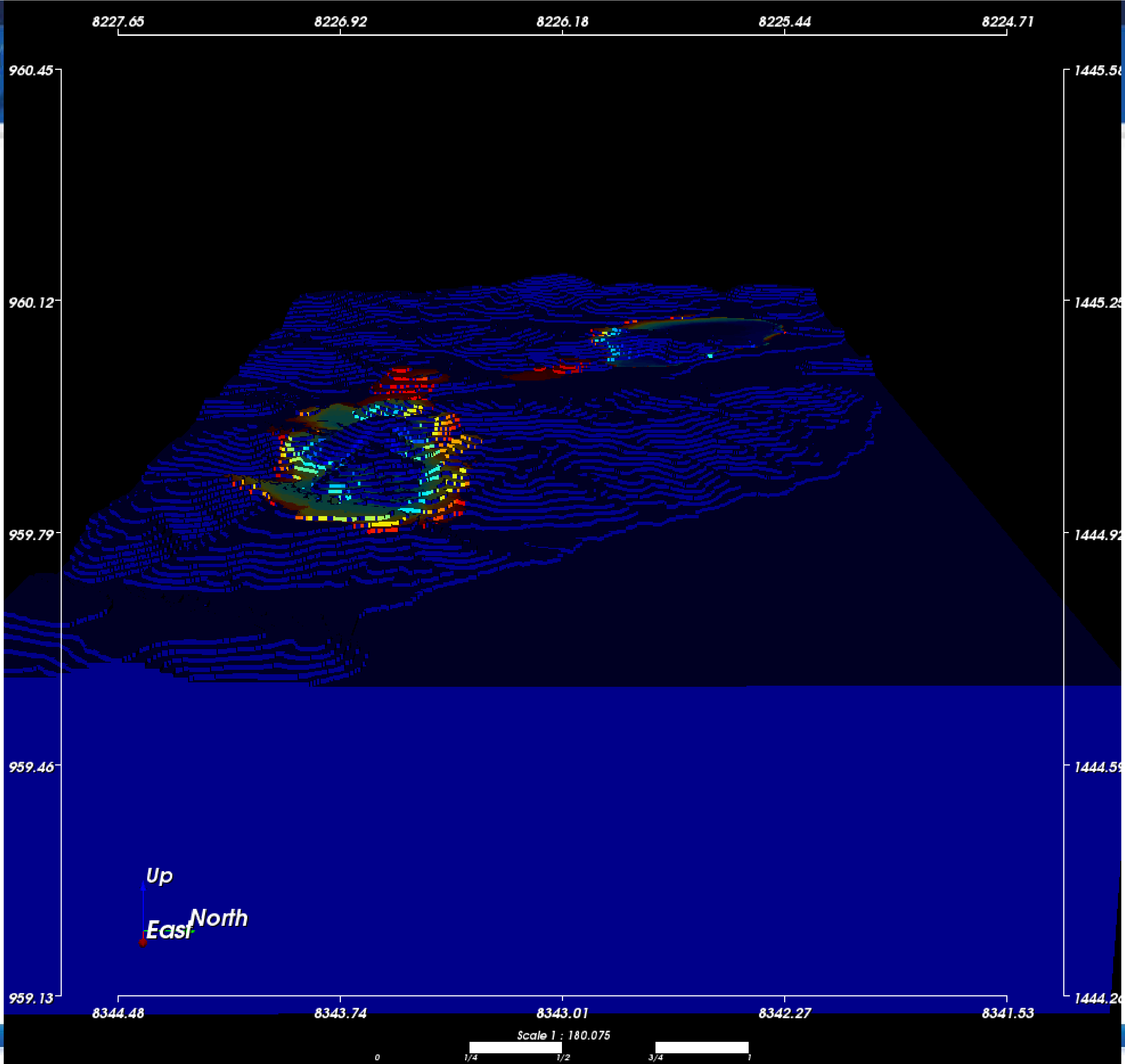

8163.66
8165.18
8166.71
8168.23
8169.75
848.61
847.90
847.18
846.47
845.75
1355.12
1354.41
1353.69
1352.98
1352.26
Up
North
East
8313.48
8315.00
8316.53
8318.05
8319.57
Scale 1 : 188.022
0
1/4
1/2
3/4
1



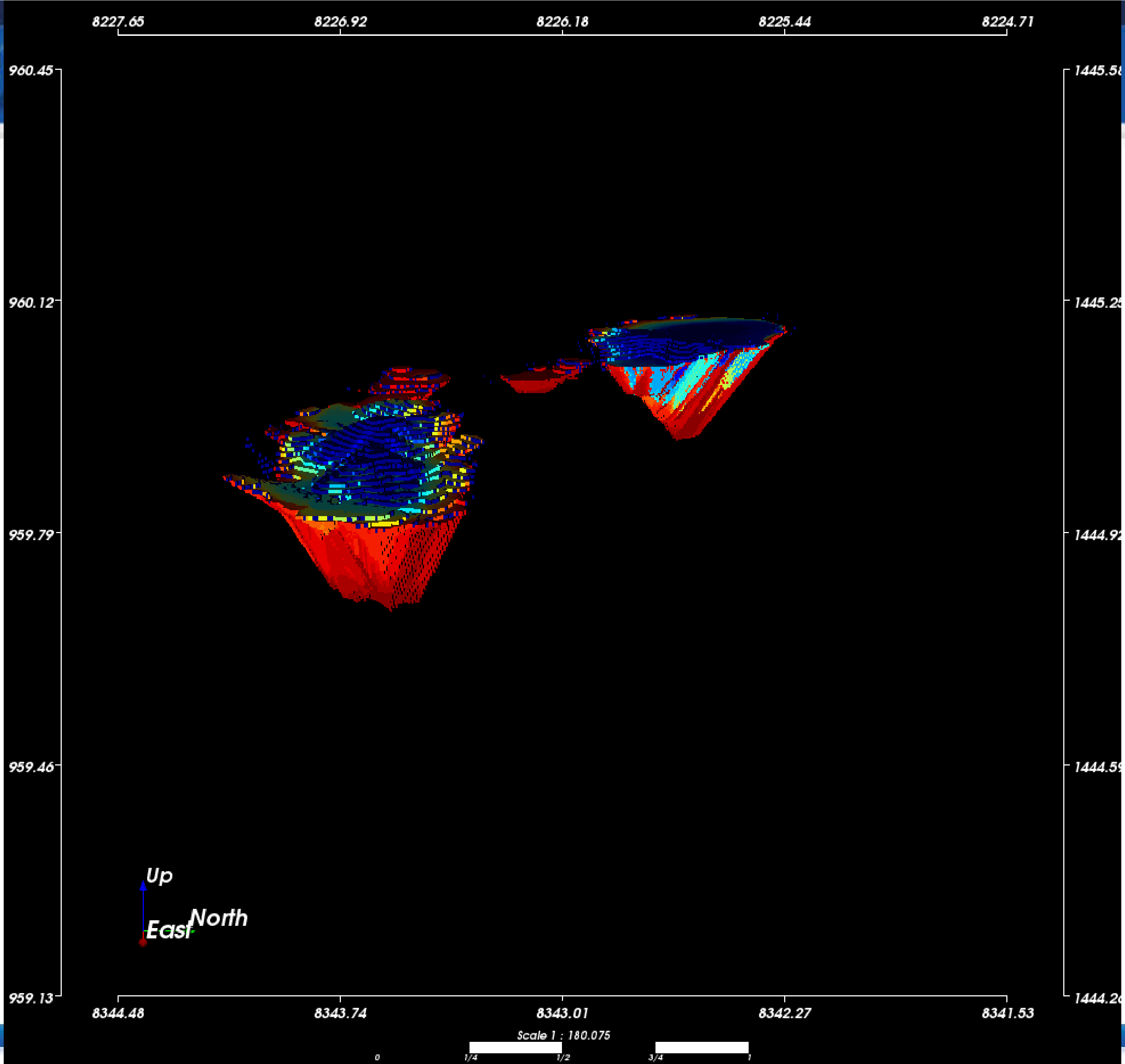
8163.66
8165.18
8166.71
8168.23
8169.75
848.61
847.90
847.18
846.47
845.75
Up
North
East
8313.48
8315.00
8316.53
8318.05
8319.57
Scale 1 : 188.022
0
1/4
1/2
3/4
1



8163.66
8165.18
8166.71
8168.23
8169.75
848.61
847.90
847.18
846.47
845.75
1355.12
1354.41
1353.69
1352.98
1352.26
Up
North
East
8313.48
8315.00
8316.53
8318.05
8319.57
Scale 1 : 188.022



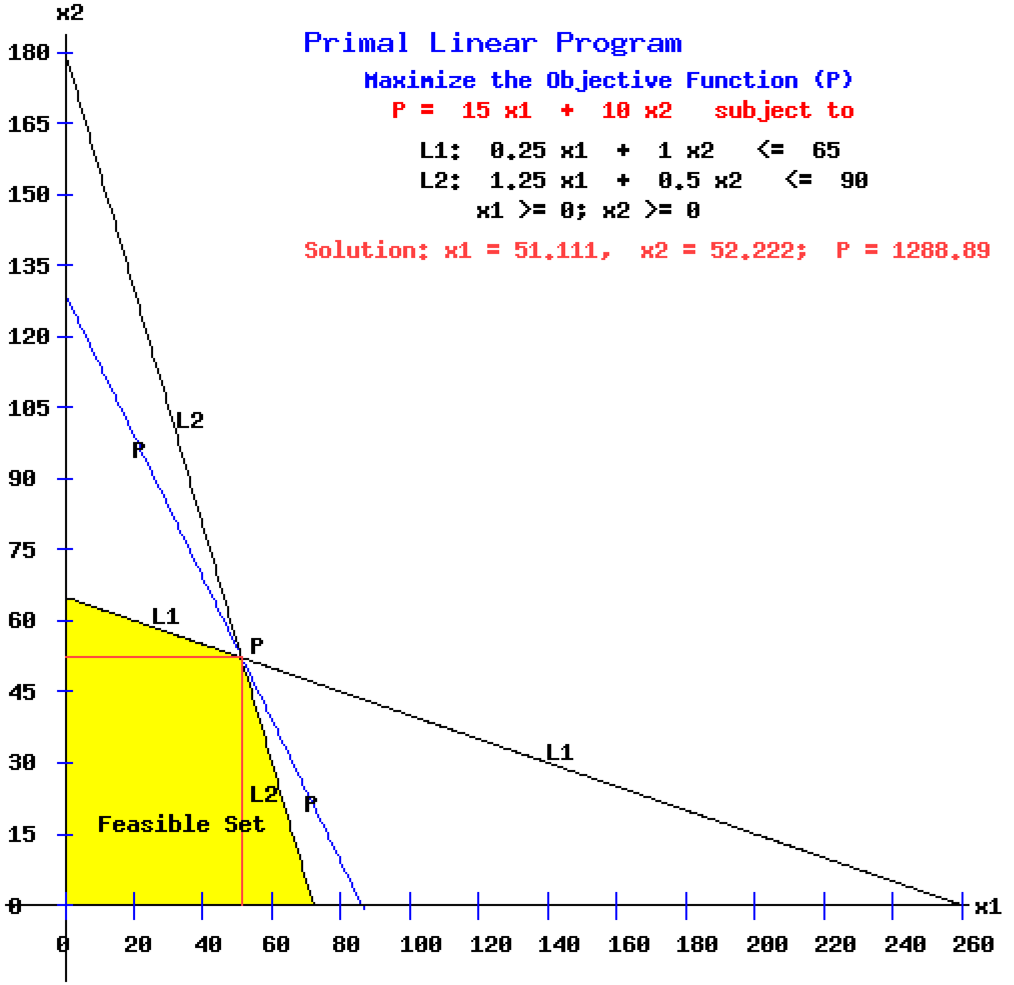
8227.65
8226.92
8226.18
8225.44
8224.71
960.45
960.12
959.79
959.46
959.13
1445.58
1445.25
1444.92
1444.59
1444.26
Up
East
North
8344.48
8343.74
8343.01
8342.27
8341.53
Scale 1 : 180.075
0
1/4
1/2
3/4
1



8227.65
8226.92
8226.18
8225.44
8224.71
960.45
960.12
959.79
959.46
959.13
Up
East
North
8344.48
8343.74
8343.01
8342.27
8341.53
Scale 1 : 180.075

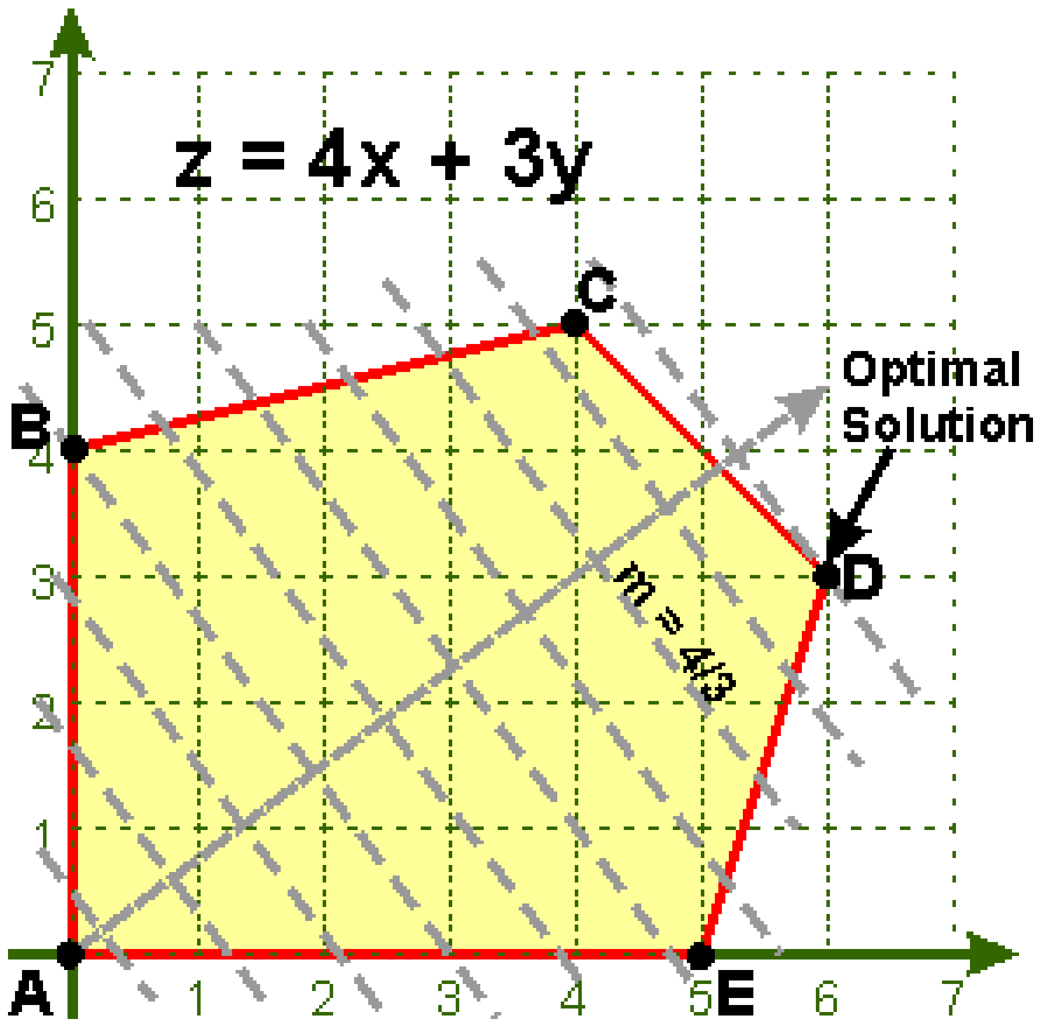
$$\begin{aligned} \textbf{max.} \quad & c^T x \\ \textbf{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

Simplex



x2
180
165
150
135
120
105
90
75
60
45
30
15
0
Primal Linear Program
Maximize the Objective Function (P)
P = 15 x1 + 10 x2 subject to
L1: 0.25 x1 + 1 x2 <= 65
L2: 1.25 x1 + 0.5 x2 <= 90
x1 >= 0; x2 >= 0
Solution: x1 = 51.111, x2 = 52.222; P = 1288.89
L2
P
L1
P
L1
L2
P
Feasible Set
x1
0
20
40
60
80
100
120
140
160
180
200
220
240
260

Interior Point



LINPACK AND MATRIX

Organized around matrix decompositions

LU	Cholesky
QR	singular value

Cholesky

- Small block model ~ 66k blocks
  - Sparse Cholesky 23000 x 23000 for one iteration
    - 529M elements
    - 207k non zero elements
  - Dense CPU single solution
    - Over 30 min
  - Sparse CPU single solution (not state of the art)
    - Few seconds
- CHOLMOD (suitesparse from NVIDIA)
  - 3x speedup over CPU multithread implementation

Cholesky

$$A = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{pmatrix} \equiv LL^T$$

$$= \begin{pmatrix} l_{11}^2 & l_{21}l_{11} & l_{31}l_{11} \\ l_{21}l_{11} & l_{21}^2 + l_{22}^2 & l_{31}l_{21} + l_{32}l_{22} \\ l_{31}l_{11} & l_{31}l_{21} + l_{32}l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{pmatrix}$$

$$l_{kk} = \sqrt{a_{kk} - \sum_{j=1}^{k-1} l_{kj}^2}$$

$$l_{ik} = \frac{1}{l_{kk}} \left(a_{ik} - \sum_{j=1}^{k-1} l_{ij} l_{kj} \right)$$

Cholesky

```
procedure CHOLESKY(A)
int i, j, k;
for k := 0 to n − 1 do
    A[k, k] := √A[k, k];    /* Obtain the square root of the diagonal element. */
    for j := k + 1 to n − 1 do
        A[k, j] := A[k, j]/A[k, k];    /* The division step. */
    end for
    for i := k + 1 to n − 1 do
        for j := i to n − 1 do
            A[i, j] := A[i, j] - A[k, i] × A[k, j];    /* The elimination step. */
        end for
    end for
end for
```

Single Thread

```
int chol_gold(const Matrix A, Matrix U) {
    unsigned int i, j, k;
    unsigned int size = A.num_rows * A.num_columns;

    // Copy the contents of the A matrix into the working matrix U
    for (i = 0; i < size; i++)
        U.elements[i] = A.elements[i];

    // Perform the Cholesky decomposition in place on the U matrix
    for (k = 0; k < U.num_rows; k++) {
        // Take the square root of the diagonal element
        U.elements[k * U.num_rows + k] = sqrt(U.elements[k * U.num_rows + k]);
        if (U.elements[k * U.num_rows + k] <= 0) {
            printf("Cholesky decomposition failed. \n");
            return 0;
        }

        // Division step
        for (j = (k + 1); j < U.num_rows; j++)
            U.elements[k * U.num_rows + j] /= U.elements[k * U.num_rows + k]; // Division step

        // Elimination step
        for (i = (k + 1); i < U.num_rows; i++)
            for (j = i; j < U.num_rows; j++)
                U.elements[i * U.num_rows + j] -= U.elements[k * U.num_rows + i] * U.elements[k * U.num_rows + j];
    }

    // As the final step, zero out the lower triangular portion of U
    for (i = 0; i < U.num_rows; i++)
        for (j = 0; j < i; j++)
            U.elements[i * U.num_rows + j] = 0.0;
    return 1;
}
```

pthread

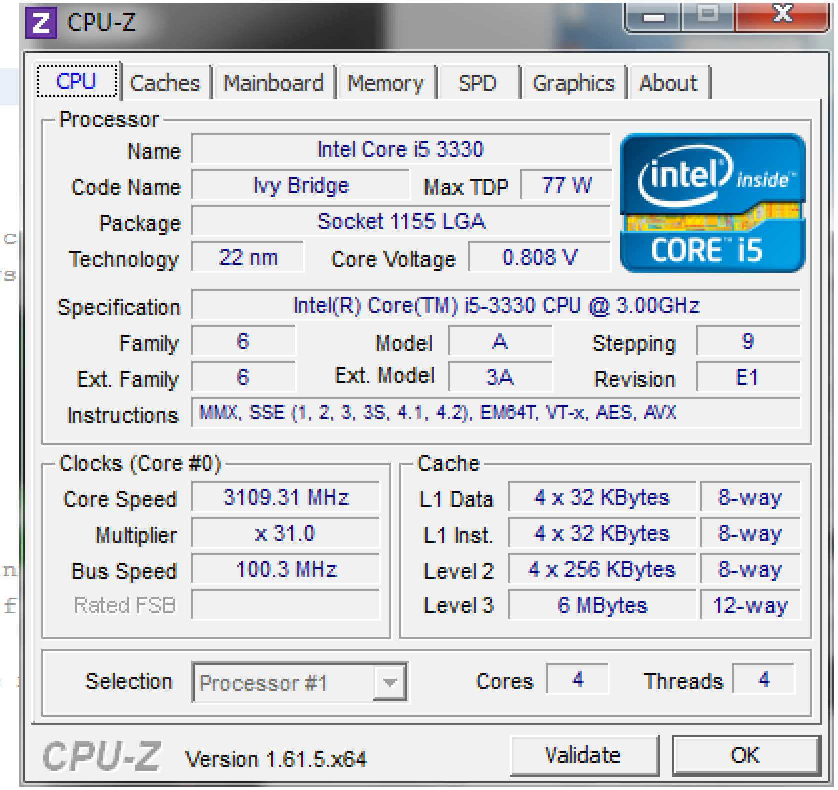
```c
#ifndef _MATRIX_H_
#define _MATRIX_H_

// Thread block size
#define MATRIX_SIZE 2048
#define NUM_PTHREADS 4
#define NUM_OMPTHREADS  NUM_PTHREADS

// Matrix dimensions
#define NUM_COLUMNS MATRIX_SIZE // Number of c
#define NUM_ROWS MATRIX_SIZE // Number of rows

// Matrix Structure declaration
typedef struct {
        //width of the matrix represented
    unsigned int num_columns;
        //height of the matrix represented
    unsigned int num_rows;
        //number of elements between the begin
        // rows in the memory layout (useful f
    unsigned int pitch;
        //Pointer to the first element of the
    float* elements;
} Matrix;


#endif // _MATRIX_H_
```



pthread

```
// Perform the Cholesky decomposition in place on the U matrix
for(k = 0; k < U.num_rows; k++)
{
        //Only one thread does squre root and division
        if(id==0)
        {
                // Take the square root of the diagonal element
                U.elements[k * U.num_rows + k] = sqrt(U.elements[k * U.num_rows + k]);
                if(U.elements[k * U.num_rows + k] <= 0){
                                printf("Cholesky decomposition failed. \n");
                                return 0;
                }

                // Division step
                for(j = (k + 1); j < U.num_rows; j++)
                {
                        U.elements[k * U.num_rows + j] /= U.elements[k * U.num_rows + k]; // Division step
                }
        }

        //Sync threads!!!!!
        sync_pthreads(barrier, id);

        //For this k iteration, split up i
        //Size of i range originally
        int isize = U.num_rows - (k + 1);
        int items_per_thread, items_last_thread;
        range_splitter(isize, NUM_PTHREADS, &items_per_thread, &items_last_thread);
        //Divy up work
        //Elim work
        int elimi_start, elimi_end;
        int offset = (k + 1); //To account for not starting at i=0 each time
```

pthread

```
// Perform the Cholesky decomposition in place on the U matrix
for(k = 0; k < U.num_rows; k++)
{
        //Only one thread does squre root and division
        if(id==0)
        {
                // Take the square root of the diagonal element
                U.elements[k * U.num_rows + k] = sqrt(U.elements[k * U.num_rows + k]);
                if(U.elements[k * U.num_rows + k] <= 0){
                                printf("Cholesky decomposition failed. \n");
                                return 0;
                }

                // Division step
                for(j = (k + 1); j < U.num_rows; j++)
                {
                        U.elements[k * U.num_rows + j] /= U.elements[k * U.num_rows + k]; // Division step
                }
        }

        //Sync threads!!!!!
        sync_pthreads(barrier, id);

        //For this k iteration, split up i
        //Size of i range originally
        int isize = U.num_rows - (k + 1);
        int items_per_thread, items_last_thread;
        range_splitter(isize, NUM_PTHREADS, &items_per_thread, &items_last_thread);
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        //Elim work
        int elimi_start, elimi_end;
        int offset = (k + 1); //To account for not starting at i=0 each time
```
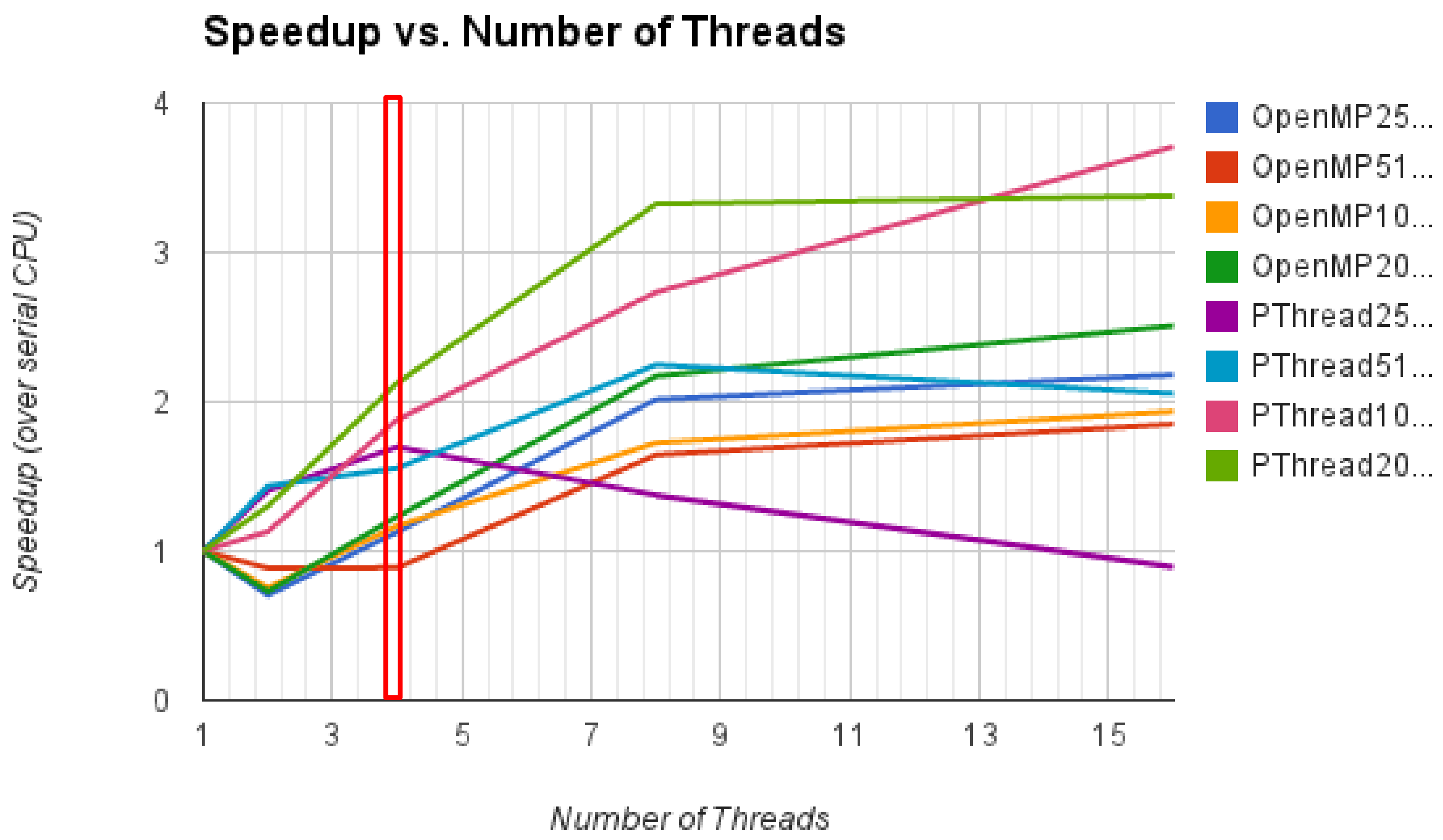
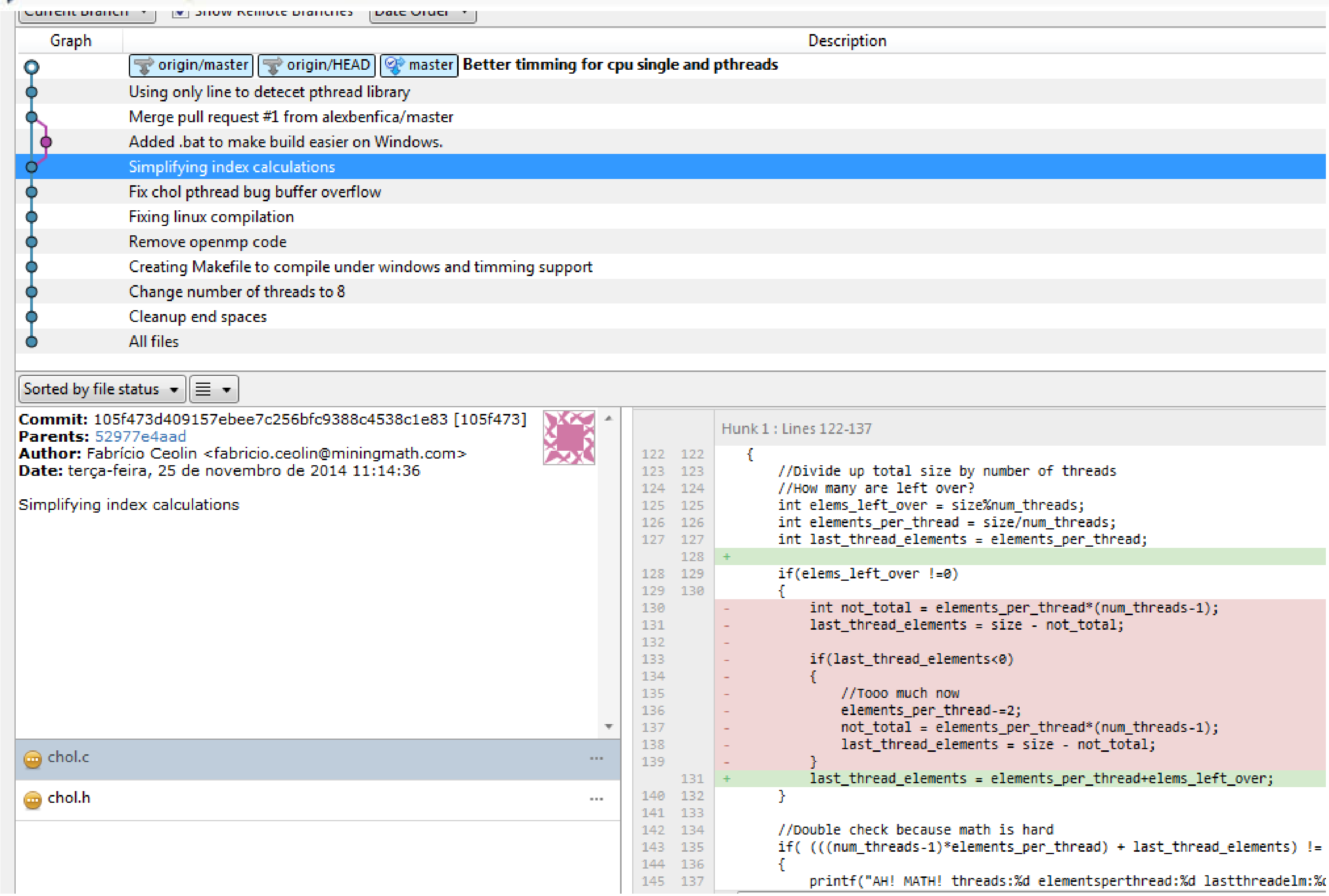

pthread

```c
// Perform the Cholesky decomposition in place on the U matrix
for(k = 0; k < U.num_rows; k++)
{
    //Only one thread does squre root and division
    if(id==0)
    {
        // Take the square root of the diagonal element
        U.elements[k * U.num_rows + k] = sqrt(U.elements[k * U.num_rows + k]);
        if(U.elements[k * U.num_rows + k] <= 0){
                printf("Cholesky decomposition failed. \n");
                return 0;
        }

        // Division step
        for(j = (k + 1); j < U.num_rows; j++)
        {
            U.elements[k * U.num_rows + j] /= U.elements[k * U.num_rows + k]; // Division step
        }
    }

    //Sync threads!!!!!
    sync_pthreads(barrier, id);

    //For this k iteration, split up i
    //Size of i range originally
    int isize = U.num_rows - (k + 1);
    int items_per_thread, items_last_thread;
    range_splitter(isize, NUM_PTHREADS, &items_per_thread, &items_last_thread);
    //Divy up work
    //Elim work
    int elimi_start, elimi_end;
    int offset = (k + 1); //To account for not starting at i=0 each time
```

Pthread: before optimizations



Pthread: our optimizations



Pthread: our optimizations

```
D:\Dropbox\Devel\Win\CudaProject\Drexel-ECEC622-Midterm1\build>CholThread.exe
Creating a 2048 x 2048 matrix with random numbers between [-.5, .5]...done.
Generating the symmetric matrix...done.
Generating the positive definite matrix...done.
Performing Cholesky decomposition on the CPU using the single-threaded versio

        Run time:    9.4130001068 s.
Cholesky decomposition on the CPU was successful.

Performing Cholesky decomposition on the CPU using the PTHREAD version.
        Run time:    3.0450000763 s.
Speedup from single to phreads = 3.091297 x.
Double checking for correctness by recovering the original matrix.
```

CUDA device query: Geforce GTS 450

```
 CUDA Device Query (Runtime API) version (CUDART static linking)

Detected 1 CUDA Capable device(s)

Device 0: "GeForce GTS 450"
  CUDA Driver Version / Runtime Version          6.5 / 6.5
  CUDA Capability Major/Minor version number:    2.1
  Total amount of global memory:                 1024 MBytes (1073741824 bytes)
  ( 4) Multiprocessors, ( 48) CUDA Cores/MP:     192 CUDA Cores
  GPU Clock rate:                                1620 MHz (1.62 GHz)
  Memory Clock rate:                             1804 Mhz
  Memory Bus Width:                              128-bit
  L2 Cache Size:                                 262144 bytes
  Maximum Texture Dimension Size (x,y,z)         1D=(65536), 2D=(65536, 65535), 3D=(2048, 2048, 2048)
  Maximum Layered 1D Texture Size, (num) layers  1D=(16384), 2048 layers
  Maximum Layered 2D Texture Size, (num) layers  2D=(16384, 16384), 2048 layers
  Total amount of constant memory:               65536 bytes
  Total amount of shared memory per block:       49152 bytes
  Total number of registers available per block: 32768
  Warp size:                                     32
  Maximum number of threads per multiprocessor:  1536
  Maximum number of threads per block:           1024
  Max dimension size of a thread block (x,y,z): (1024, 1024, 64)
  Max dimension size of a grid size    (x,y,z): (65535, 65535, 65535)
  Maximum memory pitch:                          2147483647 bytes
  Texture alignment:                             512 bytes
  Concurrent copy and kernel execution:          Yes with 1 copy engine(s)
  Run time limit on kernels:                     Yes
  Integrated GPU sharing Host Memory:            No
  Support host page-locked memory mapping:       Yes
  Alignment requirement for Surfaces:            Yes
  Device has ECC support:                        Disabled
  CUDA Device Driver Mode (TCC or WDDM):         WDDM (Windows Display Driver Model)
  Device supports Unified Addressing (UVA):      Yes
  Device PCI Bus ID / PCI location ID:           1 / 0
  Compute Mode:
     < Default (multiple host threads can use ::cudaSetDevice() with device simultaneously) >

deviceQuery, CUDA Driver = CUDART, CUDA Driver Version = 6.5, CUDA Runtime Version = 6.5, NumDevs = 1, Device0 = GeForce GTS 450
Result = PASS
```

CUDA device query: Geforce GTS 450

```
 CUDA Device Query (Runtime API) version (CUDART static linking)

Detected 1 CUDA Capable device(s)

Device 0: "GeForce GTS 450"
  CUDA Driver Version / Runtime Version          6.5 / 6.5
  CUDA Capability Major/Minor version number:    2.1
  Total amount of global memory:                 1024 MBytes (1073741824 bytes)
  ( 4) Multiprocessors, ( 48) CUDA Cores/MP:     192 CUDA Cores
  GPU Clock rate:                                1620 MHz (1.62 GHz)
  Memory Clock rate:                             1804 Mhz
  Memory Bus Width:                              128-bit
  L2 Cache Size:                                 262144 bytes
  Maximum Texture Dimension Size (x,y,z)         1D=(65536), 2D=(65536, 65535), 3D=(2048, 2048, 2048)
  Maximum Layered 1D Texture Size, (num) layers  1D=(16384), 2048 layers
  Maximum Layered 2D Texture Size, (num) layers  2D=(16384, 16384), 2048 layers
  Total amount of constant memory:               65536 bytes
  Total amount of shared memory per block:       49152 bytes
  Total number of registers available per block: 32768
  Warp size:                                     32
  Maximum number of threads per multiprocessor:  1536
  Maximum number of threads per block:           1024
  Max dimension size of a thread block (x,y,z): (1024, 1024, 64)
  Max dimension size of a grid size    (x,y,z): (65535, 65535, 65535)
  Maximum memory pitch:                          2147483647 bytes
  Texture alignment:                             512 bytes
  Concurrent copy and kernel execution:          Yes with 1 copy engine(s)
  Run time limit on kernels:                     Yes
  Integrated GPU sharing Host Memory:            No
  Support host page-locked memory mapping:       Yes
  Alignment requirement for Surfaces:            Yes
  Device has ECC support:                        Disabled
  CUDA Device Driver Mode (TCC or WDDM):         WDDM (Windows Display Driver Model)
  Device supports Unified Addressing (UVA):      Yes
  Device PCI Bus ID / PCI location ID:           1 / 0
  Compute Mode:
     < Default (multiple host threads can use ::cudaSetDevice() with device simultaneously) >

deviceQuery, CUDA Driver = CUDART, CUDA Driver Version = 6.5, CUDA Runtime Version = 6.5, NumDevs = 1, Device0 = GeForce GTS 450
Result = PASS
```

CUDA sqrt

```
int threads_per_block_sqrt = 512;
int blocks_sqrt = MATRIX_SIZE / threads_per_block_sqrt;
dim3 thread_block(threads_per_block_sqrt, 1, 1);
dim3 grid(blocks_sqrt, 1);
chol_kernel_cudaUFMG_sqrt <<<grid, thread_block>>>(gpu_u.elements);
```

CUDA sqrt kernel

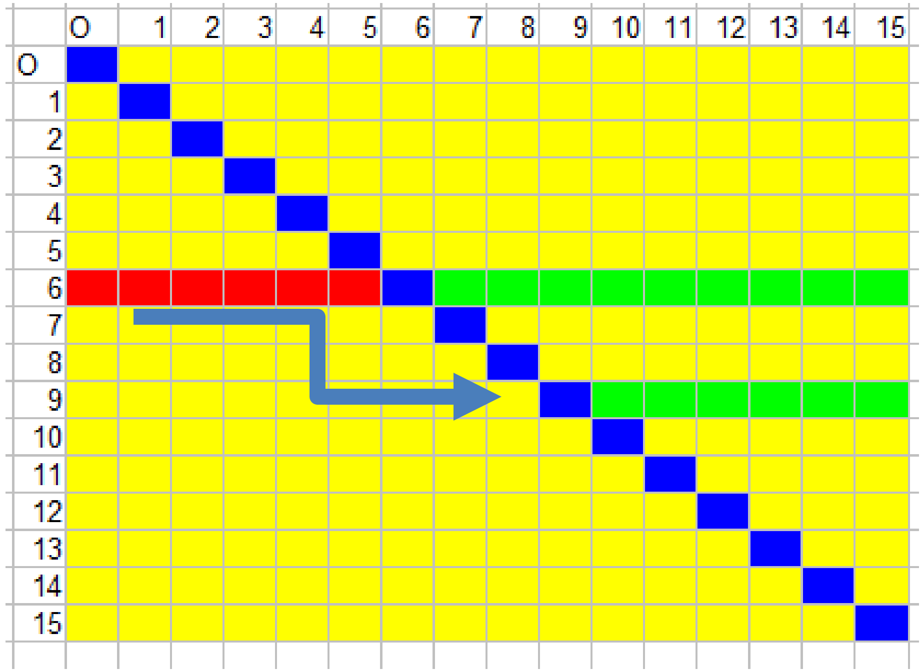
```
__global__ void chol_kernel_cudaUFMG_sqrt(float * U) {
    // Get a thread identifier
    int tx = blockIdx.x * blockDim.x + threadIdx.x;
    int tx_diag = tx * MATRIX_SIZE + tx;
    U[tx_diag] = sqrt(U[tx_diag]);
}
```

CUDA div

```
int block_x_div = 16;
int block_y_div = 16;
int thread_x_div = 4;
int thread_y_div = 4;
dim3 grid_div(block_x_div, block_y_div, 1);
dim3 thread_block_div(thread_x_div, thread_y_div, 1);
int elements_per_thread_div = ((MATRIX_SIZE * MATRIX_SIZE) / 2) /  (thread_x_div * thread_y_div * block_x_div * block_y_div);
chol_kernel_cudaUFMG_division <<<grid_div, thread_block_div >>>(gpu_u.elements, elements_per_thread_div);
```

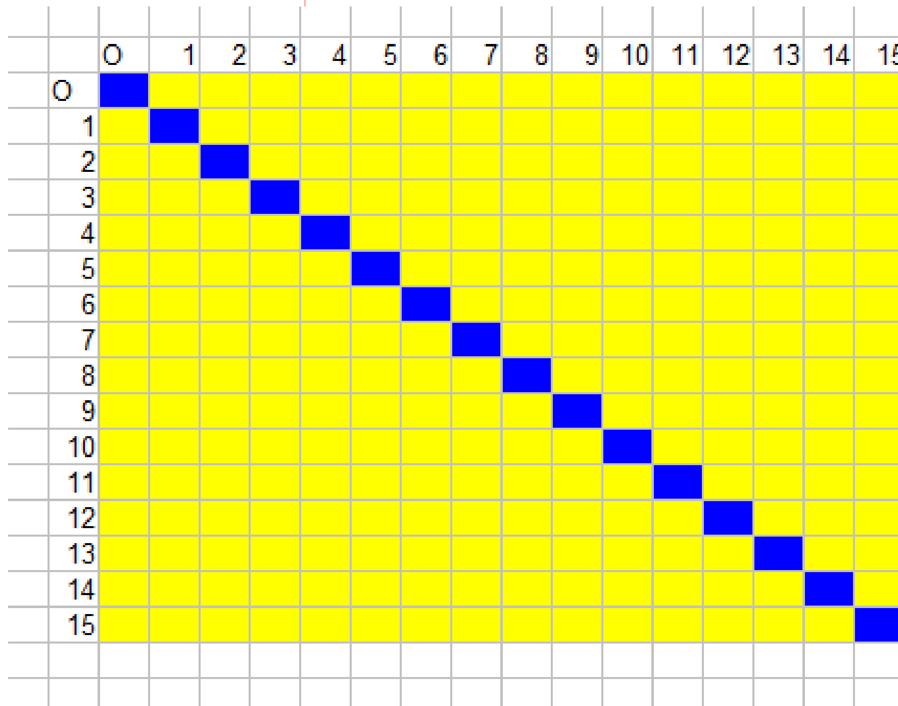
CUDA div kernel

```
__global__ void chol_kernel_cudaUFMG_division(float * U, int elem_per_thr) {
    // Get a thread identifier
    int tx = blockIdx.x * blockDim.x + threadIdx.x;
    int ty = blockIdx.y * blockDim.y + threadIdx.y;

    int tn = ty * blockDim.x * gridDim.x + tx;

    for(unsigned i=0;i<elem_per_thr;i++){
        int iel = tn * elem_per_thr + i;
        int xval = iel % MATRIX_SIZE;
        int yval = iel / MATRIX_SIZE;

        if(xval == yval){
            continue;
        }

        // if on the lower diagonal...
        if(yval > xval){
            xval = MATRIX_SIZE - xval - 1;
            yval = MATRIX_SIZE - yval - 1;
        }

        int iU = xval + yval * MATRIX_SIZE;
        int iDiag = yval + yval * MATRIX_SIZE;

        U[iU] /= U[iDiag];
    }
}
```



CUDA elimination

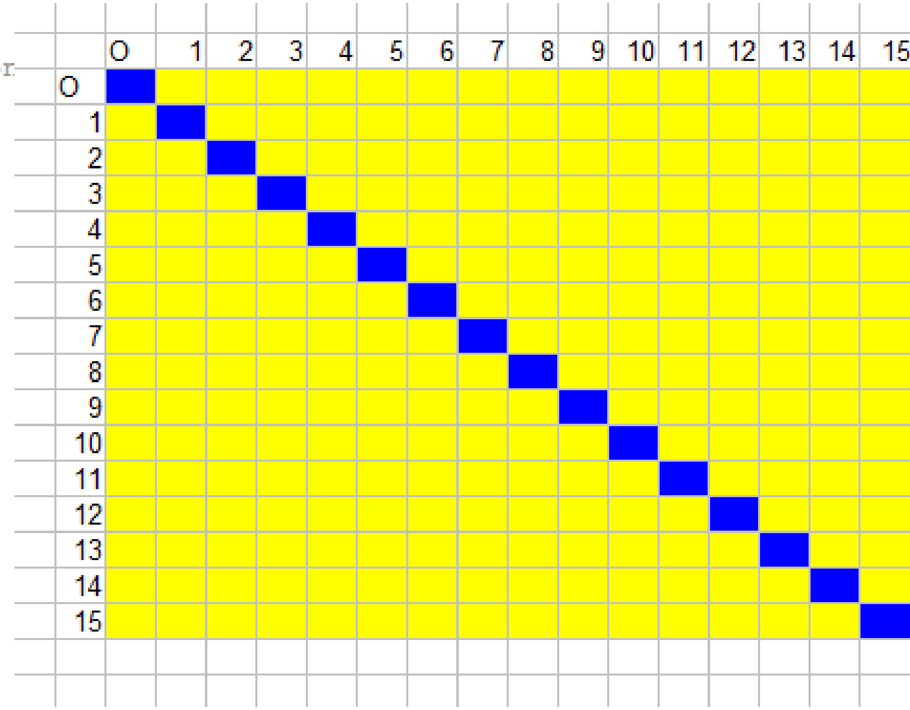
```
int block_y_eli = 1;
//Each thread within a block will take some j iterations
int thread_x_eli = 256;
int thread_y_eli = 1;

//Each kernel call will be one iteration of out K loop
for (int k = 0; k < MATRIX_SIZE; k++) {

    //Want threads to stride across memory
    //i is outer loop
    //j is inner loop
    //so threads should split the j loop
    //Each thread block will take an i iteration

    // i=k+1;i<MATRIX_SIZE
    int isize = MATRIX_SIZE - (k + 1);
    if(isize==0){
        isize++;
    }
    int block_x_eli = isize;

    //Set up the execution grid on the GPU
    dim3 thread_block(thread_x_eli, 1, 1);
    dim3 grid(block_x_eli, 1);

    //Call kernel with for this K iteration
    chol_kernel_cudaUFMG_elimination <<<grid, thread_block>>>(gpu_u.elements, k);
}
```

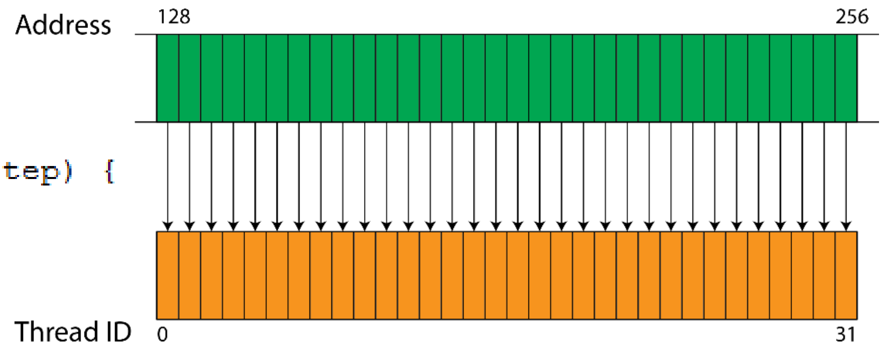



```
chol_kernel_cudaUFMG_zero <<<grid_div, thread_block_div>>>(gpu_u.elements, elements_per_thread_div);
```

CUDA elimination kernel

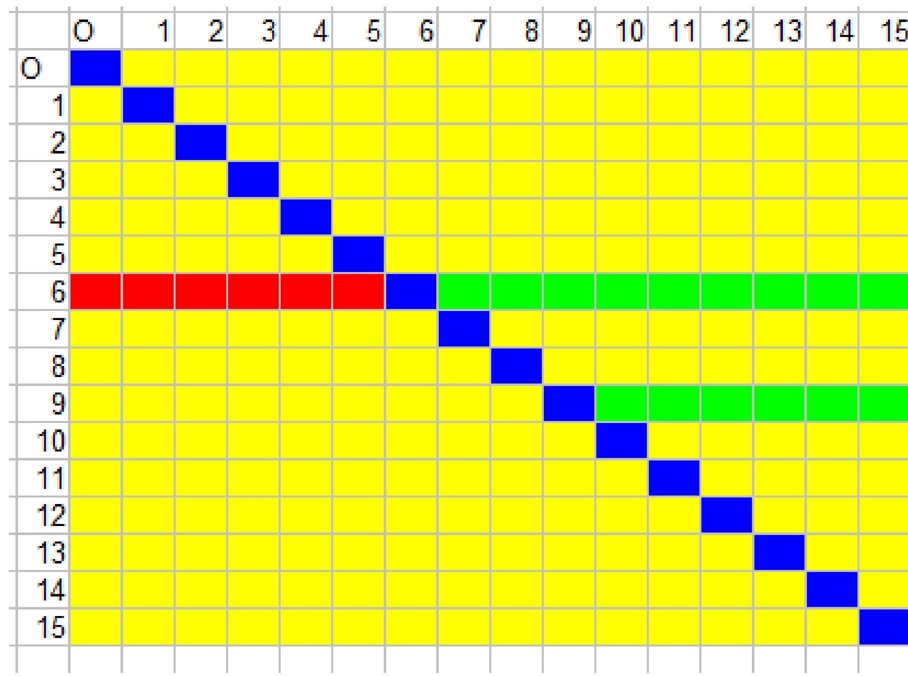
```
__global__ void chol_kernel_cudaUFMG_elimination(float * U, int k) {

    //This call acts as a single K iteration
    //Each block does a single i iteration
    int i = (k+1) + blockIdx.x;

    //Each thread does some part of j
    //Stide in units of 'stride'
    //Thread 0 does 0, 16, 32
    //Thread 1 does 1, 17, 33
    //..etc.
    int jstart = i + threadIdx.x;
    int jstep = blockDim.x;

    // Pre-calculate indexes outside loop
    int kM = k * MATRIX_SIZE;
    int iM = i * MATRIX_SIZE;
    int ki = kM + i;

    //Do work for this i iteration
    for (int j=jstart; j<MATRIX_SIZE; j+=jstep) {
        U[iM + j] -= U[ki] * U[kM + j];
    }
}
```





CUDA zero kernel

```
__global__ void chol_kernel_cudaUFMG_zero(float * U, int elem_per_thr) {
    // Get a thread identifier
    int tx = blockIdx.x * blockDim.x + threadIdx.x;
    int ty = blockIdx.y * blockDim.y + threadIdx.y;

    int tn = ty * blockDim.x * gridDim.x + tx;

    for(unsigned i=0;i<elem_per_thr;i++){
        int iel = tn * elem_per_thr + i;
        int xval = iel % MATRIX_SIZE;
        int yval = iel / MATRIX_SIZE;

        if(xval == yval){
            continue;
        }

        // if on the upper diagonal...
        if(yval < xval){
            xval = MATRIX_SIZE - xval - 1;
            yval = MATRIX_SIZE - yval - 1;
        }
        int iU = xval + yval * MATRIX_SIZE;
        U[iU] = 0;
    }

}
```



SPEEDUP Results

TYPE	SINGLE	PTHREAD	OUR PTHREAD	CUDA	OUR CUDA
SINGLE	1.00				
PTHREAD	2.20	1.00			
OUR PTHREAD	3.10	**1.41**	1.00		
CUDA	24.00	10.91	7.74	1.00	
OUR CUDA	33.00	15.00	10.65	**1.38**	1.00

Conclusion

- Improvements
  - It is possible to speedup using our algorithm with CUDA
  - No room for shared memory in our implementation
    - (We’ve tried other methods, without success)

Conclusion

- Future work
  - Pthread
    - Split sqrt and division over threads
  - Study Sparse Matrix algorithm
    - suitesparse with metis
    - Test CUDA and CPU multithread

Questions?

Fabrício Ceolin e Alex Teixeira