EQ2801/FEM3200 OPTIMAL FILTERING, 2020–09–10
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Project Assignment

In this project assignment you will work with the design of a motion tracking filter for a global positioning system (GPS) receiver. The GPS receiver is going to be used to track the motion of car driving around a race track. On the course homepage you can download a Matlab file `GPSdata.mat` with the true (position) trajectory of the car and the pseudo-range measurements recorded by the GPS receiver. Your job will be to design and tune a Kalman filter that tracks the motion of the car.

Today the GPS consists of 31 satellites that orbits the earth at an altitude of 20 000 km. These satellites constantly transmit time encoded signals that may be received by a GPS receiver at earth. The GPS receiver uses the received signals to calculated the distances between the GPS receiver and the satellites. These distance estimates are generally referred to as pseudo-range measurements since they also include an range offset caused by the offset in the GPS receiver clock. The pseudo range measurement to the i:th satellite at time k can be modeled as

$$y_k^{(i)} = \|\mathbf{p}_k^{(i)} - \mathbf{p}_k^{(rec)}\| + c\,\Delta t_k + v_k^{(i)}, \tag{1}$$

where $\mathbf{p}_k^{(i)}$ and $\mathbf{p}_k^{(rec)}$ is the position of the satellite[1] and the receiver, respectively. Further, c is the speed of light and $v_k^{(i)}$ is the measurement noise, which can be assumed zero-mean and have the variance $\mathbb{E}\{v_k^{(i)} v_l^{(j)}\} = \sigma_r^2\,\delta_{i-j,k-l}$. Moreover, the clock offset Δt_{k+1} in the receiver can be modeled as

$$\begin{bmatrix} \Delta t_{k+1} \\ \dot{\Delta} t_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta t_k \\ \dot{\Delta} t_k, \end{bmatrix} + \mathbf{w}_k^{clk}, \tag{2}$$

where T_s is the sampling period and $\mathbf{w}_k^{clk}$ is white noise with the covariance

$$\mathbf{Q}_k^{clk} = \begin{bmatrix} S_\phi T_s + \frac{T_s^3}{3} S_f & T_s^2 S_f \\ T_s^2 S_f & S_f T_s \end{bmatrix}. \tag{3}$$

Here S_ϕ and S_f is the power spectral density of the phase and frequency process noise (both of these noise processes are assumed to be white).

If the signals from four or more satellites are received then it is possible to, without introducing a model for the motion of the GPS receiver or the clock-offset, calculate a position estimate using a nonlinear least squares algorithm. The Matlab function `NonLinearLeastSquares.m` that you can download from the homepage is an implementation of such an algorithm. The function also gives an illustration of how to handle the data content in the `GPSdata.mat` file.

[1] The position of the satellite $\mathbf{p}_k^{(i)}$ is known by the receiver, but the satellites from which the GPS receiver can receive signals varies with time and the position of the receiver.

Download the material from the homepage and do the following:

i) Plot the trajectory of the car and the position estimates calculated by the GPS receiver if the nonlinear least squares algorithm is used.

ii) A survey of motion dynamic models for target tracking is presented in [1]. Combine the constant velocity or constant acceleration model described on pages 1333-1337 in the paper with the clock model in (2) and implement a Kalman filter that tracks the motion of the car.

   a) Specify the state-space and observation equations that you use in your filter.

   b) Is the stochastic process described by the state-space equations a stationary process? Does it matter?

   c) What happens if the GPS receiver stops receiving signals from the satellites? How will that affect the state covariance?

iii) Plot the position estimation errors (each coordinate axis in a separate plot) and clock offset and clock drift estimation errors. Include also the 3σ confidence intervals based on the state covariance output by the filters. Further, in the plots with the position estimation errors, also included the results of the non-linear least squares algorithm.

   a) What happens when the covariance of the process noise of the motion dynamic model becomes very small?

   b) What happens when the covariance of the process noise of the motion dynamic model becomes very large?

   c) Discuss how the innovation and the 3σ confidence intervals can be used to tune the filter?

   d) Throughout the course we have studied linear estimators (filters) that are optimal in the mean square error sense. Name at least two reasons why the implemented filter may not be optimal?

You may work 1 or 2 people in each group. Please write a technical report (not a thesis!) per group illustrating your examples and documenting your conclusions. The report and numerical results should be individually prepared by each group. Write the report in the format of a conference paper, for example using the style file and templates available at

`https://2020.ieeeicassp.org/authors/paper-kit/`.

The report should be submitted through the course web.

The `GPSdata.mat` contains the following variables.

`gps_data` : $1 \times M$ array of struct with the fields:

- `Satellite` : Name of satellite
- `Satellite_Position_NED` : Position of the satellite
- `PseudoRange` : Measured pseudo ranges

`ref_data` Struct with the fields:

- `traj_ned` : True trajectory of the vehicle
- `x_clk` : $[\Delta t_k, \dot{\Delta} t_k]$ True clock state (offset, drift)
- `Ts` : Sample period
- `PSD_clk` : $[S_\phi, S_f]$ Power spectral density of clock process noise
- `s2r` : σ_r^2 variance of range measurement error
- `c` : speed of light

References

[1] Li, X.R.; Jilkov, V.P., "Survey of maneuvering target tracking. Part I. Dynamic models, *IEEE Transactions on Aerospace and Electronic Systems, vol.39, no.4, pp.1333-1364, Oct. 2003*