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Graph search

Searching a graph can have many aims:

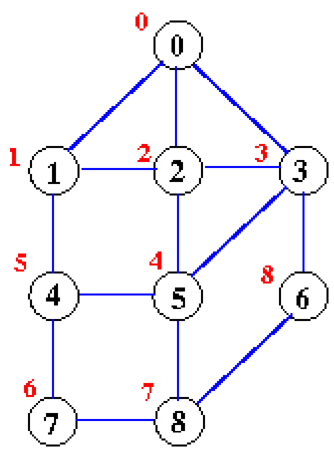
- can I reach every vertex in the graph (is it connected)?
- is one vertex reachable starting from some other vertex?
- what is the shortest path from vertex *v* to *w*?
- which vertices are reachable from a vertex? (transitive closure)
- is there a cycle that passes through all the graph? (*tour*)
- is there a tree that links all vertices? (*spanning tree*)
    - what is the *minimum* spanning tree?
- are two graphs "equivalent"? (*isomorphism*)

A search is almost never 'random': it uses an underlying strategy:

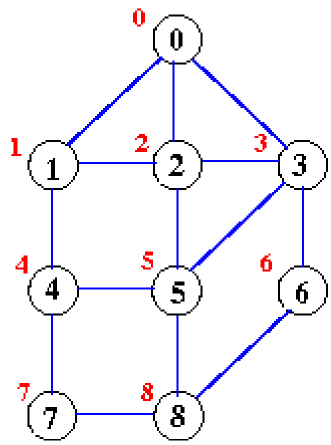
- depth-first search DFS
- breadth-first search BFS

Breadth-first versus Depth-first search

Example:



Depth–First Search



Breadth–First Search

Order is given by the 'red' labels

- in this example the label ordering is breadth-first (layer by layer)

DFS descends by selecting the first available unvisited node

- select 0
- adjacent {1,2,3}
    - select 1
    - adjacent {2, 4}
        - select 2
        - adjacent {3, 5}
            - select 3
            - adjacent {5, 6}
                - select 5
                - adjacent {4, 8}
                    - select 4
                    - adjacent {7}
                        - select 7
                        - adjacent {8}
                            - select 8
                            - adjacent {6}
                                - select 6
                                - adjacent {} no sites left unvisited

BFS descends by systematically visiting the nodes in order of level

- select 0
- adjacent {1,2,3}
    - select 1
    - adjacent {4}
    - select 2
    - adjacent {5}
    - select 3
    - adjacent {6}
        - select 4
        - adjacent {7}
        - select 5
        - adjacent {8}

        - select 6
        - adjacent {}
            - select 7
            - adjacent {}
            - select 8
            - adjacent {} no sites left unvisited

These two 'strategies' actually use the same algorithm. They differ only in their use of data structure:

- DFS uses a stack
- BFS uses a queue

Here is the pseudo-algorithm for **Depth/Breadth**-first search:

```
push the root node onto a stack/queue
while (stack/queue is not empty) {
    pop a node from the stack/queue
    if (node is a goal node)
        return 'success'
    push all children of node onto the stack/queue
}
return 'failure'
```

If the aim is not to find a goal node, but to search the whole graph:

- leave out the conditional 'return' (i.e if (node is ...)
- return when complete

Stack-Based Depth-First Search

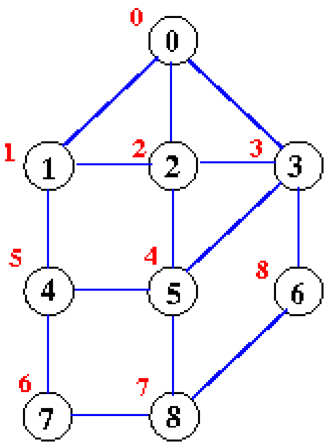
When searching we need to remember which nodes we've *visited*:

- to avoid cycles
- to make sure every node gets visited

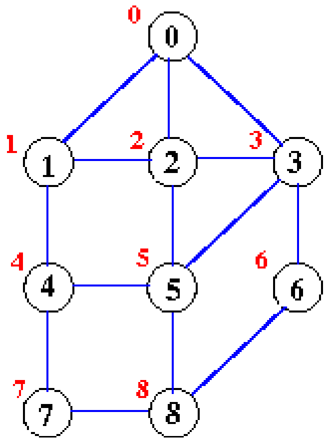
Generally an array **visited[0 .. numVertices-1]** is used

- array indices correspond to vertices
- initialise all elements to -1, meaning unvisited
- when a vertex is visited, the index is set to its 'visit order' number
    - this is simply a 'count' that gets incremented each time a new node is visited

For example, here is the earlier graph again



Depth–First Search



Breadth–First Search

The *visited* array starts as {-1,-1,-1,-1,-1,-1,-1,-1,-1}

We select the root **0** first

adjacent	**visit**	**resulting visited array**
any node	**0**	{ **0**,-1,-1,-1,-1,-1,-1,-1,-1}
1 2 3	**1**	{ 0, **1**,-1,-1,-1,-1,-1,-1,-1}
0 2 4	**2**	{ 0, 1, **2**,-1,-1,-1,-1,-1,-1}
0 1 3 5	**3**	{ 0, 1, 2, **3**,-1,-1,-1,-1,-1}
0 2 5 6	**5**	{ 0, 1, 2, 3,-1, **4**,-1,-1,-1}
2 3 4 8	**4**	{ 0, 1, 2, 3, **5**, 4,-1,-1,-1}
1 5 7	**7**	{ 0, 1, 2, 3, 5, 4,-1, **6**,-1}
4 8	**8**	{ 0, 1, 2, 3, 5, 4,-1, 6, **7**}
5 6 7	**6**	{ 0, 1, 2, 3, 5, 4, **8**, 6, 7}

Let's try a different starting vertex: this time start at vertex **5**:

adjacent	**visit**	**resulting visited array**
any node	**5**	{-1,-1,-1,-1,-1, **0**,-1,-1,-1}
2 3 4 8	**2**	{-1,-1, **1**,-1,-1, 0,-1,-1,-1}
0 1 3 5	**0**	{ **2**,-1, 1,-1,-1, 0,-1,-1,-1}
1 2 3	**1**	{ 2, **3**, 1,-1,-1, 0,-1,-1,-1}
0 2 4	**4**	{ 2, 3, 1,-1, **4**, 0,-1,-1,-1}
1 5 7	**7**	{ 2, 3, 1,-1, 4, 0,-1, **5**,-1}
4 8	**8**	{ 2, 3, 1,-1, 4, 0,-1, 5, **6**}
5 6 7	**6**	{ 2, 3, 1,-1, 4, 0, **7**, 5, 6}
3 8	**3**	{ 2, 3, 1, **8**, 4, 0, 7, 5, 6}

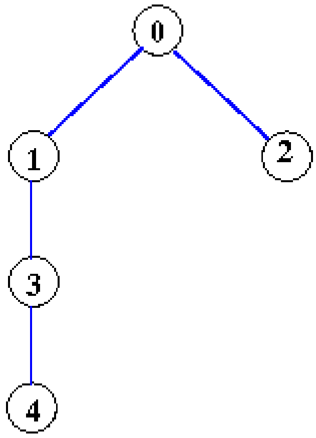
The array *visited[]* here **is the depth-first order**

- It says: { 2^{nd}, 3^{rd}, 1^{st}, 8^{th}, 4^{th}, 0^{th}, 7^{th}, 5^{th}, 6^{th}}

```
The visited array indicates the order of the search.
```

Can anything go wrong during the traversal?

- *Yes, we can hit a deadend!*



choice	visit	resulting visited array
any node	**0**	{**0**,-1,-1,-1,-1,}
1 *2*	**1**	{ 0, **1**,-1,-1,-1}
0 3	**3**	{ 0, 1,-1, **2**,-1}
1 4	**4**	{ 0, 1,-1, 2, **3**}
	finished?	

- 4 is a leaf node, we can go no further
- **there is still an unvisited vertex in the array**

How do we 'find' it?

- we need to **backtrack**
  - we go back to vertex 0, and then visit vertex 2
    - this is also a leaf node
    - ... but all nodes have been visited, so we are really finished this time

Final DFS path is visited = {0, 1, 4, 2, 3}

So we cannot expect DFS to visit every vertex in a single forward traversal

- we sometimes need to *backtrack*

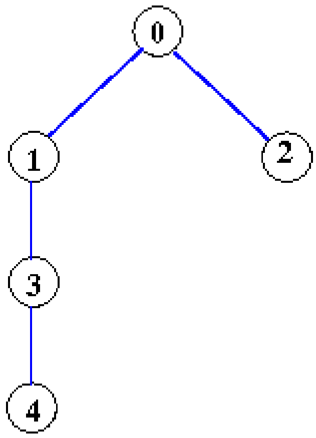
But how do we backtrack?

- we use a stack!
  - vertices are pushed onto the stack when we have 1 or more adjacent vertices to visit
  - to actually visit a vertex, we simply pop it from the stack
- *only when the stack is empty have we visited everyone!*

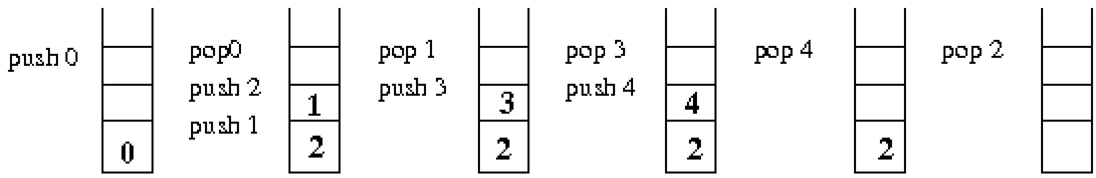
Using a stack in DFS means:

- when we *visit*, we *pop* the next vertex off the stack
- after a visit, we *push* the adjacent vertices onto the stack
- when we land on a leaf node, we cannot *push* any nodes onto the stack
  - we then *pop* a vertex instead
    - ... this is backtracking (to an earlier vertex)
- only when the stack is empty have we visited every vertex

Consider the above graph again:



The following stack operations are carried out:



dfsStack.c (calls dfsQuack() connected graphs)

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```
// dfsStack.c: traverse a graph using DFS and stacking (graph may be disconnected)
// Compile using:
//       dcc -o dfsStack dfsStack.c IOmem.c GraphAM.c Quack.c
//
#include <stdio.h>
#include <stdlib.h>
#include "Graph.h"
#include "Quack.h"
#include "IOmem.h"

#define STARTVERTEX 0   // start the depth-first search at this vertex

void dfsQuack(Graph, Vertex, int);

int main (void) {
```

```
16     int numV;
17     if ((numV = readNumV()) > 0) {
18         Graph g = newGraph(numV);
19         if (readBuildGraph(g)) {
20             showGraph(g);
21             dfsQuack(g, STARTVERTEX, numV);
22         }
23         g = freeGraph(g);
24         g = NULL;
25     }
26     else {
27         printf("Error in reading #number\n");
28         return EXIT_FAILURE;
29     }
30     return EXIT_SUCCESS;
31 }
32
33 //HANDLES CONNECTED GRAPHS ONLY
34 void dfsQuack(Graph g, Vertex v, int numV) {
35    int *visited = mallocArray(numV);
36    Quack s = createQuack();
37    push(v, s);
38    showQuack(s);
39    int order = 0;
40    while (!isEmptyQuack(s)) {
41       v = pop(s);
42       if (visited[v] == UNVISITED) {   // we visit only unvisited vertices
43          printArray("Visited: ", visited, numV);
44          visited[v] = order++;
45          for (Vertex w = numV - 1; w >= 0; w--) { // push adjacent vertices
46             if (isEdge(newEdge(v,w), g)) {        // ... in reverse order
47                push (w, s);                       // ... onto the stack
48             }
49          }
50       }
51       showQuack(s);
52    }
53    printArray("Visited: ", visited, numV);
54    free(visited);
55    return;
56 }
```

The loop formed by lines 40-52 eventually empty the stack

- ... suggesting that the traversal is complete ...
    - ... and that all nodes have been visited

This may not be true.

If the graph is disconnected then *isEdge()* on line 46 is insufficient ...

- ... there will be no edges to a disconnected part of the graph

We need to also check that every vertex has been visited before we return from the function

This involves:

1. looking for a vertex in *visited[]* that is -1
   - this vertex has not yet been visited before
2. *push* this vertex onto the stack
3. start a new traversal with this node as 'root'

The code of the 'disconnected version' of *dfsQuack()* is as follows:

dfsQuack() (disconnected graphs)

切换行号显示

```
// HANDLES DISCONNECTED GRAPHS
void dfsQuack(Graph g, Vertex v, int numV) {
   int *visited = mallocArray(numV);
   Quack s = createQuack();
   push(v, s);
   showQuack(s);
   int order = 0;
   int allVis = 0;
   while (!allVis) {       // as long as there are unvisited vertices
      while (!isEmptyQuack(s)) {
         v = pop(s);
         if (visited[v] == UNVISITED) {
            printArray("Visited: ", visited, numV);
            //printf("visited[%d] = %d\n", v, order);
            visited[v] = order++;
            for (Vertex w = numV - 1; w >= 0; w--) {
               if (isEdge(newEdge(v,w), g)) {
                  push (w, s);
               }
            }
         }
         showQuack(s);
      }
      // stack is empty, but are we finished?
      allVis = 1;
      for (Vertex w = 0; w < numV && allVis; w++) {
         if (visited[w] == UNVISITED) {
            printf("Graph is DISCONNECTED\n"); // debug
            allVis = 0;     // found an unvisited vertex
            push(w, s);     // push vertex onto stack
            showQuack(s);
         }
      }
```

```
    }
    printArray("Visited: ", visited, numV);
    free(visited);
    return;
}
```

The helper ADT IOmem

Input/output and memory management is controlled by an ADT called **IOmem**. It's interface is:

切换行号显示

```
// IOmem.h
// Interface to IOmem ADT that reads input data, builds and print graphs and manages memory.

#include <stdio.h>
#include <stdlib.h>

int readNumV();              // read an int (numV) from stdin
int readBuildGraph(Graph);   // read int pairs from stdin
int* mallocArray(int);       // malloc an array of length int * sizeof(int)
void printArray(char *, int *, int); // print an int array of length int

```

This ADT allows the amount of graph search code to be kept minimal.

We can now compile the graph search algorithm with the *Graph*, *Quack* and *IOmem* ADTs:

- we can either the *GraphAM* or *GraphAL* ADTs

  ```
  dcc dfsStack.c IOmem.c GraphAM.c Quack.c
  ```

  or

  ```
  dcc dfsStack.c IOmem.c GraphAL.c Quack.c
  ```

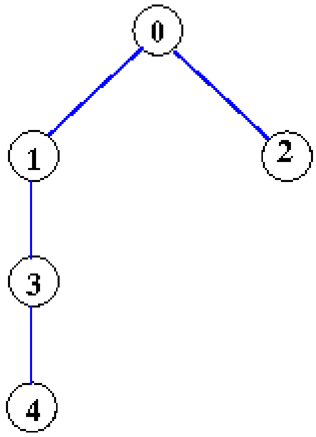
- also a choice between the array-based ADT *Quack* and linked-list version *QuackLL*
- in total, 4 combinations of *Graph* and *Quack* ADTs possible!

Test 1 of Stack-Based Depth-First Search

The input file we use is:

```
#5
0 1 0 2 1 3 3 4
```

which corresponds to the simple graph we saw before:



Executing *a.out* using this input file results in the following:

```
V=5, E=4
<0 1> <0 2>
<1 0> <1 3>
<2 0>
<3 1> <3 4>
<4 3>
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1}
Quack: <<1, 2>>
Visited: {0, -1, -1, -1, -1}
Quack: <<0, 3, 2>>
Quack: <<3, 2>>
Visited: {0, 1, -1, -1, -1}
Quack: <<1, 4, 2>>
Quack: <<4, 2>>
Visited: {0, 1, -1, 2, -1}
Quack: <<3, 2>>
Quack: <<2>>
Visited: {0, 1, -1, 2, 3}
Quack: <<0>>
Quack: << >>
Visited: {0, 1, 4, 2, 3}
```

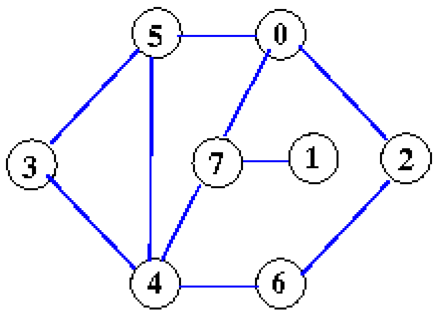
Here we see:

- the starting vertex 0 is pushed
- 0 is popped and its neighbours 1 and 2 are pushed
  - visited[0] = **0**
- 1 is popped and its neighbours 0 and 3 are pushed
  - visited[1] = **1**
- 0 is popped and ignored as it is in array *visited*
- 3 is popped and its neighbours 1 and 4 are pushed
  - visited[3] = **2**
- 1 is popped and is ignored
- 4 is popped and its neighbour 3 is pushed

    - visited[4] = **3**
- 3 is popped and is ignored
- 2 is popped and its neighbour 0 is pushed
    - visited[2] = **4**
- 0 is popped
- *quack is empty*

Test 2 of Stack-Based Depth-First Search

What about a more substantial graph:



It is represented by the input data:

```
#8
0 2 0 5 0 7 2 6 1 7 4 7 4 6 4 3 3 5 4 5
```

If we want to do a DFS starting from vertex 0 (remember: a *#define* in the code):

```
V=8, E=10
<0 2> <0 5> <0 7>
<1 7>
<2 0> <2 6>
<3 4> <3 5>
<4 3> <4 5> <4 6> <4 7>
<5 0> <5 3> <5 4>
<6 2> <6 4>
<7 0> <7 1> <7 4>
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1, -1, -1, -1}
Quack: <<2, 5, 7>>
Visited: {0, -1, -1, -1, -1, -1, -1, -1}
Quack: <<0, 6, 5, 7>>
Quack: <<6, 5, 7>>
Visited: {0, -1, 1, -1, -1, -1, -1, -1}
Quack: <<2, 4, 5, 7>>
Quack: <<4, 5, 7>>
Visited: {0, -1, 1, -1, -1, -1, 2, -1}
Quack: <<3, 5, 6, 7, 5, 7>>
Visited: {0, -1, 1, -1, 3, -1, 2, -1}
Quack: <<4, 5, 5, 6, 7, 5, 7>>
Quack: <<5, 5, 6, 7, 5, 7>>
Visited: {0, -1, 1, 4, 3, -1, 2, -1}
Quack: <<0, 3, 4, 5, 6, 7, 5, 7>>
```

```
Quack: <<3, 4, 5, 6, 7, 5, 7>>
Quack: <<4, 5, 6, 7, 5, 7>>
Quack: <<5, 6, 7, 5, 7>>
Quack: <<6, 7, 5, 7>>
Quack: <<7, 5, 7>>
Visited: {0, -1, 1, 4, 3, 5, 2, -1}
Quack: <<0, 1, 4, 5, 7>>
Quack: <<1, 4, 5, 7>>
Visited: {0, -1, 1, 4, 3, 5, 2, 6}
Quack: <<7, 4, 5, 7>>
Quack: <<4, 5, 7>>
Quack: <<5, 7>>
Quack: <<7>>
Quack: << >>
Visited: {0, 7, 1, 4, 3, 5, 2, 6}
```

Test 3 of Stack-Based Depth-First Search

Performance

- number of pushes and pops
  - should be the same (stack is empty at the end)
- number of pushes of a vertex *v* = vertex degree of *v*
- total number of pushes
  - = sum of all the vertex degrees of vertices v in the graph

    ```
      The sum of vertex degrees is equal to twice the
    number of edges.
    ```

- this means the complexity is linear in the number of edges, *O(E)*
  - **what does this mean? ...**
  - *how many edges are there?*
    - the worst case is a dense graph: $E = V*(V-1)/2$
    - so the complexity is quadratic in *V*: i.e. $O(V^2)$
    - if it is sparse, then it will be less than quadratic
- often said that DFS is *linear in the size of the graph* ...
  - ... where 'size' is the number of edges
  - ... which is another way of saying *quadratic in the number of vertices*

Recursive Depth-First Search

DFS above used our own stack to 'remember' which path it was traversing and do backtracking.

The system also has a stack, called a *call stack*, which is used to execute functions

- a function call causes a *function frame* to be pushed onto the call stack
  - ... upon a function return, the *frame* is popped off the call stack

This works even for recursive functions of course.

The call stack can be used instead of the 'stack' ADT we used above

- so when we compile we do not need the ADT quack

It works by recursion:

- a function *dfsR()* calls itself recursively ...
- in essence adjacent vertices are being *pushed* onto the system 'call' stack
  - the *for-loop* in *dfsR()* is over all all unvisited adjacent vertices
    - in the *for-loop*, *dfsR()* is called for every vertex
    - these calls *stack up* as you descend down the tree

dfsRec.c (disconnected graphs)

切换行号显示

```
// dfsRec.c: traverse a graph using DFS (graph may be disconnected)
// Compile using:
//       dcc -o dfsRec dfsRec.c IOmem.c GraphAM.c
//
#include <stdio.h>
#include <stdlib.h>
#include "Graph.h"
#include "IOmem.h"

void dfs(Graph, Vertex, int);
void dfsR(Graph, Vertex, int, int *, int *);

int main(void) {
   int numV;
   if ((numV = readNumV()) >= 0) {
       Graph g = newGraph(numV);
       if (readBuildGraph(g)) {
          showGraph(g);
          dfs(g, 0, numV); // DEPTH-FIRST SEARCH FROM NODE 0
       }
       g = freeGraph(g);
       g = NULL;
   }
   else {
      return EXIT_FAILURE;
   }
   return EXIT_SUCCESS;
}

void dfs(Graph g, Vertex v, int numV) { // a 'wrapper' for recursive dfs
   int *visited = mallocArray(numV);  // ... handles disconnected graphs
   int order = 0;
```

```
    Vertex newv = v;                        // this is the starting vertex
    int allVis = 0;                         // assume not all visited
    while (!allVis) {                       // as long as there are vertices
       dfsR(g, newv, numV, &order, visited);
       allVis = 1;                          // are all visited now?
       for (Vertex w = 0; w < numV && allVis; w++) { // look for more
          if (visited[w] == UNVISITED) {
             printf("Graph is disconnected\n"); // debug
             allVis = 0;                    // found an unvisited vertex
             newv = w;                      // next loop dfsR this vertex
          }
       }
    }
    printArray("Visited: ", visited, numV);
    free(visited);
    return;
 }

 void dfsR(Graph g, Vertex v, int numV, int *order, int *visited) {
    visited[v] = *order;                     // records the order of visit
    printf("Visiting vertex %d in order %d\n", v, *order);
    *order = *order+1;
    for (Vertex w = 0; w < numV; w++) {
       if (isEdge(newEdge(v,w), g) && visited[w]==UNVISITED) {
          dfsR(g, w, numV, order, visited);
       }
    }
    return;
 }
```

Here the function *dfs()* is called by *main()*

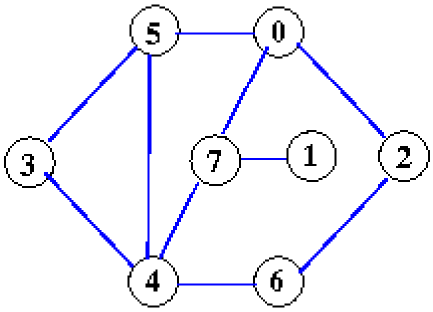
- this function is a *wrapper*
- it does 'housekeeping' (initialising *order* and the array *visited*)
- it calls the recursive function *dfsR()*

Remember: in the stack version the main function called *dfsQuack()*

- it does 'housekeeping' (initialising *order* and the array *visited*)
- *pops* and *pushes* off/on the quack until the quack is empty

Testing recursive DFS

Let's run this recursive DFS on the graph we had above:



Remember, it is represented by the input data:

```
#8
0 2 0 5 0 7 2 6 1 7 4 7 4 6 4 3 3 5 4 5
```

Compiling:

```
dcc -o dfsRec dfsRec.c IOmem.c GraphAM.c
```

notice, no *Quack* ADT, and executing

```
V=8, E=10
<0 2> <0 5> <0 7>
<1 7>
<2 0> <2 6>
<3 4> <3 5>
<4 3> <4 5> <4 6> <4 7>
<5 0> <5 3> <5 4>
<6 2> <6 4>
<7 0> <7 1> <7 4>
Visiting vertex 0 in order 0
Visiting vertex 2 in order 1
Visiting vertex 6 in order 2
Visiting vertex 4 in order 3
Visiting vertex 3 in order 4
Visiting vertex 5 in order 5
Visiting vertex 7 in order 6
Visiting vertex 1 in order 7
Visited: {0, 7, 1, 4, 3, 5, 2, 6}
```

Comparing that with the stack version, the last lines shown below:

```
.
.
.
Visited: {0, -1, 1, 4, 3, 5, 2, 6}
Quack: <<7, 4, 5, 7>>
Quack: <<4, 5, 7>>
Quack: <<5, 7>>
Quack: <<7>>
Quack: << >>
```

```
Visited: {0, 7, 1, 4, 3, 5, 2, 6}
```

In summary:

- we've seen 2 versions of depth-first search:
  - an explicit stack version *dfsStack.c* that uses a *Quack* ADT
  - a call-stack version *dfsRec.c* that uses recursion

You could argue that the stack version:

- requires much less system resources (no recursion)
- does backtracking in an *iterative* manner, so will be much faster

Globally visited

Crucial in both versions is the array *visited[]* and integer variable *order*

- *visited[]* records unvisited vertices and the *order* of visiting
- stop cycles occurring (remember, we are dealing with graphs)

Almost always implemented as global variables.

- For example, in recursive DFS, *dfsRec.c*
  - *visited[]* and *order* are initialised in the 'wrapper'
  - if global variables are used, they

GraphSearchDFSglobal

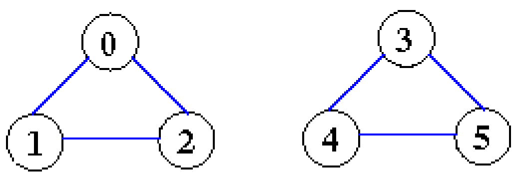
Testing a disconnected graph

We saw in *dfsRec.c* that the recursion handles disconnected graphs

Let's check that.

```
#6
0 1 0 2 1 2 3 4 3 5 4 5
```

corresponding to:



and assuming the starting vertex is 0, then *dfsRec* produces:

```
V=6, E=6
<0 1> <0 2>
<1 0> <1 2>
<2 0> <2 1>
<3 4> <3 5>
```

```
<4 3> <4 5>
<5 3> <5 4>
Visiting vertex 0 in order 0
Visiting vertex 1 in order 1
Visiting vertex 2 in order 2
Graph is disconnected
Visiting vertex 3 in order 3
Visiting vertex 4 in order 4
Visiting vertex 5 in order 5
Visited: {0, 1, 2, 3, 4, 5}
```

Notice:

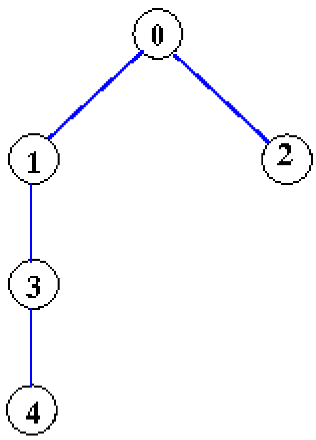
- the first (sub) graph's DFS search begins at vertex 0
- the second (sub) graph's DFS search begins at vertex 3

Breadth First Search

All adjacent vertices are visited before moving to another vertex

- each level of vertices is visited before the next level's vertices are considered

For example:



1. visit vertex 0
2. visit vertex 1 and 2
3. visit vertex 3
4. visit vertex 4

In essence, the vertices are processed ***in order*** (top to bottom, left to right)

- DFS used a stack:
  - we pushed all the adjacent vertices of a vertex onto a stack
  - so we would remember the vertices we need to 'still visit'
- BFS instead uses a queue:
  - we push all the adjacent vertices of a vertex onto a queue (not a stack)
  - but we visit them in the order they occur (as we must in a queue)
- the change in the *quack* version is trivial:
  - just 4 changes

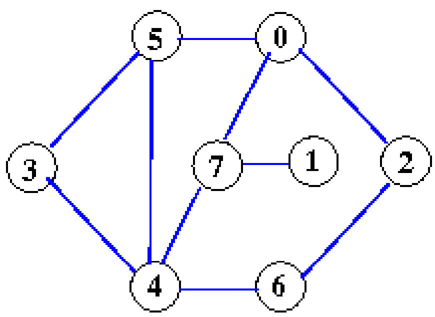
bfsQuack() (connected)

切换行号显示

```
void bfsQuack(Graph g, Vertex v, int numV) { //name change
   int *visited = mallocArray(numV);
   Quack q = createQuack();
   qush(v, q);                                //qush, not push
   showQuack(q);
   int order = 0;
   while (!isEmptyQuack(q)) {
      v = pop(q);
      if (visited[v] == UNVISITED) {
         printf("\t\t\t ... visit %d\n", v);
         visited[v] = order++;
         Vertex w;
         for (w = 0; w < numV; w++) {     //vertex order
            if (isEdge(newEdge(v,w), g)) {
               qush(w, q);                //qush, not push
            }
         }
      }
      showQuack(q);
   }
   printArray("Visited: ", visited, numV);
   free(visited);
   makeEmptyQuack(q);
   return;
}
```

What about the graph we considered above for DFS represented by the input data

```
#8
0 2 0 5 0 7 2 6 1 7 4 7 4 6 4 3 3 5 4 5
```



Starting at vertex 0, what did DFS do:

- *Visited[]* = {0, 7, 1, 4, 3, 5, 2, 6}
- corresponds to the vertices: 0 2 6 4 3 5 7 1

What does *bfs()* do:

```
Quack: <<0>>                           <== start node
Visited: {-1, -1, -1, -1, -1, -1, -1, -1}
Quack: <<2, 5, 7>>                     <== 2,5,7 pushed
Visited: {0, -1, -1, -1, -1, -1, -1, -1}
Quack: <<5, 7, 0, 6>>                  <== 0,6 quashed
Visited: {0, -1, 1, -1, -1, -1, -1, -1}
Quack: <<7, 0, 6, 0, 3, 4>>            <== 0,3,4 pushed
Visited: {0, -1, 1, -1, -1, 2, -1, -1}
Quack: <<0, 6, 0, 3, 4, 0, 1, 4>>      <== 0,1,4 pushed
Quack: <<6, 0, 3, 4, 0, 1, 4>>
Visited: {0, -1, 1, -1, -1, 2, -1, 3}
Quack: <<0, 3, 4, 0, 1, 4, 2, 4>>      <== etc
Quack: <<3, 4, 0, 1, 4, 2, 4>>
Visited: {0, -1, 1, -1, -1, 2, 4, 3}
Quack: <<4, 0, 1, 4, 2, 4, 4, 5>>
Visited: {0, -1, 1, 5, -1, 2, 4, 3}
Quack: <<0, 1, 4, 2, 4, 4, 5, 3, 5, 6, 7>>
Quack: <<1, 4, 2, 4, 4, 5, 3, 5, 6, 7>>
Visited: {0, -1, 1, 5, 6, 2, 4, 3}
Quack: <<4, 2, 4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<2, 4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<4, 5, 3, 5, 6, 7, 7>>
Quack: <<5, 3, 5, 6, 7, 7>>
Quack: <<3, 5, 6, 7, 7>>
Quack: <<5, 6, 7, 7>>
Quack: <<6, 7, 7>>
Quack: <<7, 7>>
Quack: <<7>>
Quack: << >>
Visited: {0, 7, 1, 5, 6, 2, 4, 3}
```

In the first few lines of this output we see:

- vertex 0 is qushed, then popped
- vertices 2, 5, 7 are qushed, then successively
  - 2 is popped
  - 5 is popped
  - 7 is popped
- 'qushing' means everything gets added to the end of the quack
- the order that vertices are visited is:
  - 0 2 5 7 6 3 4 1
  - note:
    - 0 is a level-0 vertex
    - 2 5 7 are level-1
    - 6 3 4 1 are level-2
    - no vertex is more than a path length of 2 away from the starting vertex

GraphSearch (2019-07-24 12:45:21由AlbertNymeyer编辑)