目录

Graph search

Searching a graph can have many aims:

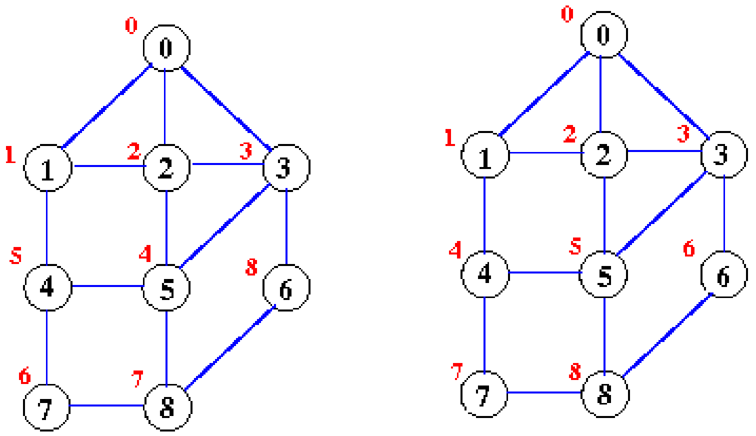
- can I reach every vertex in the graph (is it connected)?
- is one vertex reachable starting from some other vertex?
- what is the shortest path from vertex *v* to *w*?
- which vertices are reachable from a vertex? (transitive closure)
- is there a cycle that passes through all the graph? (*tour*)
- is there a tree that links all vertices? (*spanning tree*)
  - what is the *minimum* spanning tree?
- are two graphs "equivalent"? (*isomorphism*)

A search is almost never 'random': it uses an underlying strategy:

- depth-first search DFS
- breadth-first search BFS

breadth first and depth first

Example:



Depth–First Search   Breadth–First Search

Order is given by the 'red' labels

- in this example the label ordering is breadth-first (layer by layer)

DFS descends by selecting the first available unvisited node

- select 0
- connect {1,2,3}
    - select 1
    - connect {2, 4}
        - select 2
        - connect {3, 5}
            - select 3
            - connect {5, 6}
                - select 5
                - connect {4, 8}
                    - select 4
                    - connect {7}
                        - select 7
                        - connect {8}
                            - select 8
                            - connect {6}
                                - select 6
                                - connect {} no sites left unvisited

BFS descends by systematically visiting the nodes in order of level

- select 0
- connect {1,2,3}
    - select 1
    - connect {4}
    - select 2
    - connect {5}
    - select 3
    - connect {6}
        - select 4
        - connect {7}
        - select 5
        - connect {8}
            - select 6
            - connect {}

  - select 7
  - connect {}
  - select 8
  - connect {} no sites left unvisited

These two 'strategies' actually use the same algorithm. They differ only in their use of data structure:

- DFS uses a stack
- BFS uses a queue

Here is the pseudo-algorithm for **Depth/Breadth**-first search:

```
push the root node onto a stack/queue
while (stack/queue is not empty) {
    remove a node from the stack/queue
    if (node is a goal node)
        return 'success'
    push all children of node onto the stack/queue
}
return 'failure'
```

Depth-First Search, using a stack

When searching we need to remember which nodes we've *visited*:

- to avoid cycles
- to make sure every node gets visited

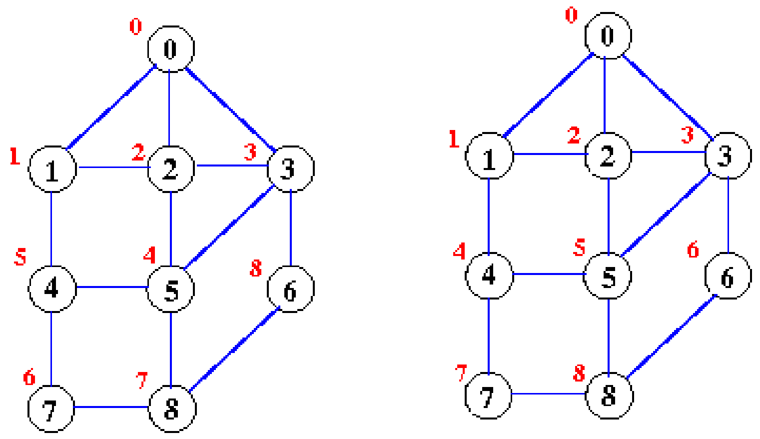
Generally a global array variable **visited[0 .. numVertices-1]** is used

- array indices correspond to vertices
- initialise all elements to -1, meaning unvisited
- when a vertex is visited, the index is set to its 'visit order' number
  - this is simply a 'count' that gets incremented each time a new node is visited

Many strategies are possible

- obvious strategy: select the smallest unvisited vertex
- less obvious: select an arbitrary unvisited vertex

For example, here is the earlier graph again



Depth-First Search   Breadth-First Search

The *visited* array starts as {-1,-1,-1,-1,-1,-1,-1,-1,-1}

We select the root **0** first

choice	visit	resulting visited array
any node	**0**	{ **0**,-1,-1,-1,-1,-1,-1,-1,-1}
1 2 3	**1**	{ 0, **1**,-1,-1,-1,-1,-1,-1,-1}
0 2 4	**2**	{ 0, 1, **2**,-1,-1,-1,-1,-1,-1}
0 1 3 5	**3**	{ 0, 1, 2, **3**,-1,-1,-1,-1,-1}
0 2 5 6	**5**	{ 0, 1, 2, 3,-1, **4**,-1,-1,-1}
2 4 6 8	**4**	{ 0, 1, 2, 3, **5**, 4,-1,-1,-1}
1 5 7	**7**	{ 0, 1, 2, 3, 5, 4,-1, **6**,-1}
4 8	**8**	{ 0, 1, 2, 3, 5, 4,-1, 6, **7**}
6 7	**6**	{ 0, 1, 2, 3, 5, 4, **8**, 6, 7}

Let's try a different starting vertex: this time start at vertex **5**:

choice	visit	resulting visited array
any node	**5**	{-1,-1,-1,-1,-1, **0**,-1,-1,-1}
2 3 4 8	**2**	{-1,-1, **1**,-1,-1, 0,-1,-1,-1}
0 1 3 5	**0**	{ **2**,-1, 1,-1,-1, 0,-1,-1,-1}
1 2 3	**1**	{ 2, **3**, 1,-1,-1, 0,-1,-1,-1}
0 2 4	**4**	{ 2, 3, 1,-1, **4**, 0,-1,-1,-1}
1 5 7	**7**	{ 2, 3, 1,-1, 4, 0,-1, **5**,-1}
4 8	**8**	{ 2, 3, 1,-1, 4, 0,-1, 5, **6**}
5 6 7	**6**	{ 2, 3, 1,-1, 4, 0, **7**, 5, 6}
3 8	**3**	{ 2, 3, 1, **8**, 4, 0, 7, 5, 6}

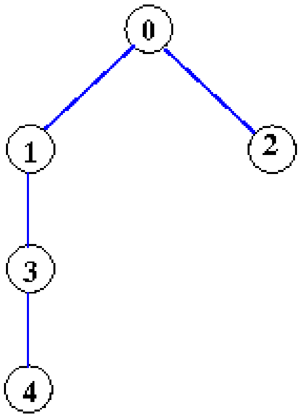
The array *visited[]* **is the depth-first order**. It says:

- visited[5] = 0^{th}
- visited[2] = 1^{st}
- visited[0] = 2^{nd}
- visited[1] = 3^{rd}
- visited[4] = 4^{th}
- visited[7] = 5^{th}
- visited[8] = 6^{th}
- visited[6] = 7^{th}
- visited[3] = 8^{th}

```
The visited array indicates the order of the search.
```

Can anything go wrong during the traversal?

- *Yes, we can hit a deadend!*



choice	visit	resulting visited array
any node	**0**	{**0**,-1,-1,-1,-1,}
1 *2*	**1**	{ 0, **1**,-1,-1,-1}
0 3	**3**	{ 0, 1,-1, **2**,-1}
1 4	**4**	{ 0, 1,-1, 2, **3**}
	deadend	

- **there is still an unvisited vertex in the array**

How do we 'find' it?

- we need to **backtrack**
  - we go back to vertex 0, and then visit vertex 1
    - again a *deadend*
    - ... but all nodes have been visited, so we are finished
  - final DFS path is visited = {0, 1, 4, 2, 3}

So we cannot expect DFS to visit <u>every</u> vertex in a <u>single forward traversal</u>

- we sometimes need to *backtrack*

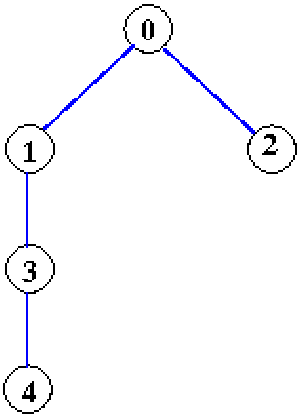
But how do we backtrack?

- we use a stack!
  - vertices are pushed onto the stack when we have 1 or more adjacent vertices to visit
  - to actually visit a vertex, we simply pop it from the stack
- *only when the stack is empty have we visited everyone!*

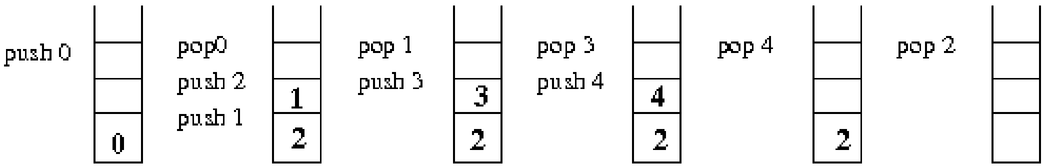
Using a stack in DFS means:

- when we *visit*, we *pop* the next vertex off the stack
- after a visit, we *push* the adjacent vertices onto the stack
- a deadend occurs when there are no more vertices, so no *push* is possible
  - we then *pop* a vertex
    - ... this vertex we saw earlier and need to visit
- only when the stack is empty have we visited <u>every</u> vertex

Consider the graph with the deadend again



The following stack operations are carried out:



Code:

切换行号显示

```
// dfsquack.c: traverse a graph using DFS and a stack implementation
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>
#include "Graph.h"
#include "Quack.h"

void dfs(Graph, Vertex, int);

#define WHITESPACE 100

int readNumV(void) { // returns the number of vertices numV or -1
   int numV;
   char w[WHITESPACE];
   scanf("%[ \t\n]s", w);  // skip leading whitespace
   if ((getchar() != '#') ||
       (scanf("%d", &numV) != 1)) {
       fprintf(stderr, "missing number (of vertices)\n");
       return -1;
   }
   return numV;
}

int readGraph(int numV, Graph g) { // reads number-number pairs until EOF
   int success = true;             // returns true if no error
   int v1, v2;
```

数据输入类型如下：
#23

output:23

用来存储
空格、制表符、回车
所组成的空白部分

如果上一个字符是 #
则继续读取后面的数字部分
正确读入就存储
否则返回错误

接下来读入下一个字符
如果是 # 就继续
否则就输出错误

读取成对的edge
直到文件最后

```
   while (scanf("%d %d", &v1, &v2) != EOF && success) {
       if (v1 < 0 || v1 >= numV || v2 < 0 || v2 >= numV) {
          fprintf(stderr, "unable to read edge\n");
          success = false;
       }
       else {
          insertE(g, newE(v1, v2));
       }
   }
   return success;
}

void dfs(Graph g, Vertex v, int numV) {
```

数字的值没有问题
就插入edge

g：图
v：第一个读取的顶点
numV：顶点的个数

```
   int *mallocArray(int numV) {
      int *v = malloc(numV * sizeof(int));
      if (v == NULL) {
         fprintf(stderr, "Out of memory\n");
         exit(1);
      }
      int i;
      for (i=0; i<numV; i++) {
         v[i] = UNVISITED;
      }
      return v;
   }
```

为-1
创建一个都是-1的数组

```
   void showArray(int *v, int numV) {
      int i;
      printf("Visited: {");
      for (i=0; i<numV; i++) {
         printf("%d", v[i]);
         if (i <= numV-2) {
            printf(", ");
         }
      }
      printf("}\n");
      return;
   }
```

打印上面那个数组，如果访问过的话，数组的值就会替换为vertex的值

```
   int *visited = mallocArray(numV);
   Quack s = createQuack();
   push(v, s);
   showQuack(s);
```

v 为第一个读取的vertex

```
   int order = 0;
   while (!isEmptyQuack(s)) {
      v = pop(s);
```

对于已经访问过的顶点，直接pop掉了，不用管了

```
      if (visited[v] == UNVISITED) {
         showArray(visited, numV);
         //printf("visited[%d]=%d\n", v, order);
         visited[v] = order++;
         Vertex w;
```

order用来记录数据的访问顺序

```
         for (w=numV-1; w>=0; w--) {
            if (isEdge(g, newE(v,w))) {
               push (w, s);
            }
         }
      }
      showQuack(s);
```

遍历v节点所对应的所有edge

```
    }
    showArray(visited, numV);
    free(visited);
    return;
}

int main (void) {
    int numV;
    if ((numV = readNumV()) >= 0) {
        Graph g = newGraph(numV);
        if (readGraph(numV, g)) {
            showGraph(g);
            dfs(g, 0, numV);
        }
        g = freeGraph(g);
        g = NULL;
    }
    else {
        return EXIT_FAILURE;
    }
    return EXIT_SUCCESS;
}
```

We compile this code using:

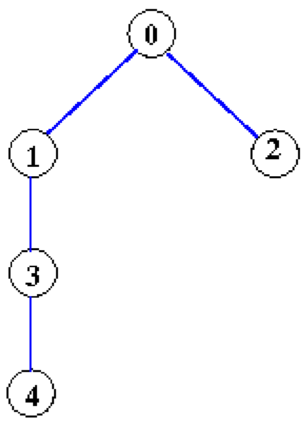
```
gcc dfsquack.c GraphAM.c Quack.c
```

If we execute this code on the graph:

```
#5
0 1 0 2
1 3
3 4
```

以文件的形式读取，否则永远也无法到达 EOF

which corresponds to the simple graph:



then the output is:

```
V=5, E=4
<0 1> <0 2>
<1 0> <1 3>
<2 0>
<3 1> <3 4>
```

```
<4 3>
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1}
Quack: <<1, 2>>
Visited: {0, -1, -1, -1, -1}
Quack: <<0, 3, 2>>
Quack: <<3, 2>>
Visited: {0, 1, -1, -1, -1}
Quack: <<1, 4, 2>>
Quack: <<4, 2>>
Visited: {0, 1, -1, 2, -1}
Quack: <<3, 2>>
Quack: <<2>>
Visited: {0, 1, -1, 2, 3}
Quack: <<0>>
Quack: << >>
Visited: {0, 1, 4, 2, 3}
```

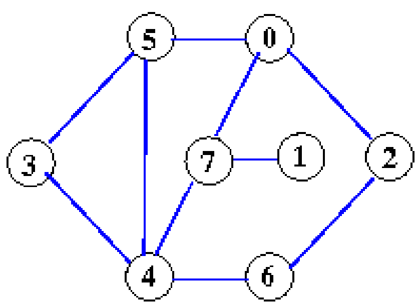
Here we see:

- the starting vertex 0 is pushed
- 0 is popped and its neighbours 1 and 2 are pushed
  - visited[0] = **0**
- 1 is popped and its neighbours 0 and 3 are pushed
  - visited[1] = **1**
- 0 is popped and ignored as it is in array *visited*
- 3 is popped and its neighbours 1 and 4 are pushed
  - visited[3] = **2**
- 1 is popped and is ignored
- 4 is popped and its neighbour 3 is pushed
  - visited[4] = **3**
- 3 is popped and is ignored
- 2 is popped and its neighbour 0 is pushed
  - visited[2] = **4**
- 0 is popped
- *quack is empty*

上述算法运行解释

The role of the stack in DFS is crucial. It facilitates backtracking.

What about a more substantial graph:



It is represented by the input data

```
8
```

```
0-2 0-5 0-7 2-6 1-7 4-7 4-6 4-3 3-5 4-5
```

If we want to do a DFS starting from vertex 0, *dfs()* generates the output:

```
V=8, E=10
0-7 0-5 0-2
1-7
2-6 2-0
3-5 3-4
4-5 4-3 4-6 4-7
5-4 5-3 5-0
6-4 6-2
7-4 7-1 7-0
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1, -1, -1, -1}
Quack: <<2, 5, 7>>
Visited: {0, -1, -1, -1, -1, -1, -1, -1}
Quack: <<0, 6, 5, 7>>
Quack: <<6, 5, 7>>
Visited: {0, -1, 1, -1, -1, -1, -1, -1}
Quack: <<2, 4, 5, 7>>
Quack: <<4, 5, 7>>
Visited: {0, -1, 1, -1, -1, -1, 2, -1}
Quack: <<3, 5, 6, 7, 5, 7>>
Visited: {0, -1, 1, -1, 3, -1, 2, -1}
Quack: <<4, 5, 5, 6, 7, 5, 7>>
Quack: <<5, 5, 6, 7, 5, 7>>
Visited: {0, -1, 1, 4, 3, -1, 2, -1}
Quack: <<0, 3, 4, 5, 6, 7, 5, 7>>
Quack: <<3, 4, 5, 6, 7, 5, 7>>
Quack: <<4, 5, 6, 7, 5, 7>>
Quack: <<5, 6, 7, 5, 7>>
Quack: <<6, 7, 5, 7>>
Quack: <<7, 5, 7>>
Visited: {0, -1, 1, 4, 3, 5, 2, -1}
Quack: <<0, 1, 4, 5, 7>>
Quack: <<1, 4, 5, 7>>
Visited: {0, -1, 1, 4, 3, 5, 2, 6}
Quack: <<7, 4, 5, 7>>
Quack: <<4, 5, 7>>
Quack: <<5, 7>>
Quack: <<7>>
Quack: << >>
Visited: {0, 7, 1, 4, 3, 5, 2, 6}
```

老师的课件代码根本不统一
应该都是凑到一起的

Multiple ADTs: quack.h and graph.h

- *dfsquack.c* above used both the *Graph* and the *Quack* abstract data types
    - we need to include both *quack.h* and *graph.h*
        - we can choose between implementations *quackAR.c* and *quackLL.c* at compile time
        - we can choose between implementations *graphAR.c* and *graphAM.c* at compile time
    - that is, use:
        - **gcc -Wall -Werror -O quackAR.c graphAM.c dfsquack.c** or

- **gcc -Wall -Werror -O quackAR.c graphAL.c dfsquack.c** or
- **gcc -Wall -Werror -O quackLL.c graphAM.c dfsquack.c** or
- **gcc -Wall -Werror -O quackLL.c graphAL.c dfsquack.c**

Performance

- number of pushes and pops
  - should be the same (stack is empty at the end)
- number of pushes of a vertex *v* = vertex degree of *v*
- total number of pushes
  - = sum of all the vertex degrees of vertices v in the graph

```
  If E is finite, then the total sum of vertex degrees
is equal to twice the number of edges.
```

  - = 2 * number of edges
- this means the complexity is linear in the number of edges, *O(E)*
  - **what does this mean? ...**
  - *how many edges are there?*
    - the worst case is a dense graph: $E = V*(V-1)/2$
      - so the complexity is linear in the number $V*(V-1)/2$ **???**
      - this means it is quadratic in *V*, or $O(V^2)$
    - if it is sparse, then it will be less than quadratic
- often said that DFS is linear in the size of the graph

Depth-First Search, using recursion

The implementation above used our own stack to 'remember' which path it was traversing and do backtracking.

Instead of our own stack, however, we can use the system stack by using recursion:

- in the literature, this is often done in combination with *global variables*
  - global variables are not permitted in this course however

切换行号显示

```c
// THIS IS FOR YOUR INFORMATION -- DO NOT USE
int order;     // global variable for the visiting order; yuk!
int *visited;  // global array of visiting orders; yuk!
               // indexed by vertex 0..g->nV

void dfs(Graph g, Vertex v) { // housekeeping function
   int i;
   visited = malloc(g->nV*sizeof(int));
   // CHECK FOR NULL OMITTED AS THIS CODE SHOULD NOT BE USED
   for (i = 0; i < g->nV; i++) {
      visited[i] = UNVISITED;
   }
   order = 0;
```

```
   dfsR(g, v);
}

void dfsR(Graph g, Vertex v) { // the actual recursive DFS
   visited[v] = order++;
   Vertex w;
   for (w = 0; w < g->nV; w++) { // THIS LINE 'REPLACES' THE STACK ABOVE
      if (g->edges[v][w] && visited[w] == UNVISITED) {
         dfsR(g, w);
      }
   }
}
```

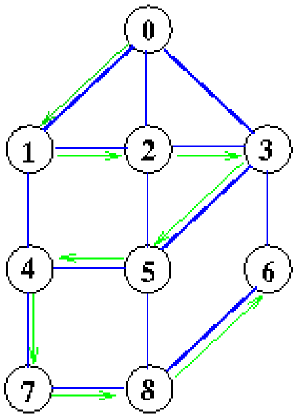
A version without global variables is:

切换行号显示

```
#define UNVISITED -1
void dfs(Graph g, Vertex v, int numV) {
   int *mallocArray(int numV) {
      int *array = malloc(numV * sizeof(int));// l
      if (array == NULL) {                    // o
         fprintf(stderr, "Out of memory\n");  // c
         exit(1);                             // a
      }                                       // l
      int i;                                  // f
      for (i=0; i<numV; i++) {                // u
         array[i] = UNVISITED;                // n
      }                                       // c
      return array;                           // t
   }
   void showArray(int *array, int numV) {
      int i;                                  // l
      printf("Visited: {");                   // o
      for (i=0; i<numV; i++) {                // c
         printf("%d", array[i]);              // a
         if (i <= numV-2) {                   // l
            printf(", ");                     // f
         }                                    // u
      }                                       // n
      printf("}\n");                          // c
      return;                                 // t
   }
   int *visited = mallocArray(numV);
   int order = 0;
   dfsR(g, v, numV, &order, visited);
   showArray(visited, numV);
   free(visited);
   return;
}

void dfsR(Graph g, Vertex v, int numV, int *order, int *visited) {
   visited[v] = *order;
   *order = *order+1;
```

```
    Vertex w;
    for (w=0; w < numV; w++) {
       if (isEdge(g, newE(v,w)) && visited[w]==UNVISITED) {
          dfsR(g, w, numV, order, visited);
       }
    }
    return;
}
```

The graph (that compared DFS and BFS) from above:



has the input file is:

```
9
0-1 0-2 0-3 1-4 1-2 2-3 2-5 3-5 3-6 4-5 4-7 5-8 7-8 6-8
```

and assuming the starting vertex is 0, the output is:

```
Visited: {0, 1, 2, 3, 5, 4, 8, 6, 7}
```

This is same DFS order that we found before, and is here indicated by the green arrows.

animation of DFS

Depth-First Search on disconnected graphs, using recursion

If the graph is disconnected, then the DFS algorithm above will not work:

- it uses edges to get from one vertex to another

Disconnected graphs are sometimes referred to as *forests* of graphs

- each 'tree' in the 'forest' is a graph

切换行号显示

```
void dfsDisc(Graph g, Vertex v, int numV) { // handles disconnected graphs
   int *mallocArray(int numV) {
      ... as above
   }
```

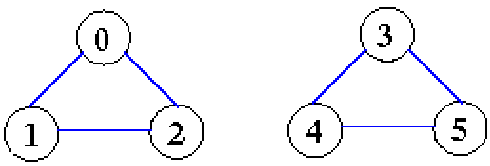
```
   void showArray(int *array, int numV) {
      ... as above
   }
   int *visited = mallocArray(numV);
   int order = 0;
   Vertex newv = v;                 // this is the starting vertex
   int finished = 0;
   while (!finished) {              // as long as there are vertices

      dfsR(g, newv, numV, &order, visited);

      Vertex w;
      finished = 1;                 // assume all vertices visited
      for (w = 0; w < numV && finished; w++) { // look for a new vertex
         if (visited[w] == UNVISITED) {
            finished = 0;           // found an unvisited vertex
            newv = w;
         }
      }
   }
   showArray(visited, numV);
   free(visited);
   return;
}
```

判断下 visited 数组是否已经满了，如果没有，从 -1 的顶点开始继续操作，并将 finished 赋值为 0，然后继续循环操作

If the (disconnected) input graph is:

```
6
0-1 0-2 1-2 3-4 3-5 4-5
```

corresponding to:



and assuming the starting vertex is 0, then the DFS is:

```
Visited: {0, 1, 2, 3, 4, 5}
```

Notice:

- the first (sub) graph's DFS search begins at vertex 0
- the second (sub) graph's DFS search begins at vertex 3

Cycle Detection (using DFS)

A graph that does not contain a cycle is called a tree, so

- asking whether a graph contains no cycles is equivalent to
- asking whether a graph is a tree

The *recursive* DFS algorithm, *dfsR()* we saw earlier is:

切换行号显示

```
void dfsR(Graph g, Vertex v, int numV, int *order, int *visited) {
   visited[v] = *order;
   *order = *order+1;
   Vertex w;
   for (w=0; w < numV; w++) {
      if (isEdge(g, newE(v,w)) && visited[w]==UNVISITED) {
         dfsR(g, w, numV, order, visited);
      }
   }
   return;
}
```

首先为数组赋值，并自增
其次定义分支的顶点
然后通过遍历，将没有访问
的分支节点recursive

To search for a cycle, the function *main()* calls *searchForCycle()*:

- which does housekeeping and in turn
- calls the recursive function *hasCycle()*

The function *hasCycle()* is a simple modification of *dfsR()*. The changes are:

- introduce a variable *found* to terminate the search if a cycle is found
  - this is a sort of early-exit
  - this variable must be passed up the recursive calls
- separate the 2 conditions in the for-loop
  - if *w* is adjacent to *v* then
    - if *w* is UNVISITED then recurse (in other words, keep searching)
    - else *w* has been visited before and **we have a cycle**
      - (assuming we are not going backwards along the same path)
        - to avoid going backwards, we pass 2 vertices to *hasCycle()*

The program is as follows:

切换行号显示

```
void searchForCycle(Graph g, int v, int numV) {
   int *mallocArray(int numV) {
      // as before
   }
   void showArray(int *vis, int numV) {
      // as before
   }
   int *visited = mallocArray(numV);
   int order = 0;

   if (hasCycle(g, numV, v, v, &order, visited)) {
      printf("found a cycle\n");
   }
```

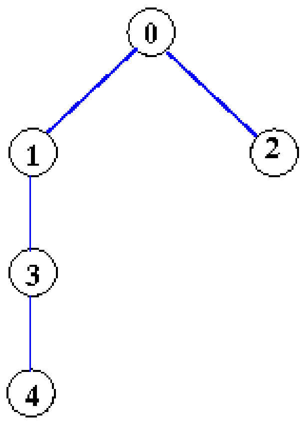
注意代码替换

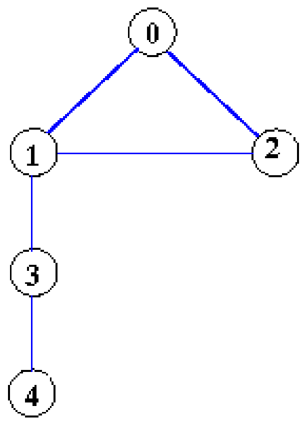
并且此函数为调用函数

```
   else {
      printf("no cycle found\n");
   }
   showArray(visited, numV);
   free(visited);
   return;
}

int hasCycle(Graph g, int numV, Vertex fromv, Vertex v, int *order, int *visited) {
   int retval = 0;
   visited[v] = *order;
   *order = *order+1;
   Vertex w;
   for (w=0; w<numV && !retval; w++) {
      if (isEdge(g, newE(v,w))) {
         if (visited[w]==UNVISITED) {
            printf("traverse edge %d-%d\n", v, w);
            retval = hasCycle(g, numV, v, w, order, visited);
         }
         else {
            if (w != fromv) { // exclude the vertex we've just come from
               printf("traverse edge %d-%d\n", v, w);
               retval = 1;
            }
         }
      }
   }
   return retval;
}
```

原函数只访问没有被访问的，但是cycle需要访问那些已经被访问的是否还会被访问到

If we input the graph on the left below:





a program that calls this function will output:

```
found a cycle
```

Alternatively, if the input is the graph on the right, it will output:

```
no cycle found
```

Eulerian cycles (using DFS)

Eulerian path是指正好包含每个边正好一次的path，可能多次经过某个顶点

An Eulerian **path** in a graph is a path that includes every edge exactly once

- this path may include many visits to the same vertex

In turn, an Eulerian **cycle** is a special case of a Eulerian path in which the start and end points are the same Eulerian cycle是指首尾顶点是相同的

Eulerian Cycle：所有都是偶数度
No Eulerian Cycle：有一个奇数度

Eulerian Path：有两个奇数度，其他都是偶数度

Animation of the *Konigsburg Bridge Problem*

- http://www.youtube.com/watch?v=3bUvjajakGM

Graph animation of finding an Euler cycle

- http://www.cs.sunysb.edu/~skiena/combinatorica/animations/euler.html

Well-known properties of Eulerian paths and cycles:

所有顶点拥有偶数的度，并且是连通图

```
A connected graph has an Eulerian cycle if all its vertices
have even degree.
```

```
A connected graph has no Eulerian cycle if any of its vertices
has odd degree.
```

连通图如果任何一个顶点不是偶数的度

```
A connected graph has an Eulerian path if it has exactly 2
vertices of odd degree, and all the rest have even degree.
```

```
A connected graph has no Eulerian path if it has more than 2
vertices of odd degree.
```

正好两个顶点有奇数的度

So, a graph with more than 2 vertices of odd degree has no Eulerian cycle or path.

```
If a graph contains a Eulerian path, it cannot contain a
Eulerian cycle, and vice versa.
```

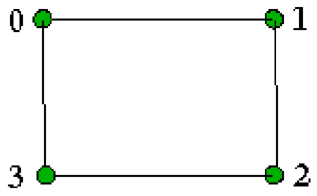
Eulerian cycle 和 Eulerian path只能有一个

```
A graph that consists of vertices all with even degree is
often called an '''''Eulerian graph'''''
```

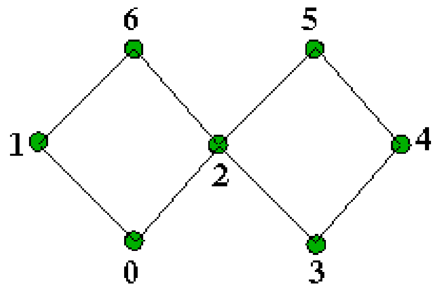
There is a simple algorithm to find an Eulerian cycle in an Eulerian graph

- it uses a DFS, hence requires a stack (to backtrack when you reach a deadend).

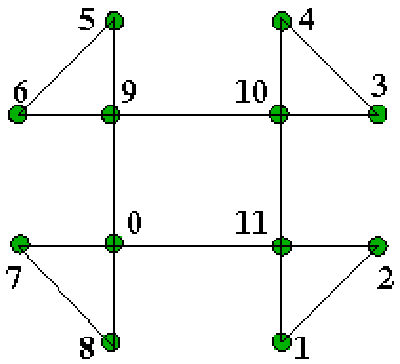
Examples of Eulerian graphs:



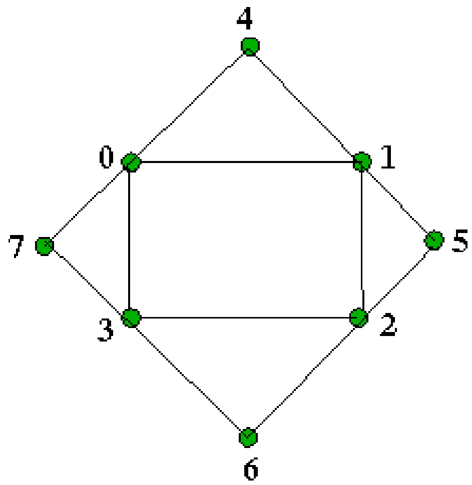
'box'



'bow'



'propbows'



'concsquares'

Eulerian Cycle Algorithm

Assume graph is connected and is Eulerian.

这个Eulerian Cycle只是得到从0出发的循环
如果有循环，则第一个pop为0，最后一个pop也为0

1. Read the Eulerian graph and initialise a stack.
2. Choose any vertex *v* and push *v*.
3. While the stack is not empty:
   - if the top of stack vertex has one of more adjacent vertices
     - select arbitrarily the largest vertex *w*
     - push *w*
     - remove edge *v*-*w*
   - else pop the top vertex and print it

The sequence of vertices that is printed is an Eulerian cycle. How does the algorithm work?

- we traverse the graph by pushing vertices on the stack and removing edges that lead to them
  - the vertex that is pushed is the neighbour of the previous vertex that is pushed
- we continue to push vertices as long as we can
- if we have a vertex on the stack with no neighbours, we start to pop and print

The function *findEulerCycle()* below implements the algorithm:

切换行号显示

```
void findEulerCycle(Graph g, int numV, Vertex startv) {
```

这里虽然看上去多此一举
主要是为了获取 v 的值

```
   Quack s = createQuack();
   printf("Eulerian cycle: ");

   push(startv, s);
   while (!isEmptyQuack(s)) {
      Vertex v = pop(s); // v is the top of stack vertex and ...
      push(v, s);        // ... the stack has not changed
      Vertex w;
      if ((w = getAdjacent(g, numV, v)) >= 0) {
         push(w, s);     // push a neighbour of v onto stack
         removeE(g, newE(v, w)); // remove edge to neighbour
      }
      else {
         w = pop(s);
         printf("%d ", w);
      }
   }
   putchar('\n');
}

Vertex getAdjacent(Graph g, int numV, Vertex v) {
   // returns the Largest Adjacent Vertex if it exists, else -1
   Vertex w;
   Vertex lav = -1; // the adjacent vertex
   for (w=numV-1; w>=0 && lav==-1; w--) {
      Edge e = newE(v, w);
      if (isEdge(g, e)) {
         lav = w;
      }
   }
   return lav;
}
```

如果没找到邻接点，则会返回-1

找到最大的邻接点push
然后不停的push
直到没有邻接点了
开始pop
直到发现又有邻接点了
push 。。 pop。。
直到stack空了为止

用于遍历，总节点数

当前的节点

如果没有找到邻接点则会返回-1

For example: for the **box** graph above:

```
push 0
push 3 and remove 0-3
push 2 and remove 3-2
push 1 and remove 2-1
push 0 and remove 1-0 <-- at this point all the edges have been removed, and 0 is isolated
pop 0 and print 0
pop 1 and print 1
pop 2 and print 2
pop 3 and print 3
pop 0 and print 0
```

- **Eulerian cycle: 0 1 2 3 0**
- *It is not obvious why you need a stack here: 5 pushes were followed by 5 pops.*

We can reach a deadend during the traversal

- if the top vertex on the stack has no adjacent vertices, the traversal has reached a deadend
    - (remember that we are removing edges as we traverse the graph)
- but there may be vertices on the stack that **do** have branches to vertex neighbours
    - if there is one, then a vertex neighbour is pushed and the traversal continues
    - this is a process of **back-tracking** of course
- the process stops when branches have been taken
    - this will happen when all edges have been removed
    - every vertex on the stack will have been popped and printed
        - we may see the same vertex many times, but each edge is traversed just once

In the next example, for the **bow** graph above, we see backtracking in action:

```
push 0
push 2 and remove 0-2
push 6 and remove 2-6
push 1 and remove 6-1
push 0 and remove 1-0
pop 0 and print 0
pop 1 and print 1
pop 6 and print 6
push 5 and remove 2-5
push 4 and remove 5-4
push 3 and remove 4-3
push 2 and remove 3-2
pop 2 and print 2
pop 3 and print 3
pop 4 and print 4
pop 5 and print 5
pop 2 and print 2
pop 0 and print 0
```

- **Eulerian cycle: 0 1 6 2 3 4 5 2 0**

You may need to backtrack many times of course. Consider the **propbows** graph above:

```
push 0
pushed 11 and remove edge 0-11
pushed 10 and remove edge 11-10
pushed 9 and remove edge 10-9
pushed 6 and remove edge 9-6
pushed 5 and remove edge 6-5
pushed 9 and remove edge 5-9
pushed 0 and remove edge 9-0
pushed 8 and remove edge 0-8
pushed 7 and remove edge 8-7
pushed 0 and remove edge 7-0
popped 0 and print 0
popped 7 and print 7
popped 8 and print 8
popped 0 and print 0
popped 9 and print 9
popped 5 and print 5
popped 6 and print 6
popped 9 and print 9
pushed 4 and remove edge 10-4
```

```
pushed 3 and remove edge 4-3
pushed 10 and remove edge 3-10
popped 10 and print 10
popped 3 and print 3
popped 4 and print 4
popped 10 and print 10
pushed 2 and remove edge 11-2
pushed 1 and remove edge 2-1
pushed 11 and remove edge 1-11
popped 11 and print 11
popped 1 and print 1
popped 2 and print 2
popped 11 and print 11
popped 0 and print 0
```

- **Eulerian cycle: 0 7 8 0 9 5 6 9 10 3 4 10 11 1 2 11 0**

Path searching (using DFS)

You could want to search for a path between two given vertices:

- the starting vertex we already have
- add the goal vertex as a parameter and test 'is there a path' in the *dfs()* function.

切换行号显示

```
int isPath(Graph g, Vertex v, Vertex goalv, int numV, int *order, int *visited) {
   int found = 0;
   visited[v] = *order;
   *order = *order+1;
   if (v == goalv) {
      found = 1;
   }
   else {
      Vertex w;
      for (w=0; w < numV && !found; w++) {
         if (isEdge(g, newE(v,w))) {
            if (visited[w] == UNVISITED) {
               found = isPath(g, w, goalv, numV, order, visited);
               printf("path %d-%d\n", w, v);
            }
         }
      }
   }
   return found;
}
```

连续遍历，不走重复的点
直到goalv

- this function does a DFS traversal, exactly like *dfsR()* ...
  - ... but at the same time, it searches for a path to a vertex *goalv*

So, instead of the call to *dfsR()* (in *dfs()*):

切换行号显示

```
dfsR(g, v, numV, &order, visited);
```

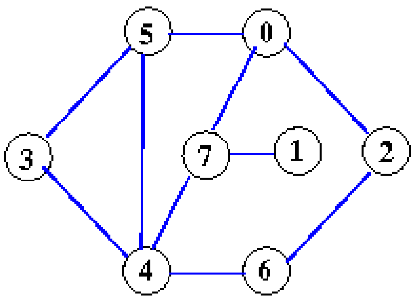
we call the function *isPath()*:

切换行号显示

```
#define STARTV 0
#define GOALV 3
if (isPath(g, STARTV, GOALV, numV, &order, visited)) { //notice the STARTV and GOALV arguments
   printf("found path\n");
}
else {
   printf("no path\n");
}
```

- Here a #define is used to name the start and goal vertices: this could better be input interactively of course.

If we input the graph:



then the output is:

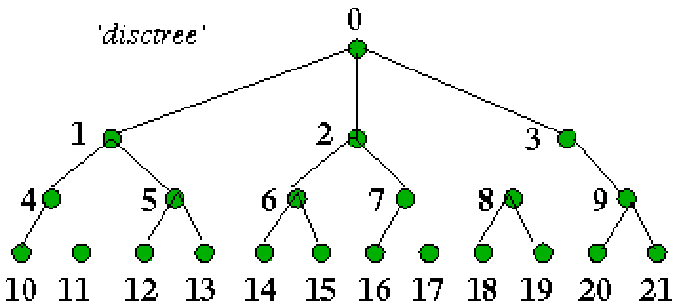
```
path 3-4
path 4-6
path 6-2
path 2-0
found path
Visited: {0, -1, 1, 4, 3, -1, 2, -1}
```

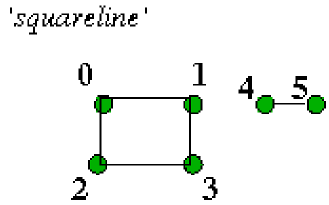
- Here we can see which vertices were visited

Reachability Analysis

In many problems we are interested in knowing which vertices are *reachable* from some start vertex.

- for example, in the graphs:





some of the vertices are unreachable from the start vertex 0, others are not

- if the graph is undirected, it is obviously disconnected
- (if the graph is directed then it may not be disconnected of course)

A DFS algorithm can be used to find all the reachable vertices

- simply run the algorithm from the start vertex
- on conclusion, check the visited array
  - any vertex (except the start vertex) that is unvisited is unreachable

An alternative method is to use so-called 'fixed-point' computation.

1. initialise:
   - a reachable set comprising of just the start vertex
   - every other vertex is considered unreachable
2. check every unreachable vertex *v*
   - if there is an edge from a vertex in the reachable set to *v*
     - then add *v* to the reachable set
3. repeat the previous step until the reachable set does not change
   - if the reachable set does not change, terminate

When the set does not change, we have reached a 'fixed point'

- the set of vertices in the reachable set can be reached from the start vertex
- all other vertices cannot be reached

Example: consider the graph *squareline* above

- let R be the set of reachable vertices, and the start vertex be 0
  - initially $R = \{0\}$
- consider vertices 1..5
  - 1 is adjacent to 0, add to R
  - 2 is adjacent to 0, add to R
  - $R = \{0, 1, 2\}$
  - R has changed, so repeat
- consider vertices 3..5
  - 3 is adjacent to 1, add to R
  - $R = \{0, 1, 2, 3\}$
  - R has changed, so repeat
- consider vertices 4 and 5
  - neither vertex is adjacent to a vertex in R
  - R does not change

- terminate the algorithm

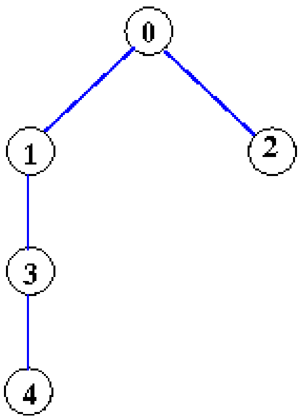
Notice that you do not needs to use a stack here, or recursion. There is no backtracking.

Breadth First Search

Consider all edges of one vertex before moving to another vertex

- each level of vertices is visited before the next level's vertices are considered

For example:



1. visit vertex 0
2. visit vertex 1 and 2
3. visit vertex 3
4. visit vertex 4

In essence, the vertices are processed ***in order*** (top to bottom, left to right)

- DFS used a stack:
    - we pushed all the adjacent vertices of a vertex onto a stack
    - so we would remember the vertices we need to backtrack to (i.e. 'try' later on)
- BFS instead uses a queue:
    - we push all the adjacent vertices of a vertex onto a queue (just like DFS)
    - but we visit them in the order they occur (as we must in a queue)
- the change in the *quack* version is trivial:
    - just 4 changes

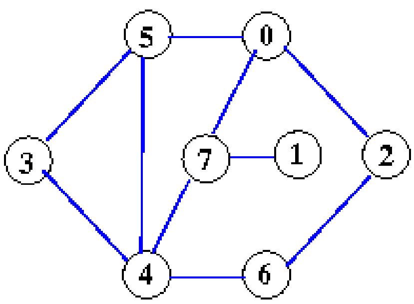
切换行号显示

```
void bfs(Graph g, Vertex v, int numV) { // CHANGE 1: NAME DFS => BFS
   int *mallocArray(int numV) {
      int *array = malloc(numV * sizeof(int));// l
      if (array == NULL) {                   // o
         fprintf(stderr, "Out of memory\n");  // c
         exit(1);                            // a
      }                                      // l
      int i;                                 // f
      for (i=0; i<numV; i++) {               // u
         array[i] = UNVISITED;               // n
      }                                      // c
```

```
      return array;                            // t
   }
   void showArray(int *array, int numV) {
      int i;                                   // l
      printf("Visited: {");                    // o
      for (i=0; i<numV; i++) {                 // c
         printf("%d", array[i]);               // a
         if (i <= numV-2) {                    // l
            printf(", ");                      // f
         }                                     // u
      }                                        // n
      printf("}\n");                           // c
      return;                                  // t
   }
   int *visited = mallocArray(numV);
   Quack s = createQuack();
   qush(v, s);                        // CHANGE 2: PUSH => QUSH
   showQuack(s);
   int order = 0;
   while (!isEmptyQuack(s)) {
      v = pop(s);
      if (visited[v] == UNVISITED) {
         showArray(visited, numV);
         visited[v] = order++;
         Vertex w;
         for (w=0; w<numV; w++) {     // CHANGE 3: ORDER: SMALL TO LARGE
            if (isEdge(g, newE(v,w))) {
               qush(w, s);            // CHANGE 4: PUSH -> QUSH
            }
         }
      }
      showQuack(s);
   }
   showArray(visited, numV);
   free(visited);
   return;
}
```

What about the graph we considered above for DFS represented by the input data

```
8
0-2 0-5 0-7 2-6 1-7 4-7 4-6 4-3 3-5 4-5
```



Starting at vertex 0, the visited sequence of *dfs()* was {0, 7, 1, 4, 3, 5, 2, 6}.

- this corresponds to the order of vertices: 0 2 6 4 3 5 7 1

What does *bfs()* do:

```
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1, -1, -1, -1}
Quack: <<2, 5, 7>>
Visited: {0, -1, -1, -1, -1, -1, -1, -1}
Quack: <<5, 7, 0, 6>>
Visited: {0, -1, 1, -1, -1, -1, -1, -1}
Quack: <<7, 0, 6, 0, 3, 4>>
Visited: {0, -1, 1, -1, -1, 2, -1, -1}
Quack: <<0, 6, 0, 3, 4, 0, 1, 4>>
Quack: <<6, 0, 3, 4, 0, 1, 4>>
Visited: {0, -1, 1, -1, -1, 2, -1, 3}
Quack: <<0, 3, 4, 0, 1, 4, 2, 4>>
Quack: <<3, 4, 0, 1, 4, 2, 4>>
Visited: {0, -1, 1, -1, -1, 2, 4, 3}
Quack: <<4, 0, 1, 4, 2, 4, 4, 5>>
Visited: {0, -1, 1, 5, -1, 2, 4, 3}
Quack: <<0, 1, 4, 2, 4, 4, 5, 3, 5, 6, 7>>
Quack: <<1, 4, 2, 4, 4, 5, 3, 5, 6, 7>>
Visited: {0, -1, 1, 5, 6, 2, 4, 3}
Quack: <<4, 2, 4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<2, 4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<4, 5, 3, 5, 6, 7, 7>>
Quack: <<5, 3, 5, 6, 7, 7>>
Quack: <<3, 5, 6, 7, 7>>
Quack: <<5, 6, 7, 7>>
Quack: <<6, 7, 7>>
Quack: <<7, 7>>
Quack: <<7>>
Quack: << >>
Visited: {0, 7, 1, 5, 6, 2, 4, 3}
```

In the first few lines of this output we see:

- vertex 0 is qushed, then popped
- vertices 2, 5, 7 are qushed, then successively
  - 2 is popped
  - 5 is popped
  - 7 is popped
- 'qushing' means everything gets added to the end of the quack
- the order that vertices are visited is:
  - 0 2 5 7 6 3 4 1
  - note:
    - 0 is a level-0 vertex
    - 2 5 7 are level-1
    - 6 3 4 1 are level-2
    - no vertex is more than a path length of 2 away from the starting vertex

Path searching (using BFS)

You could want to search for a path between two given vertices, *start* and *goal* say:

- the starting vertex we already have
- add the goal vertex as a parameter and test for it in the *bfs()* function

But, we don't follow any 'path' during BFS (as we did in DFS): we traverse by level:

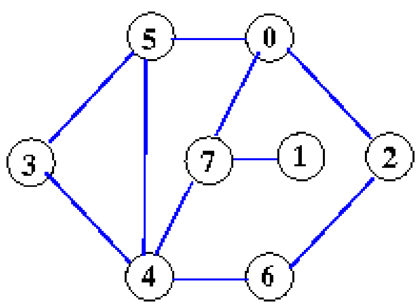
- whenever we 'visit' a node, we must remember its parent
  - we store the parent of each vertex in an array called *parent[]*
- we hence need 2 arrays, *visited[]* and *parent[]*
- when we find the goal node, we're finished
  - the path 'backwards' to the goal node will be stored in *parent[]*
- we print the path from the *start* to the *goal* stored in *parent[]*

切换行号显示

```
void searchPath(Graph g, Vertex start, Vertex goal, int numV) {
   int *mallocArray(int numV) {
      int *array = malloc(numV * sizeof(int));// l
      if (array == NULL) {                    // o
         fprintf(stderr, "Out of memory\n");  // c
         exit(1);                             // a
      }                                       // l
      int i;                                  // f
      for (i=0; i<numV; i++) {                // u
         array[i] = UNVISITED;                // n
      }                                       // c
      return array;                           // t
   }
   void showArray(int *array, int numV) {
      int i;                                  // l
      printf("Visited: {");                   // o
      for (i=0; i<numV; i++) {                // c
         printf("%d", array[i]);              // a
         if (i <= numV-2) {                   // l
            printf(", ");                     // f
         }                                    // u
      }                                       // n
      printf("}\n");                          // c
      return;                                 // t
   }
   void printPath(int parent[], int numV, Vertex goal) {
      // local function discussed below
   }
   int *visited = mallocArray(numV);
   int *parent = mallocArray(numV);      // need extra array to store parents
   Quack q = createQuack();
   qush(start, q);
   int order = 0;
   visited[start] = order++;
   int found = 0;
   while (!isEmptyQuack(q) && !found) {
      Vertex x = pop(q);
      Vertex y;
      for (y = 0; y<numV && !found; y++) {
         if (isEdge(g, newE(x,y))) {      // for
```

```
adjacent vertex y ...
                if (visited[y]==UNVISITED) {  // ... if y is unvisited ...
                   qush(y, q);                // ... queue y
                   visited[y] = order++;      // y is now visited
                   parent[y] = x;             // y's parent is x
                   if (y == goal) {           // if y is the goal ...
                      found = 1;              // ... SUCCESS! now get out
                   }
                }
             }
          }
       }
       if (found) {
          printf("SHORTEST path from %d to %d is ", start, goal);
          printPath(parent, numV, goal);
          putchar('\n');
       }
       else {
          printf("no path found\n");
       }
       free(visited);
       free(parent);
       makeEmptyQuack(q);
       return;
}
```

If we input the graph:



then the output is:

```
SHORTEST path from 0 to 6 is 6<--2<--0
```

The array *parent[]* stores the parent of every visited node in the graph, but:

- the start vertex has no parent
- unvisited nodes have no parents

To print the path:

- we print *goal*
- we print *parent[goal]*

- we print *parent[parent[goal]]*
- we print *parent[parent[parent[goal]]]*
- we print *parent[parent[parent[parent[goal]]]]*
- ...
- until we find the *start* vertex

That's what the function below does:

切换行号显示

```
void printPath(int parent[], int numV, Vertex v) {
   printf("%d", v);
   if (0<=v && v<numV) {
      Vertex p = parent[v];
      while (p != UNVISITED) {
          printf("<--%d", p);
          p = parent[p];
      }
   }
   else {
      fprintf(stderr, "printPath: illegal vertex in parent[]\n");
   }
}
```

其他顶点都有 parent
只有初始顶点没有，所以其parent就是默认的 -1

As we start with *goal* and work backwards, we print the path in reverse direction.

animation of BFS

GraphSearch (2019-07-18 18:02:19由AlbertNymeyer编辑)