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# Abstract data types

An abstract data type (ADT) is a data type that

- is defined by the operations that may be performed on it
- the data in the ADT is accessible only via operations ADT不能被用户看见，只能通过函数来调用
- a user of an ADT cannot access or even see the data (it's hidden)

In the **specification** of an ADT, we must:

- define the data in some way
- define a set of functions to manipulate the data



wikipedia definition of ADTs

In mathematics:

- below is a 'signature' of an ADT for STACK, which uses the format:
  - 'operator': 'type' x 'type' --> 'type'

```

createstack:                --> STACK
push      : STACK x INTEGER --> STACK
pop       : STACK           --> INTEGER | {undefined}
empty     : STACK           --> BOOLEAN

```

- this signature specifies the STACK data type completely

In programming

- the 'signature' above is called an **interface**
- there is also an **implementation** of the **interface**
- and there is the **client** that uses the **interface**

## Terminology

- an ADT is accessed (only) through its **interface**
- an **interface** is a contract between the **client** and the **implementation**
- an ADT 'architecture' is:

implementation: ADT.c  
interface: ADT.h  
client: test.c

```
client <=> interface <=> implementation
```

声明

- an **interface** is a set of **prototypes** that specifies the ADT
  - e.g. a *header file* such as `fredADT.h`
- a **client** uses the interface to carry out a task
  - the client contains the *main()* function
  - e.g. a client could be called `client.c`
- the **implementation** is the actual ADT
  - e.g. it could be called `fredADT.c`
- specifically, in terms of the architecture, we have:

```
client.c <=> fredADT.h <=> fredADT.c
```

Importance of ADTs:

- *abstraction*: hides complexity and detail from a client
- *flexibility*: change the implementation without changing the client
- *decomposition*: decompose the problem into 'component' tasks
- *structure*: structuring for readability and maintainability
- *security*: restricts the ability of hostile clients **corrupting data**

## A Point data type

Consider the following data type for a Cartesian point, defined in *point.h*

切换行号显示

```
1 // point.h
2 #include <math.h>
3 typedef struct point Point;
4 struct point{
5     float x; float y;
6 };
7 float distance(Point, Point); // returns distance between
two points
8 void move(Point, float, float); // moves a point by a certain
amount
9
```

and its implementation

切换行号显示

```
1 // point.c: an implementation of a point
```

```

2    #include "point.h"
3    float distance(Point a, Point b) {
4        float dx = a.x - b.x;
5        float dy = a.y - b.y;
6        return sqrt(dx*dx + dy*dy);
7    }
8    void move(Point *a, float dx, float dy) {
9        a->x += dx;
10       a->y += dy;
11    }

```

and its client

切换行号显示

```

1    // clientPoint.c: a client of the point data type
2    #include <stdio.h>
3    #include <stdlib.h>
4    #include "point.h"
5    int main(int argc, char *argv[]) {
6        Point r, s;
7        float d;
8
9        r.x = 5;
10       r.y = 5;
11       s.x = 4;
12       s.y = 6;
13       d = distance(r, s);
14       printf("1st distance: %0.1f\n", d);
15       move(&r, -1, +2);
16       d = distance(r, s);
17       printf("2nd distance: %0.1f\n", d);
18       return EXIT_SUCCESS;
19    }

```

You compile the 'implementation' and 'client' by:

```

prompt$ gcc point.c clientPoint.c
prompt$ ./a.out
1st distance: 1.4
2nd distance: 1.0

```

**Is Point an Abstract Data Type? NO**, because:

- the declaration of *Point* is visible to the client
  - even worse, *clientPoint.c* sets the values inside the type (in lines 9-12)
  - *Point* is not abstract: it forms part of the code

## Point ADT

Here is a *Point* ADT:

切换行号显示

```

1    // pointADT.h: an interface for the Point ADT
2    typedef struct point *Point;    // notice that Point is a

```

```

pointer to a struct
3  float distance(Point, Point);    // returns distance between
two points
4  void  move(Point, float, float); // moves a point by certain
amount
5  Point create(float, float);      // create a new Point
6

```

and an implementation of the interface

切换行号显示

```

1  // pointADT.c
2  #include <stdio.h>
3  #include <stdlib.h>
4  #include <math.h>
5  #include "pointADT.h"
6
7  struct point {
8      float x;
9      float y;
10 };
11
12 Point create(float xpos, float ypos) {
13     Point p;
14     p = malloc(sizeof(struct point)); // returns a pointer to
the struct
15     if (p == NULL) {
16         fprintf(stderr, "No memory\n");
17         exit(1);
18     }
19     p->x = xpos;
20     p->y = ypos;
21     return p;
22 }
23
24 float distance(Point a, Point b) {
25     float dx = a->x - b->x;
26     float dy = a->y - b->y;
27     return sqrt(dx*dx + dy*dy);
28 }
29 void move(Point a, float dx, float dy) {
30     a->x += dx;
31     a->y += dy;
32 }

```

and to finish up, a client that uses the *Point* ADT

切换行号显示

```

1  // clientPointADT.c: a client of the Point ADT
2  #include <stdio.h>
3  #include <stdlib.h>
4  #include "pointADT.h"
5
6  int main() {
7     Point r, s;
8     float d;
9
10     r = create(5,5);
11     s = create(4,6);

```

```

12     d = distance(r,s);
13     printf("1st distance: %0.1f\n", d);
14     move(r, -1, +2);
15     d = distance(r,s);
16     printf("2nd distance: %0.1f\n", d);
17     return EXIT_SUCCESS;
18 }

```

- note the design is much cleaner
- we cannot see inside *struct point* pointed to by *Point*

We compile using:

```
gcc pointADT.c clientPointADT.c
```

The output is the same as for the previous version of *Point*.

## Stacks and queues

Stacks mimic many real-world behaviours:

- moving boxes
- putting plates away in the cupboard and then setting the table
- placing and removing many rings on a (single) finger
- putting on more than one pair of socks because it is cold and then taking them off
- many cars going down a one-way street that is blocked and having to back out again
- calling functions in C
- ... all are examples of *LAST IN, FIRST OUT (LIFO)*
  - *notice that in LIFO, you add and remove from the same end*

Queues in fact are even more common-place than stacks:

- the checkout at a supermarket
- people standing at a ticket window
- people queueing to go onto a bus
- cars queueing to go onto a ferry
- objects flowing through a pipe (where they cannot overtake each other)
- messages on an answering machine
- ... all are examples of *FIRST IN, FIRST OUT (FIFO)*
  - *notice, you add to one end, remove from the other*

Example of a stack (with top-of-stack on the left):

Operation	Resulting Stack	Return Value
<b>push(1)</b>	1	
<b>push(2)</b>	2 1	
<b>push(3)</b>	3 2 1	
<b>pop()</b>	2 1	3

<b>push(4)</b>	4 2 1	
----------------	-------	--

### How do we implement and use a stack?

- We can implement a *stack* using:
  - an array
    - fast, easiest to implement (usually fixed length)
  - a linked list
    - slow, but easier on memory (memory is added only when needed)

An ADT for a stack may require a small *library* of functions:

- create a stack
- make the stack empty (i.e. destroy it)
- push data onto the stack
- pop data off the stack,
- is the stack empty
- get the height of the stack
- print or show the contents of a stack
- print or show the top element on the stack
- ...

Normally, need to create an ADT for a stack, and another ADT for a queue

- instead in this course, create an ADT called **Quack** that does both

## Quack

Quack = stack + queue  
Quack 可以实现 stack 和 queue 的所有功能

### Quack interface

The interface is below:

切换行号显示

```

1 // quack.h: an interface definition for a queue/stack
2 #include <stdio.h>
3 #include <stdlib.h>
4
5 typedef struct node *Quack;
6
7 Quack createQuack(void); // create and return Quack
8 void push(int, Quack); // put the given integer onto the
quack
9 int pop(Quack); // pop and return the top element on
the quack
10 int isEmptyQuack(Quack); // return 1 if Quack is empty, else
0
11 void makeEmptyQuack(Quack); // remove all the elements on Quack
12 void showQuack(Quack); // print the contents of Quack, from
the top down
13

```

- note I have dropped the *ADT* suffix off the name

## Quack implementation

Most popular and efficient is to use an array:

- the first element in the array is the bottom of the stack
- *push* puts an element at the top
- *pop* takes an element from the top

The array 'grows' as the elements are pushed onto the quack

- normally the size of the array is fixed, to *HEIGHT* say
- the top is just an index in the array
  - **underflow** occurs when you *pop* and the top = -1 (= empty quack)
  - **overflow** occurs when you call *push* and the top = HEIGHT (= full quack)

underflow : 比最小还小  
overflow : 比最大还大

切换行号显示

```

1 // quack.c: an array-based implementation of a quack
2 #include "quack.h"
3
4 #define HEIGHT 1000
5
6 struct node {
7     int array[HEIGHT];
8     int top;    // used by both pop and push
9 };            就是记录数组的长度
               top是数组最大元素的索引值
               一个元素就是0
10
11 Quack createQuack(void) {
12     Quack qs;
13     qs = malloc(sizeof(struct node));
14     if (qs == NULL) {
15         fprintf(stderr, "createQuack: no memory, aborting\n");
16         exit(1); // should pass control back to the caller
17     }
18     qs->top = -1;
19     return qs;
20 }
21
22 void push(int data, Quack qs) {
23     if (qs == NULL) {
24         fprintf(stderr, "push: quack not initialised\n");
25     }
26     else {    数据已经满了,就不能再加了
27         if (qs->top >= HEIGHT-1) {
28             fprintf(stderr, "push: quack overflow\n");
29         }
30         else {    把数据追加在最后
31             ++qs->top;
32             qs->array[qs->top] = data;
33         }
34     }
35     return;
36 }
37
38 int pop(Quack qs) { // return top element, or 0 if error
39     int retval = 0;
40     if (qs == NULL) {
41         fprintf(stderr, "pop: quack not initialised\n");
42     }

```

```

43     else {
44         if (isEmptyQuack(qs)) {
45             fprintf(stderr, "pop: quack underflow\n");
46         }
47         else {
48             retval = qs->array[qs->top]; // top element on stack
49             --qs->top;
50         }
51     }
52     return retval;
53 }
54
55 void makeEmptyQuack(Quack qs) {
56     if (qs == NULL) {
57         fprintf(stderr, "makeEmptyQuack: quack not
initialised\n");
58     }
59     else {
60         while (!isEmptyQuack(qs)) {
61             pop(qs);
62         }
63     }
64     return;
65 }
66
67 int isEmptyQuack(Quack qs) {
68     int empty = 0;
69     if (qs == NULL) {
70         fprintf(stderr, "isEmptyQuack: quack not initialised\n");
71     }
72     else {
73         empty = qs->top < 0;
74     }
75     return empty;
76 }
77
78 void showQuack(Quack qs) {
79     if (qs == NULL) {
80         fprintf(stderr, "showQuack: quack not initialised\n");
81     }
82     else {
83         printf("Quack: ");
84         if (qs->top < 0) {
85             printf("<< >>\n");
86         }
87         else {
88             int i;
89             printf("<<"); // start with a <<
90             for (i = qs->top; i > 0; --i) {
91                 printf("%d, ", qs->array[i]); // print each element
92             }
93             printf("%d>>\n", qs->array[0]); // last element
94         }
95     }
96     return;
97 }

```

因为数据追加的时候是在后面，  
因此pop也是后面，相当于top  
所以打印的时候，从后往前打印

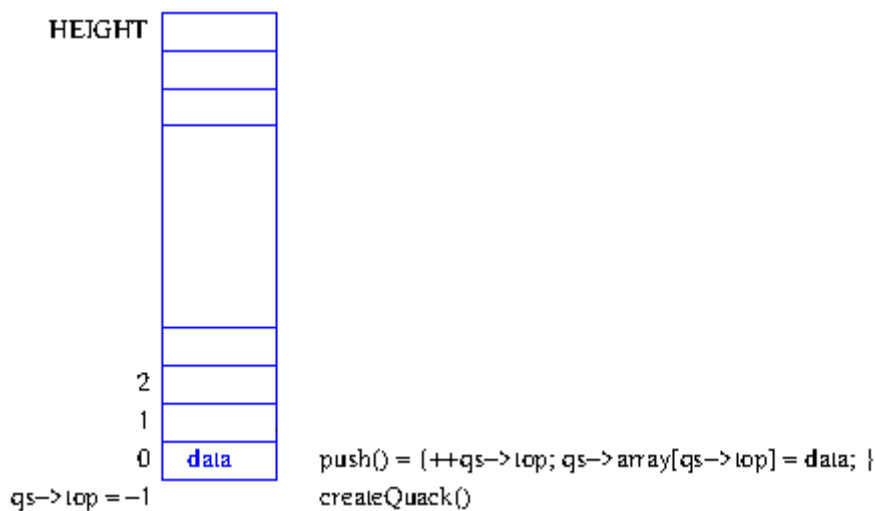
- note I have dropped the *ADT* suffix off the name to match the interface

Below we see a picture of an array-based quack. Notice,

- the fixed height



- `createQuack()` initialises the top-of-quack index to -1
- `push()` increments the top-of-stack index and places data at that location in the array
- `pop()` returns the element at the top-of-stack and decrements the index



Summarising, we have an ADT consisting of **quack.h** and **quack.c**

## Quack clients

### Client 1. black-box unit tester

'Black-box' testing is

- a method of testing a 'unit' from the outside
- the tester is just a *client*
- it can call only interface functions
  - this severely limits what can be tested
  - sometimes (often) 'secret' functions are included in ADTs that provide more information
    - including extra functions for BB testing is usually called 'instrumentation'
    - they are often security holes!
- (note 'white-box' testing inserts code (into the ADT) to verify correct behaviour)

One aim of BB testing is to verify every error message

- *Can you generate every error message?*
  - often errors are not handled 'gracefully':
    - with an `exit(1)` for example

BB testing can only ever test some of the behaviours

切换行号显示

```
1 // blackbox.c: black box unit tester for a quack
2 #include <stdio.h>
3 #include <stdlib.h>
4 #include <assert.h>
5 #include "quack.h"
```

```

6
7 int main(int argc, char *argv[]){
8     Quack s = NULL;
9     Quack t;
10
11     printf("Test #1: Init stack is empty: ");
12     s = createQuack();
13     assert(isEmptyQuack(s));
14     printf("passed\n");
15
16     printf("Test #2: Push, stack not empty: ");
17     push(1, s);
18     assert(!isEmptyQuack(s));
19     printf("passed\n");
20
21     printf("Test #3: Push again, pop twice, then empty: ");
22     push(2, s);
23     pop(s);
24     pop(s);
25     assert(isEmptyQuack(s));
26     printf("passed\n");
27
28     printf("Test #4: TESTING ERROR: quack underflow\n");
29     pop(s);
30     return EXIT_SUCCESS;
31 }

```

```

prompt$ gcc quack.c blackbox.c
prompt$ ./a.out
Test #1: Init stack is empty: passed
Test #2: Push, stack not empty: passed
Test #3: Push again, pop twice, then empty: passed
Test #4: TESTING ERROR: quack underflow
pop: quack underflow

```

Note:

- the quack is an *abstract* structure
  - the client cannot see how the quack has been implemented
- error handling is problematical:
  - *what does a client do with errors that are returned?*
  - *what is a 'serious' error, what is a 'usage' error?*
  - *how do you test that every error message is correct?*

## Client 2. reversing a string

Pushing the characters in a string onto a stack and then popping them off again until the stack is empty will reverse the order.

- for example: the string "abcde"
  - push('a'); push('b'); ...push('e'); will result in the stack

```
<<e,d,c,b,a>>
```

where *e* is at the top

- pop(); pop(); ... pop(); will result in 'e' being popped, then 'd', etc

切换行号显示

```

1 // revarg.c: reverse the chars in the first command-line
argument
2 #include <stdio.h>
3 #include <stdlib.h>
4 #include "quack.h"
5
6 int main(int argc, char *argv[]) {
7     Quack s = NULL;
8
9     if (argc >= 2) {
10         char *inputc = argv[1];
11         s = createQuack();
12         while (*inputc != '\0') {
13             push(*inputc++, s);
14         }
15         while (!isEmptyQuack(s)) {
16             printf("%c", pop(s));
17         }
18         putchar('\n');
19     }
20     return EXIT_SUCCESS;
21 }

```

对于获取字符串的每一个字符，不需要知道字符串的长度  
可以通过判断是否到达'\0'来实现

相当于\*(inputc++)，通过地址对数组赋值，终止条件就是找到字符串的结尾'\0'

Compiling and running:

```

prompt$ gcc quack.c revarg.c
prompt$ ./a.out stressed
desserts

```

### Client 3. a postfix calculator

Reminder:

- **infix notation:**  $A * (B + C) / D$
- **prefix notation:**  $/ * A + B C D$  (also called *Polish notation*)
- **postfix notation:**  $A B C + * D /$  (also called *reverse Polish notation*)

Given an expression in *postfix* notation, return its value

- for example, evaluating the following postfix expression:

2 1 3 + 2 5 \* \* 7 + \*

遇到数字就push，遇到运算符，就pop两次，然后将计算的结果在push，直到最后

means

(2 (( (1 3 +) (2 5 \*) \*) 7 +) \*)

which equals 94

How do we program this? Using a stack, and reading each character.

- when we get an operator character:
  - **pop** the top 2 elements off the quack

- operate (i.e. '+' or '\*') on the two elements
- **push** the result back onto the quack
- when we get a 'number', **push** it onto the quack ...
  - but the number may be many digits long
    - need to translate this string to a number
      - a *while-loop* reads each digit:
        - converts it to its numerical value
        - adds it to  $10 * \text{previous value}$

切换行号显示

```

1 // postfix.c
2 // a calculator for command-line postfix expressions
3 // reports an error if the expression contains anything but +,
*, integer, space, tab
4 #include <stdio.h>
5 #include <stdlib.h>
6 #include "quack.h"
7
8 #define PLUSCHAR '+'
9 #define MULTCHAR '*'
10
11 int main(int argc, char *argv[]) {
12     Quack s = NULL;
13     int error = 0;
14     int operandFound = 0;
15
16     if (argc >= 2) {
17         char *inputc = argv[1];
18         s = createQuack();
19         while (*inputc != '\0') { 判断是否到达字符串终点
20             int sum;
21             switch (*inputc) {
22                 case PLUSCHAR: push(pop(s) + pop(s), s); 遇到+号, 执行加法
23                     break;
24                 case MULTCHAR: push(pop(s) * pop(s), s); 遇到*号, 执行乘法
25                     break;
26                 case '0':
27                 case '1':
28                 case '2': 遇到数字, 说明可以操作, operandFound = 1
29                 case '3': 由于存在多位数字, 因此需要通过读取数字来得到最终的值
30                 case '4': 例如读取213
31                 case '5': 2, sum = 0*10 + 2 = 2
32                 case '6': 1, sum = 2*10 + 1 = 21
33                 case '7': 3, sum = 21*10 + 3 = 213
34                 case '8': 类似这样获取数字的实际结果
35                 case '9': operandFound = 1; case 里面的数字, 表示遇到第一个数字,
36                     sum = 0; 之后就是通过while循环获取其他数字
37                     while ((*inputc >= '0') && (*inputc <= '9')) {
38                         sum = 10 * sum + (*inputc - '0'); // notice
39                     }
40                     inputc++;
41                     push(sum, s);
42                     inputc--; // the while-loop reads one too many
43                     break; 最后多走了一步
44                 case ' ':
45                 case '\t':
46                     break;
47                 default: fprintf(stderr, "Invalid character %c\n",

```

```

*inputc);
48     }
49     inputc++;
50     }
51     if (operandFound) {
52         if (!isEmptyQuack(s)) { // stack must contain the
result
53             printf("%d\n", pop(s));
54         }
55         else {
56             fprintf(stderr, "Error: stack empty, no result\n");
57             error = 1;
58         }
59     }
60     if (!isEmptyQuack(s)) { // stack must now be empty
61         fprintf(stderr, "Error: extra operand(s)\n");
62         error = 1;
63     }
64     if (error) {
65         return EXIT_FAILURE;
66     }
67 }
68 return EXIT_SUCCESS;
69 }

```

If we want to use the array version of quack, compile using:

```

prompt$ gcc quack.c postfix.c
prompt$ ./a.out "2 3 +"
5
prompt$ ./a.out "2 4 5 + *"
18
prompt$ ./a.out "1 2 3 4 5 * + * +"
47
prompt$ ./a.out ""
prompt$ ./a.out
prompt$

```

What's the program doing?

- operands are operated on as they are put onto the quack
- e.g., quack contents for "1 2 3 4 5 \* + \* +":

```

push 1 <<1>>
push 2 <<2,1>>
push 3 <<3,2,1>>
push 4 <<4,3,2,1>>
push 5 <<5,4,3,2,1>>
oper * <<20,3,2,1>>
oper + <<23,2,1>>
oper * <<46,1>>
oper + <<47>>

```

## Stacks versus queues

If the data structure is a stack, then a *push* puts an element onto the **top**:

Operation	Resulting Stack	Return Value
-----------	-----------------	--------------

<b>push(1)</b>	1	
<b>push(2)</b>	2 1	
<b>push(3)</b>	3 2 1	
<b>push(4)</b>	4 3 2 1	
<b>pop()</b>	3 2 1	4
<b>pop()</b>	2 1	3
<b>pop()</b>	1	2
<b>pop()</b>		1

栈：在数列的队尾  
push，在数列的队尾  
pop  
  
队列：在数列的对头  
push，在数列的队尾  
pop  
  
输出队列是从后往前  
输出

where the stack read left-to-right is top-to-bottom.

If the data structure is a queue, then a 'push' puts an element onto the **bottom**:

Operation	Resulting Queue	Return Value
<b>push(1)</b>	1	
<b>push(2)</b>	1 2	
<b>push(3)</b>	1 2 3	
<b>push(4)</b>	1 2 3 4	
<b>pop()</b>	2 3 4	1
<b>pop()</b>	3 4	2
<b>pop()</b>	4	3
<b>pop()</b>		4

Stack：从顶上往下压，push 从  
顶上压  
pop 从顶上拿  
  
顶上插入，顶上删除  
push：top压栈，向下使劲  
pop：top  
  
栈的应用实例：  
手电筒插入电池  
叠盘子  
  
Queue：push队尾，pop从对头离  
开  
  
队尾插入，对头删除  
push：bottom，向上使劲  
pop：top  
  
队列应用实例：排队

You can see that the numbers are popped off in reverse order.

The difference between a *stack push* and a *queue push* is which end is used

- in a stack, a *push* and *pop* both work at the top
- in a queue, a *pop* takes from the head (i.e. top), a *queue push* adds to the tail (i.e. bottom)

We'll call a *queue push* a **qush** from now on:

- a **qush** inserts an element at the bottom of the data structure
- in contrast to a **push**, which inserts an element at the top

The same *pop* is used for a stack and a queue:

Some operations do not know or care whether the ADT is a stack or a queue ...

- *createQuack*
- *isEmptyQuack*
- *showQuack*

If you want to model a stack in an application, *qush* must not be used

If you want to model a queue in an application, *push* must not be used

In real-life, *qush* and *push* can be mixed to model behaviour

- e.g. a supermarket queue, sometimes someone at the tail of the queue gets served first

The function `qush()` has not been implemented yet in the ADT

- to implement it we need to add a function *qush()* to *quack.c*
- and add its prototype to *quack.h*

**This is an exercise for this week.**

#### Client 4. separate stack and queue

Here is a simple program that mixes a stack and a queue

- both are declared as *quacks*
  - but one uses *push* and *pop*
  - the other uses *qush* and *pop*

切换行号显示

```
1 // separateQuack.c: have both a
  program
2 #include <stdio.h>
3 #include "quack.h"
4
5 int main(void) {
6     Quack s = NULL;
7     Quack q = NULL;
8
9     s = createQuack();
10    q = createQuack();
11
12    push(1, s);
13    push(2, s);
14    printf("pop from s produces %d\n", pop(s));
15    printf("pop from s produces %d\n", pop(s));
16
17    qush(1, q);
18    qush(2, q);
19    printf("pop from q produces %d\n", pop(q));
20    printf("pop from q produces %d\n", pop(q));
21
22    return EXIT_SUCCESS;
23 }
```

```
//used as queue, push element from bottom
void qush(int data, Quack qs){
    if (qs == NULL){
        fprintf(stderr, "qush: quack not initialised\n");
    }
    else {
        if (qs->top >= HEIGHT - 1) {
            fprintf(stderr, "qush: quack overflow\n");
        }
        else {
            for (int i = qs->top + 1; i > 0; i--) {
                qs->array[i] = qs->array[i-1];
            }
            qs->array[0] = data;
            qs->top++;
        }
    }
    return;
}
```

Assuming we have implemented *qush*, compiling and executing:

```
prompt$ gcc quack.c separateQuack.c
prompt$ ./a.out
pop from s produces 2
pop from s produces 1
```

```
pop from q produces 1
pop from q produces 2
```

Notice that the pushed/qushed integers are popped off in opposite order in the two data structures.

### Client 5. mixed stack and queue

You can also mix 'pushes' and 'qushes' on the one quack:

切换行号显示

```
1 // mixedQuack.c: mix qush and push in a quack
2 #include <stdio.h>
3 #include "quack.h"
4
5 int main(void) {
6     Quack s = NULL;
7
8     s = createQuack();
9
10    printf("push 1 and 2\n");
11    push(1, s);
12    push(2, s);
13    printf("qush 3 and 4\n");
14    qush(3, s);
15    qush(4, s);
16    showQuack(s);
17    printf("pop produces %d\n", pop(s));
18    printf("pop produces %d\n", pop(s));
19    printf("pop produces %d\n", pop(s));
20    printf("pop produces %d\n", pop(s));
21    return EXIT_SUCCESS;
22 }
```

- ... interesting, but maybe hard to think of an application

### Client 6. 'circular' queue

This is an application that uses a queue only.

This is the 1<sup>st</sup> century Jewish historian/philosopher/mathematician **Flavius Josephus**





In the book:

- *Matters Mathematical* (Chelsea Publishing, 1978) by *Herstein and Kaplansky* there is a story/legend about *Flavius Josephus*
  - *Flavius* was in a group of **10** people that were surrounded by the Romans
  - the group decided that they would rather die than surrender
  - *Flavius* suggested forming a ring, and going around the ring clockwise repeatedly
    - ... killing every **3<sup>rd</sup>** person
    - ... until just one person remained
    - ... who would kill himself
  - *Flavius* placed himself at position **4**

```

      2  3
    1      4 <-- Flavius
  10      5
    9      6
      8  7

```

Starting at 1, you eliminate every 3<sup>rd</sup> person, people would be removed in the order:

```
3 6 9 2 7 1 8 5 10
```

The last remaining person was the 4<sup>th</sup>, Flavius.

This strategy can be implemented using a queue.

## 切换行号显示

```

1 // Josephus.c: use a queue to simulate a ring of n people, and
2 // eliminate every mth person until there is just a single
person remaining
3 #include <stdio.h>
4 #include <stdlib.h>
5 #include "quack.h"
6
7 int main(int argc, char *argv[]) {
8     Quack q = NULL;
9
10    int n, m;
11    if ((argc != 3) ||
12        (sscanf(argv[1], "%d", &n) != 1) ||
13        (sscanf(argv[2], "%d", &m) != 1)) {
14        fprintf(stderr, "Usage: %s total eliminate\n", argv[0]);
15        return EXIT_FAILURE;
16    }
17    q = createQuack();
18    int i;
19    for (i=1; i<=n; i++) { // populate the queue
20        qush(i, q);        // top = '1' and bottom = 'n'
21    }
22    showQuack(q);
23    int person=0;
24    while (!isEmptyQuack(q)) { // continue until empty
25        for (i=0; i<m-1; i++) { // skip m-1 people
26            qush(pop(q), q);    // move from front to back
27        }
28        person = pop(q);        // if this person ...
29        if (!isEmptyQuack(q)) { // ... is not last one ...
30            printf("byebye %d\n", person); // eliminate him
31        }
32    }
33    printf("%d is the only person left\n", person);
34    return EXIT_SUCCESS;
35 }

```

判断是否为数字

pop之后，从一端移动到另一端，实现转圈

Compiling and executing with arguments 10 and 3:

```

prompt$ dcc quack.c Josephus.c
prompt$ ./a.out 10 3
Quack: <<1, 2, 3, 4, 5, 6, 7, 8, 9, 10>>
goodbye 3
goodbye 6
goodbye 9
goodbye 2
goodbye 7
goodbye 1
goodbye 8
goodbye 5
goodbye 10
Quack << >>
4 is the only person left

```

The <<>> notation shows the contents of the queue initially and at the end.

Notice:

- how the queue was initialised

- how people are 'skipped'
  - a person is 'popped' then 'qushed', which moves him from the head to the tail (keeps them alive)
- **the queue itself is not circular, the behaviour is circular**
  - the program loops until the queue is empty
  - after skipping  $m$  people, the next person is 'popped'
    - if that person
      - is not the last, they stay 'popped', and print *goodbye*
      - is the last, the quack must be empty, and the loop terminates with this person

'Reality' check

- Actually there were 39 soldiers in the group, every 7<sup>th</sup> was eliminated, and Flavius was 17<sup>th</sup>
- Flavius decided not to commit suicide
- The Romans were impressed by Flavius
- He ended up joining the Romans, legend has it

The dilemma that *Josephus* had 2000 years ago, children can have every day!

- when children need to choose, or decide who will get something, they often use rhymes
- example 1
  - with a flower that has  $n$  petals, pick the petals and repeat until there is just one petal left:

```
she loves me,
she loves me not,
she loves me,
she loves me not,
...
```

- this is *Josephus ring* solution with  $m=2$
- example 2
  - go around a group of  $n$  children looking for someone to remove from the group:

```
1      2      3      4
EEny MEEny MIny MOE
5      6      7      8
CATCH a TIGer BY the TOE
9      10     11     12
IF he HOLlers LET him GO
13     14     15     16
EEny MEEny MIny MOE
17     18     19     20
'O' 'U' 'T' SPELLS
21
OUT!
```

- *Josephus ring* is being used:  $m=21$

ADTs (2019-06-19 14:11:41由AlbertNymeyer编辑)