目录

Graph search

Searching a graph can have many aims:

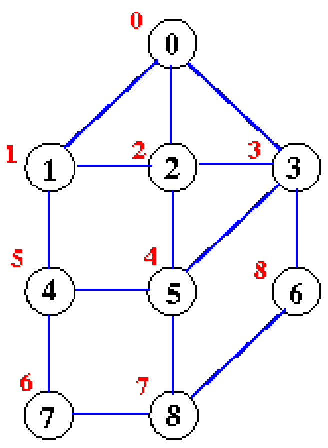
- can I reach every vertex in the graph (is it connected)?
- is one vertex reachable starting from some other vertex?
- what is the shortest path from vertex *v* to *w*?
- which vertices are reachable from a vertex? (transitive closure)
- is there a cycle that passes through all the graph? (*tour*)
- is there a tree that links all vertices? (*spanning tree*)
    - what is the *minimum* spanning tree?
- are two graphs "equivalent"? (*isomorphism*)

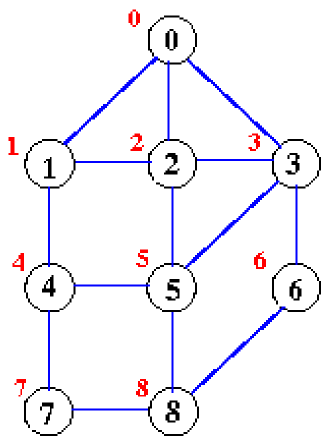
A search is almost never 'random': it uses an underlying strategy:

- depth-first search DFS
- breadth-first search BFS

breadth first and depth first

Example:


Depth–First Search


Breadth–First Search

Order is given by the 'red' labels

- in this example the label ordering is breadth-first (layer by layer)

DFS descends by selecting the first available unvisited node

- select 0
- connect {1,2,3}
    - select 1
    - connect {2, 4}
        - select 2
        - connect {3, 5}
            - select 3
            - connect {5, 6}
                - select 5
                - connect {4, 8}
                    - select 4
                    - connect {7}
                        - select 7
                        - connect {8}
                            - select 8
                            - connect {6}
                                - select 6
                                - connect {} no sites left unvisited

BFS descends by systematically visiting the nodes in order of level

- select 0
- connect {1,2,3}
    - select 1
    - connect {4}
    - select 2
    - connect {5}
    - select 3
    - connect {6}
        - select 4
        - connect {7}
        - select 5

            - connect {8}
                - select 6
                - connect {}
                    - select 7
                    - connect {}
                    - select 8
                    - connect {} no sites left unvisited

These two 'strategies' actually use the same algorithm. They differ only in their use of data structure:

- DFS uses a stack
- BFS uses a queue

Here is the pseudo-algorithm for **Depth/Breadth**-first search:

```
push the root node onto a stack/queue
while (stack/queue is not empty) {
    remove a node from the stack/queue
    if (node is a goal node)
        return 'success'
    push all children of node onto the stack/queue
}
return 'failure'
```

Depth-First Search, using a stack

When searching we need to remember which nodes we've *visited*:

- to avoid cycles
- to make sure every node gets visited

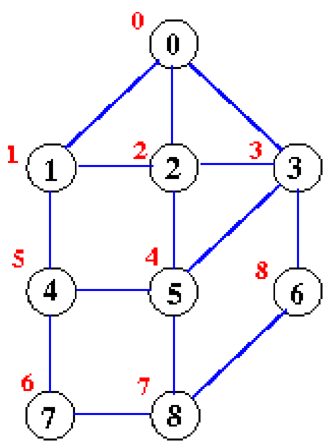
Generally a global array variable **visited[0 .. numVertices-1]** is used

- array indices correspond to vertices
- initialise all elements to -1, meaning unvisited
- when a vertex is visited, the index is set to its 'visit order' number
    - this is simply a 'count' that gets incremented each time a new node is visited

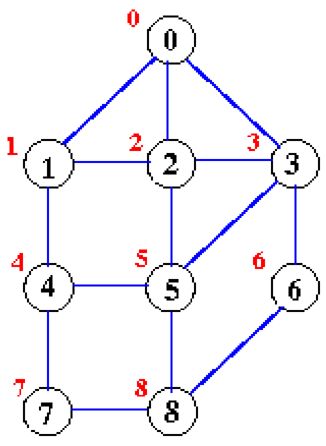
Many strategies are possible

- obvious strategy: select the smallest unvisited vertex
- less obvious: select an arbitrary unvisited vertex

For example, here is the earlier graph again



Depth–First Search



Breadth–First Search

The *visited* array starts as {-1,-1,-1,-1,-1,-1,-1,-1,-1}

We select the root **0** first

choice	visit	resulting visited array
any node	**0**	{ **0**,-1,-1,-1,-1,-1,-1,-1,-1}
1 2 3	**1**	{ 0, **1**,-1,-1,-1,-1,-1,-1,-1}
0 2 4	**2**	{ 0, 1, **2**,-1,-1,-1,-1,-1,-1}
0 1 3 5	**3**	{ 0, 1, 2, **3**,-1,-1,-1,-1,-1}
0 2 5 6	**5**	{ 0, 1, 2, 3,-1, **4**,-1,-1,-1}
2 4 6 8	**4**	{ 0, 1, 2, 3, **5**, 4,-1,-1,-1}
1 5 7	**7**	{ 0, 1, 2, 3, 5, 4,-1, **6**,-1}
4 8	**8**	{ 0, 1, 2, 3, 5, 4,-1, 6, **7**}
6 7	**6**	{ 0, 1, 2, 3, 5, 4, **8**, 6, 7}

Let's try a different starting vertex: this time start at vertex **5**:

choice	visit	resulting visited array
any node	**5**	{-1,-1,-1,-1,-1, **0**,-1,-1,-1}
2 3 4 8	**2**	{-1,-1, **1**,-1,-1, 0,-1,-1,-1}
0 1 3 5	**0**	{ **2**,-1, 1,-1,-1, 0,-1,-1,-1}
1 2 3	**1**	{ 2, **3**, 1,-1,-1, 0,-1,-1,-1}
0 2 4	**4**	{ 2, 3, 1,-1, **4**, 0,-1,-1,-1}
1 5 7	**7**	{ 2, 3, 1,-1, 4, 0,-1, **5**,-1}
4 8	**8**	{ 2, 3, 1,-1, 4, 0,-1, 5, **6**}
5 6 7	**6**	{ 2, 3, 1,-1, 4, 0, **7**, 5, 6}
3 8	**3**	{ 2, 3, 1, **8**, 4, 0, 7, 5, 6}

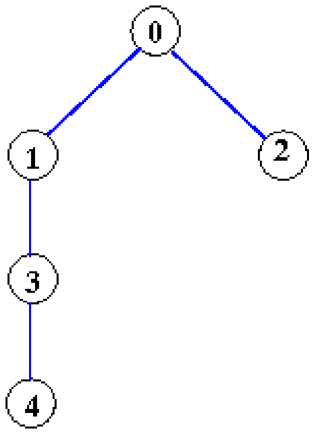
The array *visited[]* **is the depth-first order**. It says:

- visited[5] = 0th
- visited[2] = 1st
- visited[0] = 2nd
- visited[1] = 3rd
- visited[4] = 4th
- visited[7] = 5th
- visited[8] = 6th
- visited[6] = 7th
- visited[3] = 8th

```
The visited array indicates the order of the search.
```

Can anything go wrong during the traversal?

- *Yes, we can hit a deadend!*



choice	visit	resulting visited array
any node	**0**	{**0**,-1,-1,-1,-1,}
1 2	**1**	{ 0, **1**,-1,-1,-1}
0 3	**3**	{ 0, 1,-1, **2**,-1}
1 4	**4**	{ 0, 1,-1, 2, **3**}
	deadend	

- **there is still an unvisited vertex in the array**

How do we 'find' it?

- we need to **backtrack**
  - we go back to vertex 0, and then visit vertex 1
    - again a *deadend*
    - ... but all nodes have been visited, so we are finished
  - final DFS path is visited = {0, 1, 4, 2, 3}

So we cannot expect DFS to visit every vertex in a single forward traversal

- we sometimes need to *backtrack*

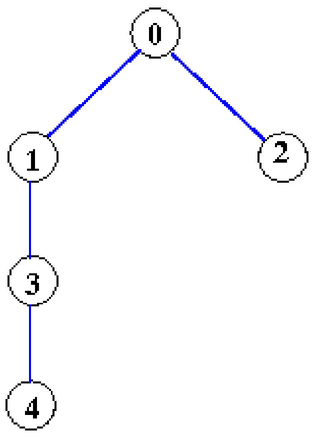
But how do we backtrack?

- we use a stack!
  - vertices are pushed onto the stack when we have 1 or more adjacent vertices to visit
  - to actually visit a vertex, we simply pop it from the stack
- *only when the stack is empty have we visited everyone!*

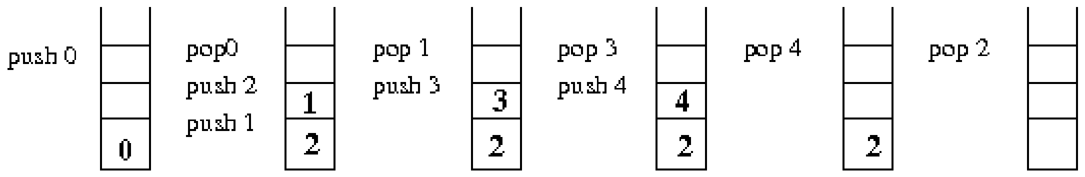
Using a stack in DFS means:

- when we *visit*, we *pop* the next vertex off the stack
- after a visit, we *push* the adjacent vertices onto the stack
- a deadend occurs when there are no more vertices, so no *push* is possible
  - we then *pop* a vertex
    - ... this vertex we saw earlier and need to visit
- only when the stack is empty have we visited every vertex

Consider the graph with the deadend again



The following stack operations are carried out:



Code:

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```
// dfsquack.c: traverse a graph using DFS and a stack implementation
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>
```

```c
#include "Graph.h"
#include "Quack.h"

void dfs(Graph, Vertex, int);

#define WHITESPACE 100

int readNumV(void) { // returns the number of vertices numV or -1
   int numV;
   char w[WHITESPACE];
   scanf("%[ \t\n]s", w);  // skip leading whitespace
   if ((getchar() != '#') ||
       (scanf("%d", &numV) != 1)) {
       fprintf(stderr, "missing number (of vertices)\n");
       return -1;
   }
   return numV;
}

int readGraph(int numV, Graph g) { // reads number-number pairs until EOF
   int success = true;             // returns true if no error
   int v1, v2;
   while (scanf("%d %d", &v1, &v2) != EOF && success) {
       if (v1 < 0 || v1 >= numV || v2 < 0 || v2 >= numV) {
          fprintf(stderr, "unable to read edge\n");
          success = false;
       }
       else {
          insertE(g, newE(v1, v2));
       }
   }
   return success;
}

void dfs(Graph g, Vertex v, int numV) {
   int *mallocArray(int numV) {
      int *v = malloc(numV * sizeof(int));
      if (v == NULL) {
         fprintf(stderr, "Out of memory\n");
         exit(1);
      }
      int i;
      for (i=0; i<numV; i++) {
         v[i] = UNVISITED;
      }
      return v;
   }
   void showArray(int *v, int numV) {
      int i;
      printf("Visited: {");
      for (i=0; i<numV; i++) {
         printf("%d", v[i]);
         if (i <= numV-2) {
            printf(", ");
         }
      }
      printf("}\n");
      return;
   }

   int *visited = mallocArray(numV);
   Quack s = createQuack();
   push(v, s);
   showQuack(s);
   int order = 0;
   while (!isEmptyQuack(s)) {
      v = pop(s);
      if (visited[v] == UNVISITED) {
         showArray(visited, numV);
         //printf("visited[%d]=%d\n", v, order);
         visited[v] = order++;
         Vertex w;
         for (w=numV-1; w>=0; w--) {
            if (isEdge(g, newE(v,w))) {
               push (w, s);
            }
         }
      }
      showQuack(s);
   }
   showArray(visited, numV);
   free(visited);
   return;
}

int main (void) {
    int numV;
    if ((numV = readNumV()) >= 0) {
        Graph g = newGraph(numV);
        if (readGraph(numV, g)) {
            showGraph(g);
            dfs(g, 0, numV);
        }
        g = freeGraph(g);
        g = NULL;
    }
    else {
        return EXIT_FAILURE;
    }
    return EXIT_SUCCESS;
}
```

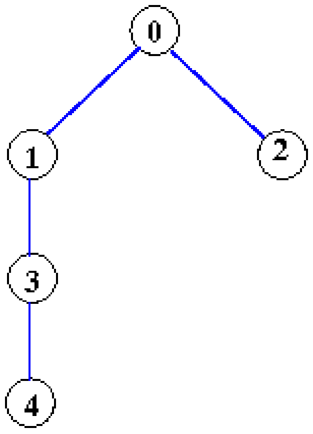
We compile this code using:

```
gcc dfsquack.c GraphAM.c Quack.c
```

If we execute this code on the graph:

```
#5
0 1 0 2
1 3
3 4
```

which corresponds to the simple graph:



then the output is:

```
V=5, E=4
<0 1> <0 2>
<1 0> <1 3>
<2 0>
<3 1> <3 4>
<4 3>
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1}
Quack: <<1, 2>>
Visited: {0, -1, -1, -1, -1}
Quack: <<0, 3, 2>>
Quack: <<3, 2>>
Visited: {0, 1, -1, -1, -1}
Quack: <<1, 4, 2>>
Quack: <<4, 2>>
Visited: {0, 1, -1, 2, -1}
Quack: <<3, 2>>
Quack: <<2>>
Visited: {0, 1, -1, 2, 3}
Quack: <<0>>
```

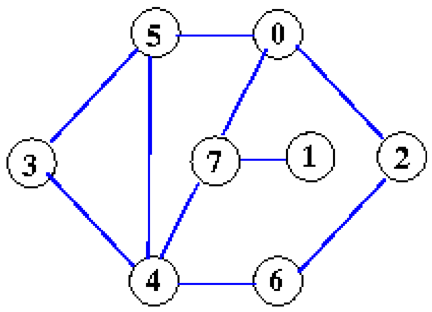
```
Quack: << >>
Visited: {0, 1, 4, 2, 3}
```

Here we see:

- the starting vertex 0 is pushed
- 0 is popped and its neighbours 1 and 2 are pushed
  - visited[0] = **0**
- 1 is popped and its neighbours 0 and 3 are pushed
  - visited[1] = **1**
- 0 is popped and ignored as it is in array *visited*
- 3 is popped and its neighbours 1 and 4 are pushed
  - visited[3] = **2**
- 1 is popped and is ignored
- 4 is popped and its neighbour 3 is pushed
  - visited[4] = **3**
- 3 is popped and is ignored
- 2 is popped and its neighbour 0 is pushed
  - visited[2] = **4**
- 0 is popped
- *quack is empty*

The role of the stack in DFS is crucial. It facilitates backtracking.

What about a more substantial graph:



It is represented by the input data

```
8
0-2 0-5 0-7 2-6 1-7 4-7 4-6 4-3 3-5 4-5
```

If we want to do a DFS starting from vertex 0, *dfs()* generates the output:

```
V=8, E=10
0-7 0-5 0-2
1-7
2-6 2-0
3-5 3-4
4-5 4-3 4-6 4-7
```

```
5-4 5-3 5-0
6-4 6-2
7-4 7-1 7-0
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1, -1, -1, -1}
Quack: <<2, 5, 7>>
Visited: {0, -1, -1, -1, -1, -1, -1, -1}
Quack: <<0, 6, 5, 7>>
Quack: <<6, 5, 7>>
Visited: {0, -1, 1, -1, -1, -1, -1, -1}
Quack: <<2, 4, 5, 7>>
Quack: <<4, 5, 7>>
Visited: {0, -1, 1, -1, -1, -1, 2, -1}
Quack: <<3, 5, 6, 7, 5, 7>>
Visited: {0, -1, 1, -1, 3, -1, 2, -1}
Quack: <<4, 5, 5, 6, 7, 5, 7>>
Quack: <<5, 5, 6, 7, 5, 7>>
Visited: {0, -1, 1, 4, 3, -1, 2, -1}
Quack: <<0, 3, 4, 5, 6, 7, 5, 7>>
Quack: <<3, 4, 5, 6, 7, 5, 7>>
Quack: <<4, 5, 6, 7, 5, 7>>
Quack: <<5, 6, 7, 5, 7>>
Quack: <<6, 7, 5, 7>>
Quack: <<7, 5, 7>>
Visited: {0, -1, 1, 4, 3, 5, 2, -1}
Quack: <<0, 1, 4, 5, 7>>
Quack: <<1, 4, 5, 7>>
Visited: {0, -1, 1, 4, 3, 5, 2, 6}
Quack: <<7, 4, 5, 7>>
Quack: <<4, 5, 7>>
Quack: <<5, 7>>
Quack: <<7>>
Quack: << >>
Visited: {0, 7, 1, 4, 3, 5, 2, 6}
```

Multiple ADTs: quack.h and graph.h

- *dfsquack.c* above used both the *Graph* and the *Quack* abstract data types
  - we need to include both *quack.h* and *graph.h*
    - we can choose between implementations *quackAR.c* and *quackLL.c* at compile time
    - we can choose between implementations *graphAR.c* and *graphAM.c* at compile time
  - that is, use:
    - **gcc -Wall -Werror -O quackAR.c graphAM.c dfsquack.c** or
    - **gcc -Wall -Werror -O quackAR.c graphAL.c dfsquack.c** or
    - **gcc -Wall -Werror -O quackLL.c graphAM.c dfsquack.c** or
    - **gcc -Wall -Werror -O quackLL.c graphAL.c dfsquack.c**

Performance

- number of pushes and pops
    - should be the same (stack is empty at the end)
- number of pushes of a vertex *v* = vertex degree of *v*
- total number of pushes
    - = sum of all the vertex degrees of vertices v in the graph

      ```
        If E is finite, then the total sum of vertex degrees
      is equal to twice the number of edges.
      ```

    - = 2 * number of edges
- this means the complexity is linear in the number of edges, *O(E)*
    - **what does this mean? ...**
    - *how many edges are there?*
        - the worst case is a dense graph: *E = V*(V-1)/2*
            - so the complexity is linear in the number *V*(V-1)/2* **???**
            - this means it is quadratic in *V*, or $O(V^2)$
        - if it is sparse, then it will be less than quadratic
- often said that DFS is linear in the size of the graph

Depth-First Search, using recursion

The implementation above used our own stack to 'remember' which path it was traversing and do backtracking.

Instead of our own stack, however, we can use the system stack by using recursion:

- in the literature, this is often done in combination with *global variables*
    - global variables are not permitted in this course however

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```
// THIS IS FOR YOUR INFORMATION -- DO NOT USE
int order;     // global variable for the visiting order; yuk!
int *visited;  // global array of visiting orders; yuk!
               // indexed by vertex 0..g->nV

void dfs(Graph g, Vertex v) { // housekeeping function
   int i;
   visited = malloc(g->nV*sizeof(int));
   // CHECK FOR NULL OMITTED AS THIS CODE SHOULD NOT BE USED
   for (i = 0; i < g->nV; i++) {
      visited[i] = UNVISITED;
   }
   order = 0;
   dfsR(g, v);
}

```

```
void dfsR(Graph g, Vertex v) { // the actual recursive DFS
   visited[v] = order++;
   Vertex w;
   for (w = 0; w < g->nV; w++) { // THIS LINE 'REPLACES' THE STACK ABOVE
      if (g->edges[v][w] && visited[w] == UNVISITED) {
         dfsR(g, w);
      }
   }
}
```

A version without global variables is:

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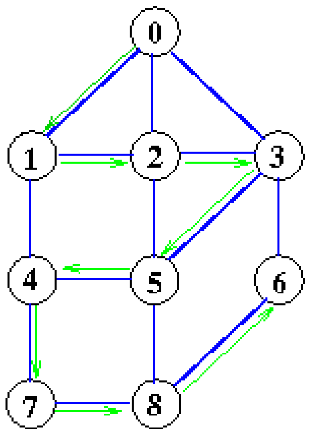
```
#define UNVISITED -1
void dfs(Graph g, Vertex v, int numV) {
   int *mallocArray(int numV) {
      int *array = malloc(numV * sizeof(int));// l
      if (array == NULL) {                     // o
         fprintf(stderr, "Out of memory\n");   // c
         exit(1);                              // a
      }                                        // l
      int i;                                   // f
      for (i=0; i<numV; i++) {                 // u
         array[i] = UNVISITED;                 // n
      }                                        // c
      return array;                            // t
   }
   void showArray(int *array, int numV) {
      int i;                                   // l
      printf("Visited: {");                    // o
      for (i=0; i<numV; i++) {                 // c
         printf("%d", array[i]);               // a
         if (i <= numV-2) {                    // l
            printf(", ");                      // f
         }                                     // u
      }                                        // n
      printf("}\n");                           // c
      return;                                  // t
   }
   int *visited = mallocArray(numV);
   int order = 0;
   dfsR(g, v, numV, &order, visited);
   showArray(visited, numV);
   free(visited);
   return;
}

void dfsR(Graph g, Vertex v, int numV, int *order, int *visited) {
   visited[v] = *order;
   *order = *order+1;
   Vertex w;
   for (w=0; w < numV; w++) {
      if (isEdge(g, newE(v,w)) && visited[w]==UNVISITED)
{
         dfsR(g, w, numV, order, visited);
      }
   }
   return;
}
```

The graph (that compared DFS and BFS) from above:



has the input file is:

```
9
0-1 0-2 0-3 1-4 1-2 2-3 2-5 3-5 3-6 4-5 4-7 5-8 7-8 6-8
```

and assuming the starting vertex is 0, the output is:

```
Visited: {0, 1, 2, 3, 5, 4, 8, 6, 7}
```

This is same DFS order that we found before, and is here indicated by the green arrows.

animation of DFS

Depth-First Search on disconnected graphs, using recursion

If the graph is disconnected, then the DFS algorithm above will not work:

- it uses edges to get from one vertex to another

Disconnected graphs are sometimes referred to as *forests* of graphs

- each 'tree' in the 'forest' is a graph

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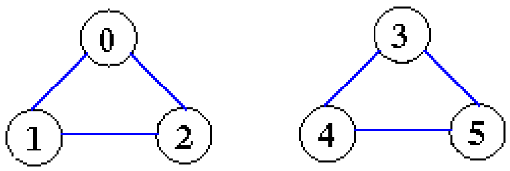
```
void dfsDisc(Graph g, Vertex v, int numV) { // handles disconnected graphs
   int *mallocArray(int numV) {
      ... as above
   }
   void showArray(int *array, int numV) {
      ... as above
   }
   int *visited = mallocArray(numV);
   int order = 0;
   Vertex newv = v;                       // this is the starting vertex
   int finished = 0;
   while (!finished) {                    // as long as there are vertices

      dfsR(g, newv, numV, &order, visited);

      Vertex w;
      finished = 1;                       // assume all vertices visited
      for (w = 0; w < numV && finished; w++) { // look for a new vertex
         if (visited[w] == UNVISITED) {
            finished = 0;                 // found an unvisited vertex
            newv = w;
         }
      }
   }
   showArray(visited, numV);
   free(visited);
   return;
}
```

If the (disconnected) input graph is:

```
6
0-1 0-2 1-2 3-4 3-5 4-5
```

corresponding to:



and assuming the starting vertex is 0, then the DFS is:

```
Visited: {0, 1, 2, 3, 4, 5}
```

Notice:

- the first (sub) graph's DFS search begins at vertex 0
- the second (sub) graph's DFS search begins at vertex 3

Cycle Detection (using DFS)

A graph that does not contain a cycle is called a tree, so

- asking whether a graph contains no cycles is equivalent to
- asking whether a graph is a tree

The *recursive* DFS algorithm, *dfsR()* we saw earlier is:

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```
void dfsR(Graph g, Vertex v, int numV, int *order, int *visited) {
   visited[v] = *order;
   *order = *order+1;
   Vertex w;
   for (w=0; w < numV; w++) {
      if (isEdge(g, newE(v,w)) && visited[w]==UNVISITED) {
         dfsR(g, w, numV, order, visited);
      }
   }
   return;
}
```

To search for a cycle, the function *main()* calls *searchForCycle()*:

- which does housekeeping and in turn
- calls the recursive function *hasCycle()*

The function *hasCycle()* is a simple modification of *dfsR()*. The changes are:

- introduce a variable *found* to terminate the search if a cycle is found
  - this is a sort of early-exit
  - this variable must be passed up the recursive calls
- separate the 2 conditions in the for-loop
  - if *w* is adjacent to *v* then
    - if *w* is UNVISITED then recurse (in other words, keep searching)
    - else *w* has been visited before and **we have a cycle**
      - (assuming we are not going backwards along the same path)
        - to avoid going backwards, we pass 2 vertices to *hasCycle()*

The program is as follows:

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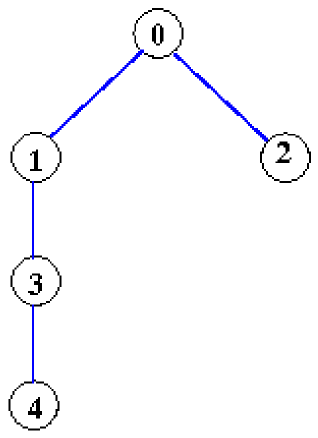
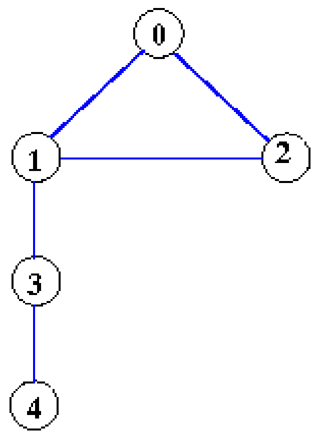
```
void searchForCycle(Graph g, int v, int numV) {
   int *mallocArray(int numV) {
      // as before
   }
   void showArray(int *vis, int numV) {
      // as before
   }
   int *visited = mallocArray(numV);
   int order = 0;

   if (hasCycle(g, numV, v, v, &order, visited)) {
      printf("found a cycle\n");
   }
   else {
      printf("no cycle found\n");
   }
   showArray(visited, numV);
   free(visited);
   return;
}

int hasCycle(Graph g, int numV, Vertex fromv, Vertex v, int *order, int *visited) {
   int retval = 0;
   visited[v] = *order;
   *order = *order+1;
   Vertex w;
   for (w=0; w<numV && !retval; w++) {
      if (isEdge(g, newE(v,w))) {
         if (visited[w]==UNVISITED) {
            printf("traverse edge %d-%d\n", v, w);
            retval = hasCycle(g, numV, v, w, order, visited);
         }
         else {
            if (w != fromv) { // exclude the vertex we've just come from
               printf("traverse edge %d-%d\n", v, w);
               retval = 1;
            }
         }
      }
   }
   return retval;
}
```

If we input the graph on the left below:





a program that calls this function will output:

```
found a cycle
```

Alternatively, if the input is the graph on the right, it will output:

```
no cycle found
```

Eulerian cycles (using DFS)

An Eulerian **path** in a graph is a path that includes every edge exactly once

- this path may include many visits to the same vertex

In turn, an Eulerian **cycle** is a special case of a Eulerian path in which the start and end points are the same

Animation of the *Konigsburg Bridge Problem*

- http://www.youtube.com/watch?v=3bUvjajakGM

Graph animation of finding an Euler cycle

- http://www.cs.sunysb.edu/~skiena/combinatorica/animations/euler.html

Well-known properties of Eulerian paths and cycles:

```
A connected graph has an Eulerian cycle if all its vertices
have even degree.
```

```
A connected graph has no Eulerian cycle if any of its vertices
has odd degree.
```

```
A connected graph has an Eulerian path if it has exactly 2
vertices of odd degree, and all the rest have even degree.
```

```
A connected graph has no Eulerian path if it has more than 2
vertices of odd degree.
```

So, a graph with more than 2 vertices of odd degree has no Eulerian cycle or path.

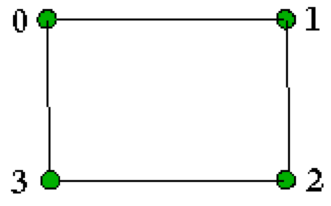
```
If a graph contains a Eulerian path, it cannot contain a
Eulerian cycle, and vice versa.
```

```
A graph that consists of vertices all with even degree is
often called an '''''Eulerian graph'''''
```

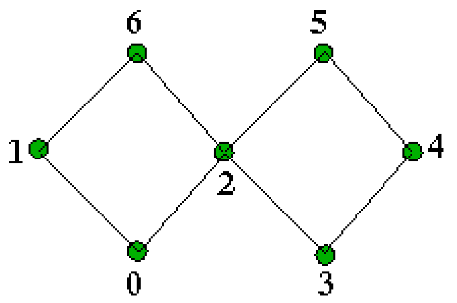
There is a simple algorithm to find an Eulerian cycle in an Eulerian graph

- it uses a DFS, hence requires a stack (to backtrack when you reach a deadend).

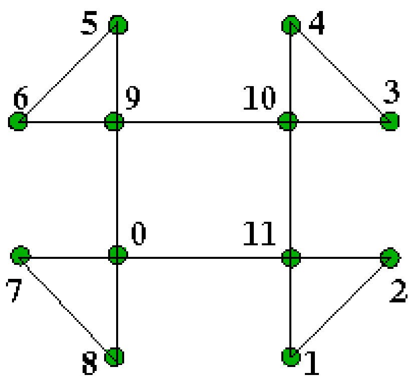
Examples of Eulerian graphs:



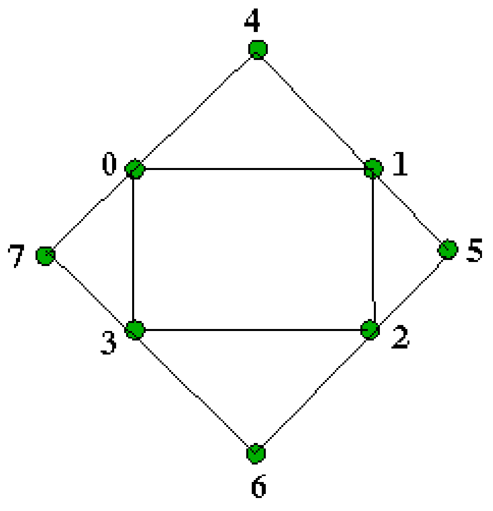
'box'



'bow'



'propbows'



'concsquares'

Eulerian Cycle Algorithm

Assume graph is connected and is Eulerian.

1. Read the Eulerian graph and initialise a stack.

2. Choose any vertex *v* and push *v*.
3. While the stack is not empty:
    - if the top of stack vertex has one of more adjacent vertices
        - select arbitrarily the largest vertex *w*
        - push *w*
        - remove edge *v*-*w*
    - else pop the top vertex and print it

The sequence of vertices that is printed is an Eulerian cycle. How does the algorithm work?

- we traverse the graph by pushing vertices on the stack and removing edges that lead to them
    - the vertex that is pushed is the neighbour of the previous vertex that is pushed
- we continue to push vertices as long as we can
- if we have a vertex on the stack with no neighbours, we start to pop and print

The function *findEulerCycle()* below implements the algorithm:

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```
void findEulerCycle(Graph g, int numV, Vertex startv) {
   Quack s = createQuack();
   printf("Eulerian cycle: ");

   push(startv, s);
   while (!isEmptyQuack(s)) {
      Vertex v = pop(s); // v is the top of stack vertex and ...
      push(v, s);        // ... the stack has not changed
      Vertex w;
      if ((w = getAdjacent(g, numV, v)) >= 0) {
         push(w, s);     // push a neighbour of v onto stack
         removeE(g, newE(v, w)); // remove edge to neighbour
      }
      else {
         w = pop(s);
         printf("%d ", w);
      }
   }
   putchar('\n');
}

Vertex getAdjacent(Graph g, int numV, Vertex v) {
   // returns the Largest Adjacent Vertex if it exists, else -1
   Vertex w;
   Vertex lav = -1; // the adjacent vertex
   for (w=numV-1; w>=0 && lav==-1; w--) {
      Edge e = newE(v, w);
```

```
        if (isEdge(g, e)) {
           lav = w;
        }
     }
     return lav;
  }
```

For example: for the **box** graph above:

```
push 0
push 3 and remove 0-3
push 2 and remove 3-2
push 1 and remove 2-1
push 0 and remove 1-0 <-- at this point all the edges have
been removed, and 0 is isolated
pop 0 and print 0
pop 1 and print 1
pop 2 and print 2
pop 3 and print 3
pop 0 and print 0
```

- **Eulerian cycle: 0 1 2 3 0**
- *It is not obvious why you need a stack here: 5 pushes were followed by 5 pops.*

We can reach a deadend during the traversal

- if the top vertex on the stack has no adjacent vertices, the traversal has reached a deadend
  - (remember that we are removing edges as we traverse the graph)
- but there may be vertices on the stack that **do** have branches to vertex neighbours
  - if there is one, then a vertex neighbour is pushed and the traversal continues
  - this is a process of **back-tracking** of course
- the process stops when branches have been taken
  - this will happen when all edges have been removed
  - every vertex on the stack will have been popped and printed
    - we may see the same vertex many times, but each edge is traversed just once

In the next example, for the **bow** graph above, we see backtracking in action:

```
push 0
push 2 and remove 0-2
push 6 and remove 2-6
push 1 and remove 6-1
push 0 and remove 1-0
pop 0 and print 0
pop 1 and print 1
pop 6 and print 6
push 5 and remove 2-5
push 4 and remove 5-4
push 3 and remove 4-3
push 2 and remove 3-2
```

```
pop 2 and print 2
pop 3 and print 3
pop 4 and print 4
pop 5 and print 5
pop 2 and print 2
pop 0 and print 0
```

- **Eulerian cycle: 0 1 6 2 3 4 5 2 0**

You may need to backtrack many times of course. Consider the **propbows** graph above:

```
push 0
pushed 11 and remove edge 0-11
pushed 10 and remove edge 11-10
pushed 9 and remove edge 10-9
pushed 6 and remove edge 9-6
pushed 5 and remove edge 6-5
pushed 9 and remove edge 5-9
pushed 0 and remove edge 9-0
pushed 8 and remove edge 0-8
pushed 7 and remove edge 8-7
pushed 0 and remove edge 7-0
popped 0 and print 0
popped 7 and print 7
popped 8 and print 8
popped 0 and print 0
popped 9 and print 9
popped 5 and print 5
popped 6 and print 6
popped 9 and print 9
pushed 4 and remove edge 10-4
pushed 3 and remove edge 4-3
pushed 10 and remove edge 3-10
popped 10 and print 10
popped 3 and print 3
popped 4 and print 4
popped 10 and print 10
pushed 2 and remove edge 11-2
pushed 1 and remove edge 2-1
pushed 11 and remove edge 1-11
popped 11 and print 11
popped 1 and print 1
popped 2 and print 2
popped 11 and print 11
popped 0 and print 0
```

- **Eulerian cycle: 0 7 8 0 9 5 6 9 10 3 4 10 11 1 2 11 0**

Path searching (using DFS)

You could want to search for a path between two given vertices:

- the starting vertex we already have
- add the goal vertex as a parameter and test 'is there a path' in the *dfs()* function.

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```
int isPath(Graph g, Vertex v, Vertex goalv, int numV, int *order, int *visited) {
   int found = 0;
   visited[v] = *order;
   *order = *order+1;
   if (v == goalv) {
      found = 1;
   }
   else {
      Vertex w;
      for (w=0; w < numV && !found; w++) {
         if (isEdge(g, newE(v,w))) {
            if (visited[w] == UNVISITED) {
               found = isPath(g, w, goalv, numV, order, visited);
               printf("path %d-%d\n", w, v);
            }
         }
      }
   }
   return found;
}
```

- this function does a DFS traversal, exactly like *dfsR()* ...
    - ... but at the same time, it searches for a path to a vertex *goalv*

So, instead of the call to *dfsR()* (in *dfs()*):

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```
   dfsR(g, v, numV, &order, visited);
```

we call the function *isPath()*:

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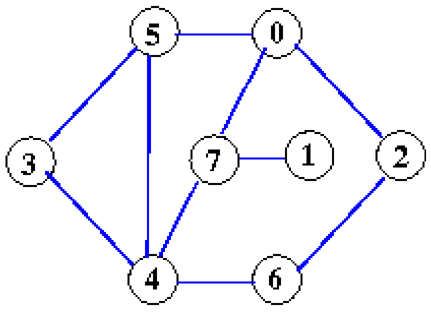
```
   #define STARTV 0
   #define GOALV 3
   if (isPath(g, STARTV, GOALV, numV, &order, visited)) { //notice the STARTV and GOALV arguments
      printf("found path\n");
   }
   else {
      printf("no path\n");
   }
```

- Here a #define is used to name the start and goal vertices: this could better be input interactively of course.

If we input the graph:



then the output is:

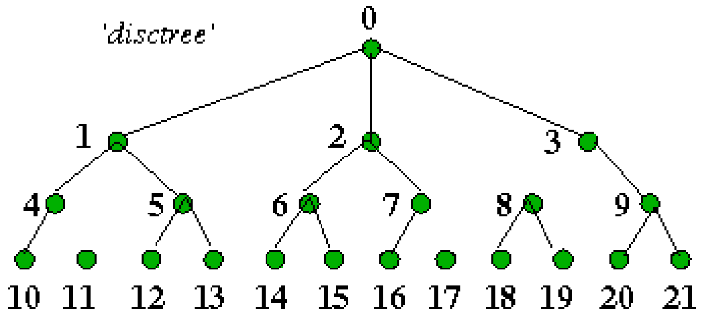
```
path 3-4
path 4-6
path 6-2
path 2-0
found path
Visited: {0, -1, 1, 4, 3, -1, 2, -1}
```

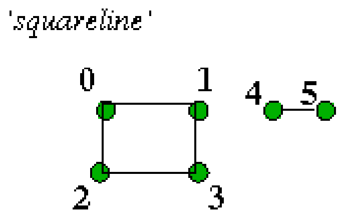
- Here we can see which vertices were visited

Reachability Analysis

In many problems we are interested in knowing which vertices are *reachable* from some start vertex.

- for example, in the graphs:





some of the vertices are unreachable from the start vertex 0, others are not

- if the graph is undirected, it is obviously disconnected
- (if the graph is directed then it may not be disconnected of course)

A DFS algorithm can be used to find all the reachable vertices

- simply run the algorithm from the start vertex
- on conclusion, check the visited array
  - any vertex (except the start vertex) that is unvisited is unreachable

An alternative method is to use so-called 'fixed-point' computation.

1. initialise:
    - a reachable set comprising of just the start vertex
    - every other vertex is considered unreachable
2. check every unreachable vertex *v*
    - if there is an edge from a vertex in the reachable set to *v*
        - then add *v* to the reachable set
3. repeat the previous step until the reachable set does not change
    - if the reachable set does not change, terminate

When the set does not change, we have reached a 'fixed point'

- the set of vertices in the reachable set can be reached from the start vertex
- all other vertices cannot be reached

Example: consider the graph *squareline* above

- let *R* be the set of reachable vertices, and the start vertex be 0
    - initially *R* = {0}
- consider vertices 1..5
    - 1 is adjacent to 0, add to *R*
    - 2 is adjacent to 0, add to *R*
    - *R* = {0, 1, 2}
    - *R* has changed, so repeat
- consider vertices 3..5
    - 3 is adjacent to 1, add to *R*
    - *R* = {0, 1, 2, 3}
    - *R* has changed, so repeat
- consider vertices 4 and 5
    - neither vertex is adjacent to a vertex in *R*
    - *R* does not change
- terminate the algorithm

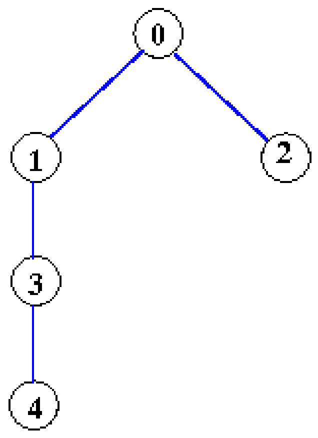
Notice that you do not needs to use a stack here, or recursion. There is no backtracking.

Breadth First Search

Consider all edges of one vertex before moving to another vertex

- each level of vertices is visited before the next level's vertices are considered

For example:



1. visit vertex 0
2. visit vertex 1 and 2
3. visit vertex 3
4. visit vertex 4

In essence, the vertices are processed ***in order*** (top to bottom, left to right)

- DFS used a stack:
  - we pushed all the adjacent vertices of a vertex onto a stack
  - so we would remember the vertices we need to backtrack to (i.e. 'try' later on)
- BFS instead uses a queue:
  - we push all the adjacent vertices of a vertex onto a queue (just like DFS)
  - but we visit them in the order they occur (as we must in a queue)
- the change in the *quack* version is trivial:
  - just 4 changes

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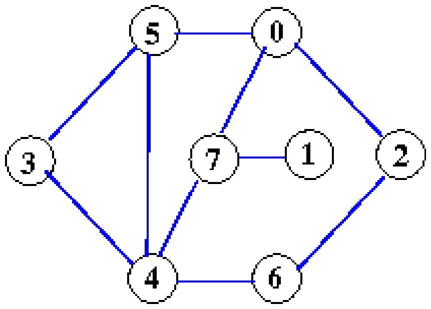
```
void bfs(Graph g, Vertex v, int numV) { // CHANGE 1: NAME DFS => BFS
   int *mallocArray(int numV) {
      int *array = malloc(numV * sizeof(int));// l
      if (array == NULL) {                    // o
         fprintf(stderr, "Out of memory\n");  // c
         exit(1);                             // a
      }                                       // l
      int i;                                  // f
      for (i=0; i<numV; i++) {                // u
         array[i] = UNVISITED;                // n
      }                                       // c
      return array;                           // t
   }
   void showArray(int *array, int numV) {
      int i;                                  // l
      printf("Visited: {");                   // o
      for (i=0; i<numV; i++) {                // c
         printf("%d", array[i]);              // a
         if (i <= numV-2) {                   // l
            printf(", ");                     // f
         }                                    // u
      }                                       // n
      printf("}\n");                          // c
      return;                                 // t
   }
   int *visited = mallocArray(numV);
   Quack s = createQuack();
   qush(v, s);                          // CHANGE 2: PUSH
=> QUSH
   showQuack(s);
   int order = 0;
   while (!isEmptyQuack(s)) {
      v = pop(s);
      if (visited[v] == UNVISITED) {
         showArray(visited, numV);
         visited[v] = order++;
         Vertex w;
         for (w=0; w<numV; w++) {       // CHANGE 3: ORDER:
SMALL TO LARGE
            if (isEdge(g, newE(v,w))) {
               qush(w, s);              // CHANGE 4: PUSH
-> QUSH
            }
         }
      }
      showQuack(s);
   }
   showArray(visited, numV);
   free(visited);
   return;
}
```

What about the graph we considered above for DFS represented by the input data

```
8
0-2 0-5 0-7 2-6 1-7 4-7 4-6 4-3 3-5 4-5
```



Starting at vertex 0, the visited sequence of *dfs()* was {0, 7, 1, 4, 3, 5, 2, 6}.

- this corresponds to the order of vertices: 0 2 6 4 3 5 7 1

What does *bfs()* do:

```
Quack: <<0>>
Visited: {-1, -1, -1, -1, -1, -1, -1, -1}
Quack: <<2, 5, 7>>
Visited: {0, -1, -1, -1, -1, -1, -1, -1}
Quack: <<5, 7, 0, 6>>
Visited: {0, -1, 1, -1, -1, -1, -1, -1}
Quack: <<7, 0, 6, 0, 3, 4>>
Visited: {0, -1, 1, -1, -1, 2, -1, -1}
Quack: <<0, 6, 0, 3, 4, 0, 1, 4>>
Quack: <<6, 0, 3, 4, 0, 1, 4>>
Visited: {0, -1, 1, -1, -1, 2, -1, 3}
Quack: <<0, 3, 4, 0, 1, 4, 2, 4>>
Quack: <<3, 4, 0, 1, 4, 2, 4>>
Visited: {0, -1, 1, -1, -1, 2, 4, 3}
Quack: <<4, 0, 1, 4, 2, 4, 4, 5>>
Visited: {0, -1, 1, 5, -1, 2, 4, 3}
Quack: <<0, 1, 4, 2, 4, 4, 5, 3, 5, 6, 7>>
Quack: <<1, 4, 2, 4, 4, 5, 3, 5, 6, 7>>
Visited: {0, -1, 1, 5, 6, 2, 4, 3}
Quack: <<4, 2, 4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<2, 4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<4, 4, 5, 3, 5, 6, 7, 7>>
Quack: <<4, 5, 3, 5, 6, 7, 7>>
Quack: <<5, 3, 5, 6, 7, 7>>
Quack: <<3, 5, 6, 7, 7>>
Quack: <<5, 6, 7, 7>>
Quack: <<6, 7, 7>>
Quack: <<7, 7>>
Quack: <<7>>
Quack: << >>
Visited: {0, 7, 1, 5, 6, 2, 4, 3}
```

In the first few lines of this output we see:

- vertex 0 is qushed, then popped
- vertices 2, 5, 7 are qushed, then successively
  - 2 is popped
  - 5 is popped
  - 7 is popped
- 'qushing' means everything gets added to the end of the quack
- the order that vertices are visited is:
  - 0 2 5 7 6 3 4 1
  - note:
    - 0 is a level-0 vertex
    - 2 5 7 are level-1
    - 6 3 4 1 are level-2
    - no vertex is more than a path length of 2 away from the starting vertex

Path searching (using BFS)

You could want to search for a path between two given vertices, *start* and *goal* say:

- the starting vertex we already have
- add the goal vertex as a parameter and test for it in the *bfs()* function

But, we don't follow any 'path' during BFS (as we did in DFS): we traverse by level:

- whenever we 'visit' a node, we must remember its parent
    - we store the parent of each vertex in an array called *parent[]*
- we hence need 2 arrays, *visited[]* and *parent[]*
- when we find the goal node, we're finished
    - the path 'backwards' to the goal node will be stored in *parent[]*
- we print the path from the *start* to the *goal* stored in *parent[]*

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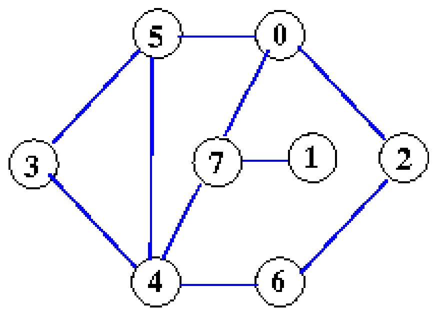
```c
void searchPath(Graph g, Vertex start, Vertex goal, int numV) {
   int *mallocArray(int numV) {
      int *array = malloc(numV * sizeof(int));// l
      if (array == NULL) {                    // o
         fprintf(stderr, "Out of memory\n");  // c
         exit(1);                             // a
      }                                       // l
      int i;                                  // f
      for (i=0; i<numV; i++) {                // u
         array[i] = UNVISITED;                // n
      }                                       // c
      return array;                           // t
   }
   void showArray(int *array, int numV) {
      int i;                                  // l
      printf("Visited: {");                   // o
      for (i=0; i<numV; i++) {                // c
         printf("%d", array[i]);              // a
         if (i <= numV-2) {                   // l
            printf(", ");                     // f
         }                                    // u
      }                                       // n
      printf("}\n");                          // c
      return;                                 // t
   }
   void printPath(int parent[], int numV, Vertex goal) {
      // local function discussed below
   }
   int *visited = mallocArray(numV);
   int *parent = mallocArray(numV);      // need extra array to store parents
   Quack q = createQuack();
   qush(start, q);
   int order = 0;
   visited[start] = order++;
   int found = 0;
   while (!isEmptyQuack(q) && !found) {
      Vertex x = pop(q);
      Vertex y;
      for (y = 0; y<numV && !found; y++) {
         if (isEdge(g, newE(x,y))) {      // for adjacent vertex y ...
            if (visited[y]==UNVISITED) {  // ... if y is unvisited ...
               qush(y, q);                // ... queue y
               visited[y] = order++;      // y is now visited
               parent[y] = x;             // y's parent is x
               if (y == goal) {           // if y is the goal ...
                  found = 1;              // ... SUCCESS! now get out
               }
            }
         }
      }
   }
   if (found) {
      printf("SHORTEST path from %d to %d is ", start, goal);
      printPath(parent, numV, goal);
      putchar('\n');
   }
   else {
      printf("no path found\n");
   }
   free(visited);
   free(parent);
   makeEmptyQuack(q);
   return;
}
```

If we input the graph:



then the output is:

```
SHORTEST path from 0 to 6 is 6<--2<--0
```

The array *parent[]* stores the parent of every visited node in the graph, but:

- the start vertex has no parent
- unvisited nodes have no parents

To print the path:

- we print *goal*
- we print *parent[goal]*
- we print *parent[parent[goal]]*
- we print *parent[parent[parent[goal]]]*
- we print *parent[parent[parent[parent[goal]]]]*
- ...
- until we find the *start* vertex

That's what the function below does:

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```
void printPath(int parent[], int numV, Vertex v) {
   printf("%d", v);
   if (0<=v && v<numV) {
      Vertex p = parent[v];
      while (p != UNVISITED) {
          printf("<--%d", p);
          p = parent[p];
      }
   }
   else {
      fprintf(stderr, "printPath: illegal vertex in parent[]\n");
   }
}
```

As we start with *goal* and work backwards, we print the path in reverse direction.

animation of BFS

GraphSearch (2019-07-18 18:02:19由AlbertNymeyer编辑)