N = 4 SYM Loop Notes

Name: Alexander Boccaletti **Updated Date: 12.05.2022**
KU ID: bct232 **Supervisor**: Chi Zhang

(a) **Basics**

Applying the Feynman-diagram formalism to determine scattering amplitudes at arbitrary order of of perturbation theory, the results are expressed in terms of a non-trivial linear combination of product of external 4-momenta p_i^μ and polarisation vectors ϵ_i^μ, depending on the process being considered. Thus, generally the amplitude calculation will yield a result of the form $A_n = A(p_1, ..., p_n)$. Given the momenta p_i^μ we map the momenta as

$$p_i^\mu \to (p_i)_{\alpha\ddot{\alpha}} = p_i^\mu(\sigma_\mu)_{\alpha\ddot{\alpha}} = \begin{pmatrix} p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & p^0 + p^3 \end{pmatrix} \tag{1}$$

Hence, such matrices $(p_i)_{\alpha\ddot{\alpha}}$ are hermitian. This makes sense as each p^μ has 4 independent components and the $2x2$ hermitians $p_{\alpha\ddot{\alpha}}$ also have 4 independent components. Also, $\det p = -p^2 = -m^2$ Now let's consider $SL(2,\mathbb{C}) \ni L_1, L_2$,

$$L = L_1 L_2 \implies \det L = \det L_1 \det L_2 = 1 \cdot 1 = 1 \implies L \in SL(2,\mathbb{C}) \tag{2}$$

So, its a group. Given a matix p, we can establish the mapping

$$p' = L^\dagger p L \implies (p')^\dagger = p \tag{3}$$

Thus, every $SL(2,\mathbb{C}) \ni L$ defines a transformation on 4-vectos p_i^μ, but it is not injective, since $SL(2,\mathbb{C})$ double covers $SO(3,1)$

At high energies, the discussion simplifies as the rest energy can be neglected; thus, giving $\det(p_i) = -p_i^2 = 0$, so that the 4-momentum vector is null-like (light-like). Since, the determinant is 0, the matrix has an eigenvalue of 0; thus, it is a $2x2$ with rank 1 $\iff$ $p_{\alpha\ddot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\ddot{\alpha}}$ But real momenta $\tilde{\lambda} = \lambda^*$; hence, the two are not independent. In order to keep them independent, we can choose two ways.
We can analytically continue to complex momenta $\implies$ $\tilde{\lambda} \neq \lambda^*$. Alternatively, we can keep the momentum real and change the signature of the metric so that we consider $SO(2,2)$ instead of $SO(1,3)$

$$(-,+,+,+) \to (-,-,+,+) \tag{4}$$

which also eliminates the constraint on the Weyl-spinors. If we combine the 2 appoaches, we consider light-like complex momenta throughout and this naturally yields the conformal algebra; the formalism yields us the conformal group $SO(4,2)$.The conformal algebra of d-dim Minkowski spacetime with the Lorentz group $SO(1, d-1)$ is the Lie algebra of $SO(2, d)$. Thus, we can descibe conformally compactified Minkowski space as the SO(2, 4)-invariant

$$T^2 + V^2 - W^2 - X^2 - Y^2 - Z^2 = 0 \tag{5}$$

with $\mathbb{RP}^5 \ni T, V, W, X, Y, Z$