

Name: Alex Book

ID: 108073300

CSCI 3104, Algorithms
Problem Set 6a (10 points)

Profs. Hoenigman & Agrawal
Fall 2019, CU-Boulder

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words**. Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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1. (1 pt) What do the edge weights of a graph G in a maximum-flow network represent?

Solution.

The edge weights of graph G represent the capacity of the edges.

2. (2 pts) What are the two conditions that must be met for network flow?

Solution.

The conditions are as follows:

1. Every node (except the source and sink) must have the same amount of flow coming into and out of it.
 2. The flow carried by an edge must be non-negative and less than or equal to the edge's capacity ($0 \leq \text{flow}(e) \leq \text{capacity}(e)$).
3. (2 pts) What do the edge weights in the residual graph G_f represent? Include both forward and backward edges.

Solution.

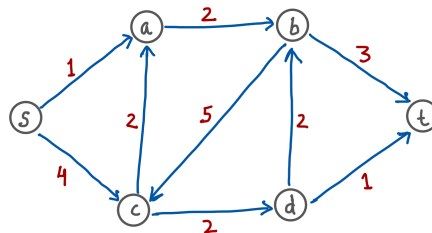
The edge weights in G_f represent the following:

1. Forward edge weights are the remaining units of capacity that can handle additional flow units.
2. Backward edge weights are how much flow can be pushed backward through the edge (equal to the amount of flow currently being pushed forward through the edge).

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4. (5 pts) Based on the following network and the given edge capacities answer the following.



- (a) (1 pts) Can the max flow be 5 ($\text{capacity}(e_{sa}) + \text{capacity}(e_{sc})$)? Justify your answer in one sentence.

Solution.

No, it can only be 4, as there can only 4 possible flow units pushed into t (the sink).

- (b) (2 pts) For the graph, identify one simple $s - t$ path and the bottleneck edge value on that path. Also report the maximum allowed flow on this $s - t$ path.

Solution.

Consider the path $s \rightarrow a \rightarrow b \rightarrow t$. The bottleneck edge is $s \rightarrow a$, having a value of 1. Therefore, the maximum possible flow allowed on this path is 1.

- (c) (2 pts) Assuming all $f(e)$ are initially 0 where f represents flow, what are the residual capacities on the forward and backward edges of G_f after one iteration of the Ford-Fulkerson algorithm. Use the simple path you identified in Part b.

Solution.

Edge	Forward weight	Backward weight
$s \rightarrow a$	0	1
$a \rightarrow b$	1	1
$b \rightarrow t$	2	1
$s \rightarrow c$	4	0
$c \rightarrow d$	2	0
$d \rightarrow t$	1	0
$c \rightarrow a$	2	0
$b \rightarrow c$	5	0
$d \rightarrow b$	2	0