

Name: Alex Book

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CSCI 3104, Algorithms  
Problem Set 4a (11 points)

Profs. Hoenigman & Agrawal  
Fall 2019, CU-Boulder

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Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words**. Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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1. (1 pt) What is the definition of a Minimum Spanning Tree (MST)?

*Solution.*

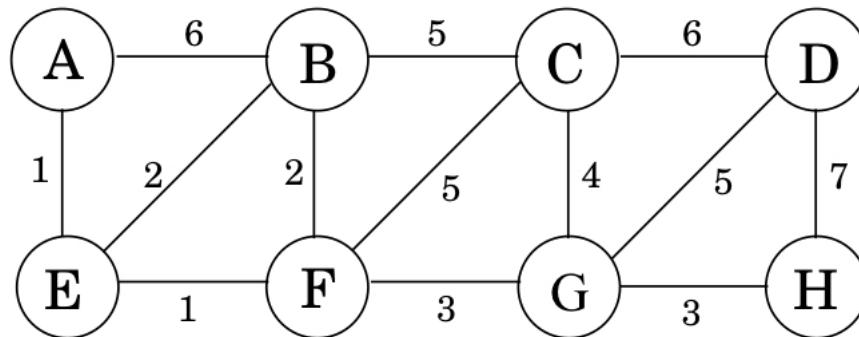
Given a connected, undirected graph  $G=(V,E)$ , a spanning tree is the minimum weight subgraph that is an undirected tree and contains all vertices in  $G$ .

2. (1 pt) Describe in one or two sentences, a greedy rule for constructing an MST.

*Solution.*

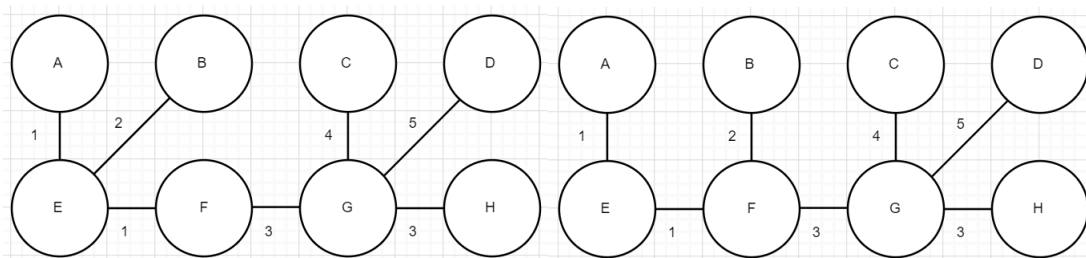
Choose the minimum edge weight such that no cycle is created and there are less than or equal to  $n-1$  total edges selected, where  $n$  is the number of vertices in the graph.

3. (3 pts) How many unique MSTs does the following graph have :



*Solution.*

The graph has two unique MSTs, each of which have a weight of 19.



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4. (3 pts) Suppose that you have calculated the MST of an undirected graph  $G = (V, E)$  with positive edge weights.

If you increase each edge weight by 2, will the MST change? Prove that it cannot change or give a counterexample if it changes. (Note: Your proof, if there is one, can be a simple logical argument.)

*Solution.*

Using the greedy rule presented in problem 2, adding 2 to every edge weight wouldn't change the selection of the MST, as the minimum edge weights would remain as such, since all edges are scaled up equally.

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5. (3 pts) Suppose that you have calculated the shortest paths to all vertices from a fixed vertex  $s \in V$  of an undirected graph  $G = (V, E)$  with positive edge weights. If you increase each edge weight by 2, will the shortest paths from  $s$  change? Prove that it cannot change or give a counterexample if it changes. (Note: Just as in Part a, your proof can be a simple logical argument.)

*Solution.*

The following is a counterexample:

