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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 5a (11 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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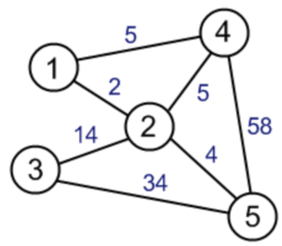
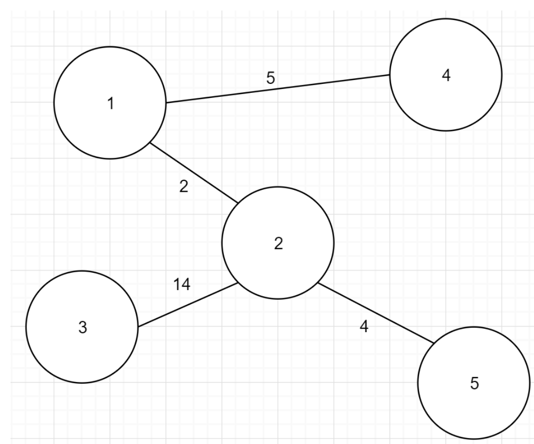
1. (7 pts) Consider the following weighted graph.



   (a) (2 pts) Run Dijkstra's algorithm on this graph to obtain a tree of shortest paths. Use vertex 1 as the source vertex.

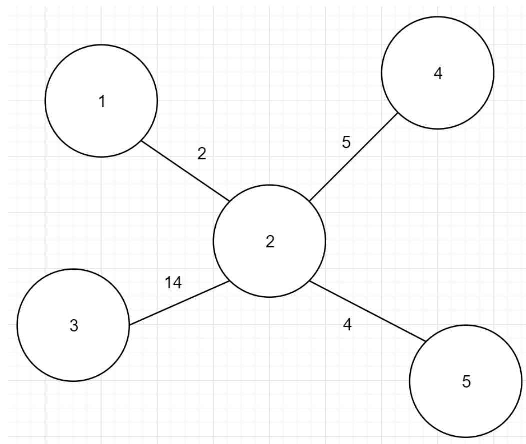
   Solution.
   Start at V_1
   V_4=0+5=5 (update because 5<∞)
   V_2=0+2=2 (update because 2<∞)
   Visited: V_1
   Visit V_2
   V_4=2+5=7 (ignore because 7>5)
   V_5=2+4=6 (update because 6<∞)
   V_3=2+14=16 (update because 16<∞)
   Visited: V_1, V_2
   Visit V_4
   V_5=5+58=63 (ignore because 63>6)
   Visited: V_1, V_2, V_4
   Visit V_5
   V_3=6+34=40 (ignore because 40>16)
   Visited: V_1, V_2, V_4, V_5
   Visit V_3
   No unvisited adjacent vertices
   Visited: V_1, V_2, V_4, V_5, V_3
   All vertices have been visited
   V_1=0, V_2=2, V_3=16, V_4=5, V_5=6



(b) (2 pts) Run Kruskal's algorithm on this graph to obtain a minimum spanning tree.

Solution.

Choose $E_{1,2}$
Choose $E_{2,4}$ (abritrarily break tie)
$E_{1,4}$ would cause a cycle, so don't add it to the MST
Choose $E_{2,5}$
Choose $E_{2,3}$
$E_{3,5}$ would cause a cycle, so don't add it to the MST
$E_{4,5}$ would cause a cycle, so don't add it to the MST
All edges considered, so return the MST:



(c) (1 pts) Is the tree of shortest paths produced by Dijkstra's algorithm a minimum spanning tree? Justify your answer.

Solution.

Yes, as although it doesn't include all the same edges, it has the same total weight as the MST produced by Kruskal's (a weight of 25).

(d) (2 pts) Find two vertices u and v, where the $u - v$ path in the Kruskal tree is not a shortest $u - v$ path.

Solution.

Let vertex u be vertex 1 and vertex v be vertex 4. The 1-4 path in the Kruskal tree is made of $E_{1,2}$ and $E_{2,4}$, having a weight of 2+5=7. However, the shortest 1-4 path in the graph is made of only $E_{1,4}$, having a weight of 5.

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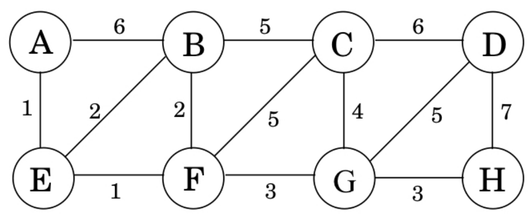
2. (1 pt) Provide a brief description of what the *find(v)* and *union(A,B)* features of the union-find algorithm produce.

   Solution.
   The *find(v)* feature produces the connected component to which vertex v belongs. The *union(A,B)* feature joins two connected components A and B, making them one larger connected component, taking the name of whichever was individually larger (had more vertices), ties being broken arbitrarily.

3. (3 pts) Identify three edges in the following graph G that won't be included in any MST of G. Provide a 3-4 sentence explanation of your answer.



   Solution.
   The edges $E_{D,H}$, $E_{D,C}$, and $E_{A,B}$ won't be included in any MST of G. This is because each of them is the largest edge in a cycle. To choose these edges, simply look at the largest edges in the graph, and if they form a cycle, remove them from the possibility of being in an MST.