1. What are the mechanical degrees of freedom options for buying a Kinova Gen3 Robotic arm? What advantage does one option have over the other? Which do you think is more expensive?

   The options are 6 or 7 degrees of freedom. The benefit of having 7 over 6 is that you can maneuver once "piece" of the robot and stay in the same state reached by the other 6 pieces, whereas with only 6 degrees of freedom, moving any of the 6 pieces would result in a state change. The benefit of having 6 over 7 is simplicity (in terms of programming, as well as ideating about applications). I think the 7 DoF robot is likely more expensive, as the production cost is likely more.

2. What are the independent environmental degrees of freedom for orientable robots on the X-Y plane?

   They are x, y, and theta (angle of rotation).

3. (a) Calculate the angle between vectors $(0.966,0.2588,0)^T$ and $(0.2588, 0.966,0)^T$

   $a \cdot b = |a| * |b| * \cos(\theta)$
   $0 = \sqrt{.966^2 + .2588^2} * \sqrt{(-.2588)^2 + .966^2} * \cos(\theta)$
   $\theta = 90$ degrees, $\frac{\pi}{2}$ radians.

   (b) Provide a third vector that forms a unit-length coordinate system with the other two.

   $(0, 0, \sqrt{.966^2 + .2588^2})$

4. (a) Write out the entries of a rotation matrix ${}^{B}_{A}R$ assuming basis vectors X_A, Y_A, Z_A, and X_B, Y_B, Z_B.

$$
\begin{aligned}
{}^{A}_{B}R &= [{}^{A}X_B, {}^{A}Y_B, {}^{A}Z_B] \\
&= \begin{bmatrix} X_B \cdot X_A & Y_B \cdot X_A & Z_B \cdot X_A \\ X_B \cdot Y_A & Y_B \cdot Y_A & Z_B \cdot Y_A \\ X_B \cdot Z_A & Y_B \cdot Z_A & Z_B \cdot Z_A \end{bmatrix}
\end{aligned}
$$

(b) Express ${}^{B}X = [0, 1, 0]^T$ in frame $\{A\}$.

$$
\begin{aligned}
{}^{A}P &= \begin{bmatrix} X_A - X_B \\ Y_A - Y_B \\ Z_A - Z_B \end{bmatrix} \\
{}^{A}X &= {}^{A}_{B}R * {}^{B}X + {}^{A}P \\
&= \begin{bmatrix} X_B \cdot X_A & Y_B \cdot X_A & Z_B \cdot X_A \\ X_B \cdot Y_A & Y_B \cdot Y_A & Z_B \cdot Y_A \\ X_B \cdot Z_A & Y_B \cdot Z_A & Z_B \cdot Z_A \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} X_A - X_B \\ Y_A - Y_B \\ Z_A - Z_B \end{bmatrix} \\
&= \begin{bmatrix} Y_B X_A + X_A - X_B \\ Y_B Y_A + Y_A - Y_B \\ Y_B Z_A + Z_A - Z_B \end{bmatrix}
\end{aligned}
$$

(c) Write out the entries of rotation matrix ${}^{B}_{A}R$.

$$
{}^{B}_{A}R = \begin{bmatrix} X_A X_B & Y_A X_B & Z_A X_B \\ X_A Y_B & Y_A Y_B & Z_A Y_B \\ X_A Z_B & Y_A Z_B & Z_A Z_B \end{bmatrix}
$$

5. Consider a tricycle with two independent standard wheels in the rear and a steerable, actuated front-wheel. Assume r to be the radius of the front wheel and l to be the distance between the front and rear axle. Chose a suitable coordinate system, use ϕ as the steering wheel angle, and $\dot{\omega}$ as angular velocity (only the front-wheel is driven). Provide the forward kinematics equations for the tricycle.

   I will use the point between the back two wheels as the origin/center of the tricycle.
   $\dot{x} = \dot{\omega} r \cos(\phi)$
   $\dot{\Theta} = \frac{\dot{\omega} r \sin(\phi)}{l}$

   The tricycle can't skid left or right, so there is no necessary $\dot{y}$ update function.

6. A robot using a local coordinate frame B detects an object Q at position $(8, -4)$. In coordinate frame A, the robot shows odometry readings of $(6, 2, 2.26893)$. Using a homogenous transform, find the position of Q in coordinate frame A.

$$
\begin{aligned}
\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}Q \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(2.26893) & -\sin(2.26893) & 6 \\ \sin(2.26893) & \cos(2.26893) & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 3.92 \\ 10.7 \\ 1 \end{bmatrix}
\end{aligned}
$$

   Position of Q in coordinate frame A: $\begin{bmatrix} 3.92 \\ 10.7 \end{bmatrix}$