CSCI 3434, CU Boulder, Fall 2020 **Alex Book**
Collaborators: None **PS3**

Important reminders from the syllabus:

- **Do not use external sources** beyond the materials linked to on the course website to solve these problems.
- You are encouraged to work with ($\leq$ 2) other students, but you must **write your solutions independently**.
- Be sure to **list your collaborators** by name clearly at top of your submission, or “no collaborators” if none.

Some of these problems are from the book. The letters correspond to the section of the book: the [H]omeworks starting page 301, and [M]iscellaneous [E]xercises starting page 315. For example, H1.2 denotes Homework 1, problem 2.

Standards are now given in parentheses (S7 == Standard 7).

1. Induction (S1)
NOTE: I have demonstrated mastery of S1 twice, and will be skipping this problem.

2. Pumping lemma and closure properties (S6)
H4.1(d)
Prove it twice: once with the pumping lemma, and once with closure properties.
The set PAREN of balanced strings of parentheses ( ). For example, the string ((( ) ( )) ( )) is in PAREN, but the string) (( ) is not.

a. Pumping lemma: Spell out each step, especially your choice of x, y, z (which may depend on k) and i (which may depend on k and u, v, w).

Assume PAREN is regular.
For $k \geq 0$, let $x = \epsilon, y = (^k, z =)^k$.
$y = uvw$ and $|v| \neq 0$, so v must contain at least one (.
Take $i = 0 : xuv^iwz = xuv^0wz = (^{k-|v|})^k$
$|v| > 0, \text{so} (^{k-|v|})^k \notin$ PAREN.
Therefore, PAREN is not regular.

b. Closure properties: Here you can use any closure property of regular languages proven in the book / discussed in class, such as concatenation, union, intersection, asterate, reverse, image or inverse image of homomorphism, etc.

Let $A = (^*)^*$. Similar to the language a^*b^*, A is regular.
Suppose PAREN is regular. Therefore, $A\cap$PAREN should be regular as well. However, $A\cap$PAREN$= \{(^n)^n | n \geq 0\}$, which, similar to a^nb^n, is not regular.
Therefore, PAREN is not regular.

3. Classification (S8)
ME.37(a,d,g,i,j)
For the justification, when showing a language is regular you can demonstrate using the usual constructions (DFA,NFA,regexp) and do not need to prove that your construction works. You could also use closure properties and known regular languages. To justify the claim that a language is non-regular, you can use either the pumping lemma or closure properties.

(a) $\{a^n b^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$

This can be represented as the following regexp: $a^*(bb)^*$

(d) $\{a^{p-1} \mid p \text{ is prime}\}$

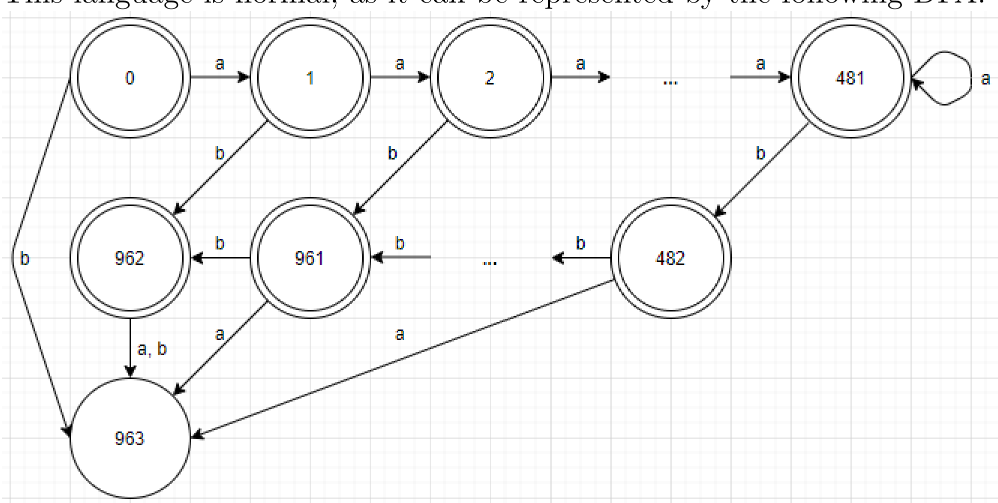
Assume $\{a^{p-1} | p \text{ is prime}\}$ is regular.
For $k = p - 1 \geq 0, x = \epsilon, y = a^k, z = \epsilon$.
With $y = uvw$, let $u = a^d, v = a^e, w = a^f$ (s.t. $d + e + f = k = p - 1$).
$xuv^iwz = a^d a^{ei} a^f = a^{d+ei+f} = a^{k+e(i-1)} = a^{p-1+e(i-1)} = a^{(p+e(i-1))-1}$
So, $(p + e(i - 1))$ must be prime for all $i \geq 0$.
Take $i = p + 1 : (p + e(i - 1)) = (p + e(p + 1 - 1)) = p + ep = p(e + 1)$
$p(e + 1)$ is not prime, so $\{a^{p-1} | p \text{ is prime}\}$ is not regular.

(g) $\{a^n b^{n+481} \mid n \geq 0\}$

Assume that the given language A is regular.
$\{a^{481}\}$ is regular, as it is finite.
$\{a^{481}\}A = \{a^n b^n | n \geq 481\}$ must be regular under concatenation.
$\{a^n b^n | 0 \leq n < 481\}$ is regular, as it is finite.
$\{a^n b^n | n \geq 481\} \cup \{a^n b^n | n < 481\} = \{a^n b^n | n \geq 0\}$ must be regular under union.
However, $\{a^n b^n | n \geq 0\}$ is a known non-regular language, so A must be non-regular.

(i) $\{a^n b^m \mid n \geq m \text{ and } m \leq 481\}$

This language is normal, as it can be represented by the following DFA:



(j) $\{a^n b^m \mid n \geq m \text{ and } m \geq 481\}$

Assume $A = \{a^n b^m | n \geq m \text{ and } m \geq 481\}$ is regular.
For $k \geq 0$, let $x = a^{481}, y = a^k, z = b^k b^{481}$.
$y = uvw$ and $|v| \neq 0$, so v must contain at least one a.
Take $i = 0 : xuv^i wz = xuv^0 wz = a^{481} a^{k-|v|} b^k b^{481} \notin A$
Therefore, $A = \{a^n b^m | n \geq m \text{ and } m \geq 481\}$ is not regular.

4. DFA minimization (S7)
H4.3(right)
Complete (a-c) for just the right automaton, not both. For (a) just a set of states is fine. For (b), list the classes like "$1 \approx 2 \approx 3$; $4 \approx 5$; 6; $7 \approx 8$".

a. There are no unreachable states.

b. $1 \approx 2, 3 \approx 4 \approx 8, 5 \approx 6 \approx 7$

c.

	a	b
12F	38	57
348	567	12
567	12	348

5. Novel construction (S9), proof (S10)
H3.3
Hint: This is a tricky one; see ME.26 and its solution in the book.