Some of these problems are from the book. The letters correspond to the section of the book: the [H]omeworks starting page 301, and [M]iscellaneous [E]xercises starting page 315. For example, H1.2 denotes Homework 1, problem 2.

1. Problems completed on Automata Tutor

2. Product construction
   H1.2

   (a)

	a	b
11	21	32
21	31	12
31	11	22
12	21	32
22	31	12
32 F	11	22

   (b)

	a	b
11	21	32
21	31	12
31 F	11	22
12 F	21	32
22 F	31	12
32 F	11	22

3. Induction on strings
   H1.3
   Let $M = (Q, \Sigma, \delta, s, F)$ be an arbitrary DFA. Prove by induction on $|y|$ that for all strings $x, y \in \Sigma*$ and $q \in Q$,

$$\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y),$$

   where $\hat{\delta}$ is the extended version of δ defined on all strings described in Lecture 3.

   Base Case:
   Consider the case where $|y| = 0$. This is equivalent to $y = \epsilon$.
   By definition, the null string ϵ serves as an identity for string concatenation.

$$\begin{aligned}
\hat{\delta}(q, xy) &= \hat{\delta}(q, x\epsilon) && \text{by substituting } \epsilon \text{ in for } y \\
&= \hat{\delta}(q, x) && \text{by definition of } \epsilon \\
&= \hat{\delta}(\hat{\delta}(q, x), \epsilon) && \text{by definition of } \hat{\delta} \text{ in lecture 3 (3.1)} \\
&= \hat{\delta}(\hat{\delta}(q, x), y) && \text{by substituting } y \text{ in for } \epsilon
\end{aligned}$$

   Inductive Hypothesis:
   Assume that for $|y| = k$, where $k \geq 1$, $\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y)$.

   Inductive Step:
   Let $|y| = k + 1$, so $y \neq \epsilon$. Therefore, y can be represented as the concatenation za where $z \in \Sigma^*$ and $a \in \Sigma$.

$$\begin{aligned}
\hat{\delta}(q, xy) &= \hat{\delta}(q, xza) && \text{by substituting } za \text{ in for } y \\
&= \hat{\delta}(\hat{\delta}(q, xz), a) && \text{by definition of } \hat{\delta} \text{ in lecture 3 (3.2, page 16)} \\
&= \hat{\delta}(\hat{\delta}(\hat{\delta}(q, x), z), a) && \text{by Inductive Hypothesis} \\
&= \hat{\delta}(\hat{\delta}(q, x), za) && \text{by definition of } \hat{\delta} \text{ in lecture 3 (3.2, page 16)} \\
&= \hat{\delta}(\hat{\delta}(q, x), y) && \text{by substituting } y \text{ in for } za
\end{aligned}$$

4. Subset construction
   ME.3

   (a) abbbb

   (b)

state	a	b
$\varnothing$	$\varnothing$	$\varnothing$
{s}	{s,t,u,v}	$\varnothing$
{s,t}	{s}	{s}
{s,t,u}	{s}	{s,t}
{s,t,u,v}	{s}	{s,t,u}

5. Novel construction, proof
   H2.2
   Construction:
   For the construction of the DFA/NFA accepting rev(A), let us start with the DFA/NFA accepting A, called M_1. We will first switch the start and final states, then we will reverse the direction of the edges (effectively reversing each transition a→b to become b→a). Let us call this new DFA/NFA M_2. For the desired proof, we will induct on the length of an arbitrary string $x \in A$ from the original language A.

   Base Case:
   Let $|x| = 1$. If such a string were to be accepted by M_1, there would be a one-step path from a start state to an end state ($q_1 \to f_1$). In the creation of M_2, this path would still exist, but be pointing in the opposite direction ($q_2 \to f_2$). The reverse of a string of length one is simply that same string, so it would be accepted by M_2.

   Inductive Hypothesis:
   Assume that for $|x| = k$, where $k \geq 1$, if x is accepted by M_1, rev(x) is accepted by M_2.

   Inductive Step:
   Let $|x| = k + 1$. x can therefore be seen as za, where $z \in \Sigma^*$ and $a \in \Sigma$. By the definition of rev() on strings, rev(x) = rev(za) = az. We need to show that if za is accepted by M_1, az is accepted by M_2.

   By our inductive hypothesis, if z is accepted by M_1, rev(z) is accepted by M_2. So, we must inspect the relation between the acceptance of a as the last character of za, and a as the first character of az.

   In M_1, the acceptance of a can be represented by the edge between $\hat{\delta}(q_1, z)$ and f_1 (note: $q_1, f_1 = f_2, q_2$, as defined in the **Construction** step). In the creation of M_2, this edge is reversed, now spanning from q_2 to $\hat{\delta}(q_1, z)$ (this specific node is effectively the same in M_1 and M_2, simply changing from being connected to an accept state in M_1 to being connected to a start state in M_2). In this 'final' edge's existence in M_1, a guarantee is established that the equivalent reversed edge will exist in M_2. This reversed edge means that the acceptance of a as the last character of za guarantees the acceptance of a as the first character of az.

   Therefore, if if za is accepted by M_1, az is accepted by M_2. If A is regular, rev(A) must also be regular.