Homework 9
Colorado CSCI 5454

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People I studied with for this homework: Cole Sturza
Other external resources used: N/A

Problem 1

Note: n is the number of dimensions in which each point in represented.

Part a

There are 2^n points (2-d square has 4 points, 3-d cube has 8 points, etc.).

Dimensionality of data before transform:

$$n \times 2^n$$

Dimensionality of data after transform (using $d = O(\log n)$ as provided in lecture notes):

$$n \log(2) \times 2^n$$

There is no decrease in dimensionality from doing a JL-transform, so we wouldn't use it before analyzing the points.

Part b

There are $2n$ points (2-d circle has 4 points, 3-d sphere has 6 points, etc.).

Dimensionality of data before transform:

$$n \times 2n$$

Dimensionality of data after transform (using $d = O(\log n)$ as provided in lecture notes):

$$\begin{aligned} &\log(2n) \times 2n \\ =&(\log(2) + \log(n)) \times 2n \end{aligned}$$

There is a decrease in dimensionality from doing a JL-transform, so we would indeed use it before analyzing the points.

Problem 2

Part a

$$
\begin{aligned}
A_k &= U_k D_k V_k^T \\
\text{Let } Y_k &= D_k V_k^T \\
&= \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & \sigma_k \end{bmatrix} \cdot \begin{bmatrix} v_1(1) & v_1(2) & \dots & v_1(d) \\ v_2(1) & v_2(2) & \dots & v_2(d) \\ \vdots & \vdots & \ddots & \vdots \\ v_k(1) & v_k(2) & \dots & v_k(d) \end{bmatrix} \\
Y_k(i,\ell) &= \sum_{t=1}^{k} d(i,t)v(t,\ell) \\
&= \sigma_i v_i(\ell)
\end{aligned}
$$

The transition to the bottom line above is true because when $i \neq t, d(i,t) = 0$, and when $i = t, d(i,t) = \sigma_i$.

$$
\begin{aligned}
A_k &= U_k D_k V_k^T \\
&= U_k Y_k \\
A_k(j,\ell) &= \sum_{i=1}^{k} u(j,i)y(i,\ell) \\
&= \sum_{i=1}^{k} u_i(j)\sigma_i v_i(\ell) \\
&= \sum_{i=1}^{k} \sigma_i u_i(j) v_i(\ell)
\end{aligned}
$$

Part b

$$
\begin{aligned}
[\sigma_i u_i v_i^T](j,\ell) &= \sigma_i u_i(j) v_i(\ell) \\
[\sum_{i=1}^{k} \sigma_i u_i v_i^T](j,\ell) &= \sum_{i=1}^{k} \sigma_i u_i(j) v_i(\ell) \\
[\sum_{i=1}^{k} \sigma_i u_i v_i^T](j,\ell) &= A_k(j,\ell) \\
A_k &= \sum_{i=1}^{k} \sigma_i u_i v_i^T
\end{aligned}
$$

Part c

$$
\begin{aligned}
A_k &= \sum_{i=1}^{k} \sigma_i u_i v_i^T \\
A_{k+1} &= A_k + \sigma_{k+1} u_{k+1} v_{k+1}^T
\end{aligned}
$$

Part d

$$
\begin{aligned}
A &= \sum_{i=1}^{r} \sigma_i u_i v_i^T \\
A(j,\ell) &= \sum_{i=1}^{r} \sigma_i u_i(j) v_i(\ell) \\
R_k &= A - A_k \\
R_k(j,\ell) &= A(j,\ell) - A_k(j,\ell) \\
R_k(j,\ell) &= \sum_{i=1}^{r} \sigma_i u_i(j) v_i(\ell) - \sum_{i=1}^{k} \sigma_i \\
&= \sum_{i=k+1}^{r} \sigma_i u_i(j) v_i(\ell)
\end{aligned}
$$