Homework 1
Colorado CSCI 5654

Alex Book

January 27, 2021

Problem 1

$$
\begin{aligned}
\text{min. } & 3x_1 - 5x_2 \\
\text{s.t. } & 4x_1 + x_2 \geq -4 \\
& 2x_1 - x_2 \geq -8 \\
& x_1 + 2x_2 \leq 4 \\
& x_1 \geq 0
\end{aligned}
$$

$$\downarrow$$

$$
\begin{aligned}
\text{max. } & -3x_1 + 5x_2^+ - 5x_2^- \\
\text{s.t. } & -4x_1 - x_2^+ + x_2^- \leq 4 \\
& -2x_1 + x_2^+ - x_2^- \leq 8 \\
& x_1 + 2x_2^+ - 2x_2^- \leq 4 \\
& x_1, x_2^+, x_2^- \geq 0
\end{aligned}
$$

Problem 2

$$\begin{aligned}
\text{max.} \quad & -3x_1 + 5x_2^+ - 5x_2^- \\
\text{s.t.} \quad & -4x_1 - x_2^+ + x_2^- \leq 4 && \leftarrow w_1 \\
& -2x_1 + x_2^+ - x_2^- \leq 8 && \leftarrow w_2 \\
& x_1 + 2x_2^+ - 2x_2^- \leq 4 && \leftarrow w_3 \\
& x_1, x_2^+, x_2^- \geq 0
\end{aligned}$$

$$\downarrow$$

$$\begin{aligned}
\text{max.} \quad & -3x_1 + 5x_2^+ - 5x_2^- \\
\text{s.t.} \quad & w_1 = 4 + 4x_1 + x_2^+ - x_2^- \\
& w_2 = 8 + 2x_1 - x_2^+ + x_2^- \\
& w_3 = 4 - x_1 - 2x_2^+ + 2x_2^- \\
& x_1, x_2^+, x_2^-, w_1, w_2, w_3 \geq 0
\end{aligned}$$

Initial dictionary:

w_1	4 + $4x_1 + x_2^+ - x_2^-$
w_2	8 + $2x_1 - x_2^+ + x_2^-$
w_3	4 - $1x_1 - 2x_2^+ + 2x_2^-$
z	0 - $3x_1 + 5x_2^+ - 5x_2^-$

Entering variable: x_2^+, Leaving variable: w_3
$x_2^+ = 2 - .5x_1 - .5w_3 + x_2^-$

Dictionary after pivoting:

w_1	6 + $3.5x_1 - .5w_3 + 0x_2^-$
w_2	6 + $2.5x_1 + .5w_3 + 0x_2^-$
x_2^+	2 - $.5x_1 - .5w_3 + x_2^-$
z	10 - $5.5x_1 - 2.5w_3 + 0x_2^-$

Problem 3

See attached files for code and solutions. Solutions are copied below for convenience:

Investment Number	Units Purchased
1	0
2	0
3	467.1442
4	0
5	0
6	0
7	0
8	0
9	0
10	147.6378
11	0
12	53.7634
13	0
14	459.6856
15	0

Total profits: 8,513.50

This problem is both feasible and bounded.

Problem 4

a.

Need to prove $A(\lambda x_1 + (1-\lambda)x_2) \leq b$

$$\begin{aligned}
&A(\lambda x_1 + (1-\lambda)x_2) \\
=&\lambda A x_1 + A x_2 - \lambda A x_2 \\
\leq&\lambda b + b - \lambda b = b
\end{aligned}$$

b.

Need to prove $c^T(\lambda x_1 + (1-\lambda)x_2) = z^*$

$$\begin{aligned}
&c^T(\lambda x_1 + (1-\lambda)x_2) \\
=&\lambda c^T x_1 + c^T x_2 - \lambda c^T x_2 \\
=&\lambda z^* + z^* - \lambda z^* \\
=&z^*
\end{aligned}$$

Problem 5

a.

$a_1x_1 - a_5x_5 \leq 0$
$a_5x_5 - a_3x_3 \leq 0$

b.

$.4\Sigma_{i=1}^{n} a_ix_i - a_1x_1 \leq 0 \xrightarrow{simplify} .4\Sigma_{i=2}^{n} a_ix_i - .6a_1x_1 \leq 0$
$a_1x_1 - .6\Sigma_{i=1}^{n} a_ix_i \leq 0 \xrightarrow{simplify} .4a_1x_1 - .6\Sigma_{i=2}^{n} a_ix_i \leq 0$

c.

Such a constraint allows solutions along at least one of the n axes. The set of points between solutions that lie on separate axes contains points that aren't on any axis (there are points on the line between two solutions that are not themselves solutions). Thus this constraint is not convex, and LP formulation is not possible.

Problem 6

$$\begin{aligned} x_1 - 10w_1 &\leq 0 \\ x_2 - 15w_2 &\leq 0 \\ x_1 - (1 - w_2)10 &\leq 0 \\ x_2 - (1 - w_1)15 &\leq 0 \end{aligned}$$