# Logic for Computer Science

*Project Report*

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## 1. Introduction

This report is about solving the knight's tour problem using a SAT solver. We'll need a variable to keep track of the step the program is in. The variable $s$ (for step) ranges from 0 to $M \times N - 1$.

We now have a way to add clauses to our solver. Each $(s, i, j)$ tuple describes a proposition. The following proposition means "The knight is in cell (i, j) at step $s$" : $x_{(s,i,j)}$. We also map each $(s, i, j)$ to one unique ID with a dictionary.

In this document, $i \in [0, M]$ and $j \in [0, N]$ and $(i_0, j_0)$ is the starting cell.

## 2. First Question

We define the boolean variable $x_{s,i,j}$ which is true if the knight is in cell $(i, j)$ at step $s$. The indices range as follows: steps $s \in [0, M \times N - 1]$, rows $i \in [0, M - 1]$, and columns $j \in [0, N - 1]$.

We encode the problem using the following constraints derived directly from the CNF clauses in our code:

a) Initial Position: At step $s = 0$, the knight must be at the given position $(i_0, j_0)$.

$$x_{0,i_0,j_0}$$

b) Valid Position at Each Step: For every step $s$, the knight must be in exactly one cell. We split this into two parts:
   - At least one cell:

$$\bigwedge_s \left( \bigvee_{i,j} x_{s,i,j} \right)$$

   - At most one cell (Pairwise Exclusion): For every pair of distinct cells, the knight cannot be in both.

$$\bigwedge_s \left( \bigwedge_{(i,j)\neq(i',j')} \left( \neg x_{s,i,j} \vee \neg x_{s,i',j'} \right) \right)$$

c) Legal Moves (Transitions): For every step $s < M \times N - 1$, if the knight is at $(i, j)$, it must move to a valid neighbor in the next step.

$$\bigwedge_{s=0}^{MN-2} \bigwedge_{i,j} \left( \neg x_{s,i,j} \vee \bigvee_{(i',j')\in \text{ Moves}(i,j)} x_{s+1,i',j'} \right)$$

This is equivalent to the implication $x_{s,i,j} \Rightarrow \bigvee x_{s+1,i',j'}$.

d) Visit Each Cell Exactly Once: Every cell $(i, j)$ must be visited at exactly one time step.

- At least once:

$$\bigwedge_{i,j} \left( \bigvee_{s} x_{s,i,j} \right)$$

- At most once (Pairwise Exclusion): For any cell, it cannot be visited at two different steps $s$ and $s'$.

$$\bigwedge_{i,j} \left( \bigwedge_{s \neq s'} \left( \neg x_{s,i,j} \vee \neg x_{s',i,j} \right) \right)$$

# 3. Second Question

# 4. Third Question

# 5. Fourth Question

# 6. Fifth Question