Logic for Computer Science

Project Report

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1. Introduction

This report is about solving the knight's tour problem using a SAT solver. We'll need a variable to keep track of the step the program is in. The variable s (for step) ranges from 0 to $M \times N - 1$.

We now have a way to add clauses to our solver. Each (s, i, j) tuple describes a proposition. The following proposition means "The knight is in cell (i, j) at step s" : $x_{(s,i,j)}$. We also map each (s, i, j) to one unique ID with a dictionary.

In this document, $i \in [0, M]$ and $j \in [0, N]$ and (i_0, j_0) is the starting cell.

2. First Question

We define the proposition $x_{(s,i,j)}$ as true if the knight is in cell (i, j) at step s. We have the following constraints:

a) At step $s = 0$, the knight is at (i_0, j_0).

$$x_{(0,i_0,j_0)}$$

b) At every step s, the knight must be in exactly one cell. This requires two clauses: "at least one" and "at most one".

$$\bigwedge_s \left(\bigvee_{i,j} x_{(s,i,j)} \right) \quad \text{and} \quad \bigwedge_s \left(\bigwedge_{(i,j) \neq (i',j')} \left(\neg x_{(s,i,j)} \vee \neg x_{(s,i',j')} \right) \right)$$

c) If the knight is at (i, j) at step s, it must be at a valid neighbor $(i', j') \in \text{Moves}(i, j)$ at step $s + 1$.

$$\bigwedge_{s,i,j} \left(x_{(s,i,j)} \Rightarrow \bigvee_{(i',j') \in \text{ Moves}(i,j)} x_{(s+1,i',j')} \right)$$

d) Every cell (i, j) must be occupied at exactly one time step s.

$$\bigwedge_{i,j} \left(\bigvee_s x_{(s,i,j)} \right) \quad \text{and} \quad \bigwedge_{i,j} \left(\bigwedge_{s \neq s'} \left(\neg x_{(s,i,j)} \vee \neg x_{(s',i,j)} \right) \right)$$

3. Second Question

4. Third Question

To count the number of solutions, we use a loop with Blocking Clauses:

1. Run the SAT solver.
2. If a model (solution) S is found, increment the counter.
3. Add a new clause to the solver that invalidates S. The clause is the negation of the conjunction of all true variables in S:

$$\neg\left(\bigwedge_{(s,i,j)\in S} x_{(s,i,j)}\right) \equiv \bigvee_{(s,i,j)\in S} \neg x_{(s,i,j)}$$

4. Repeat until the solver returns “UNSAT”.

5. Fourth Question

6. Fifth Question