ANALISI 1 del 23 Settembre

1.0

Binomio di Newton: Dimostrazione

$$P(n) = \left\{ (a+b)^n = \sum_{K=0}^{n} \binom{n}{K} a^{n-K} b^K \right\}$$

1) $$P(0) = \left\{ (a+b)^0 = \sum_{K=0}^{0} \binom{0}{K} a^{-K} b^K = \binom{0}{0} a^0 + b^0 = 1 \right\} \checkmark$$

$$P(1) = \left\{ (a+b)^1 = \sum_{K=0}^{1} \binom{1}{K} a^{1-K} b^K = \binom{1}{0} a^1 + \binom{1}{1} b^1 = a+b \right\} \checkmark$$

2) $$P(n+1) = \sum_{K=0}^{n+1} \binom{n+1}{K} a^{n+1-K} b^K = (a+b)^{n+1}$$

$$(a+b)^{n+1} = (a+b)\underbrace{(a+b)^n}_{P(n)} = (a+b)\sum_{K=0}^{n} \binom{n}{K} a^{n-K} b^K = a\sum_{K=0}^{n} \binom{n}{K} a^{n-K} b^K + b\sum_{K=0}^{n} \binom{n}{K} a^{n-K} b^K =$$

$$= \sum_{K=0}^{n} \binom{n}{K} a^{n+1-K} b^K + \sum_{K=0}^{n} \binom{n}{K} a^{n-K} b^{K+1} = \sum_{K=0}^{n} \binom{n}{K} a^{n+1-K} b^K + \sum_{h=1}^{n+1} \binom{n}{h-1} a^{n+1-h} b^h =$$

$h = K+1$

$$= \sum_{K=0}^{n} \binom{n}{K} a^{n+1-K} b^K + \sum_{h=1}^{n} \binom{n}{h-1} a^{n+1-h} b^h + \binom{n+1-1}{n+1-1} a^{\cancel{n+1-n-1}} b^{n+1} =$$

$n : \{1, 2, \dots\}$
$K : 0 \quad 1 \quad 2 \dots n$
$h : 1 \quad 2 \quad 3 \dots n+1$

$$= \sum_{K=0}^{n} \binom{n}{K} a^{n+1-K} b^K + \sum_{h=1}^{n} \binom{n}{h-1} a^{n+1-h} b^h + b^{n+1} = \binom{n}{0} a^{n+1} + \sum_{K=1}^{n} \binom{n}{K} a^{n+1-K} b^K + \sum_{h=1}^{n} a^{n+1-h} b^h + b^{n+1} =$$

$$= a^{n+1} + \sum_{K=1}^{n} \left[\binom{n}{K} a^{n+1-K} b^K + \binom{n}{K-1} a^{n+1-K} b^K \right] + b^{n+1} = a^{n+1} + \sum_{K=1}^{n} \left[\binom{n}{K} + \binom{n}{K-1} \right] a^{n+1-K} b^K + b^{n+1} =$$

$$= a^{n+1} + \sum_{K=1}^{n} \binom{n+1}{K} a^{n+1-K} b^K + b^{n+1} = \binom{n+1}{0} a^{n+1-0} b^0 + \sum_{K=1}^{n} \binom{n+1}{K} a^{n+1-K} b^K + \binom{n+1}{n+1} a^0 b^{n+1} = \sum_{K=0}^{n+1} a^{n+1-K} b^K$$

DIMOSTRATO!

1.1

$(2,3) \sim (1,2) \iff 2+2 = 3+1 = 4$

!! $2-3 = -1 = 1-2$!!

$(5,8) \sim (3,6) \iff 5+6 = 8+3$

!! $5-8 = -3 = 3-6$!!

1.2

Dimostrazione $- \times - = +$

$2 \cdot (-3) = -3-3 = -6$

$(-2)(-3) = ?$ Dim: $0 = b \cdot 0 = b(a-a) = ba + b(-a) = ba - ba = ab - (-a)(-b) = 0$

$\hookrightarrow \underline{ab = (-a)(-b)}$

$\Downarrow$

$\underline{2 \cdot 3 = (-2) \cdot (-3) = 6}$

1.3

Dimostrazione $\sqrt{2} \notin \mathbb{Q}$ (per assurdo)

Supponiamo $\exists\, m, n \in \mathbb{Z}, n \neq 0 \mid x = \frac{m}{n} \Rightarrow m, n$ non hanno fattori comuni.
allora $\exists\, x^2 = \frac{m^2}{n^2} = 2 \in \mathbb{Q} \Rightarrow m^2 = 2n^2 \Rightarrow \exists\, K \mid m^2 = 4K^2 \Rightarrow \underline{m^2 = 4K^2 = 2n^2} \rightarrow$ fattori comuni $= 2$

$\hookrightarrow$ ASSURDO: per Hp m ed n non hanno fattori comuni