CLASSIFICAZIONE DEL PUNTO DI EQUILIBRIO DI UN SISTEMA LINEARE AUTONOMO BIDIMENS.

$$\begin{cases} \dot{x} = ax + by \\ \dot{y} = cx + dy \end{cases} \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det A \neq 0$$

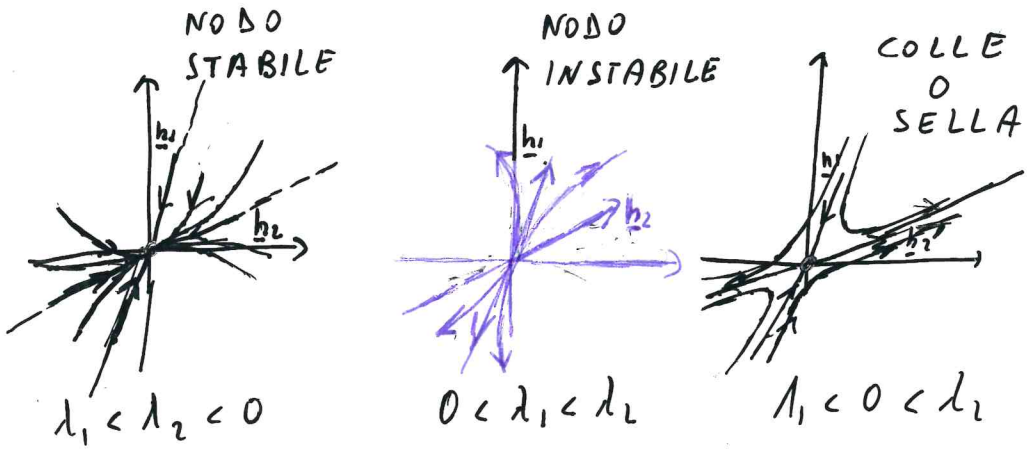
λ_1, λ_2 autovalori di A.

CASO $\lambda_1 \neq \lambda_2$ $\quad \lambda_1, \lambda_2 \in \mathbb{R}$ $\quad \lambda_1 < \lambda_2$

$$\underline{\varphi}(t) = c_1 \underline{h}_1 e^{\lambda_1 t} + c_2 \underline{h}_2 e^{\lambda_2 t} = (x(t), y(t))$$

$$\frac{y(t)}{x(t)} = \frac{c_1 h_{12} e^{(\lambda_1 - \lambda_2)t} + c_2 h_{22}}{c_1 h_{11} e^{(\lambda_1 - \lambda_2)t} + c_2 h_{21}} \nearrow \frac{h_{22}}{h_{21}} \text{ per } t \to +\infty \quad c_2 \neq 0$$

$$\nrightarrow \frac{h_{12}}{h_{11}} \qquad c_2 = 0$$



$\lambda_1 < \lambda_2 < 0$ $\qquad 0 < \lambda_1 < \lambda_2$ $\qquad \lambda_1 < 0 < \lambda_2$

2) CASO AUTOVALORI COMPLESSI CONIUGATI

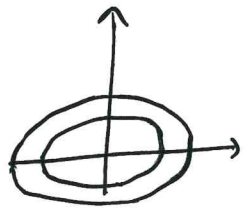
$$\underline{\varphi}(t) = c_1 \underline{h}_1 e^{(\alpha + i\beta)t} + c_2 \bar{\underline{h}}_1 e^{(\alpha - i\beta)t} =$$

$$= e^{\alpha t} \left(\left(c_1 \underline{h}^1 + c_2 \underline{h}^2 \right) \cos \beta t + \left(c_1 \underline{h}^1 - c_2 \underline{h}^2 \right) \sin \beta t \right)$$

$\alpha = 0$ soluzioni periodiche - orbite ellittiche.

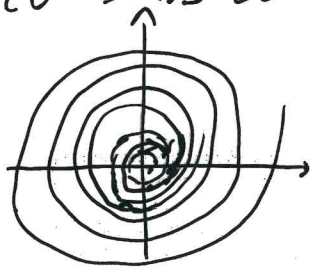
$E(x,y) = cx^2 + 2axy + by^2$ per a, b, c opportuni

$\underline{0}$ STABILE (CENTRO)



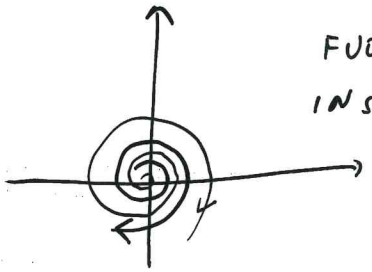
$\alpha < 0 \quad (\alpha > 0) \qquad \|\varphi(t)\| \to 0 \quad (\to +\infty)$ per $t \to +\infty$

FUOCO STABILE AS.



$\alpha < 0$

FUOCO INSTABILE



$\alpha > 0$

) CASO AUTOVALORE DOPPIO λ

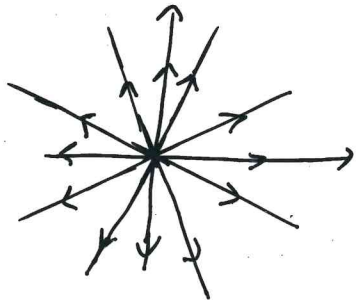
3a) λ REGOLARE : DEVE ESSERE

$$r \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0 \quad \text{cioè} \quad b = c = 0$$

$$a = d = \lambda \ (\neq 0)$$

$$\underline{\varphi}(t) = c_1 \underline{h}_1 e^{\lambda t} + c_2 \underline{h}_2 e^{\lambda t} \qquad \underline{h}_1, \underline{h}_2 \text{ autovettori}$$

$$= \underline{c}\, e^{\lambda t}$$



NODO A STELLA

OGNI SEMIRETTA USCENTE DA $\underline{0}$ È UN' ORBITA

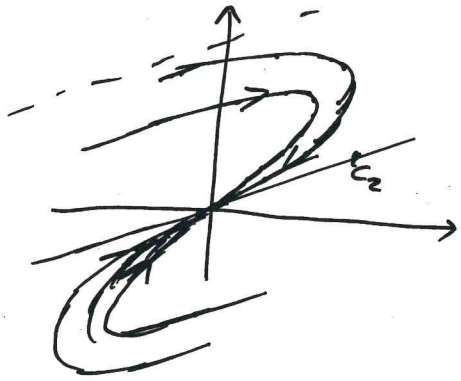
$\lambda > 0$ INSTABILE

($\lambda < 0$ AS. STAB)

3b) λ non regolare.

$$\underline{\varphi}(t) = \underline{c}_1 e^{\lambda t} + \underline{c}_2 t e^{\lambda t}$$

per $t \to +\infty$, $\underline{c}_2 \neq 0$, $\dfrac{y(t)}{x(t)} \to \dfrac{c_{22}}{c_{21}}$



caso $\lambda < 0$

NODO (A UNA TANGENTE)

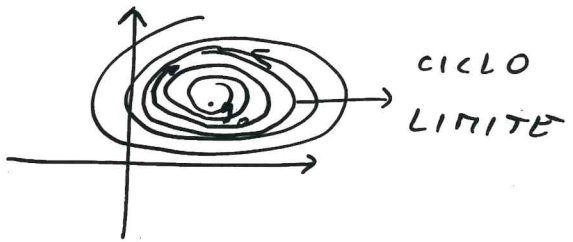
QUALCHE INFORMAZIONE IN PIÙ SUL NON LINEARE

SPIRALI → SPIRALI

NODI → NODI o SPIRALI

CENTRI →

↘ CENTRI



CICLO LIMITE