

1) Determinare il carattere di $\sum_{n=1}^{\infty} \left(\cos \tan \frac{1}{n} - \frac{1}{n} \right)$
 $\sum_{n=1}^{\infty} \cos \tan \frac{1}{n} \rightarrow +\infty$, $\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow +\infty$ \hookrightarrow Serie armonica: converge quando $a_n = l < b_n = \infty$. Non possiamo dire nulla se $a_n = 0, b_n = 0$.
 $T_2(\cos \tan \frac{1}{n}) = -\frac{1}{2} \cos^2 \tan \frac{1}{n} + \frac{1}{n} \cos \tan \frac{1}{n}$
 $T_2(\cos \tan \frac{1}{n}) = \frac{1}{n} - \frac{1}{2} \left(\tan \frac{1}{n} \right)^2 + o\left(\frac{1}{n}\right) \rightarrow \frac{1}{n} - \frac{1}{2} \tan^2 \frac{1}{n} = \frac{1}{n} - \frac{1}{2} \left(\frac{1}{n} \right)^2 + o\left(\frac{1}{n}\right) = -\frac{1}{2} \frac{1}{n^2} + o\left(\frac{1}{n}\right) \rightarrow$ converge.

2) $\int (3x-1)^3 dx = \frac{1}{3} \int 3(3x-1)^2 dx = \frac{3 \cdot 3x^3}{3} + c$
 $\int x \sqrt{5-x^2} dx = -\frac{1}{3} \int 3x \sqrt{5-x^2} dx = -\frac{1}{3} \int \frac{5-x^2}{3} dx = -\frac{1}{9} \left(5x - \frac{x^3}{3} \right) + c = -\frac{5x}{9} + \frac{x^3}{27} + c$
 $\int \frac{\ln^2 x}{x} dx = \frac{\ln^3 x}{3} + c$
 $\int \frac{1}{x \ln x} dx = \ln |\ln x| + c$
 $\int \tan(x) dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{\cos x}{\cos x} dx = -\ln |\cos x| + c$
 $\int 3x^2 e^{x^3} dx = \frac{3}{10} \int 10x^2 e^{x^3} dx = \frac{3}{10} e^{x^3} + c$
 $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = 2 \int \frac{\sin \sqrt{x}}{2\sqrt{x}} dx = -2 \cos \sqrt{x} + c$
 $\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + c$
 $\int \frac{1}{x} = \ln |x|$
 $\int \frac{1}{x^2} = -\frac{1}{x}$
 $\int 2x(\ln x)^2 = (\ln x)^2 \int 2x dx = (\ln x)^2 \cdot x^2 = (x \ln x)^2 - \int x \ln x dx = (x \ln x)^2 - \left[\frac{x^2 \ln x}{2} - \int \frac{x^2}{2} dx \right] = (x \ln x)^2 - \left[\frac{x^2 \ln x}{2} - \frac{x^3}{6} \right] + c$
 $\int \frac{1}{x} = \ln |x|$
 $\int \frac{1}{x^2} = -\frac{1}{x}$
 $\int \frac{1}{x^3} = -\frac{1}{2x^2}$