1) $\sum_{n=2}^{+\infty} \frac{1}{n^2-1} = \sum_{n=2}^{+\infty} \frac{1}{(n+1)(n-1)} = \sum_{n=2}^{+\infty} \frac{1}{2}\left(\frac{1}{(n+1)} - \frac{1}{(n-1)}\right) = \frac{1}{2} \lim_{K\to\infty} \sum_{n=2}^{K}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) = \frac{1}{2} \lim_{K\to+\infty}\left(\sum_{n=2}^{K}\frac{1}{n-1} - \sum_{n=2}^{K}\frac{1}{n+1}\right) = \frac{1}{2} \lim_{K\to+\infty}\left(\sum_{j=0}^{K-2}\frac{1}{j+1} - \sum_{n=2}^{K}\frac{1}{n+1}\right) = \frac{1}{2} \lim_{K\to+\infty}\left(1 + \frac{1}{2} + \cancel{\sum_{j=2}^{K-2}\frac{1}{j+1}} - \cancel{\sum_{n=2}^{K-2}\frac{1}{n+1}} - \frac{1}{K} - \frac{1}{K+1}\right) = \frac{1}{2} \lim_{K\to+\infty}\left(1+\frac{1}{2}\right) = \frac{3}{4}$

↳ converge: confr. as. $\frac{1}{n^2} \sim \frac{1}{n^2-1} \Rightarrow \frac{1}{n^2}$ conv. $\Rightarrow \frac{1}{n^2-1}$ conv.

2) $\sum_{n=1}^{+\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n^2+n}} = \sum_{n=1}^{+\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n(n+1)}} = \sum_{n=1}^{+\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = [\,\dots\,]$

3) $\sum_{n=1}^{+\infty} \ln\left(1+\frac{1}{n}\right) = [\,\dots\,] = +\infty$

4) $\sum_{n=1}^{+\infty} \frac{\cos(n\frac{\pi}{2})}{\sqrt{n}} = \sum_{K=1}^{+\infty} \frac{(-1)^K}{\sqrt{2K}} \Rightarrow$ per criterio di Leibniz converge

$\frac{\cos(n\frac{\pi}{2})}{\sqrt{n}} \begin{cases} 0 & n \text{ dispari} \\ \frac{(-1)^K}{\sqrt{2K}} & n = 2K \end{cases}$

5) $\sum_{n=1}^{+\infty} (-1)^n \frac{\tan\frac{1}{n}}{10^n \ln n} \Rightarrow \left|(-1)^n \frac{\tan\frac{1}{n}}{10^n \ln n}\right| \sim \left|\frac{1}{n10^n \ln n}\right| \Rightarrow$ converge abs

$0 \le \left|\frac{1}{n10^n \ln n}\right| \le \frac{1}{10^n} \Rightarrow$ per confronto converge abs

6) $\sum_{n=1}^{+\infty} (-1)^n \frac{1}{n}\sin\frac{1}{n} \Rightarrow \left|(-1)^n \frac{1}{n}\sin\frac{1}{n}\right| \sim \frac{1}{n^2} \Rightarrow$ converge abs

$\sum_{n=1}^{+\infty} (-1)^n \frac{n}{\sqrt{n^4+n}} \Rightarrow \left|(-1)^n \frac{n}{\sqrt{n^4+n}}\right| \sim \frac{1}{n} \Rightarrow$ non converge abs.

$\frac{n}{\sqrt{n^4+n}} \Rightarrow$ restrizione di $f(x) \Rightarrow f'(x) = \dots = \frac{-2x^4+x}{2(x^4+x)^{3/2}} < 0$ definitivamente $\Rightarrow$ monotona decrescente def. $\Rightarrow$ per Leibniz converge