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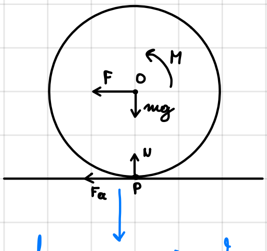
### 11.6.1 CONDIZIONI PER IL ROTOLAMENTO PURO

Sul corpo agiscono un momento e una forza applicata sul centro di massa. È necessaria anche una forza d'attrito di natura statica (a causa della condizione di rotolamento puro):




$$\begin{cases} F + F_a = m a_{cm} \\ N - mg = 0 \end{cases} \longrightarrow \underset{\text{(rotolamento puro)}}{a_{cm} = -\alpha R} \Rightarrow \begin{cases} F + F_a = m a_{cm} \\ N = mg \end{cases} \text{ — } \dots$$

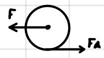
$$F_a R - M = -\frac{I a_{cm}}{R}$$

$$F_a R - M = I\alpha$$

il verso non è noto, dipende dal segno di $F_a$ nei calcoli.

$$\dots \longrightarrow \begin{cases} a_{cm} = \dfrac{F + F_a}{m} \\ F_a R - M = -\dfrac{I}{R}\left(\dfrac{F+F_a}{m}\right) \end{cases} \longrightarrow \dots \longrightarrow \begin{cases} a_{cm} = \dfrac{FR^2 + MR}{I + mR^2} \\ F_a = \dfrac{mMR - IF}{I + mR^2} \end{cases}$$

Consideriamo il caso $- F \neq 0,\ M = 0$: $\quad F_a = -\dfrac{IF}{I + mR^2}$




↳ attrito statico → $|F_A| < \mu_s m g$ → $F \leq \mu_s m g \left(1 + \dfrac{mR^2}{I}\right)$

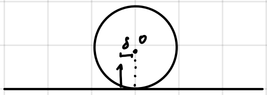
↳ se F è maggiore non siamo più in rotolamento puro

$- F = 0,\ M \neq 0$: $\quad F_a = \dfrac{mMR}{I + mR^2} > 0$

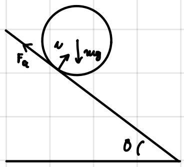
↳ $|F_a| < \mu_s N$ → $M \leq \dfrac{\mu_s g}{R}(I + mR^2)$

### 11.6.2 ATTRITO VOLVENTE

La forza di attrito, come detto prima, è statico ed è applicato, non solo su un punto, ma su una superficie. L'attrito viene chiamato attrito volvente e ha come momento: $M = N\delta$




#### ESERCIZIO




$\mu_s$ | rot puro?

$v$ alla fine del piano

$$\begin{cases} mg \sin\theta - F_a = ma \\ N - mg\cos\theta = 0 \end{cases} \longrightarrow \begin{cases} mg\sin\theta - F_a = ma \\ F_a R = I\, a/R \end{cases} \rightarrow mg\sin\theta - F_a = m\,\frac{F_a R^2}{I}$$

$$F_A R = I\alpha = I\,\frac{a}{R}$$

$$F_a = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}} = \frac{mg\sin\theta}{3}$$

$I_{disco} = \frac{1}{2} mR^2$

$$|F_a| \leq \mu_s N \rightarrow \mu_s \geq \frac{\tan\theta}{3}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} \rightarrow \dots \rightarrow v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{4}{3}gh}$$

## ESERCITAZIONE

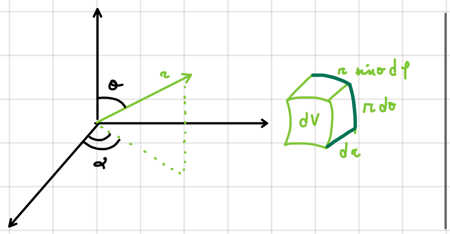
### ESERCIZIO 1

$$I = \rho\int r^2 dV = \int_0^R\int_0^\pi\int_0^{2\pi} \rho\, r^2\sin^2\theta\, r^2\sin\theta\, dr\, d\theta\, d\alpha = 2\pi\rho\int_0^R r^4 dr\int_0^\pi \sin^3\theta\, d\theta = 2\pi\rho\frac{1}{5}R^5\int_0^\pi \sin^3\theta\, d\theta = \dots = 2\pi\rho\frac{1}{5}R^5\cdot\frac{4}{3} =$$

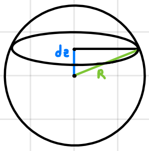
$$= 2\pi\frac{M}{\frac{4}{3}\pi R^3}\frac{1}{5}R^5\frac{4}{3} = \frac{2}{5}MR^2$$

→ $dm = \rho\, dV$

→ $dV = r^2\sin\theta\, dr\, d\theta\, d\alpha$



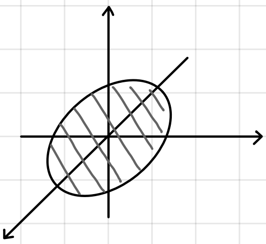
## Metodo alternativo:




$$dI = \frac{1}{2} dm (R^2 - z^2)$$
$$dm = \rho dV = \rho \pi (R^2 - z^2) dz$$

$$\longrightarrow I = \int_{-R}^{R} \frac{1}{2} \rho \pi (R^2 - z^2)^2 dz = \frac{1}{2} \rho \pi \int_{-R}^{R} (R^2 - z^2)^2 dz = \frac{1}{2} \rho \pi \left[ R^4 z + \frac{1}{5} z^5 - \frac{2}{3} R^2 z^3 \right]_{-R}^{R} =$$
$$= \cdots = \frac{2}{5} M R^2$$

## ESERCIZIO 2



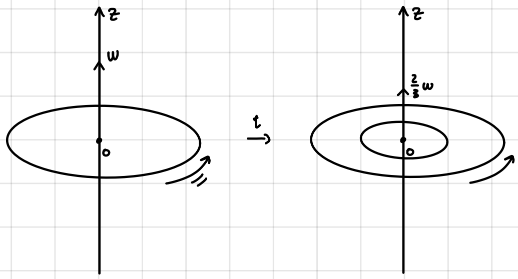
$$\sigma = \frac{M}{\pi R^2}$$

coordinate polari

$$ds = r\, dr\, d\theta \rightarrow dI_x = (r \sin\theta)^2 dm = f(r, \theta)\, dr\, d\theta$$ X (se voglio fare 2 integrali, sennò uso th asse perp.)

$$dI_z = dm\, x^2 + dm\, y^2 = dI_x + dI_y = 2 dI_x \Rightarrow I_x = \frac{1}{2} I_z = \frac{1}{4} M R^2$$
$$dI_x = dI_y$$

## ESERCIZIO 3

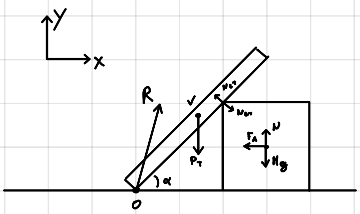



$I_i = \frac{1}{2} M R^2$ $\quad \omega_i = \omega$ $\quad M = \sigma \pi R^2$ $\quad r\,?$

$I_F = \frac{1}{2} M R^2 + \frac{1}{2} m r^2$ $\quad \omega_F = \frac{2}{3} \omega$ $\quad m = \sigma \pi r^2$

$$\vec{M} = \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L} \text{ cost}$$
$$\Downarrow$$
$$I_i \omega_i = I_F \omega_F$$
$$\Downarrow$$
$$I_F = I_i \frac{\omega_F}{\omega_i} = \frac{3}{2} I_i \longrightarrow \frac{I_f}{I_i} = \frac{\frac{1}{2} M R^2 + \frac{1}{2} m r^2}{\frac{1}{2} M R^2} = 1 + \frac{\cancel{\sigma \pi} r^2 r^2}{\cancel{\sigma \pi} R^2 R^2} = 1 + \frac{r^4}{R^4} = \frac{3}{2}$$
$$\hookrightarrow r = \sqrt[4]{\frac{1}{2}}\, R$$

## ESERCIZIO 4




$\alpha = 45°$

$OV = \frac{\ell}{\sqrt{2}}$ $\quad M = 1\,Kg$ $\quad \mu_s = 0{,}2$ $\quad (\vec{N}_{TC} = -\vec{N}_{CT})$

$$m \mid \vec{F}_{TOT} = \vec{M}_{TOT} = 0\ ?$$

$$T: \begin{cases} R_x - \frac{N_{CT}}{\sqrt{2}} = 0 \\ \frac{N_{CT}}{\sqrt{2}} - mg + R_y = 0 \\ -\frac{\ell}{2} \frac{mg}{\sqrt{2}} + \frac{\ell}{\sqrt{2}} N_{CT} = 0 \end{cases} \longrightarrow \begin{aligned} R_y &= mg - \frac{N_{CT}}{\sqrt{2}} \\ R_x &= \frac{N_{CT}}{\sqrt{2}} \\ N_{CT} &= \frac{mg}{2} \end{aligned}$$

$$C: \begin{cases} -F_a + \frac{N_{TC}}{\sqrt{2}} = 0 \\ N - \frac{N_{TC}}{\sqrt{2}} - Mg = 0 \end{cases} \longrightarrow \begin{aligned} F_A &= \frac{N_{TC}}{\sqrt{2}} = \frac{mg}{2\sqrt{2}} \leq \mu_s N \\ N &= Mg + \frac{N_{TC}}{\sqrt{2}} \end{aligned}$$
$$\Downarrow$$
$$\frac{mg}{2\sqrt{2}} \leq \mu_s Mg + \mu_s \frac{mg}{2\sqrt{2}}$$
$$\vdots$$
$$m \leq \frac{2\sqrt{2}\mu_s}{1 - \mu_s} M \Longrightarrow m_{MAX} = \ldots = 0{,}7\,Kg$$