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$A^{-1} = \frac{1}{|A|} \cdot A^*$ $\quad A^* = [C_{ji}]$ $\quad r(A) = \dim(R(A)) = \dim(C(A))$

$U_1 + U_2 = \{ v \in V \mid v = u_1 + u_2,\ u_1 \in U_1,\ u_2 \in U_2 \}$ $\quad \dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$

$V = \bigoplus U_i \Rightarrow V = \sum U_i,\ U_k \cap U_{\hat{k}} = \{\bar{0}\}$ $\quad A = [u_{1|B} | \dots | u_{n|B}] \to X = \sum t_i A_{c(i)} \to [A|X] \to [S|X']$

$Im(f) \sim C(F)$ $\quad Ker(f) \sim Ker(F)$ $\quad K(f) = \dim(Ker(f))$ $\quad r(f) = \dim(Im(f))$

$F_{|B_V B_W} = [f(v_1)_{|B_W} | \dots | f(v_n)_{|B_W}]$ $\quad \dim(V) = r(f) + K(f)$ $\quad$ iniet. $r(f) = \dim(V)$

$M_{B \to B'} = [v_{1|B'} | \dots | v_{n|B'}]$ $\quad M_{B' \to B} = M_{B \to B'}^{-1}$ $\quad$ suriet. $r(f) = \dim(W)$

$V_{|B'} = M_{B \to B'} \cdot V_{|B}$ $\quad F_{|B'_V B'_W} = M_{B_W \to B'_W} \cdot F_{|B_V B_W} \cdot M_{B'_V \to B_V}$ $\quad$ biiet. $r(f) = \dim(V) =$

$F_{|B'_V} = M_{B'_V \to B_V}^{-1} \cdot F_{|B_V} \cdot M_{B'_V \to B_V}$ $\quad = \dim(W)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = B_{|B_0} - A_{|B_0}$ $\quad A\ [u_{1|B} | \dots | u_{n|B}]$ $\quad X = Q_{|B_0} + \sum t_i A_{ci} \to [A | X - Q_{|B_0}] \to [S|X']$

$//\ r(\tilde{A}|\tilde{B}) \overset{?}{=} r(\tilde{A}) = n-q$

$\times\ r(\tilde{A}|\tilde{B}) = r(\tilde{A}) > n-q$ $\quad [\tilde{A}|\tilde{B}] = \left[\begin{array}{c|c} A & B \\ \hline A' & B \end{array}\right]$

sgh. $r(\tilde{A}|B) > r(\tilde{A}) > n-q$

$[A|B] \to$ rapp. alg di S, $p = \dim(S)$

$[A'|B'] \to$ rapp. alg di T, $q = \dim(T)$

$n = \dim(A)$

A simile a B se $\exists S: A = S^{-1} \cdot B \cdot S$, A simile a B se hanno autovalori coincidenti

A simile a $B \to$ $r(A) = r(B)$, $Tr(A) = Tr(B)$, $|A| = |B|$, $P_A(\lambda) = P_B(\lambda)$ (radici e coefficienti)

$V_\lambda = Ker(A - \lambda I)$ $\quad P_A(\lambda) = |A - \lambda I| \to C_0 = |A|,\ C_{n-1} = (-1)^{n-1} \cdot Tr(A),\ C_n = (-1)^n$

$\langle u, v \rangle_E = u^T_{|B} \cdot v_{|B}$ $\quad \langle A, B \rangle_F = Tr(A^T \cdot B)$ $\quad \langle x, y \rangle_G = x^T_{|B} \cdot G_{|B} \cdot y_{|B}$ $\quad G_{|U} = [\langle u_i, u_j \rangle]$ $\quad G_{|B'} = M^T_{B'B} G_{|B} M_{B'B}$

$\|u+v\|^2 = \|u\|^2 + \|v\|^2 + 2\langle u, v \rangle$ $\quad U^\perp = \{ v \in V \mid \langle u, v \rangle = 0 \}$ $\quad P_U(v) = \sum \frac{\langle v, u_i \rangle}{\|u_i\|^2} u_i \to v = \sum \frac{\langle v, v_i \rangle}{\|v_i\|^2} v_i$

GRAM-SCHMIDT:

$\tilde{u}_1 = u_1$

$\tilde{u}_i = u_i - \sum_{k=1}^{i-1} \frac{\langle \tilde{u}_k, u_i \rangle}{\|\tilde{u}_k\|^2} \tilde{u}_k$

$|\langle u, v \rangle| \le \|u\| \|v\| \Rightarrow \widehat{v_1 v_2} = \arccos\left(\frac{\langle v_1, v_2 \rangle}{\|v_1\| \|v_2\|} \right)$ $\quad \|u+v\| \le \|u\| + \|v\|$

B è orientata come B' se $|M_{BB'}| > 0$ $\quad \widehat{\widehat{UV}} = \begin{cases} \widehat{UV} & |[u|v]| > 0 \\ -\widehat{UV} & |[u|v]| < 0 \end{cases}$

$w = v \otimes u \Rightarrow \|w\| = \sqrt{G(v,u)}$, $w \in \{v, u\}^\perp$, $\{v, u, w\}$ è pos. or. $\Rightarrow w = \left| \left[v_{|B} | u_{|B} | e_n \right] \right|$ $\quad |\langle \tilde{v}, w \rangle| = \sqrt{G(u, v, \tilde{v})}$

$\langle v, u \otimes w \rangle = \langle w, v \otimes u \rangle$, $u_1 \otimes (u_2 \otimes u_3) = \langle u_1, u_3 \rangle \cdot u_2 - \langle u_1, u_2 \rangle \cdot u_3$, $u_1 \otimes (u_2 \otimes u_3) + u_2 \otimes (u_3 \otimes u_1) + u_3 \otimes (u_1 \otimes u_2) = 0$

$f \in End(V)$, isometria $\to f \in O(V)$ $\quad O \subset GL(V)$ $\quad O(n; \mathbb{R}) \sim O(V)$ $\quad Q \in O(n; \mathbb{R}) \iff Q \cdot Q^T = I$ $\quad (|Q| = \pm 1)$

$SL = \{ A \in GL(n; \mathbb{K}) \mid |A| = 1 \} \to SO = SL \cap O \to SO(n; \mathbb{R}) \sim SO(V)$, $SO(V) \to$ rot.

se: $\begin{cases} f = Id_V \quad V_+ = V \ \} \ f = -Id_V \quad V_- = V \ \} \ |f| = 1,\ f \ne Id_V \Rightarrow \dim(V_+) = 1 \wedge \text{ruota in } V_+^\perp \text{ di } Tr(f) = 1 + 2\cos\alpha \\ |f| = -1,\ f \ne -Id_V \Rightarrow \dim(V_-) = 1 \wedge \text{riflette lungo } V_- \wedge \text{ruota in } V_-^\perp \text{ di } Tr(f) = -1 + 2\cos\alpha \end{cases}$

se: $|f| = 1 \Rightarrow$ ruota di $\cos\alpha = \frac{1}{2} Tr(f)$ $\}$ $|f| = -1 \Rightarrow$ riflette lungo V_+

f^* di f se $\langle f(v), w \rangle_W = \langle f^*(w), v \rangle_V$ $\quad F^* = F^T$ $\quad Im(f) = Ker(f^*)^\perp$, $Im(f^*) = Ker(f)^\perp$

se $f^* = f$ $\quad f$ è autoaggiunto $\Rightarrow f \in S(V)$ $\quad S(V) \sim S(n; \mathbb{R})$

f è ort. diag. se $\exists Q \in O(n; \mathbb{R}): D = Q^T \cdot F \cdot Q$ $\quad$ Spettrale: f è autoaggiunta se e solo se f è ort. diagon.

λ_1, λ_2 dist. $\Rightarrow V_{\lambda_1} \perp V_{\lambda_2}$ $\quad \hookrightarrow A$ è ort. diag. $\iff A \in S(n; \mathbb{R})$

$d(P, Q) = \|\overrightarrow{PQ}\|$ $\quad \widehat{P_1Q_1\, P_2Q_2} = \widehat{\overrightarrow{P_1Q_1}\, \overrightarrow{P_2Q_2}}$ $\quad d(S, P) = \sqrt{\frac{G(u_1 \dots u_n, \overrightarrow{QP})}{G(u_1 \dots u_n)}} = \|\overrightarrow{P_S(P) P}\|$

Se S è iperpiano: $\quad$ rapp alg: $|[u_{1|B} | \dots | u_{n|B} | P_{|B_0}]| = |[u_{1|B} | \dots | u_{n|B} | Q_{|B_0}]|$

se ha rapp. alg. $\sum a_i x_i = b \Rightarrow d(S; P) = \frac{|\sum a_i \bar{x}_i - b|}{\sqrt{\sum a_i^2}}$

S e T sottospazi: $\quad d(S, T) = \sqrt{\frac{G(v_1, \dots, v_n, \overrightarrow{QP})}{G(v_1, \dots, v_n)}}$ con $\{v_1, \dots, v_n\}$ base di $U + W$, $P \in S$, $Q \in T$

$\hookrightarrow \times \Rightarrow d(S, T) = 0$ $\quad // \Rightarrow d(S, T) = d(S, P)\ \forall P \in T$ $\quad$ sgh. $\Rightarrow d(S, T) = d(S, T')$ $\quad T \subseteq T'$, $T' // S$

$r(C)$	$r(A)$	I_3	I_2	I_1I_3	*Forma canonica*	*Moduli*	*Nome*
3	2	$\neq 0$	>0	<0	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}=1$	$\alpha,\beta>0$	*Ellisse con punti reali*
3	2	$\neq 0$	>0	>0	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}=-1$	$\alpha,\beta>0$	*Ellisse privo di punti reali*
3	2	$\neq 0$	<0		$\frac{\tilde{x}^2}{\alpha^2}-\frac{\tilde{y}^2}{\beta^2}=1$	$\alpha,\beta>0$	*Iperbole*
3	1	$\neq 0$	$=0$		$\tilde{x}^2-2\rho\tilde{y}=0$	$\rho>0$	*Parabola*
2	2	$=0$	>0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}+\tilde{y}^2=0$	$\alpha>0$	*Coppia di rette incidenti con un unico punto reale*
2	2	$=0$	<0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}-\tilde{y}^2=0$	$\alpha>0$	*Coppia di rette incidenti con infiniti punti reali*
2	1	$=0$	$=0$	$=0$	$\frac{\tilde{x}^2}{\alpha^2}=1$	$\alpha>0$	*Coppia di rette parallele con infiniti punti reali*
2	1	$=0$	$=0$	$=0$	$\frac{\tilde{x}^2}{\alpha^2}=-1$	$\alpha>0$	*Coppia di rette parallele prive di punti reali*
1	1	$=0$	$=0$	$=0$	$\tilde{x}^2=0$		*Retta doppia*

$r(C)$	$r(A)$	I_4	I_3	I_2	I_1I_3	*Forma canonica*	*Moduli*	*Nome*
4	3	<0	$\neq 0$	>0	>0	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}+\frac{\tilde{z}^2}{\gamma^2}=1$	$\alpha,\beta,\gamma>0$	*Ellissoide con punti reali*
4	3	>0	$\neq 0$	>0	>0	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}+\frac{\tilde{z}^2}{\gamma^2}=-1$	$\alpha,\beta,\gamma>0$	*Ellissoide privo di punti reali*
4	3	>0	$\neq 0$	≤ 0		$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}-\frac{\tilde{z}^2}{\gamma^2}=1$	$\alpha,\beta,\gamma>0$	*Iperboloide ad una falda od iperbolico*
4	3	>0	$\neq 0$		≤ 0			
4	3	<0	$\neq 0$	≤ 0		$\frac{\tilde{x}^2}{\alpha^2}-\frac{\tilde{y}^2}{\beta^2}-\frac{\tilde{z}^2}{\gamma^2}=1$	$\alpha,\beta,\gamma>0$	*Iperboloide a due falde od ellittico*
4	3	<0	$\neq 0$		≤ 0			
4	2	<0	$=0$	>0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}+\tilde{y}^2-2\rho\tilde{z}=0$	$\alpha,\rho>0$	*Paraboloide ellittico*
4	2	>0	$=0$	<0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}-\tilde{y}^2-2\rho\tilde{z}=0$	$\alpha,\rho>0$	*Paraboloide iperbolico*
3	3	$=0$	$\neq 0$	>0	>0	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}+\tilde{z}^2=0$	$\alpha,\beta>0$	*Cono con un unico punto reale*
3	3	$=0$	$\neq 0$	≤ 0		$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}-\tilde{z}^2=0$	$\alpha,\beta>0$	*Cono con infiniti punti reali*
3	3	$=0$	$\neq 0$		≤ 0			
3	2	$=0$	$=0$	>0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}=1$	$\alpha,\beta>0$	*Cilindro ellittico con punti reali*
3	2	$=0$	$=0$	>0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}+\frac{\tilde{y}^2}{\beta^2}=-1$	$\alpha,\beta>0$	*Cilindro ellittico privo di punti reali*
3	2	$=0$	$=0$	<0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}-\frac{\tilde{y}^2}{\beta^2}=1$	$\alpha,\beta>0$	*Cilindro iperbolico*
3	1	$=0$	$=0$	$=0$	$=0$	$\tilde{x}^2-2\rho\tilde{y}=0$	$\rho>0$	*Cilindro parabolico*
2	2	$=0$	$=0$	>0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}+\tilde{y}^2=0$	$\alpha>0$	*Coppia di piani incidenti con una retta di punti reali*
2	2	$=0$	$=0$	<0	$=0$	$\frac{\tilde{x}^2}{\alpha^2}-\tilde{y}^2=0$	$\alpha>0$	*Coppia di piani incidenti con infiniti punti reali*
2	1	$=0$	$=0$	$=0$	$=0$	$\frac{\tilde{x}^2}{\alpha^2}=1$	$\alpha>0$	*Coppia di piani paralleli con infiniti punti reali*
2	1	$=0$	$=0$	$=0$	$=0$	$\frac{\tilde{x}^2}{\alpha^2}=-1$	$\alpha>0$	*Coppia di piani paralleli privi di punti reali*
1	1	$=0$	$=0$	$=0$	$=0$	$\tilde{x}^2=0$		*Piano doppio*

$$\sum_{i=1}^{n} a_{ii}x_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_ix_j + 2\sum_{i=1}^{n} b_i x_i + c = 0$$

$$A\begin{bmatrix} a_{ii} & \cdots & a_{ij} \\ \vdots & \ddots & \vdots \\ a_{ji} & \cdots & a_{nn}\end{bmatrix} \quad B\begin{bmatrix} b_1 \\ \vdots \\ b_n\end{bmatrix} \quad C\left[\begin{array}{c|c} A & B \\ \hline B^T & c\end{array}\right]$$

$$X^T\cdot A\cdot X + 2B^T\cdot X + c = 0$$

$$Y^T\cdot C\cdot Y = 0 \qquad \left(Y=\begin{bmatrix} X \\ \hline 1\end{bmatrix}\right)$$

$$\tilde{A} = Q^T\cdot A\cdot Q \qquad \tilde{B} = Q^T(A\cdot T + B)$$

$$\tilde{C} = T^T\cdot A\cdot T + 2B^T\cdot T + c$$

$$F = \left[\begin{array}{c|c} Q & T \\ \hline 0 & 1\end{array}\right] \to \tilde{C} = F^T\cdot C\cdot F$$

$$r(A) = r(\tilde{A}), \quad r(C) = r(\tilde{C})$$

$$P_A(\lambda) = P_{\tilde{A}}(\lambda), \quad |C| = |\tilde{C}|$$

in $\mathbb{E}^2$: $I_1 = \mathrm{Tr}(A)$ $\quad I_3 = |C|$

$\quad I_2 = |A|$

in $\mathbb{E}^3$: $I_1 = \mathrm{Tr}(A)$ $\quad I_3 = |A|$

$\quad I_2 = \mathrm{II}_2(P_A)$ $\quad I_4 = |C|$

$$\begin{cases} \lambda_1 x_1^2 + \dots + \lambda_n x_n^2 = 0 & r(C) = r(A) \\ \lambda_1 x_1^2 + \dots + \lambda_n x_n^2 + c = 0 & r(C) = r(A)+1 \\ \lambda_1 x_1^2 + \dots + \lambda_r x_r^2 + 2\rho x_{r+1} = 0 & r(C) = r(A)+2 \end{cases}$$

$$\tilde{C}\left[\begin{array}{ccc|c} \lambda_1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \lambda_n & 0 \\ \hline 0 & \cdots & 0 & \tilde{c}\end{array}\right] \vee \left[\begin{array}{cccc|c} \lambda_1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & \lambda_r & & 0 \\ 0 & \cdots & & 0 & \rho \\ \hline 0 & \cdots & 0 & \rho & 0\end{array}\right]$$

$X = Q\cdot\tilde{X} + T$!!

$\lambda_n = \lambda_n \Rightarrow$ rotazione

C: $[A|-B]$

r: $X = C + t v_n$ (v_n a.v di A)

1) $Q_1 \cdot A_1 \in \mathbb{D} = Q_1^T\cdot A Q_1 \Rightarrow A_1 = \dots, \; B_1 = Q^T\cdot B, \; c_1 = c$

2) $T_2 \cdot \begin{cases} -\frac{(B_1)_{i1}}{\lambda} & 1\leq i\leq r \\ 0 & r+1\leq i\leq n\end{cases} \Rightarrow A_2 = A_1, \; B_2 = A_1\cdot T_2 + B_1, \; c = -\sum_{1}^{r}\frac{(B_1)_{i1}^2}{\lambda_i} - c_1$

3) $Q_3 \left[\mathrm{I}_{nr} \,\Big|\, \frac{B_2}{\|B_2\|_F} \; M_1 \dots M_{n-r-1}\right] \Rightarrow A_3 = Q_3^T A_2 Q_3, \; B_3 = Q_3^T\cdot B_2, \; c_3 = c_2$

4) T: $a_{1,r+1} = -\frac{c_3}{2\rho} \Rightarrow A_4 = A_3, \; B_3 = B_4, \; c_4 = 0$ $\quad$! $\rho = \|B_2\|_F$!

$\Rightarrow Q = Q_1\cdot Q_3 \quad T = Q_1(Q_3\cdot T_4 + T_2)$

ROT. INTORNO AD ASSE

$$\begin{cases} Q\in\gamma & Q\in\gamma,\ P\in\mathbb{E}^3 \\ \|\vec{OQ}\|^2 = \|\vec{OP}\|^2 & v \text{ dir. asse} \\ \langle v, \vec{QP}\rangle = 0 & O\in\text{asse}\end{cases}$$

(elimino $Q=(x_0,y_0,z_0)$)

CILINDRO GEN. DA γ // a r:

$$\begin{cases} Q\in\gamma & Q\in\gamma \\ r' & r' \,//\, r \text{ passante per } Q\end{cases}$$

(elimino $Q=(x_0,y_0,z_0)$ e t)

CONO CON DIR. γ

$$\begin{cases} P_0\in\gamma \\ \vec{VP} = t\,\vec{VP_0}\end{cases}$$

$P_0\in\gamma$, $P\in\mathbb{E}^3$ $\quad V\in\mathbb{E}^3$

(elimino $P_0=(x_0,y_0,z_0)$ e t)