ANALISI 1 23 settembre

1.0 Binomio Newton

$$P(n) = \left\{ (a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \right\}$$

$$P(0) = \left\{ (a+b)^n = \sum_{k=0}^{0} \binom{n}{k} a^{n-k} b^k = \binom{0}{0} a^0 b^0 = 1 \right\}$$

$$P(1) = \left\{ \begin{aligned} (a+b)^n &= \sum_{k=0}^{1} \binom{n}{k} a^{n-k} b^k = \binom{1}{0} a^1 b^0 + \binom{1}{1} a^0 b^1 = \\ &= a+b \end{aligned} \right\}$$

Per ipotesi $P(n)$ è vera $\Longrightarrow$ allora deve esser vera $P(n+1)$

↳ dimostra che $P(n+1) = (a+b)^{n+1} = \sum_{k=0}^{n+1} \binom{n+1}{k} a^{n-k} b^k$

$$(a+b)^{n+1} = (a+b)\underbrace{(a+b)^n}_{\text{per Hp}} = (a+b)\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k =$$

$$= a\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k + b\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k =$$

$$= \sum_{k=0}^{n} a^{n+1-k}b^k + \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^{k+1}$$

$$= \sum_{k=0}^{n+1} a^{n+1-k}b^k + \sum_{h=0}^{n+1} a^{n+1-h}b^h =$$

$$h = k-1$$

$$\binom{n}{0}a^{n+1} + \sum_{k=1}^{n}\binom{n}{k}a^{n+1-k}b^k + \sum_{k=1}^{n}\binom{n}{k}a^{n+1-k}b^k$$

$$+ \binom{n}{n}a^0 b^{n+1}$$

$$= a^{n+1} + \sum_{k=1}^{n} \underbrace{\left[\binom{n}{k} + \binom{n}{k-1}\right]}_{\hookrightarrow \binom{n+1}{k}} a^{n+1-k} b^{k} + b^{k-1}$$

$$= a^{n+1} + b^{k-1} + \sum_{k=1}^{n} \binom{n+1}{k} a^{n+1-k} b^{k} = [\dots]$$

1.1

$(2,3) \sim (1,2) \Leftrightarrow 2 + 2 = 3 + 1$ ‖ $(5,8) \sim (3,6) \Leftrightarrow 5 + 6 = 8 + 3$

$$\begin{aligned} &2 - 3 \\ &= \quad -1 \\ &1 - 2 \end{aligned} \qquad\qquad \begin{aligned} &5 - 8 \\ &= \quad -3 \\ &3 - 6 \end{aligned}$$

1.2

$2 \cdot (-3) = -3 - 3 = -6$

$(-2) \cdot (-3) =$

DIMOSTRAZIONE

$0 = b \cdot 0 = b \cdot (a + (-a))$

$= b \cdot a + b(-a) = b \cdot a - b \cdot a = 0$

$(-2)(-3) - (-2)(-3) = (-2)(-3) + (2)(-3) = (-2)(-3) - (2 \cdot 3) = 0$

$(-2)(-3) - 6 = 0$

$(-2)(-3) = 6$

1.3

Per assurdo: $\exists\, m, n \in \mathbb{Z},\ n \neq 1 \mid x = \frac{m}{n} \Rightarrow$ m ed n non hanno fattori comuni

$\hookrightarrow x^2 = \frac{m^2}{n^2} = 2 \Rightarrow m^2 = 2n^2 \Rightarrow \exists K \mid m^2 = 4K^2$

$\hookrightarrow m^2 = 4K^2 = 2n^2$

$\hookrightarrow$ ASSURDO: m ed n non hanno divisori