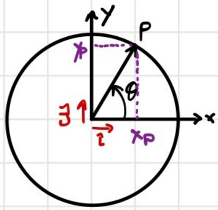
Esercitazioni di Meccanica

03/03/22

RECAP



dati: $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t)$, R

$$\vec{P} = (P-O) = x_P(\theta(t))\hat{i} + y_P(\theta(t))\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$\vec{v} = \frac{d(P-O)}{dt} = \frac{\partial x_P}{\partial t}\frac{\partial\theta}{\partial t}\hat{i} + \frac{\partial y_P}{\partial t}\frac{\partial\theta}{\partial t}\hat{j} = -R\dot{\theta}\sin\theta\,\hat{i} + R\dot{\theta}\cos\theta\,\hat{j}$$

$$= R\dot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= R\dot{\theta}\,\hat{t} \rightarrow |\hat{t}| = 1 \quad \hat{t} \perp \vec{P}$$

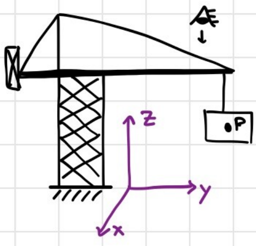
$$\vec{a} = \frac{d\vec{v}}{dt} = (-R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta)\hat{i} + (R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta)\hat{j}$$

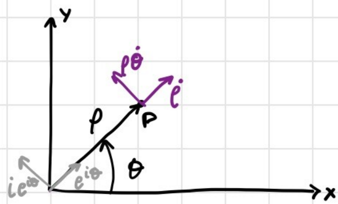
$$= R\ddot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) + R\dot{\theta}^2(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

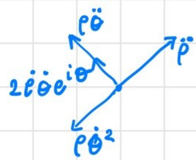
$$= \underbrace{R\ddot{\theta}\,\hat{t}}_{\vec{a}_t} + \underbrace{R\dot{\theta}^2\,\hat{n}}_{\vec{a}_n} \Rightarrow |\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

$$\vec{P} = Re^{i\theta} \quad ; \quad \vec{v} = \underbrace{iR\dot{\theta}e^{i\theta}}_{+\pi/2} \quad ; \quad \vec{a} = \underbrace{iR\ddot{\theta}e^{i\theta}}_{+\pi/2} - R\dot{\theta}^2e^{i\theta}$$

ESERCIZIO 1





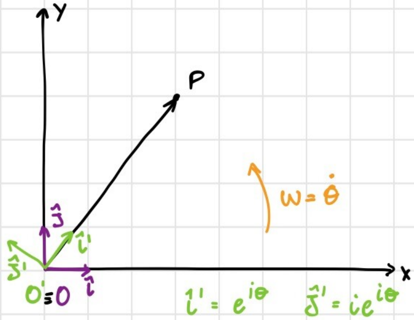


$$\vec{P} = \rho e^{i\theta}$$

$$\vec{v} = \dot{\rho}e^{i\theta} + i\rho\dot{\theta}e^{i\theta}$$

$$\vec{a} = \ddot{\rho}e^{i\theta} + \dot{\rho}i\dot{\theta}e^{i\theta} + i\dot{\rho}\dot{\theta}e^{i\theta} + i\rho\ddot{\theta}e^{i\theta} - \rho\dot{\theta}^2e^{i\theta}$$

$$= \ddot{\rho}e^{i\theta} + 2i\dot{\rho}\dot{\theta}e^{i\theta} + i\rho\ddot{\theta}e^{i\theta} - \rho\dot{\theta}^2e^{i\theta}$$



$\omega_{GRU} = \omega_{O'}$

Per TMR:

$$\vec{v} = \vec{v}_P^{\,T} + \vec{v}_P^{\,R} = \vec{\omega} \wedge (P-O) + \dot\rho\, \hat{i}'$$

$$= \omega \hat{k} \wedge (\rho\, \hat{i}') + \dot\rho\, \hat{i}' = \underbrace{\omega\rho\, \hat{j}'}_{i\dot\theta\rho e^{i\theta}} + \underbrace{\dot\rho\, \hat{i}'}_{\dot\rho e^{i\theta}}$$

$$\vec{a} = \vec{a}_P^{\,T} + \vec{a}_P^{\,R} + \vec{a}_P^{\,C} =$$

$$= \vec{a}_P^{\,T,t} + \vec{a}_P^{\,T,n} + \vec{a}_P^{\,R} + \vec{a}_P^{\,C}$$

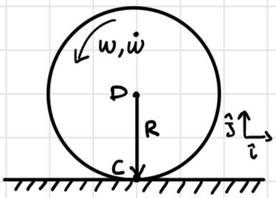
$$= \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge (\vec{\omega} \wedge (P-O)) + \ddot\rho\, \hat{i}' + 2\, \vec{\omega} \wedge \vec{v}_P^{\,R}$$

$$\omega \hat{k} \wedge (\omega \hat{k} \wedge (\rho\, \hat{i}')) \qquad 2\, \omega \hat{k} \wedge \dot\rho\, \hat{i}'$$

$$\omega \hat{k} \wedge (\omega \rho\, \hat{j}') \qquad \sim$$

$$= \underbrace{\dot\omega\rho\, \hat{j}'}_{i\ddot\theta\rho e^{i\theta}} - \underbrace{\omega^2\rho\, \hat{i}'}_{-\rho\dot\theta^2 e^{i\theta}} + \underbrace{\ddot\rho\, \hat{i}'}_{\ddot\rho e^{i\theta}} + \underbrace{2\omega\dot\rho\, \hat{j}'}_{2i\dot\rho\dot\theta e^{i\theta}}$$

ESERCIZIO 2

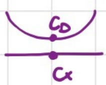


$$\begin{cases} V_{Cx}^{G} = V_{Cx}^{D} = 0 \\ V_{Cy} = 0 \end{cases}$$

DATI: $R, \omega, \dot\omega$

INCOGNITE: $\vec{a}_C$

$\vec{a}_D = -R\dot\omega\, \hat{i}$

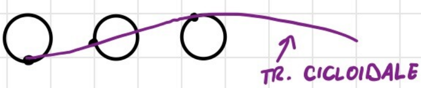


$$\vec{a}_C = \vec{a}_D + \dot{\vec{\omega}} \wedge (C-D) + \vec{\omega} \wedge (\vec{\omega} \wedge (C-D))$$

$$= \vec{a}_D + \dot\omega R\, \hat{i} + \vec{\omega} \wedge (\omega R\, \hat{i})$$

$$= \vec{a}_D + \dot\omega R\, \hat{i} + \omega^2 R\, \hat{j}$$

$$= \cancel{-R\dot\omega\, \hat{i}} + \cancel{\dot\omega R\, \hat{i}} + \omega^2 R\, \hat{j}$$



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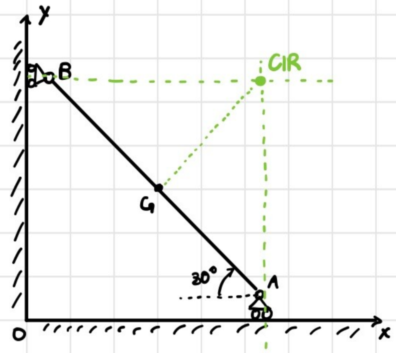
ESERCIZIO 1



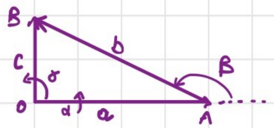
$\overline{AB} = 0,7\,m$

$x_A = 0,606\,m \qquad \beta = 150°$

$v_a = 5\,m/s \qquad a_a = 0,3\,m s$

v_B, v_G? a_B, a_G?

→ 1gdl => abbiamo bisogno solo di 1 informazione nota



$(B-O) = (A-O) + (B-A)$ ← eq di chiusura

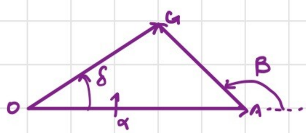
↳ $c e^{i\gamma} = a e^{i\alpha} + b e^{i\beta}$

$ic = a + b e^{i\beta} \rightarrow \begin{cases} 0 = a + b\cos\beta \\ c = 0 + b\sin\beta \end{cases}$

c, β: INCOGNITE

$\rightarrow \begin{cases} 0 = \dot{a} - b\dot{\beta}\sin\beta & \rightarrow \dot{\beta} = 14,3\ rad/s \\ \dot{c} = b\dot{\beta}\cos\beta & \rightarrow \dot{c} = -8,68\ m/s \end{cases}$

$\rightarrow \begin{cases} 0 = \ddot{a} - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta & \rightarrow \ddot{\beta} = 354\ rad/s^2 \\ \ddot{c} = b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta & \rightarrow \ddot{c} = -286,5\ m/s^2 \end{cases}$



$(G-O) = (A-O) + (G-A)$

↳ $d e^{i\delta} = a + \frac{b}{2} e^{i} \rightarrow \begin{cases} x_G = a + \frac{b}{2}\cos\beta \\ y_G = \frac{b}{2}\sin\beta \end{cases}$

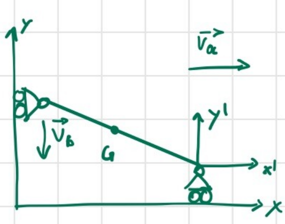
$\rightarrow \begin{cases} \dot{x}_G = \dot{a} - \frac{b}{2}\dot{\beta}\sin\beta = 2,498\ m/s \\ \dot{y}_G = \frac{b}{2}\dot{\beta}\cos\beta = -4,33\ m/s \end{cases}$

$\rightarrow \begin{cases} \ddot{x}_G = \ddot{a} - \frac{b}{2}\ddot{\beta}\sin\beta - \frac{b}{2}\dot{\beta}^2\cos\beta = 0,33\ m/s^2 \\ \ddot{y}_G = \frac{b}{2}\ddot{\beta}\cos\beta - \frac{b}{2}\dot{\beta}^2\sin\beta = -143,1\ m/s \end{cases}$

STESSO ESERCIZIO RISOLTO CON I MOTI RELATIVI

$\vec{v}_B = \vec{v}_{B,TR} + \vec{v}_{B,REL}$

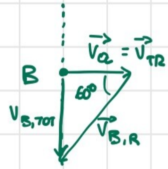
$= \vec{v}_a + \dots$

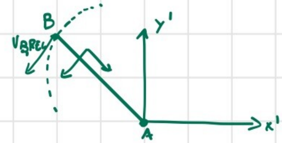


$\rightarrow \vec{V}_{B,TOT} = \vec{v}_{\alpha}\sqrt{3}$

$\vec{V}_{B,REL} = \omega \wedge (B-A)$

$\rightarrow |V_{B,REL}| = \omega \overline{BA}$

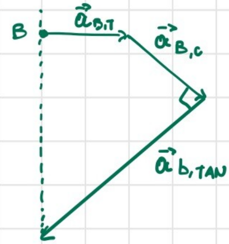




$\vec{a}_{B,TOT} = \vec{a}_{B,T} + \vec{a}_{B,R} + \vec{a}_{B,COR}$

$= \vec{a}_{\alpha} + a_{B,C} + a_{B,TAN} + 0$

$= \vec{a}_{\alpha} +$



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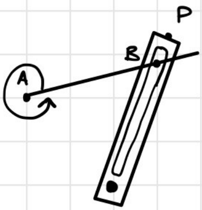
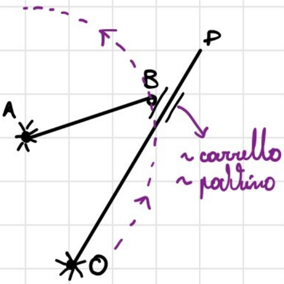
ESERCIZIO 1



$\longrightarrow$



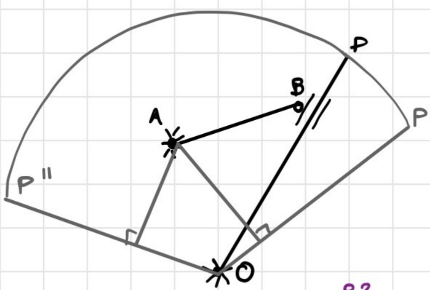
NOTI:

$\overline{AB}, \overline{OP}, \overline{OA}$

$\alpha = 0°$, $\dot{\alpha} = cost$

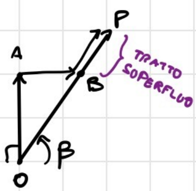
INCOGNITE:

$\vec{\omega}_{OP}$, $\dot{\vec{\omega}}_{OP}$, $\vec{V}_P$, $\dot{\vec{V}}_P$



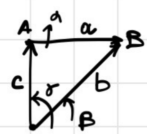
2 cerniere + 1 manicotto

$\hookrightarrow 2\cdot 3 - 2\cdot 2 - 1\cdot 1 = 1$ GDL



$$\underset{B?}{(P-O)} = \underset{PB?}{(P-B)} + \underset{\overline{AB}\text{ noto},\ \alpha\text{ noto}}{(B-A)} + (A-O)$$

$\Longrightarrow$



$\rightarrow (B-O) = (B-A) + (A-O)$

$$b e^{i\beta} = a e^{i\underset{0°}{\alpha}} + c e^{i\underset{90°}{\gamma}}$$

non si deriva l'espressione con $\alpha = 0°$ perché vale solo per l'atto di moto iniziale

$$\begin{cases} b\cos\beta = a\cancel{\cos\alpha} + \cancel{c\cos\gamma} \\ b\sin\beta = \cancel{a\sin\alpha} + c\,\cancel{\sin\gamma} \end{cases} \rightarrow \tan\beta = \frac{c}{a}$$

$$b e^{i\beta} = a e^{i\alpha} + c e^{i\gamma}$$

$$\dot b e^{i\beta} + i\dot\beta b e^{i\beta} = \cancel{\dot a e^{i\alpha}} + i\dot\alpha a e^{i\alpha} + \cancel{\dot c e^{i\gamma}} + \cancel{i\dot\gamma c e^{i\gamma}} \rightarrow \begin{cases} \dot b\cos\beta - \dot\beta b\sin\beta = -\dot\alpha a\sin\alpha \\ \dot b\sin\beta + \dot\beta b\cos\beta = \dot\alpha a\cos\alpha \end{cases}$$

$$= i\dot\alpha a e^{i\alpha}$$

$$\rightarrow \underbrace{\begin{bmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{bmatrix}}_{J} \begin{bmatrix} \dot b \\ \dot\beta \end{bmatrix} = \dot\alpha a \begin{Bmatrix} -\sin\alpha \\ \cos\alpha \end{Bmatrix}$$

$$\det(J) = b\cos^2\beta + b\sin^2\beta = b$$

$$\rightarrow \dot b = \frac{\begin{vmatrix} -\dot\alpha a\sin\alpha & -b\sin\beta \\ \dot\alpha a\cos\alpha & b\cos\beta \end{vmatrix}}{\det(J)} = \frac{a\cancel{b}\dot\alpha\sin\beta}{\cancel{b}} = a\dot\alpha\sin\beta$$

$$\dot\beta = \bullet\bullet\bullet =$$

METODO DI CRAMER

$$\ddot b e^{i\beta} + \dot b\dot\beta i e^{i\beta} + i\ddot\beta b e^{i\beta} + i\dot\beta\dot b - \dot\beta^2 b e^{i\beta} = i\cancel{\ddot\alpha a e^{i\alpha}} - \dot\alpha^2 a e^{i\alpha}$$

⋮ troviamo Jacobiano etc...

$$\vec v_P = \cancel{\vec v_O} + \vec\omega \wedge (P-O) = (\dot\beta\hat k)\wedge(\overline{PO}\cos\beta\,\hat\imath + \overline{PO}\sin\beta\,\hat\jmath)$$

$$= \dot\beta\overline{PO}\cos\beta\,\hat\jmath - \dot\beta\overline{PO}\sin\beta\,\hat\imath$$

$$= \dot\beta\overline{PO}\,\hat t$$

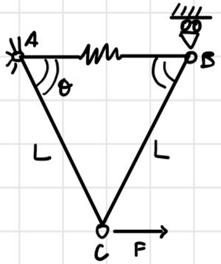
$$\vec a_P = \cancel{\vec a_O} + \dot{\vec\omega}\wedge(P-O) - \vec\omega\wedge(\vec\omega\wedge(P-O)) = \dots$$

Calcolare $\vec v_B$ e $\vec a_B$ coi moti relativi... sti cazzi

5/04/22

ESERCIZIO 1

Legge di Hook = $F_e = K \cdot \Delta l$

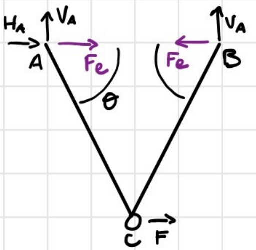


DATI: L, F, K, l_0

TROVARE: 1) reazioni vincolari, θ_{eq}

2) azioni interne

$$gdl = 2 \cdot 3 - 2 \cdot 2 - 1 = 1 \rightarrow \text{ipostatica}$$



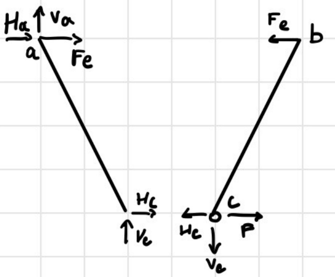
$$l = 2L\cos\theta \rightarrow F_e = K(2L\cos\theta - l_0)$$

ΣM_A: $FL\sin\theta + V_B 2L\cos\theta = 0 \quad \Rightarrow \quad V_B = -F/2 \tan\theta$

ΣF_x: $H_A + \underbrace{\cancel{F_e} - \cancel{F_e}}_{\text{forza interna}} + F = 0 \quad \Rightarrow \quad H_A = -F$

ΣF_y: $V_A + V_B = 0 \quad \Rightarrow \quad V_A = -V_B = F/2 \tan\theta$

eq extra perchè θ_{eq} è incognito



$\Sigma M_c = 0$

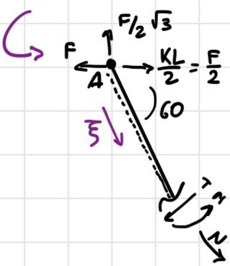
$\rightarrow F_e l \sin\theta + V_B L\cos\theta = 0$

$K(2L\cos\theta - l_0)\cancel{L}\,\cancel{\sin\theta} - F/2\,\cancel{\tan\theta}^{\sin\theta}\,\cancel{L}\,\cancel{\cos\theta} = 0$

$-F/2 + K(2L\cos\theta - l_0) = 0$

$\Rightarrow \theta_{eq} = \arccos\left(F + 2Kl_0 / 4KL\right) = 60°$

$\theta_{eq} = 60°$; $l_0 = L/2$; $F = KL$



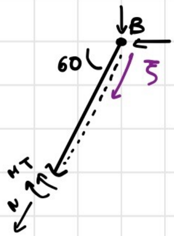
$\Sigma M_A = 0 \rightarrow -T\xi + M = 0$

$\Sigma F_{//N} = 0 \rightarrow N + (F/2 - F)\cos 60° - F/2\sqrt{3}\cos 30° = 0$

$\rightarrow N - F/4 - 3/4\,F \rightarrow N = F$

$\Sigma F_{//T} = 0 \rightarrow T + (F - F/2)\cos 30° - F/2\sqrt{3}\sin 30° = 0$

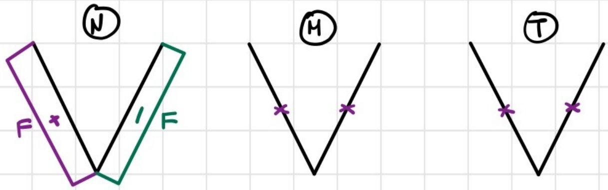
$\rightarrow T + \cancel{F/2}\,\sqrt{3}/2 - \cancel{F/2}\,\cancel{\sqrt{3}}\,1/2 = 0$



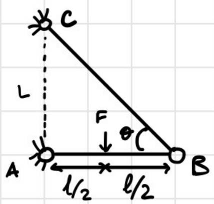
$$\Sigma F_{//N} = 0 \rightarrow N + \frac{F}{2}\cos 60° + \frac{F}{2}\sqrt{3}\cos 30° = 0$$

$$\Sigma F_{//T} = 0 \rightarrow T + \frac{F}{2}\sin 60° - \frac{F}{2}\sqrt{3}\cos 30° = 0$$

$$\Sigma M_B = 0 \rightarrow -M + T\bar{z} = 0$$

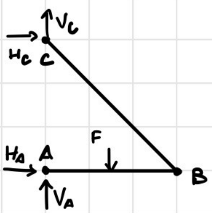


ESERCIZIO 2



NOTI: l, F, $\theta = 45°$

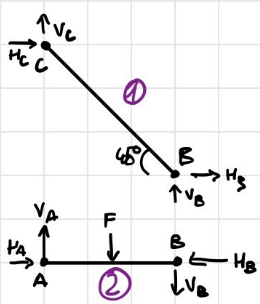
gdl $= 2\cdot 3 - 3 \cdot 2 = 0 \rightarrow$ isostatica



$$\Sigma F_x = H_C + H_A = 0 \rightarrow H_C = -H_A = F/2$$

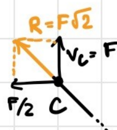
$$\Sigma F_y = V_A + V_C - F = 0 \rightarrow V_A + V_C = F$$

$$\Sigma M_A = -F\, L/2 - H_C L = 0 \rightarrow H_C = -F/2$$



① $\Sigma M_B = 0 \rightarrow -H_C L - V_C L = 0 \Rightarrow H_C = -V_C$

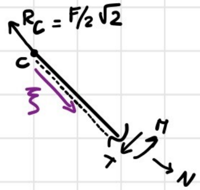
$\Rightarrow V_C = +F/2$



+ niente forze interne

$\Rightarrow$ biella

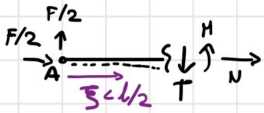
② $\Sigma F_y = 0 \rightarrow V_A = V_B + F \rightarrow V_A = F/2$



$\Sigma F_{//N} = 0 \rightarrow N = R_C$

$\Sigma F_{//T} = 0 \rightarrow T = 0$

$\Sigma M_C = 0 \rightarrow -T\xi + M = 0 \rightarrow M = 0$

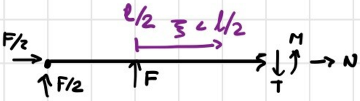


$\Sigma F_x = 0 \rightarrow N = -F/2$

$\Sigma F_y = 0 \rightarrow F/2 - T = 0 \rightarrow T = -F/2$

$\Sigma M_A = 0 \rightarrow -T\xi + M = 0 \rightarrow M = -F/2\,\xi$

$\rightarrow \begin{cases} M(0) = 0 \\ M(l/2) = -Fl/4 \end{cases}$



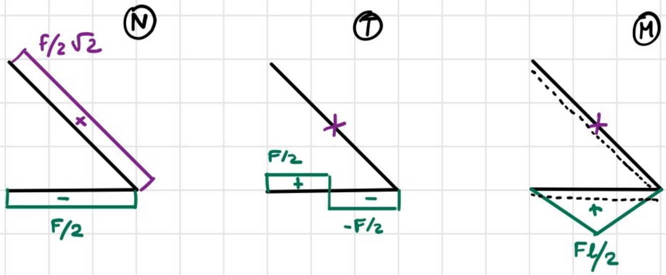
$\Sigma F_x = 0 \rightarrow N = -F/2$

$\Sigma F_y = 0 \rightarrow F/2 - F - T = 0 \rightarrow T = -F/2$

$\Sigma M_A = 0 \rightarrow -F\,l/2 - T(\xi + l/2) + M = 0$

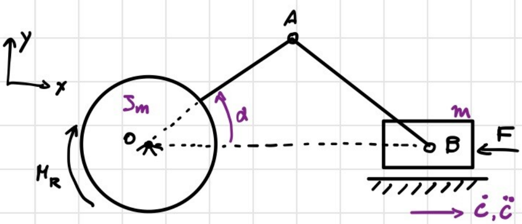
$\rightarrow M = Fl/4 - F/2\,\xi$

$\Rightarrow \begin{cases} M(0) = Fl/2 \\ M(l/2) = 0 \end{cases}$



21/04/22

ESERCIZIO 1



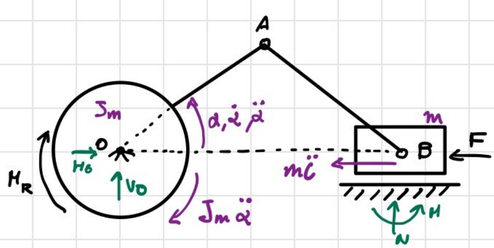
$a = \overline{OA}$

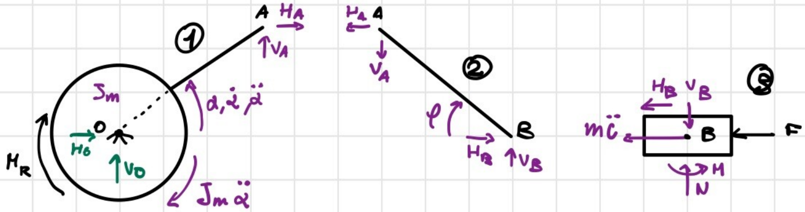
$b = \overline{AB}$

$c = \overline{OB}$

NOTI: $\alpha, \dot{\alpha}, \ddot{\alpha}, F$

M_R ?





Con d'Alembert

$$
① \begin{cases} H_O = -H_A \\ V_O = -V_A \\ \sum M_O : \; -M_r - J_m\ddot\alpha + V_A\, a \cos\alpha - H_A\, a \sin\alpha = 0 \end{cases}
$$

$$
② \begin{cases} H_A = H_B \\ V_A = V_B \\ \sum M_a = H_B\, b \sin\varphi + V_B\, b \cos\varphi = 0 \end{cases}
$$

$$
③ \begin{cases} -H_B - m\ddot{c} - F = 0 \quad \Rightarrow \quad H_B = -(m\ddot{c} + F) \\ N = V_B \\ M = 0 \end{cases}
$$

$$
\Rightarrow \quad \begin{aligned} & H_A = H_B = -H_O = -(F + m\ddot{c}) \\ & V_A = V_B = -V_O = N = H_B \tan\varphi = (-F + m\ddot{c}) \cos\varphi \\ & M_R = \cdots \end{aligned}
$$

Con il bilancio

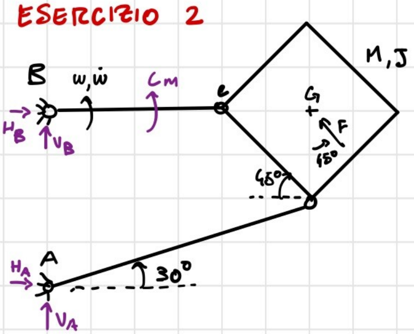
$$
\begin{aligned}
\sum W_k = dE_c/dt &= \vec{M_R} \cdot \vec{\dot\alpha} + \vec{F} \cdot \vec{v_B} \\
&= (-M_R \hat{k}) \cdot (\dot\alpha \hat{k}) + (-F \hat{\imath}) \cdot (\dot{c} \hat{\imath}) \\
&= -M_R \dot\alpha - F\dot{c}
\end{aligned}
$$

$$
E_c = \tfrac{1}{2} m v_B^2 + \tfrac{1}{2} J_m \dot\alpha^2 = \tfrac{1}{2} m (\vec{v_B} \cdot \vec{v_B}) + \tfrac{1}{2} J_m (\vec{\dot\alpha} \cdot \vec{\dot\alpha})
$$

$$
\begin{aligned}
dE_c/dt &= \tfrac{1}{2} m (\vec{a}_B \cdot \vec{v}_B) + \tfrac{1}{2} m (\vec{v}_B \cdot \vec{a}_B) + \cdots \\
&= m \vec{a_B} \cdot \vec{v_B} + \cdots \\
&= m\, a_B v_B + J_m \dot\alpha \ddot\alpha
\end{aligned}
$$

$$
\Rightarrow \; -M_R \dot\alpha - F\dot{c} = m\ddot{c}\dot{c} + J_m \dot\alpha\ddot\alpha \;\rightarrow\; M_R = -\left(\frac{F\dot{c} + m\dot{c}\ddot{c} + J_m \dot\alpha \ddot\alpha}{\dot\alpha} \right)
$$

ESERCIZIO 2



NOTI:

$\overline{AB}, \overline{BC}, \overline{CD}, \overline{AD}$

$M, J, \omega, \dot{\omega}$ F

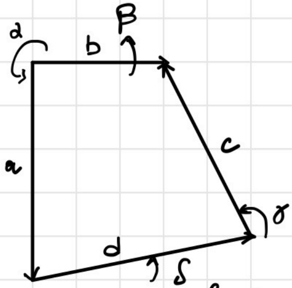
INCOGNITE:

C_M

H_A, V_A, H_B, V_B

GDL: $3 \cdot 3 - 2 \cdot 4 = 1$ gdl

CINEMATICA



COST	b	a, α	c	d
VAR	β		γ → 30°	δ → 135°

$$b e^{i\beta} = a e^{i\alpha} + c e^{i\gamma} + d e^{i\delta}$$

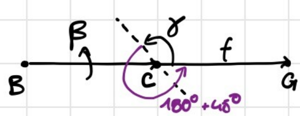
$$i\dot{\beta} b e^{i\beta} = i\dot{\gamma} c e^{i\gamma} + i\dot{\delta} d e^{i\delta}$$

$$\begin{cases} -\dot{\beta} b \sin\beta = -\dot{\gamma} c \sin\gamma + -\dot{\delta} d \sin\delta \\ \dot{\beta} b \cos\beta = \dot{\gamma} c \cos\gamma + \dot{\delta} d \cos\delta \end{cases} \rightarrow \begin{bmatrix} -c\sin\gamma & -d\sin\delta \\ c\cos\gamma & d\cos\delta \end{bmatrix} \begin{bmatrix} \dot{\gamma} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\beta} b \end{bmatrix}$$

$$(i\ddot{\beta} b - \dot{\beta}^2 b) e^{i\beta} = (i\ddot{\gamma} c - \dot{\gamma}^2 c) e^{i\gamma} + (i\ddot{\delta} - \dot{\delta}^2 d) e^{i\delta}$$

...

BARICENTRO



$$\eta = \gamma + 180° + 45° \rightarrow \dot{\eta} = \dot{\gamma} \quad \ddot{\eta} = \ddot{\gamma}$$

$$(G - B) = (G - C) + (C - B)$$
$$= f e^{i\eta} + b e^{i\beta} \rightarrow \begin{cases} x_G = f \cos\eta + b \cos\beta \\ y_G = f \sin\eta + b \sin\beta \end{cases}$$

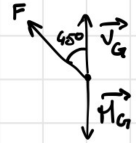
$$\vec{v}_G = i\dot{\eta} f e^{i\eta} + i\dot{\beta} b e^{i\beta} \rightarrow \begin{cases} \dot{x}_G = -f\dot{\eta} \overset{=0}{\cancel{\sin\eta}} - \dot{\beta} b \overset{=0}{\cancel{\sin\beta}} = 0 \\ \dot{y}_G = f\dot{\eta} \cos\eta + \dot{\beta} b \cos\beta = 4{,}55 \text{ m/s} \end{cases}$$

$$\vec{a}_G = (i\ddot{\eta} f - \dot{\eta}^2 f) e^{i\eta} + (i\ddot{\beta} b - \dot{\beta}^2 b) e^{i\beta}$$

$$\hookrightarrow \begin{cases} \ddot{x}_G = -\ddot{\eta} f \cancel{\sin\eta} - \dot{\eta} f \cos\eta - \cancel{\ddot{\beta} b \sin\beta} - \dot{\beta}^2 b \cos\beta = -280 \ m/s^2 \\ \ddot{y}_G = \ddot{\eta} f \cos\eta - \cancel{\dot{\eta} f \sin\eta} + \ddot{\beta} b \cos\beta - \cancel{\dot{\beta}^2 b \sin\beta} = 144{,}4 \ m/s^2 \end{cases}$$

$$a_g = \sqrt{\ddot{x}_G^2 + \ddot{y}_G^2} = 315 \ m/s^2 \qquad \varphi = \arctan\left(\frac{\ddot{y}_G}{\ddot{x}_G}\right) = 153^\circ$$

BILANCIO POTENZE $\quad \sum W = dE_c/dt$

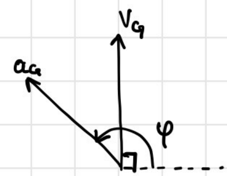


$$W = \vec{C}_m \times \vec{\omega} + M\vec{g} \times \vec{v}_G + \vec{F} \times \vec{v}_G$$
$$= C_m \omega + M g \dot{y}_G \cos 180^\circ + F \dot{y}_G \cos 45^\circ =$$
$$= C_m \omega + (-Mg + \sqrt{2}F/2)\dot{y}_G$$

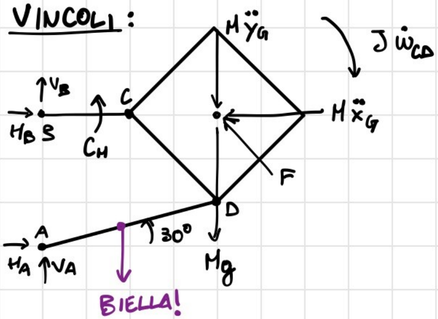
$$E_c = 1/2 \, M v_G^2 + 1/2 \, J \omega_{CD}^2$$

$$dE_c/dt = 1/2 \, M(\vec{v}_G \cdot \vec{a}_G) + 1/2 \, M(\vec{v}_G \cdot \vec{a}_G) + 1/2 \, J(\vec{\omega}_{CD} \cdot \dot{\vec{\omega}}_{CD}) + 1/2 \, J(\vec{\omega}_{CD} \cdot \dot{\vec{\omega}}_{CD})$$
$$= M(\vec{a}_G \cdot \vec{v}_G) + J(\vec{\omega}_{CD} \cdot \dot{\vec{\omega}}_{CD})$$
$$= M a_G v_G \cos(153^\circ - 90^\circ) + J \dot{\gamma} \ddot{\gamma} = 5693{,}4 \ W$$

$$C_m = \frac{1}{\omega}\left(dE_c/dt + M g v_G + \sqrt{2}/2 \, F v_G\right) = 160{,}4 \ Nm$$



VINCOLI:



EQ. DINAMICO:

$$\sum M_B: \ (-Mg - M\ddot{y}_G + \sqrt{2}/2 \, F)\overline{BG} + C_H - J_M \dot{\omega}_{CD} + R_A \overline{AB} \cos 30$$

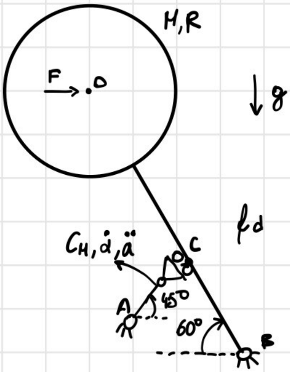
$$\Rightarrow R_A = 1244 \ N \rightarrow \begin{cases} H_A = R_A \cos 30 \\ V_A = R_A \cos 60 \end{cases}$$

$$\sum F_x = 0: \quad H_B - M\ddot{x}_G - \sqrt{2}/2 \, F + R_A \cos 30 = 0$$
$$\rightarrow H_B = -8842 \ N$$

$$\sum F_y = 0: \quad V_B - M(g + \ddot{y}_G) + \sqrt{2}/2 \, F + R_A \sin 60 = 0$$
$$\rightarrow V_B = 880 \ N$$

03/05/22

ESERCIZIO 1



$M = 10\,kg$

$R = 0{,}1\,m$

$\overline{AC} = 0{,}3\,m = a$

$\overline{BD} = 0{,}76\,m$

$\overline{CB} = 0{,}4\,m = b$

$F = 100\,N$

$\dot\alpha = 10\,rad/s$

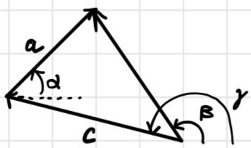
$f_d = 0{,}2$

$\vec v_D$? $\vec a_D$?

C_m ?

H_A, V_A ?

CINEMATICA



$$b e^{i\beta} = a e^{i\alpha} + c e^{i\gamma}$$

COST		a	c,γ
VAR	b,β	α	

↑ assegnato

$$(\dot b + i\dot\beta b)e^{i\beta} = i\dot\alpha a e^{i\alpha}$$

$$\begin{cases} \dot b\cos\beta - \dot\beta b\sin\beta = -\dot\alpha a\sin\alpha \\ \dot b\sin\beta + \dot\beta b\cos\beta = \dot\alpha a\cos\alpha \end{cases}$$

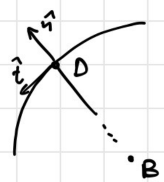
$$\begin{bmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{bmatrix}\begin{bmatrix} \dot b \\ \dot\beta \end{bmatrix} = \dot\alpha a\begin{bmatrix} -\sin\alpha \\ \cos\alpha \end{bmatrix}$$

$\rightarrow \dot b \sim 2{,}90\,m/s$

$\rightarrow \dot\beta \sim 1{,}94\,rad/s$

$$[i\dot\beta(\dot b + i\dot\beta b) + (\ddot b + i\ddot\beta b + i\dot\beta \dot b)]e^{i\beta} = (\cancel{i\ddot\alpha a} - \dot\alpha^2 a)e^{i\alpha}$$

$\Rightarrow \dots \Rightarrow \ddot b, \ddot\beta$

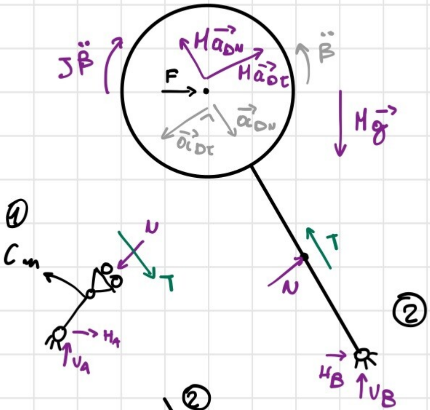


$$\vec v_D = \cancel{\vec v_B} + \vec\omega \wedge (D-B) = \boxed{\dot\beta\,\overline{DB}\,\hat t}$$

$$\vec a_D = \cancel{\vec a_B} + \dot{\vec\omega} \wedge (D-B) - \omega^2(D-B)$$

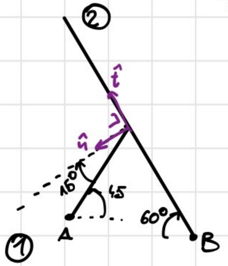
$$= \boxed{\ddot\beta\,\overline{DB}\,\hat t - \dot\beta^2\,\overline{DB}\,\hat n}$$

DINAMICA

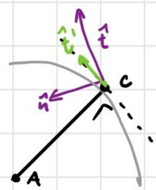


$$\vec{T} = -f_D\,|N|\,\frac{\vec{v}_{12}}{|\vec{v}_{12}|}$$

$$\vec{v}_{12} = \vec{v}_c^{\,1} - \vec{v}_c^{\,2}$$
$$= 2,9\,\hat{t} + \cancel{0,776\,\hat{n}} - \cancel{0,776\,\hat{n}}$$
$$= 2,9\,\hat{t} \ \ m/s$$



$$\vec{v}_c^{\,2} = \dot{\beta}\, b\, \hat{n} \approx 0,776 \ \ m/s$$
$$\vec{v}_c^{\,1} = a\,\dot{\alpha}\,\hat{t}' = \dot{\alpha}\, a \cos 15°\, \hat{t} + \dot{\alpha}\, a \sin 15°\, \hat{t}$$
$$= 2,9\,\hat{t} + 0,776\,\hat{n} \ \ m/s$$



② $\sum M_B^2 = 0 \quad \rightarrow \quad -J\ddot{\beta} - F\cos 60°\,\overline{DB} - M a_D^t\,\overline{DB} + Mg \sin 30° - N b = 0$

$\Longrightarrow \quad N = \cdots$

$|\vec{T}| = f_D\,|N| = \cdots$

$$\sum W_k = dE_c/dt = \vec{C}_m \cdot \dot{\vec{\alpha}} + M\vec{g}\cdot\vec{v}_D + \vec{F}\cdot\vec{v}_D + \vec{T}\cdot\vec{v}_{12}$$
$$= C_m\dot{\alpha} + Mg\,\overline{DB}\cos 60° - F\dot{\beta}\,\overline{DB}\cos 30 - T v_{12}$$

$$E_C = \tfrac{1}{2} M v_D^2 + \tfrac{1}{2} J \omega_{BD}^2$$
$$\rightarrow dE_c/dt = M\vec{a}_D\cdot\vec{v}_D + J\,\vec{\omega}_{BD}\cdot\dot{\vec{\omega}}_{BD}$$
$$= M\left(\ddot{\beta}\,\overline{DB}\,\hat{t} - \cancel{\dot{\beta}^2\,\overline{DB}\,\hat{n}}\right)\cdot\left(\dot{\beta}\,\overline{DB}\,\hat{t}\right) + J\dot{\beta}\ddot{\beta}$$
$$= M\ddot{\beta}\dot{\beta}\,\overline{DB}^2 + J\dot{\beta}\ddot{\beta} = \left(M\overline{DB}^2 + J\right)\dot{\beta}\ddot{\beta}$$

$$\Longrightarrow \quad C_m\dot{\alpha} + Mg\,\overline{DB}\cos 60° - F\dot{\beta}\,\overline{DB}\cos 30 - T v_{12} = \left(M\overline{DB}^2 + J\right)\dot{\beta}\ddot{\beta}$$

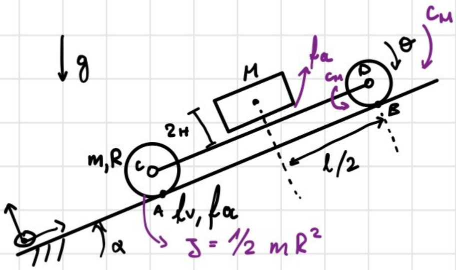
$\rightarrow$ $C_M = \cdots$

① $\sum F_x: \quad H_A - N\cos 30 + T\cos 60 = 0 \quad \cdots \quad \rightarrow \quad H_A = \cdots$

$\sum F_y: \quad V_A - T\sin 60 - N\sin 30 = 0 \quad \cdots \quad \rightarrow \quad V_A = \cdots$

05/05/2022

ESERCIZIO 1



$\alpha = 10°$

$R = 0{,}3\ m$

$\overline{CD} = l = 2{,}5\ m$

$m = 20\ Kg$

$M = 500\ Kg$

$h = 0{,}3\ m$

$C_M = 500\ Nm$

$f_{a,RUOTA} = 0{,}8$

$f_{a,M} = 0{,}5$

$f_v = 0{,}01$

$\ddot{\theta}$?

aderenza ruote?

aderenza massa?

a_{MAX} | M non scivoli?

Hp aderenza:

$$\sum W_K = dE_c/dt$$

$$E_c = \frac{1}{2} m v_C^2 + \frac{1}{2} J \omega_C^2 + \frac{1}{2} m v_D^2 + \frac{1}{2} J \omega_D^2 + \frac{1}{2} M v_M^2$$

$\hookrightarrow \vec{v}_C = \vec{v}_D = \vec{v}_M = \dot{\theta} R \hat{i} \qquad \vec{\omega}_C = \vec{\omega}_D = -\dot{\theta}\hat{k}$

$$\Rightarrow E_c = mR^2\dot{\theta}^2 + J\dot{\theta}^2 + \frac{1}{2} M\dot{\theta}^2 R^2 = \left(mR^2 + J + \frac{M}{2}R^2\right)\dot{\theta}^2$$

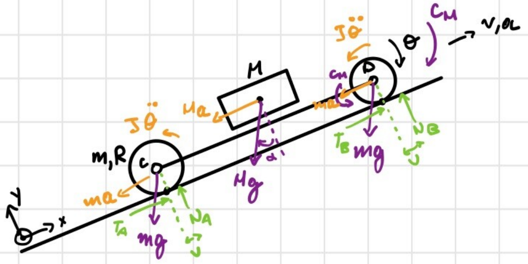
$$dE_c/dt = 2\left(mR^2 + \frac{1}{2}mR^2 + \frac{M}{2}R^2\right)\dot{\theta}\ddot{\theta} = \cdots = (3m + M)R^2\dot{\theta}\ddot{\theta}$$

poiché $\vec{a} /\!/ \vec{v}$ e $\vec{\omega} /\!/ \dot{\vec{\omega}}$ possiamo semplicemente derivare

$$\sum W_K = m\vec{g}\cdot\vec{v}_C + m\vec{g}\cdot\vec{v}_D + M\vec{g}\cdot\vec{v}_M - u|N_A||\dot{\theta}| - u|N_B||\dot{\theta}| + \vec{C}_M\cdot\vec{\omega}_D$$
$$= -(M+2m)g\dot{\theta}R\sin\alpha + C_M\dot{\theta} - (|N_A| + |N_B|)u|\dot{\theta}|$$

$$(3m+M)R^2\cancel{\dot{\theta}}\ddot{\theta} = -(M+2m)g\cancel{\dot{\theta}}R\sin\alpha + C_M\cancel{\dot{\theta}} - (N_A + N_B)u\cancel{\dot{\theta}}$$

$\rightarrow \ddot{\theta} = \cdots = 4134\ rad/s^2$

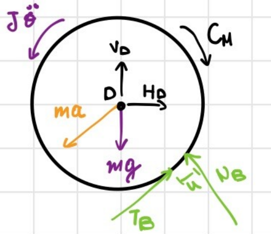


$$\Sigma F_y: \quad N_A + N_B - (2m + M) g \cos\alpha = 0$$
$$\rightarrow N_A + N_B = (2m + M) g \cos\alpha = 5217 \text{ N}$$

ADERENZA RUOTE

C'è aderenza se: $\begin{cases} |T_A| \leq f_a |N_A| \\ |T_B| \leq f_B |N_B| \end{cases}$

$$\Sigma M_{A'}: \underbrace{(mg \cos\alpha\, u + mg \sin\alpha\, R + J\ddot{\theta} + ma R)}_{\text{RUOTA C}} +$$
$$+ \underbrace{(Ma(h+R) + Mg \sin\alpha (h+R) - Mg \cos(l/2 - u))}_{\text{MASSA M}} +$$
$$+ \underbrace{(mg \sin\alpha\, R + ma R - mg \cos\alpha (l - u) + J\ddot{\theta} + N_B\, l)}_{\text{RUOTA D}} = 0$$
$$\hookrightarrow N_B = 2232 \text{ N} \implies N_A = 5217 \text{ N} - 2232 \text{ N} = 2985 \text{ N}$$

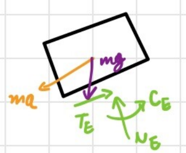


$$\Sigma M_D: \quad T_B R + N_B u - C_M + J\ddot{\theta} = 0$$
$$\implies T_B = 1632 \text{ N} \checkmark$$

globale: $\Sigma F_x: \quad T_A + T_B = (2m + M)(a + g \sin\alpha) \implies T_A = -42 \text{ N} \checkmark$

ADERENZA MASSA:

$$\Sigma F_x: \quad T_E = Ma + Mg \sin\alpha = 1472 \text{ N}$$
$$\Sigma F_y: \quad N_E = Mg \cos\alpha = 4830$$



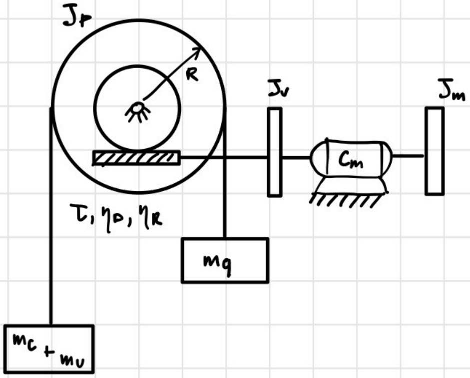
$$|T_E| \leq f_a |N_E| \checkmark$$

ACC. MAX $\quad |T_{E,L}| = f_s |N_E| \implies \Sigma F_x: \quad T_{E,L} = M a_{MAX} + Mg \sin\alpha$

$$a_{MAX} = 3{,}12 \text{ m/s}^2$$

17/05/22

ESERCIZIO 1



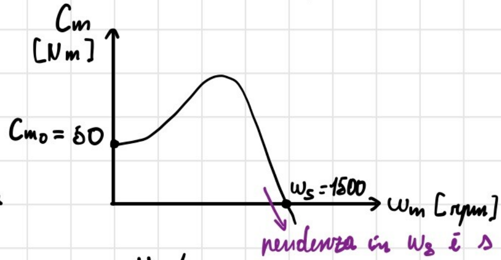
DATI

$m_c = 300$ Kg
$m_v = 320$ Kg
$m_q = 450$ Kg

$J_m = 0,03$ Kgm^2
$J_v = 0,32$ Kgm^2
$J_P = 8$ Kgm^2

$R = 0,3$ m

$\tau = 1/55$
$\eta_D = 0,8$
$\eta_R = 0,7$

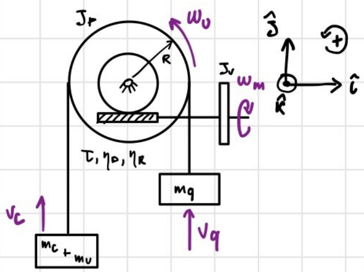


$s = -0,1$ Nm/rpm

1) a_{CABINA} in salita da fermo a pieno carico
2) condizioni di salita a regime
3) salita a regime con cabina scarica
4) discesa a regime pieno carico

1) spunto : $\begin{cases} \omega_m \simeq 0 \\ C_m = C_{m0} \end{cases}$

$$W_m + W_v + W_P = dE_c/dt$$



$\vec{v}_c = v_c \hat{J}$
$\vec{\omega}_v = -v_c/R \, \hat{K}$
$\vec{v}_q = -v_q \hat{K} = -v_c \hat{J}$
$\omega_m = \omega_v/\tau = -v_c/R\tau$

$\vec{a}_c = a_c \hat{J}$
$\dot{\omega}_v = -a_c/R$
$\vec{a}_q = -a_c \hat{J}$
$\dot{\omega}_m = -a_c/R\tau$

$$E_c = 1/2 (m_v + m_c) v_c^2 + 1/2 \, m_q v_q^2 + 1/2 \, J_P \omega_v^2 + 1/2 (J_v + J_m) \omega_m^2$$

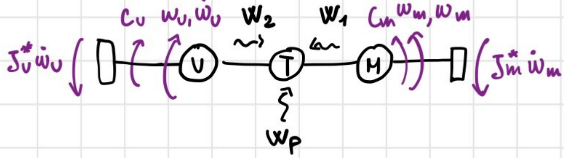
$$dE_c/dt = (m_v + m_c) \vec{v}_c \cdot \vec{a}_c + m_q \vec{v}_q \cdot \vec{a}_q + J_P \vec{\dot{\omega}}_v \cdot \vec{\omega}_v + (J_v + J_m) \vec{\dot{\omega}}_m \cdot \vec{\omega}_m$$
$$= (m_v + m_c) v_c a_c + m_q v_q a_q + J_P \dot{\omega}_v \omega_v + (J_v + J_m) \dot{\omega}_m \omega_m$$
$$= (m_v + m_c) v_c a_c + m_q v_c a_c + J_P v_c a_c / R^2 + (J_v + J_m) v_c a_c / R^2 \tau^2$$
$$= \underbrace{\left(m_v + m_c + m_q + J_P/R^2 + J_v + J_m / R^2 \tau^2\right)}_{\text{massa equivalente } m^*} v_c a_c$$

$$W_m = \vec{C}_m \cdot \vec{\omega}_m \overset{>0}{=} |C_m \omega_m|$$

$$W_U = (m_U + m_c)\vec{g}\cdot\vec{v_c} + m_q\vec{g}\cdot\vec{v_q}$$
$$= -(m_U + m_c) g v_c + m_q g v_c$$
$$= -(m_U + m_c - m_q) g v_c$$

$$W_P = -(1-\eta_?)W_I$$

↑ potenza in ingresso



$$W_1 = \vec{C_m^*}\cdot\vec{\omega_m} + (-J_m^*\vec{\dot\omega_m})\cdot\vec{\omega_m}$$
$$= C_m\omega_m - (J_m + J_V)\dot\omega_m\omega_m \lesseqgtr 0 \ ?$$

$$W_2 = \vec{C_U^*}\cdot\vec{\omega_U} + (-J_U^*\vec{\dot\omega_U})\cdot\vec{\omega_U} \quad \rightarrow \quad W_2 = W_U + W_{INERZIA\,U}$$
$$= \underbrace{-\underbrace{(m_U + m_c - m_q)}_{>0} g v_c}_{<0} + \underbrace{(m_U + m_c + m_q + J_P/R^2) v_c a_c}_{<0}$$

$$\Rightarrow W_2 < 0 \Rightarrow \text{DIRETTO}$$

$$W_P = -(1-\eta_D)W_1$$

$$\hookrightarrow \quad C_m\omega_m - (m_U + m_c - m_q) g v_c - (1-\eta_D)(C_m\omega_m - (J_m + J_V)\dot\omega_m\omega_m) = m^* v_c a_c$$

...

$$\underbrace{\left(\eta_D \frac{J_m + J_V}{R^2\tau^2} + \frac{J_P}{R^2} + m_U + m_c + m_q\right)}_{m_c^*} a_c = \underbrace{-(m_U + m_c - m_q) g}_{F_R^*} + \underbrace{\frac{\eta_D C_m}{R\tau}}_{F_m^*}$$

$$a_c = \frac{F_m^* - F_R^*}{m_c^*} = 0{,}536\ m/s^2$$

Variante più veloce:

$$W_1 + W_2 + W_P = 0$$
$$\cancel{W_1 +} W_2 - (\cancel{1-}\eta_D) W_1 = 0$$
$$\eta_D W_1 + W_2 = 0$$
$$\eta_D (C_m\omega_m - (J_V + J_m)\dot\omega_m\omega_m) = (m_U + m_c + m_q) g v_c - (m_U + m_c + m_q + J_P/R^2) v_c a_c$$

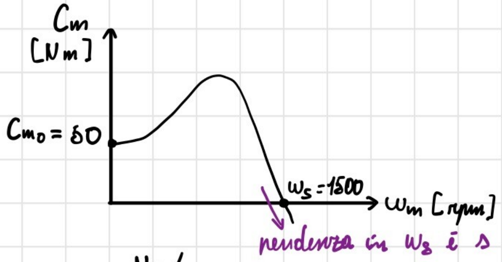
⋮

2) $\bar{C}_m, \bar{\omega}_m, \bar{W}_m$?

$$W_1 = W_m - \cancel{\frac{dE_c}{dt}}\Big|_m \rightarrow \text{REGIME} = \bar{C}_m \bar{w}_m$$
$$W_2 = W_v - \cancel{\frac{dE_c}{dt}}\Big|_v \nearrow \qquad = -(m_v + m_c - m_q)\, g\, v_c < 0$$

$\Rightarrow$ DIRETTO

$$\eta_D W_1 + W_2 = 0 \rightarrow \eta_D \bar{C}_m \bar{w}_m - (m_v + m_c - m_q)\, g\, v_c = 0$$
$$\eta_D \bar{C}_m \cancel{\bar{w}_m} - (m_v + m_c - m_q)\, g\, \tau R\, \cancel{\bar{w}_m} = 0$$
$$\vdots$$
$$\bar{C}_m = \frac{1}{\eta_D} (m_v + m_c - m_q)\, g\, \tau R = 11{,}37\ Nm$$



$$s = \frac{\Delta C_m}{\Delta w_m} = \frac{\bar{C}_m - \cancel{C_{ms}}}{\bar{w}_m - w_s}$$
$$\Rightarrow \bar{w}_m = \frac{1}{s} \bar{C}_m + w_s = 1386\ rpm$$

$$s = -0{,}1\ Nm/rpm$$

3) $W_1 = C_m w_m$

$W_2 = -(m_c \cancel{} - m_q)\, g\, v_c = \cdots > 0$ $\Rightarrow$ RETROGRADO

$W_P = -(1 - \eta_R)\, W_2$

$$\rightarrow W_1 + W_2 + W_P = 0 \Rightarrow W_1 + \cancel{W_2} - (\cancel{1} - \eta_R) W_2 = 0$$
$$W_1 + \eta_R W_2 = 0$$

$$\bar{C}_m \cancel{\bar{w}_m} = \eta_R (m_c - m_q)\, g\, R\tau\, \cancel{\bar{w}_m} = -5{,}62\ Nm$$

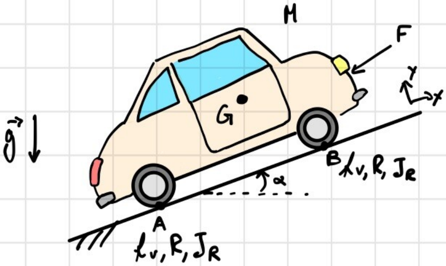
$$s = \frac{\bar{C}_m}{\bar{w}_m - w_s} = \cdots \Rightarrow \bar{w}_m = 1556\ rpm$$

4) $W_1 = \vec{C}_m \cdot \vec{w}_m \gtrless 0$

$W_2 = (m_c + m_v - m_q)\, \vec{g} \cdot \vec{v}_c$ $\Rightarrow$ RETROGRADO

$\quad = (m_c + m_v - m_q)\, g\, v_c > 0$

19/05/21



$\alpha = 5°$

$J_R = 0{,}3\ m$

$F = 800\ N$

$\eta_D = 0{,}8$

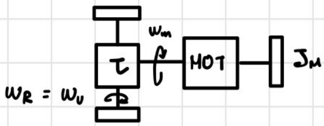
$M = 1500\ Kg$

$J_m = 0{,}2\ Kg/m^2$

$f_v = 0{,}05$

$\tau = 1/5$

$\eta_R = 0{,}7$



1) Accelerazione allo spunto, $C_{m0} = 400\ Nm$

$$W_1 = W_m - \left.\frac{dE_c}{dt}\right|_m = W_m + W_{IN,M}$$

$$= \underbrace{\vec{C}_m \cdot \vec{\omega}_m}_{>0} - \underbrace{J_m \cdot \dot{\vec{\omega}}_m \cdot \vec{\omega}_m}_{>0} \implies \text{non possiamo dire nulla}$$

$$W_2 = \underset{<0}{W_v} - \underset{<0}{\left.\frac{dE_c}{dt}\right|_v} = W_v + W_{IN,v} \implies \text{MOTO DIRETTO}$$

$$W_v = \vec{F} \cdot \vec{v} + M\vec{g} \cdot \vec{v} - (N_A + N_B)\, u\, \omega_R$$

$$= -Fv - Mg \sin\alpha\, v - Mg\cos\alpha \cdot f_v \cancel{R} \cdot v/\cancel{R}$$

$$= -[F - Mg(\sin\alpha + \cos\alpha\, f_v)]\, v < 0$$

$$\left.\frac{dE_c}{dt}\right|_v = M\vec{a} \cdot \vec{v} + 2J_R\, \dot{\vec{\omega}}_R \cdot \vec{\omega}_R$$

$$= Mav + 2J_R\, av/R^2 = (M + 2J_R/R^2)\, av > 0$$

$$a = \tau R \dot{\omega}_m \quad \begin{cases} a = R\dot{\omega}_R \\ \dot{\omega}_R = \tau \dot{\omega}_m \end{cases}$$

$$W_1 + W_2 + W_p = 0 \to W_1 + W_2 - (1 - \eta_D) W_1 \to \eta_D W_1 + W_2 = 0$$

$$\to \eta_D (C_m \omega_m - J_m \dot{\omega}_m \omega_m) - \underbrace{[F - Mg(\sin\alpha + \cos\alpha\, f_v)]}_{F^*_v} v - \underbrace{(M + 2J_R/R^2)}_{M^*_u} av = 0$$

$$\eta_D \left(\frac{C_m}{\tau R} + \frac{J_m}{\tau^2 R^2} a\right) \cancel{v} - F^*_v \cancel{v} - M^*_u a \cancel{v}$$

$$\dots$$

$$a = \frac{\eta_D\, C_m/\tau R - F^*_v}{\eta_D\, J_m/\tau^2 R + M^*_u} = 1{,}58\ m/s^2$$

2) Calcolare C_m a regime in salita

$$W_1 = W_m - \cancel{dE_c/dt|_m} = C_m \omega_m > 0 \quad \Rightarrow \quad \text{MOTO DIRETTO}$$
$$W_2 = W_U - \cancel{dE_c/dt|_U} = -F_U^* v$$

$$\eta_D W_1 + W_2 = 0 \rightarrow \eta_D C_m \cancel{\omega_m} - F_U^* \tau R \cancel{\omega_m} = 0 \rightarrow C_M = \frac{F_U^* \tau R}{\eta_D} = 211{,}2 \text{ Nm}$$

3) A partire dalla salita a regime, imponiamo $C_M = 0$. Calcola a

$$W_1 = \cancel{\vec{C_M} \cdot \vec{\omega_m}} - J_m \underbrace{\vec{\dot{\omega}}_m \cdot \vec{\omega}_m}_{<0 \rightarrow \text{decelera}} > 0 \quad \Rightarrow \quad \text{MOTO DIRETTO}$$

$$W_2 = W_U - dE_c/dt = -F_U^* v - M_U^* \underbrace{\vec{a}\,\vec{v}}_{<0}$$

$$= \underbrace{-F_U^* v}_{<0} + \underbrace{M_U^* \vec{a}\,\vec{v}}_{>0}$$

⋮

$$a = \frac{-F_U^*}{\eta_D J_m/\tau^2 R + M_U^*} = -1{,}772 \text{ m/s}^2$$

4) Fornendo $C_F = C_M = -300$ Nm, calcolare a

$$W_1 = W_m - dE_c/dt|_m = \underbrace{-C_F \omega_m}_{<0} - J_m \underbrace{\vec{\dot{\omega}}_m \cdot \vec{\omega}_m}_{<0}$$

$$W_2 = W_U - dE_c/dt|_U = \underbrace{-F_U^* v}_{<0} + \underbrace{M_U^* a v}_{>0}$$

<u>Hp</u>: moto diretto

$$W_1 = W_m - dE_c/dt|_m = -C_F \omega_m - J_m \vec{\dot{\omega}}_m \cdot \vec{\omega}_m$$
$$W_2 = W_U - dE_c/dt|_U = -F_U^* v + M_U^* a v$$

$$\eta_D W_1 + W_2 = 0 \rightarrow \eta_D(-C_F \omega_m + J_m \dot{\omega}_m \omega_m) - F_U^* v + M_U^* a v$$
$$\rightarrow \dots \rightarrow a = \dots \Rightarrow \dot{\omega}_m = 71{,}5 \text{ rad/s}$$

$$\Rightarrow W_1|_{\dot{\omega}_m} = \dots < 0 \quad \times \quad \Rightarrow \text{il moto è retrogrado}$$