1) $\mathcal{C}: -y^2+2xz-2x+2y-5=0$

$A \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $B \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ $C \left[\begin{array}{ccc|c} 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ \hline -1 & 1 & 0 & -5 \end{array}\right]$ $\begin{array}{l} I_1 = -1 \\ I_2 = -1 \\ I_3 = 1 \\ I_4 = -4 \end{array}$ $\Rightarrow$ iperbole a 2 falde

Diagonalizzo A: $[\dots] \rightarrow \lambda_1 = 1,\ \lambda_2 = -1,\ \lambda_3 = -1$. $\tilde{c} = \frac{I_4}{I_3} = -4 \Rightarrow \tilde{\mathcal{C}}: \frac{\tilde{x}^2}{4} - \frac{\tilde{y}^2}{4} - \frac{\tilde{z}^2}{4} = 1$

Calcoliamo la rototraslazione:

Q_1: Calcoliamo gli autovettori: $\dots \rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \rightarrow |Q_1|$ non è $1 \Rightarrow \begin{bmatrix} 1/\sqrt{2} & +1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$

T_1: $[A|-B] = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$\Downarrow$$

$$\begin{cases} x = 1/\sqrt{2}\,(\tilde{x}+\tilde{y}) \\ y = \tilde{z}+1 \\ z = 1/\sqrt{2}\,(\tilde{x}-\tilde{y})+1 \end{cases} \qquad (x = Q\tilde{x} + T)$$

2) $\mathcal{C}: x^2+2xy+y^2+x+3y-1=0$ in $\mathbb{E}^2$

$A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B \begin{bmatrix} 1/2 \\ 3/2 \end{bmatrix}$ $C \begin{bmatrix} 1 & 1 & 1/2 \\ 1 & 1 & 3/2 \\ 1/2 & 3/2 & -1 \end{bmatrix}$ $\begin{array}{l} I_1 = 2 \\ I_2 = 0 \\ I_3 = -3 \end{array}$ $\Rightarrow$ parabola

$r(C)=3,\ r(A)=1 \Rightarrow \lambda_1 = 2 \quad \lambda_2 = 0 \Rightarrow \tilde{\mathcal{C}}: \lambda_1\tilde{x}^2 + 2p\tilde{y} = 0 \rightarrow 2\tilde{x}^2 + 2\frac{1}{\sqrt{2}}\tilde{y} = 0$

$p = \sqrt{-\frac{I_3}{I_1}}$

$\begin{cases} x' = -\tilde{x} \\ y' = -\tilde{y} \end{cases} \rightarrow 2x'^2 - \frac{2}{\sqrt{2}}y' = 0 \Rightarrow \mathcal{C}': x'^2 \underbrace{-2\cdot\frac{1}{2\sqrt{2}}}_{p=\frac{p}{\lambda_1}} y' = 0$

Calcoliamo la rototraslazione:

Q_1: diagonalizzo A: $[\dots] \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{bmatrix} \Rightarrow A_1 \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix},\ B_1 = Q_1^T B = \begin{bmatrix} \sqrt{2} \\ 1/\sqrt{2} \end{bmatrix},\ c_1 = c$

$T_2 = \begin{bmatrix} -\frac{B_{1(11)}}{\lambda_1} \\ 0 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 0 \end{bmatrix} \rightarrow A_2 = A_1,\ B_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \end{bmatrix} \quad c_2 = -\frac{B_{1(11)}^2}{\lambda_1} + c_1 = -1-1 = -2$

$Q_3 = I$

$T_4 = \begin{bmatrix} 0 \\ -\frac{c_2}{2p} \end{bmatrix}$

$\hookrightarrow Q = Q_1 Q_3 = Q_1 \quad , \quad T = \cancel{Q_1}(Q_3 T_4 + T_2) = \dots = \begin{bmatrix} -3/2 \\ 1/2 \end{bmatrix}$

Un altro modo è:

1) fare la rotazione, sostituire dentro l'equazione originale e fare in modo di ottenere un quadrato perfetto da poter usare per per comporre la traslazione

eg: $\dots \quad 2x_1^2 + \frac{1}{\sqrt{2}}(x_1 - y_1 + 3x_1 + 3y_1) - 1 = 0 \Rightarrow 2(x_1^2 + \sqrt{2}x_1 + \frac{1}{2}) + \sqrt{2}(y_1 - \sqrt{2}) = 0 \Rightarrow 2(x_1 + 1/\sqrt{2})^2 + 2(y_1 - \sqrt{2}) = 0$

$\Rightarrow \begin{cases} \tilde{x} = x_1 + 1/\sqrt{2} \\ \tilde{y} = y_1 - \sqrt{2} \end{cases} \Rightarrow \tilde{\mathcal{C}}: 2\tilde{x}^2 + \sqrt{2}\,\tilde{y} = 0$

$\hookrightarrow \begin{cases} x_1 = \tilde{x} - 1/\sqrt{2} \\ y_1 = \tilde{y} + \sqrt{2} \end{cases} \rightarrow \begin{cases} x = \dots = 1/\sqrt{2}(\tilde{x} - \tilde{y}) - 3/2 \\ y = \dots = 1/\sqrt{2}(\tilde{x} + \tilde{y}) - 1/2 \end{cases}$