Prefisso	Simbolo	Notazione scientifica	Numero decimale	Scala corta [note 1]	Scala lunga [note 2]	Adozione [note 3]
yotta	Y	10^{24}	1 000 000 000 000 000 000 000 000	Settillione	Quadrilione	1991[1]
zetta	Z	10^{21}	1 000 000 000 000 000 000 000	Sestilione	Triliardo	1991[1]
exa	E	10^{18}	1 000 000 000 000 000 000	Quintilione	Trilione	1975[2]
peta	P	10^{15}	1 000 000 000 000 000	Quadrilione	Biliardo	1975[2]
tera	T	10^{12}	1 000 000 000 000	Trilione	Bilione	1960[3]
giga	G	10^{9}	1 000 000 000	Bilione	Miliardo	1960[3]
mega	M	10^{6}	1 000 000	Milione		1960[3]
chilo	k	10^{3}	1 000	Mille		1795
etto	h	10^{2}	100	Cento		1795
deca	da	10^{1}	10	Dieci		1795
–		10^{0}	1	Unità		–
deci	d	10^{-1}	0,1	Decimo		1795
centi	c	10^{-2}	0,01	Centesimo		1795
milli	m	10^{-3}	0,001	Millesimo		1795
micro	µ	10^{-6}	0,000001	Milionesimo		1960 [3][note 4]
nano	n	10^{-9}	0,000000001	Bilionesimo	Miliardesimo	1960[3]
pico	p	10^{-12}	0,000000000001	Trilionesimo	Bilionesimo	1960[3]
femto	f	10^{-15}	0,000000000000001	Quadrilionesimo	Biliardesimo	1964[4]
atto	a	10^{-18}	0,000000000000000001	Quintilionesimo	Trilionesimo	1964[4]
zepto	z	10^{-21}	0,000000000000000000001	Sestilionesimo	Triliardesimo	1991[1]
yocto	y	10^{-24}	0,000000000000000000000001	Settilionesimo	Quadrilionesimo	1991[1]

$T_b = 1/BR$ $\qquad B = 1/T \sim 2/T$

$P_{dBm} = 10 \log_{10} \frac{P\,[mW]}{1\,[mW]}$ $\qquad P = 10^{\frac{P_{dBm}}{10}}$ $\qquad 50\% \sim 3dB$

$x_{dB} = 10 \log_{10} x$ $\qquad x = 10^{\frac{x_{dB}}{10}}$ $\qquad \alpha = 0,115\, \alpha_{dB}$ $\qquad \alpha_{dB} = 8,686\, \alpha$

$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j 2\pi f t}\, df$ $\qquad S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j 2\pi f t}\, dt$

linearità $\quad z(t) = \alpha\, x(t) + \beta\, y(t) \rightarrow Z(f) = \alpha\, X(f) + \beta\, Y(f)$

traslazione nel tempo: $\quad s(t-\tau) \rightarrow S(f)\, e^{-j 2\pi f \tau}$

traslazione di freq.: $\quad s(t)\, e^{j 2\pi f_0 t} \rightarrow S(f - f_0)$

prodotto durata banda: $\quad B \cdot T = cost$

$|H(f)|$ non cost $\Rightarrow$ selettività in frequenza $\qquad |H_{TOT}| = \prod |H_i|$ $\qquad \lambda = \frac{v_f}{f}$

$\angle H(f)$ non lineare $\Rightarrow$ dispersione cromatica $\qquad \angle H_{TOT} = \sum \angle H_i$

↳ $\phi(f) = 2\pi\tau f$ $\quad$ ↳ fenomeno lineare!

$\tau_g = -\frac{1}{2\pi}\frac{d\phi}{df} \rightarrow \tau_g(f) \Rightarrow$ dispersione cromatica $\qquad v_g = L/\tau_g \qquad v_g < c$

$\tau_f = \frac{\phi}{2\pi f} \qquad v_f = L/\tau_f = \lambda f = \frac{c}{\sqrt{\varepsilon_r}} \qquad v_f \lessgtr c$

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \longrightarrow \gamma = \pm j\omega\sqrt{LC} = j\beta$

$\hookrightarrow \beta = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \rightarrow$ valido sempre $\qquad \lambda = \frac{2\pi}{\beta}$

$\bar{V} = V_0^+ e^{-(\alpha+j\beta)z} + V_0^- e^{(\alpha+j\beta)z}$; $\bar{I} = \frac{V_0^+}{Z_0} \dots + \frac{V_0^-}{Z_0} \dots$

$v_f = \frac{\omega}{\beta} \sim \frac{1}{\sqrt{LC}}$ $\hookrightarrow$ valido sempre $\qquad V_g = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \sim \sqrt{\frac{L}{C}} \qquad \Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$

$2\beta z_M + \theta_\Gamma = 2N\pi \qquad ROS = \frac{|V_M|}{|V_m|} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$

$\Gamma_L \neq 0 \rightarrow$ oscillazioni $\lambda/2$ di $|V|$

- $Z_L = Z_0 \rightarrow \Gamma_L = 0$
- $Z_L = \infty$ CA $\rightarrow \Gamma_L = 1 \rightarrow Z_{in} = -jZ_0 \cot(\beta l)$ (MAX su L) $\qquad \theta_L = 0$
- $Z_L = 0$ CC $\rightarrow \Gamma_L = -1 \rightarrow Z_{in} = jZ_0 \tan(\beta l)$ (MIN su L) $\qquad \theta_L = \pi$

$$Z(z) = \frac{\bar{V}(z)}{\bar{I}(z)} = \begin{cases} Z_0 \dfrac{1 + \Gamma_L e^{j2\beta z}}{1 - \Gamma_L e^{j2\beta z}} \\[2ex] Z_0 \dfrac{Z_L + jZ_0 \tan\beta z}{Z_0 + jZ_L \tan\beta z} \end{cases}$$

- $\rightarrow Z_L = Z_0 \qquad \forall l, f \qquad z(z) = Z_0$
- $\rightarrow Z_L \neq Z_0 \qquad l = k\frac{\lambda}{2} \rightarrow z(z) = Z_L$; $\quad l = \frac{\lambda}{4} + k\frac{\lambda}{2} \rightarrow z(z) = \frac{Z_0^2}{Z_L} \rightarrow Z_0 = \sqrt{Z_{in} Z_L}$

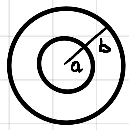
$P(z,t) = \frac{|V_0^+|^2}{2Z_0} e^{-2\alpha z} + \frac{|V_0^+|^2}{2Z_0} e^{-2\alpha z} \cos(2\omega t - 2\beta z) \rightarrow P_L = P^+ - P^- = \frac{|V_0^+|^2}{2Z_0}\left(e^{-2\alpha z} - |\Gamma|^2 e^{-2\alpha z}\right)$

$\downarrow$ P_{media} $\qquad P_{max} = \frac{|V_g|}{8Z_0}$

$\hookrightarrow P_L = P_1^+ - P_1^- = P_2^+ - P_2^- = \dots$ FRAZ. RIFL.

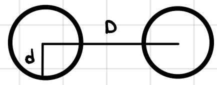
$\hookrightarrow P_L = \frac{|V_L|^2}{2Z_0}$

COAX: $Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{2\pi}\ln\frac{b}{a} \simeq \frac{60}{\sqrt{\varepsilon_r}}\ln\frac{b}{a} \qquad \beta = \frac{\omega}{c}\sqrt{\varepsilon_r} \qquad v_f = \frac{c}{\sqrt{\varepsilon_r}}$



$V_{PEAK} = R_d\, a \ln\frac{b}{a}$; $P_{max} = \frac{R_d^2 a^2 \sqrt{\varepsilon_r}\ln\frac{b}{a}}{120} \qquad LC = \varepsilon\mu$

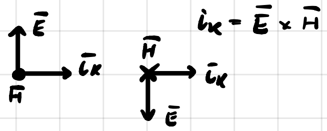
TP: $Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\pi}\ln\frac{2D}{d} \simeq \frac{120}{\sqrt{\varepsilon_r}}\ln\frac{2D}{d} \qquad \beta = \frac{\omega}{c}\sqrt{\varepsilon_r} \qquad v_f = \frac{c}{\sqrt{\varepsilon_r}}$

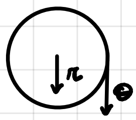


$\bar{E}(z) = \bar{E}_0^+(x,y)e^{-\gamma z} + \bar{E}_0^-(x,y)e^{\gamma z} \qquad \bar{E}(r) = \frac{\rho}{2\pi\varepsilon}\frac{1}{r}\bar{i}_r$

$\bar{H}(z) = \bar{H}_0^+(x,y)e^{-\gamma z} + \bar{H}_0^-(x,y)e^{-\gamma z} \qquad \bar{H}(r) = \frac{I}{2\pi r}\bar{i}_\theta$

$i_k = \bar{E} \times \bar{H}$





$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{E_t^+}{H_t^+} = -\frac{E_t^-}{H_t^-} \rightarrow \bar{H} = \frac{1}{\eta}\bar{i}_k \times \bar{E}$

$\bar{E} = -\eta\, \bar{i}_k \times \bar{H}$

$\hookrightarrow$ in gen $\in \mathbb{C}$; $\in \mathbb{R} \Rightarrow$ no perdite

$\gamma^2 = -\omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) = -\omega^2\mu\varepsilon_{eff}$

$\hookrightarrow \eta = \sqrt{\frac{\mu}{\varepsilon_{eff}}} \leftarrow$ perdite $\qquad \hookrightarrow \varepsilon_{eff} = \varepsilon_r - j\varepsilon_r''$

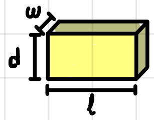
$\tan\delta = \frac{\varepsilon''}{\varepsilon} = \frac{\sigma}{\omega\varepsilon} \qquad \hookrightarrow \eta = \frac{\eta_0}{\sqrt{\varepsilon_r}}$

- buon dielettrico $\rightarrow$ $\tan\delta \ll 1$ ($\tan < 10^{-2}$) $\qquad \gamma = \alpha + j\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon} \rightarrow \sim$FASE
  - $\hookrightarrow \eta \simeq \sqrt{\frac{\mu}{\varepsilon}}$
- buon conduttore $\rightarrow$ $\tan\delta \gg 1$ ($\tan > 10^{2}$) $\qquad \alpha, \beta \propto \sqrt{f}$ $\qquad \gamma = \alpha + j\beta = (1+j)\sqrt{\pi f \mu\sigma} \rightarrow$ NO FASE $\rightarrow$ D. CROM
  - $\hookrightarrow \eta = (1+j)\frac{\alpha}{\sigma}$ $(\phi_\eta = \pi/4)$
- senza perdite $\rightarrow$ $\tan\delta = 0$ $\qquad \gamma = j\beta = j\omega\sqrt{\mu\varepsilon}$; $v_f = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n} \rightarrow$ FASE
  - $\hookrightarrow n = \sqrt{\varepsilon_r}$

EFFETTO PELLE:

$$\begin{cases} \delta = \dfrac{1}{\alpha} = \dfrac{1}{\sqrt{\pi f \mu \sigma}} \rightarrow \sigma = \infty \Rightarrow \delta = 0 \; ; \; \sigma = 0 \;\; \delta = \infty \\ r = \dfrac{1}{\sigma \omega \delta} \;\; [\Omega / m] \rightarrow \text{Cilindro: } \dfrac{1}{\sigma 2\pi r \delta} \end{cases}$$

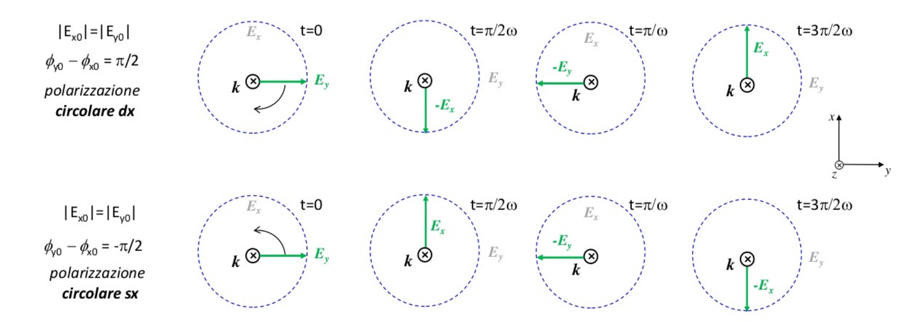
↳ $R = r\, l$



$\bar{E}(\bar{r}) = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$ (posizione)

$\bar{k} = \frac{2\pi}{\lambda}\bar{i}_k = \beta \bar{i}_k$
→ direzione z: $\bar{E}(z) = \bar{E}_0 e^{-j\beta z}$
→ generico xz: $\bar{E}(x,y) = \bar{E}_0 e^{-j(\beta_x x + \beta_z z)}$
→ generico: $\bar{E}(x,y,z) = \bar{E}_0 e^{-j(\beta_x x + \beta_y y + \beta_z z)}$

$\beta_x = \beta \cos\theta$
$\beta_z = \beta \sin\theta$
⋮



$\bar{S} = \frac{1}{2}\bar{E}\times\bar{H}^* \;\; [W\cdot m^{-2}]$

↳ $\alpha = 0 \rightarrow \bar{S} = \frac{1}{2}\frac{|E_0|^2}{\eta}\bar{i}_z$

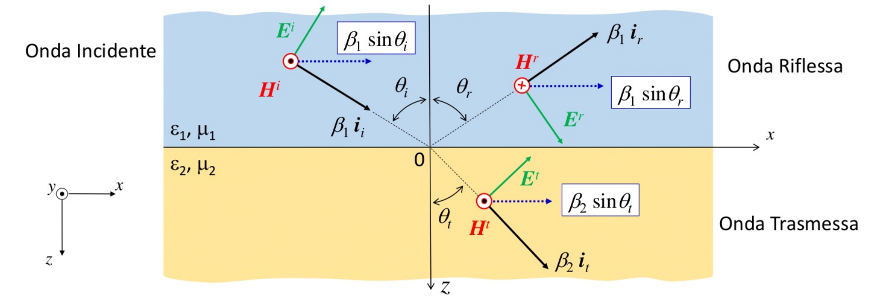
↳ $\alpha \neq 0 \rightarrow \bar{S} = \frac{1}{2}\frac{|E_0|^2}{\eta^*}e^{-2\alpha z}\bar{i}_z$

$P = Re\left\{\int_A \bar{S}\cdot\bar{i}_n\, dA\right\}$ (flusso) [W]

↳ piana uniforme: $P = SA\cos\theta$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\quad T = 1 + \Gamma \quad E_R = \Gamma E_i, \; E_T = T E_i$

↳ mezzi non magnetici ($\mu = 1$): $\Gamma = \frac{\sqrt{\varepsilon_{r2}} - \sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}} + \sqrt{\varepsilon_{r1}}} = \frac{n_2 - n_1}{n_2 + n_1}$



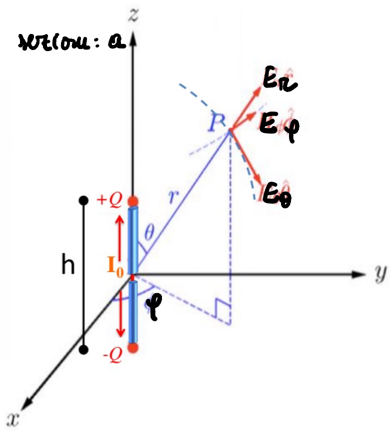
$\theta_i = \theta_r$: RIFLESSIONE

$n_1 \sin\theta_i = n_2 \sin\theta_t$: RIFRAZIONE

$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) \rightarrow \theta_i > \theta_c \Rightarrow$ RIFL.

incidenza su conduttore: $\Gamma = -1$

↳ sfasa di π



DIPOLO HERZIANO: $\frac{h}{\lambda} << 1$ $\qquad$ DIPOLO CORTO: $h < \lambda/50$

$$E_\theta = \eta\frac{I_0 h}{4\pi}\left(\frac{jk}{r} + \frac{1}{r^2} + \frac{1}{jkr^3}\right)\sin(\theta)e^{-jkr}$$

$\eta = \sqrt{\mu/\varepsilon}$ $\quad$ → $k = \omega\sqrt{\mu\varepsilon}$

$$H_\varphi = \frac{I_0 h}{4\pi}\left(\frac{jk}{r} + \frac{1}{r^2}\right)\sin(\theta)e^{-jkr}$$

$$E_r = \eta\frac{I_0 h}{2\pi}\left(\frac{1}{r^2} + \frac{1}{jkr^3}\right)\cos(\theta)e^{-jkr}$$

$$E_\varphi = H_r = H_\theta = 0$$

CAMPO LONTANO $\xrightarrow{r >> \lambda}$ $\begin{cases} E_\theta = \eta H_\varphi \\ H_\varphi = \frac{jk}{4\pi}\frac{I_0 h}{r}\sin\theta\, e^{-jkr} \end{cases}$

direzione radiale

$S = \frac{1}{2}E_\theta H_\varphi^* = \frac{\eta k^2 I_0^2 h^2}{32\pi^2 r^2}\underbrace{\sin^2\theta}_{F(\theta,\varphi)}$ → dipolo herziano! $\rightarrow P_{RAD}^{SFERA} = \frac{\eta\pi I_0^2}{3}\left(\frac{h}{\lambda}\right)^2 e^{-2\alpha r}$

↳ $R_{rad} = \frac{2\eta\pi h^2}{3\lambda^2}$

$R_{rad} = \frac{2P_{rad}}{I_0^2}$

↳ $\bar{S}(\theta,\varphi) = S_0 F(\theta,\varphi)\bar{i}_r$

$P_{rad} = \int \bar{S}\, ds = S_0 r^2 \underbrace{\int_{4\pi} F(\theta,\varphi)}_{\Omega_a}$

↳ $P_{RAD} = \frac{1}{2}R_{rad} I_0^2$

$S_i = \frac{P_{rad}}{4\pi r^2}$

↳ $S_0 = \frac{P_{rad}}{4\pi r^2}D$

$D = \frac{S_0}{S_i} = \frac{4\pi}{\Omega_a}$ $\quad S_0 = D S_i$

$$\begin{cases} \text{isotropa: } F = 1;\; D = 1;\; \Omega_a = 4\pi \\ \text{d.H.: } F = \sin^2;\; D = 1{,}5;\; \Omega_a = \frac{8\pi}{3} \\ \text{molto direttiva: } \Omega_a = \pi\left(\frac{\theta_a}{2}\right)^2 \end{cases}$$

$G = D\,\mathcal{E}$ $\qquad \mathcal{E} = \frac{R_{rad}}{R_{rad}+R_p} \le 1$ $\qquad \begin{cases} \alpha = 0 \\ \alpha \neq 0 \end{cases}$

↳ $P_{rad} = P_t\,\mathcal{E}$

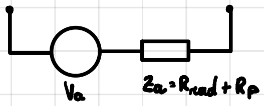
$P_{ric} = A_{eff}\,S_{ric}$

$P_{ric} = S_{ric}\,\underbrace{A_{eff}\,\mathcal{E}}_{A_\mathcal{E}}$

$\left.\begin{array}{l} A_{eff}/D = \lambda^2/4\pi \\ A_\mathcal{E}/D = \lambda^2/4\pi \end{array}\right\}$ $\quad S_{ric} = \frac{1}{2}\frac{|E_{ric}|^2}{\eta}$ $\quad P_{ric}^A = \frac{V_a^2}{8R_{rad}}$

Antenne paraboliche: A_{eff} = area geometrica $= \pi a^2$

Dipolo Hertziano: $A_{eff} = 3\lambda^2/8\pi$



$V_a = 2|E_{ric}|\sqrt{\frac{R_{rad}A_{eff}}{\eta}}$

↳ $V_a = \sqrt{8R_{rad}P_{ric}} = E_{ric}\,h$

$R_p = h \cdot r_p$ ← dipolo

$r_p = \frac{1}{2\pi a \delta}\frac{1}{\sigma}$

$P_r = P_t G_t F_t \left(\frac{\lambda}{4\pi R}\right)^2 G_r F_r$

↳ L_F

orientato nella direzione di massima rad: $F_* = 1$

$E_r = E_0 + E_1 = E_0\left(1 + a e^{-j\Delta\theta}\right)$

↳ $a = E_1/E_0$; $\Delta\theta = \beta\Delta L$ → MAX: $\Delta\theta = 2M\pi$ → costr.

MIN: $\Delta\theta = (2M+1)\pi$

↳ distruttiva

$f_r = f_0 + f_D = f_0 + f_0\frac{v\cos\theta}{c}$

$\Delta n = \frac{n_{co}-n_{cl}}{n_{co}}$ $\quad$ step-index: $n_{co} = 1{,}45$, $\Delta n \simeq 0{,}3\,\%$ $\quad \theta_0 \simeq NA = \sqrt{n_{co}^2 - n_{cl}^2}$ $\quad V = \frac{2\pi}{\lambda_0}\,a\,NA$

k_0 $\qquad$ raggio core

banda c $\qquad \alpha_R = A/\lambda_0^4$ $\;(A \simeq 0{,}7 - 0{,}9\ \text{dB}/\mu m^4)$

$\alpha_m = 0{,}2$ dB/Km con $\lambda \simeq 1550$ nm $\qquad n_{co} > n_{cl}$ $\quad \theta_i > \theta_c = \arcsin\frac{n_{cl}}{n_{co}}$

$k_0 n_{co} d \cos\theta_i + \phi = N\pi$ $\qquad f_c = \frac{c}{2d\sqrt{n_{co}^2 - n_{cl}^2}}\,N$ → guida planare!

↳ guidato se $f_0 > f_c$ o $\lambda_0 < \lambda_c$

$\beta_n = \frac{2\pi}{\lambda_0}\underbrace{n_{co}\sin\theta_N}_{n_{eff,N}}$ $\qquad \begin{cases} k_0 n_{cl} \le \beta \le k_0 n_{co} \\ \theta_c \le \theta_n \le \pi/2 \\ n_{cl} \le n_{eff,n} \le n_{co} \end{cases}$ $\quad \theta_n = \theta_c \quad \theta_n = \pi/2$

$v_{f,N} = \frac{\omega}{\beta_n} = \frac{c}{n_{eff,n}}$ → $\frac{c}{n_{co}} \le v_{f,N} \le \frac{c}{n_{cl}}$ $\quad v_g < v_f$ $\quad v_{g,N} = \frac{d\omega}{d\beta_N} = \frac{c}{\underbrace{n_{eff,N} + \omega\frac{dn_{eff,N}}{d\omega}}_{n_{g,N}}}$

$\tau_g = \frac{L}{c}\,n_g$

frequenze vicine: $v_{g,n} = \frac{2\pi\Delta f}{\Delta\beta}$

$V > 2{,}405 \Rightarrow$ multimodalità! $\quad N \simeq \frac{V^2}{2{,}46}$

$\tau = \frac{L}{v_g} \simeq L\beta_1$

$\beta(\omega) = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2$ $\qquad \begin{cases} \tau_g = L\beta_1 & \text{se } \beta_2 = 0 \\ \tau_g = L[\beta_1 + \beta_2(\omega-\omega_0)] & \text{se } \beta_2 \neq 0 \end{cases}$

limitante $\qquad \beta_2 \sim -20\ \text{ps}^2/\text{Km}$

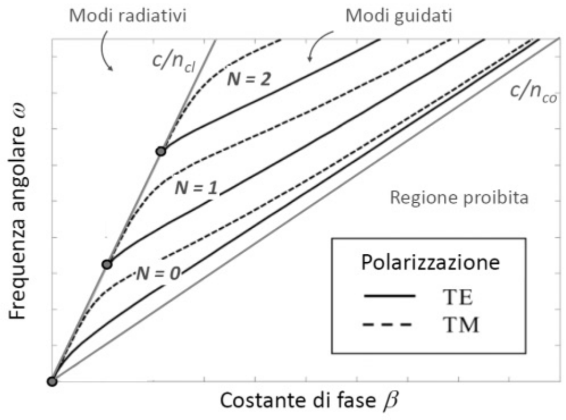
↳ Se non ci sono tanti modi considera $|n_{g,1} - n_{g,2}|$

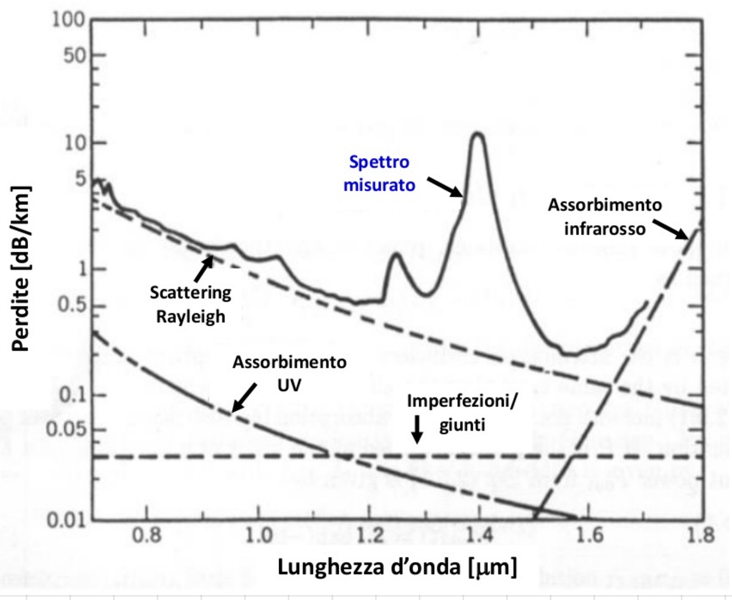
DISPERSIONE MODALE: $\Delta\tau = \frac{L}{c}(n_{co}-n_{cl}) < \frac{1}{B_s}$ → $B_s L < \frac{c}{n_{co}-n_{cl}}$

DISPERSIONE CROMATICA: $\Delta\tau = 2\pi L|\beta_2|B_s < 1/B_s$ → $B_s^2 L < 1/2\pi|\beta_2|$ → $B_s^2 L < 8\cdot10^{24}\ \text{Hz}^2\text{m}$

$T_{FWHM} = 2\sqrt{\ln 2}\,T_0 = 1{,}665\,T_0 \simeq \frac{1}{B_s}$ $\qquad T_1 = T_0\sqrt{1 + \left(\frac{L}{L_D}\right)^2}$ $\qquad L_D = T_0^2/|\beta_2|$

segnale gaussiano $\qquad$ dopo disp. cromatica





$\left.\begin{array}{l} c = 2{,}998 \cdot 10^8\ m/s \\ \varepsilon_0 = 8{,}854 \cdot 10^{-12}\ F/m \\ \mu_0 = 4\pi \cdot 10^{-7}\ H/m \end{array}\right\}$ $\quad \eta_0 = 120\pi\ \Omega$