2. (25%)

(a) $j \leftarrow 1$ $\triangleright$ 1 time
       while $j < n$ **do** $\triangleright$ n-1 times
         S_1 $\triangleright$ n-2 times
         $k \leftarrow n$ $\triangleright$ n-2 times
         while $k > j$ **do** $\triangleright$ n-j times
           S_2 $\triangleright$ n-j-1 times
           $k - -$ $\triangleright$ n-j-1 times
         $j + +$ $\triangleright$ n-2 times

The Sum of these statments can be repesented by $S(n)$:

$$S(n) = 1 + (n-1) + 2(n-2) + \sum_{j=1}^{n-1}(n - j + 2(n - j - 1))$$

$$= 3n - 4 + \sum_{j=1}^{n-1}(3n - 2j - 2)$$

$$= 3n - 4 + 3n(n-2) - 2(n-2) - 2\sum_{j=1}^{n-1} j$$

$$= 3n - 4 + 3n(n-2) - 2(n-2) - 2(\frac{(n-1)(n)}{2})$$

$$= -3n - 4 + 3n^2 - 2n + 4 - n^2 + n$$

$$= 2n^2 - 4n$$

(b) $j \leftarrow 0$ $\triangleright$ 1 time
       while $j \leq n$ **do** $\triangleright$ n+1 times
         $k \leftarrow 0$ $\triangleright$ n times
         while $k \leq j$ **do** $\triangleright$ j+1 times
           S_1 $\triangleright$ j times
           $k + +$ $\triangleright$ j times
         $j + +$ $\triangleright$ n times

Summing these statements:

$$S(n) = 1 + (n+1) + 2n + \sum_{j=0}^{n}(2j + j + 1)$$

$$= 3n + 2 + (n+1) + 3\sum_{j=0}^{n} j$$

$$= 4n + 3 + \frac{3n(n+1)}{2}$$

$$= 4n + 3 + \frac{3}{2}(n^2 + n)$$

$$= \frac{11n}{2} + \frac{3n^2}{2} + 3$$

(c) $j \leftarrow 1$ $\triangleright$ 1 time

        while $j \leq n$ **do** $\triangleright$ n times

            $k \leftarrow 1$ $\triangleright$ n-1 times

            while $k \leq j \times j$ **do** $\triangleright$ j^2 times

                S_1 $\triangleright$ $j^2 - 1$ times

                $k++$ $\triangleright$ $j^2 - 1$ times

            $j++$ $\triangleright$ n-1 times

Summing these statements:

$$S(n) = 1 + n + n - 1 + \sum_{j=1}^{n} 3j^2 - 2$$

$$= 2n - 2(n-1) + 3\sum_{j=1}^{n} j^2$$

$$= 2 + 3\frac{n(n+1)(2n+1)}{6} = 2 + \frac{n(n+1)(2n+1)}{2}$$

$$= 2 + 2n^3 + 3n^2 + n$$