2. (25%)

(a)
$j \leftarrow 1$ ▷ 1 time
while $j < n$ **do** ▷ n times
  S_1 ▷ n-1 times
  $k \leftarrow n$ ▷ n-1 times
  while $k > j$ **do** ▷ n-j+1 times
    S_2 ▷ n-j times
    $k--$ ▷ n-j times
  $j++$ ▷ n-1 times

The Sum of these statments can be repesented by $S(n)$:

$$\begin{aligned} S(n) &= 1 + n + 3(n-1) + \sum_{j=1}^{n-1}(n-j+1+2(n-j)) \\ &= 4n - 2 + \sum_{j=1}^{n-1}(3(n-j)+1) \\ &= 4n - 2 + (n-1) + 3\sum_{j=1}^{n-1}(n-j) \\ &= 5n - 3 + 3(n-1)(n) - 3\sum_{j=1}^{n-1} j \\ &= 5n - 3 + 3(n^2-n) - 3\frac{(n-1)(n)}{2} \\ &= 2n - 3 + 3n^2 - 3\frac{n^2-n}{2} \\ &= \frac{3n^2}{2} + \frac{7n}{2} - 3 \end{aligned}$$

(b)
$j \leftarrow 0$ ▷ 1 time
while $j \leq n$ **do** ▷ n+2 times
  $k \leftarrow 0$ ▷ n+1 times
  while $k \leq j$ **do** ▷ j+1 times
    S_1 ▷ j times
    $k++$ ▷ j times
  $j++$ ▷ n+1 times

Summing these statements:

$$\begin{aligned} S(n) &= 1 + (n+2) + 2n + 2 + \sum_{j=0}^{n}(2j+j+1) \\ &= 3n + 5 + (n+1) + 3\sum_{j=0}^{n} j \\ &= 4n + 6 + \frac{3n(n+1)}{2} \\ &= 4n + 6 + \frac{3}{2}(n^2+n) \\ &= \frac{11n}{2} + \frac{3n^2}{2} + 6 \end{aligned}$$

(c)

```
j ← 1                                          ▷ 1 time
while j ≤ n do                                 ▷ n+1 times
    k ← 1                                      ▷ n times
    while k ≤ j × j do                         ▷ j^2 times
        S_1                                    ▷ j^2 − 1 times
        k++                                    ▷ j^2 − 1 times
    j++                                        ▷ n times
```

Summing these statements:

$$\begin{aligned} S(n) &= 3n + 2 + \sum_{j=1}^{n}(3j^2 - 2) \\ &= 3n + 2 - 2n + 3\sum_{j=1}^{n} j^2 \\ &= 2 + n + \frac{n(n+1)(2n+1)}{2} \\ &= n^3 + \frac{3n^2}{2} + \frac{3n}{2} + 2 \end{aligned}$$