2. (25%)

(a) 
$$j \leftarrow 1$$
  $\Rightarrow$  1 time while  $j < n$  do  $\Rightarrow$  n-1 times  $S_1$   $\Rightarrow$  n-2 times  $k \leftarrow n$   $\Rightarrow$  n-2 times while  $k > j$  do  $\Rightarrow$  n-j times  $S_2$   $\Rightarrow$  n-j-1 times  $k \leftarrow n$   $\Rightarrow$  n-j-1 times  $k \leftarrow n$ 

The Sum of these statements can be represented by S(n):

$$S(n) = C_0 + \sum_{i=1}^{n-1} (C_1 + S_1 + \sum_{j=i}^{n-1} (S_2 + C_2))$$

$$= C_0 + \sum_{i=1}^{n-1} (C_1 + S_1 + (n-1-i)(S_2 + C_2))$$

$$= C_0 + (n-2)(C_1 + S_1) + (S_2 + C_2) \sum_{i=1}^{n-1} (n-1-i)$$

$$= C_0 + (n-2)(C_1 + S_1) + (S_2 + C_2) \left( n(n-2) - (n-2) - \frac{n(n-1)}{2} \right)$$

$$= C_0 + (n-2)(C_1 + S_1) + (S_2 + C_2) \left( \frac{n^2}{2} - \frac{5n}{2} + 2 \right)$$

$$= C_0 + (n-2)C_1' + C_2' \left( \frac{n^2}{2} - \frac{5n}{2} + 2 \right)$$

$$\begin{array}{lll} \text{(b)} & j \leftarrow 0 & & & \triangleright 1 \text{ time} \\ & \textbf{while } j \leq n \textbf{ do} & & \triangleright n+1 \text{ times} \\ & k \leftarrow 0 & & \triangleright n \text{ times} \\ & \textbf{while } k \leq j \textbf{ do} & & \triangleright j+1 \text{ times} \\ & S_1 & & \triangleright j \text{ times} \\ & k++ & & \triangleright j \text{ times} \\ & j++ & & \triangleright n \text{ times} \end{array}$$

Summing these statements:

$$S(n) = 1 + (n+1) + 2n + \sum_{j=0}^{n} (2j+j+1)$$

$$= 3n + 2 + (n+1) + 3\sum_{j=0}^{n} j$$

$$= 4n + 3 + \frac{3n(n+1)}{2}$$

$$= 4n + 3 + \frac{3}{2}(n^2 + n)$$

$$= \frac{11n}{2} + \frac{3n^2}{2} + 3$$