2. (25%)

(a)

$j \leftarrow 1$ ▷ 1 time
while $j < n$ **do** ▷ n-1 times
  S_1 ▷ n-2 times
  $k \leftarrow n$ ▷ n-2 times
  while $k > j$ **do** ▷ n-j times
    S_2 ▷ n-j-1 times
    $k--$ ▷ n-j-1 times
  $j++$ ▷ n-2 times

The Sum of these statments can be repesented by $S(n)$:

$$\begin{aligned}
S(n) &= 1 + (n-1) + 2(n-2) + \sum_{j=1}^{n-1}(n-j+2(n-j-1)) \\
&= 3n - 4 + \sum_{j=1}^{n-1}(3n-2j-2) \\
&= 3n - 4 + 3n(n-2) - 2(n-2) - 2\sum_{j=1}^{n-1} j \\
&= 3n - 4 + 3n(n-2) - 2(n-2) - 2(\frac{(n-1)(n)}{2}) \\
&= -3n - 4 + 3n^2 - 2n + 4 - n^2 + n \\
&= 2n^2 - 4n
\end{aligned}$$

(b)

$j \leftarrow 0$ ▷ 1 time
while $j \leq n$ **do** ▷ n+1 times
  $k \leftarrow 0$ ▷ n times
  while $k \leq j$ **do** ▷ j+1 times
    S_1 ▷ j times
    $k++$ ▷ j times
  $j++$ ▷ n times

Summing these statements:

$$\begin{aligned}
S(n) &= 1 + (n+1) + 2n + \sum_{j=0}^{n}(2j+j+1) \\
&= 3n + 2 + (n+1) + 3\sum_{j=0}^{n} j \\
&= 4n + 3 + \frac{3n(n+1)}{2} \\
&= 4n + 3 + \frac{3}{2}(n^2+n) \\
&= \frac{11n}{2} + \frac{3n^2}{2} + 3
\end{aligned}$$

(c)

$j \leftarrow 1$	▷ 1 time
while $j \leq n$ **do**	▷ n times
$\quad k \leftarrow 1$	▷ n-1 times
$\quad$ **while** $k \leq j \times j$ **do**	▷ j^2 times
$\qquad S_1$	▷ $\text{j}^2 - 1$ times
$\qquad k++$	▷ $\text{j}^2 - 1$ times
$\quad j++$	▷ n-1 times

Summing these statements:

$$\begin{aligned}
S(n) &= 1 + n + n - 1 + \sum_{j=1}^{n} 3j^2 - 2 \\
&= 2n - 2(n-1) + 3\sum_{j=1}^{n} j^2 \\
&= 2 + 3\frac{n(n+1)(2n+1)}{6} = 2 + \frac{n(n+1)(2n+1)}{2} \\
&= 2 + 2n^3 + 3n^2 + n
\end{aligned}$$