Plan for Summer 2020

Alex Bernstein
abernstein@ucsb.edu
(joint work with Alex Shkolnik)

June 22, 2020

Department of Statistics & Applied Probability
University of California, Santa Barbara

Part I: FFP and Extensions

a) Elucidate Applications for FFP in the Current Context:
  - Sensitivity Analysis (These derivatives are written in terms of the one-factor case, but should generalize):
    - Weights vs. Asset Loadings: $\frac{\partial}{\partial \beta_j} w_i$ in both cases where $i \neq j$ and $i = j$
    - Weights vs. Factor Variance: $\frac{\partial}{\partial \sigma^2} w_i$
    - Weights vs. Specific Variance: $\frac{\partial}{\partial \delta_i^2} w_i$ (0 if $i \neq j$)
    - Fixed Point vs. Asset Loadings: $\frac{\partial}{\partial \beta_i} \theta = \frac{\partial}{\partial \beta_i} \psi(\theta)$ at FP
    - Fixed Point vs. Factor Variance: $\frac{\partial}{\partial \sigma^2} \theta = \frac{\partial}{\partial \sigma^2} \psi(\theta)$ at FP
    - Fixed Point vs. Specific Variance: $\frac{\partial}{\partial \delta_i^2} \theta = \frac{\partial}{\partial \delta_i^2} \psi(\theta)$ at FP
  - Applications to other restricted QP problems

b) Complete proof for convergence of FFP algorithm for $q > 1$ factors

c) See if any progress can be made on linear objective case and/or generalizing of constraints

Lead

Typically, no explicit formula exists for a minimum-variance portfolio, which are computed via the well-developed theory of numerical convex optimization. Under a factor-covariance model, we develop a semi-explicit formula and methodology for Minimum-Variance (Markowitz) Portfolios.

Objective(Funnel)-Challenge-Action-Resolution

- Opening
  - Minimum Variance portfolios and convex optimization is well developed and studied
  - Well-understood computational theory and algorithms; less well-understood is when explicit formulae are available and sensitivity analysis can be conducted in a (relatively) computationally efficient manner
- Funnel
  - We are working in the context of minimum variance portfolios; this generalizes to other element wise-constrained quadratic programs without a linear constraint/term
- Challenge
  - We aim to show that a semi-explicit formula for minimum-variance portfolios exist for a single factor, (and can generalize to any arbitrary number of factors), and can be computed very quickly

OCAR Part II

- Action
  - We derive a fixed-point equation, the solution of which can be used to construct the minimum-variance portfolio.
  - We show that a fixed-point iteration algorithm can be used to compute the necessary fixed point, with guaranteed convergence
- Resolution-
  We are able to expand our understanding of elementwise-constrained (e.g. long-only) minimum-variance portfolios, as our formula allows us to compute partial derivatives and conduct a sensitivity analysis.

Part II: Random Matrix Theory Extensions

1. Write a good background on classical RMT results
    - What is “convergence” in the RMT sense?
    - Different regimes for convergence- CLT; Marcenko-Pastur
    - What does the matrix eigenstructure tell us?
2. Applications of spiked covariance results to eigenstructure of factor models

To Do's and Dates:

Part I:

- Clean up FFP proof- Due 6/27
- Compute listed Partial Derivatives- Due 6/27
- Work out two-factor example for generalization of proof- Due 6/29

Part II:

- RMT Background: Classical Results- Due 7/1
- Spiked Covariance Results- Due 7/6
- Simulations- Due 7/10