

# RESEARCH IN GOLDBACH'S CONJECTURE

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AMS 487

## Abstract:

Under Dr. Li's supervision and counseling on his code of finding Goldbach numbers by the sieve of Eratosthenes method, my paper indicates the application of using the sieve of Eratosthenes method to find Goldbach pairs. I also present my view of Goldbach's comet in different congruence classes.

## Introduction:

It has been almost 280 years since Goldbach's Conjecture was first proposed. It is by far one of the most famous and enduring problems still unsolved in the world. In the letter, Christian Goldbach mentioned a conjecture that every natural even number greater than two is the sum of two prime numbers.

According to Wikipedia, "T. Oliveira e Silva ran a distributed computer search that has verified the conjecture for  $n \leq 4 \times 10^{18}$  (and double-checked up to  $4 \times 10^{17}$ ) as of 2013."

### **The Sieve of Eratosthenes method:**

This method is used to find all primes up to a given natural number  $N$ . It is the process of eliminating the multiples of stacked primes. Stacked primes are a list of prime numbers less than or equal to  $\sqrt{N}$ .

Because  $N = \sqrt{N} \cdot \sqrt{N}$ , if both values are greater than  $\sqrt{N}$ , then the result will be greater than  $N$ .

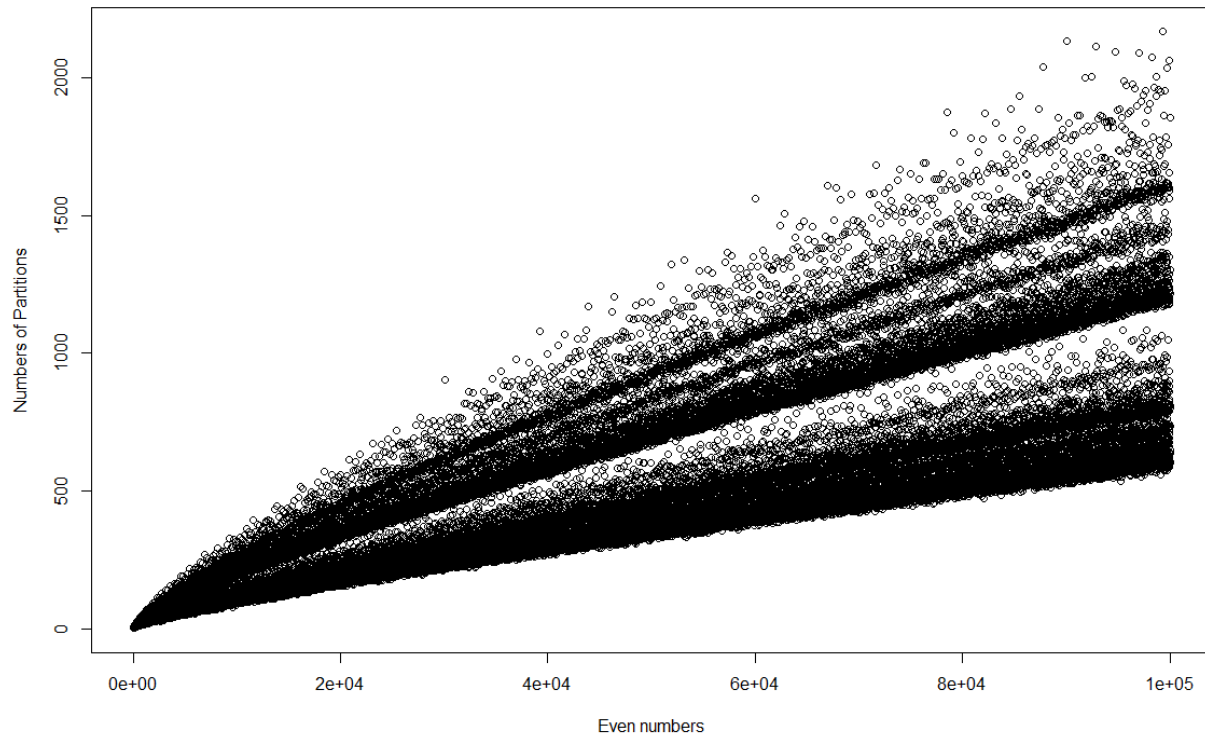
Therefore, using Sieve method is friendly memory way to generate prime numbers and check

Goldbach's Conjecture.

### **Goldbach Partitions in R:**

I used the R statistic package named "primes" to generate the list of prime numbers up to the assigned natural number  $N$ . The program will check if the result of the subtraction between an even number and a prime number less than half of that even number is in the generated prime lists. If so, the result and that prime number is a Goldbach pair. Furthermore, the program also counts the number of Goldbach's partitions each natural even number, and generates a Goldbach function, also called the Goldbach's comet. Goldbach's comet is a plot of the number of possible combinations of two primes for every even number up to the given boundary.

The following graph can probably be explained as a consequence of how the number of partitions varies between different congruence classes. It just expresses the number of partitions up to  $10^5$ , but it is clear that the number of partitions is steadily rising. That would make me assume that the probability of finding a Goldbach pair for large numbers is more than the smaller ones', which means conjecture holds.



Goldbach partitions of the first 1000000 natural even numbers, generated by R script.

According to the Prime number theory, every prime number can be written in the form of  $6m \pm 1$ .

Assume that the conjecture holds from the first  $10^5$  natural even numbers:

$$(6a + 1) + (6b - 1) = 6(a + b) + 0 \equiv 0 \pmod{6}$$

$$(6a - 1) + (6b + 1) = 6(a + b) + 0 \equiv 0 \pmod{6}$$

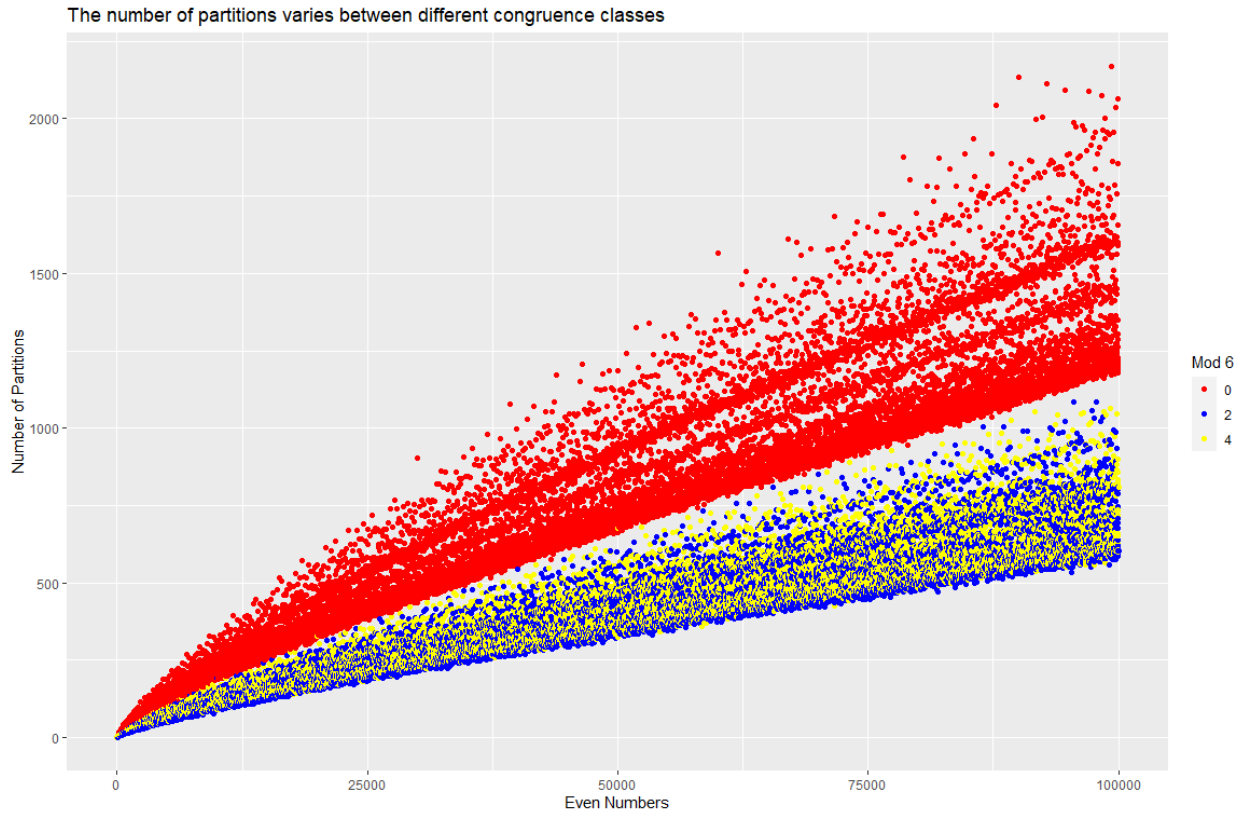
$$(6a + 1) + (6b + 1) = 6(a + b) + 2 \equiv 2 \pmod{6}$$

$$(6a - 1) + 3 = 6a + 2 \equiv 2 \pmod{6}$$

$$(6a - 1) + (6b - 1) = 6(a + b) - 2 \equiv 4 \pmod{6}$$

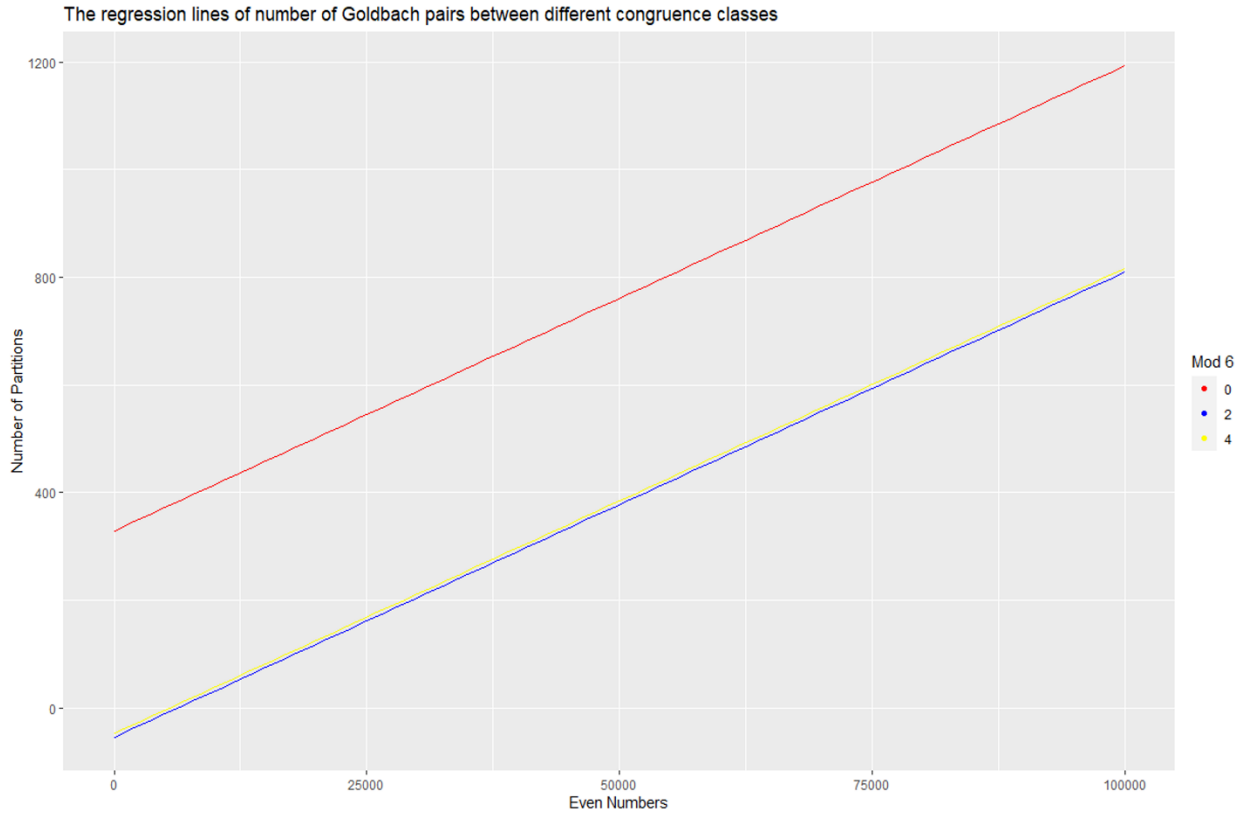
$$2 + 2 \equiv 4 \pmod{6}$$

$$(6a + 1) + 3 = 6a + 4 \equiv 4 \pmod{6}$$



The graph is generated by “ggplot2” package in R to see how the number of partitions distribute between different congruence classes of 6.

Next, I use regression analysis to see the relations between the number of partitions and the natural even number. As I expected, the even numbers in group 0 mod 6 have more Goldbach pairs than the ones in groups 2 and 4 mod 6. I also see that the slope of the 0 mod 6 group fit line is over twice the slope of the others. The slope of group 2 and 4 mod 6 fit lines are likely close.



### Code & Materials:

GitHub: <https://github.com/alexbui96/ams487>.

### References:

1. Li, Xiaolin. "sieve.cpp." May 2022.
2. "Goldbach's Conjecture." Wikipedia, Wikimedia Foundation, [https://en.wikipedia.org/wiki/Goldbach%27s\\_conjecture](https://en.wikipedia.org/wiki/Goldbach%27s_conjecture).
3. "Goldbach's comet." Wikipedia, Wikimedia Foundation, [https://en.wikipedia.org/wiki/Goldbach%27s\\_comet](https://en.wikipedia.org/wiki/Goldbach%27s_comet).
4. Härdig, Johan. Goldbach's Conjecture. Aug. 2020, <http://uu.diva-portal.org/smash/get/diva2:1471524/FULLTEXT02.pdf>.