**Goldbach’s Conjecture Project**

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AMS 325

**Project’s title:** Distribution of Goldbach’s Conjecture and Goldbach’s Weak Conjecture

**Abstract:**

This is my analysis project on Goldbach’s strong and weak conjectures. By applying the Sieve of Eratosthenes to find prime numbers up to a given upper bound number, I can easily find Goldbach pairs in a reasonable time. In programming, using the Sieve method is a friendly memory way to generate prime numbers. The Goldbach pairs of both conjectures are visualized in the residue class of 3, and multiplication classes of 3, 5, and 7, which is also called Goldbach’s comet.

**Introduction:**

1. **Goldbach’s Conjecture**

It has been almost 280 years since Goldbach’s Conjecture was first proposed. It is by far one of the most famous and enduring problems still unsolved in the world. In the letter, Christian Goldbach mentioned a conjecture that every natural even number greater than two is the sum of two prime numbers. According to Wikipedia, “T. Oliveira e Silva ran a distributed computer search that has verified the conjecture for (and double-checked up to ) as of 2013.”

1. **Goldbach’s Weak Conjecture:**

In number theory, Goldbach's weak conjecture, also known as the odd Goldbach conjecture, the ternary Goldbach problem, or the 3-primes problem, states that “Every odd number greater than 5 can be expressed as the sum of three primes.”

This conjecture is called "weak" because if Goldbach's strong conjecture is proven, then it would be also true. Goldbach’s weak conjecture was proven in 2013 by Harald Helfgott, a Peruvian mathematician. His proof has not yet appeared in a peer-reviewed publication, though was accepted for publication in the Annals of Mathematics Studies series in 2015, and has been undergoing further review and revision since.

Some state the conjecture as “Every odd number greater than 7 can be expressed as the sum of three odd primes.” I use this statement for my project analysis.

1. **The Sieve of Eratosthenes:**

In mathematics, the Sieve of Eratosthenes is an ancient algorithm for finding all prime numbers up to any given limit N. It does so by iteratively marking as composite (i.e., not prime) the multiples of each prime, starting with the first prime number, 2. Once all the multiples of each discovered prime have been marked as composites, the remaining unmarked numbers are primes. Because a composite of a number will be the product of integers smaller than or equal to , we only need to check for multiples of primes that are smaller than or equal to (Introduction - 3.1). Here is a proof for it:

*Proof for Introduction - 3.1.* Suppose a positive integer is not prime, then can be written as a product of two integer and , greater than 1 and . Assume , then

, a contradiction, since *.*

**Algorithm:**

1. **The Sieve of**

**References:**

1. “Goldbach’s conjecture” Wikipedia, Wikimedia Foundation, https://en.wikipedia.org/wiki/Goldbach%27s\_conjecture.
2. “Goldbach’s weak conjecture” Wikipedia, Wikimedia Foundation, https://en.wikipedia.org/wiki/Goldbach%27s\_weak\_conjecture.
3. “Sieve of Eratosthenes” Wikipedia, Wikimedia Foundation, https://en.wikipedia.org/wiki/Sieve\_of\_Eratosthenes.