Problem 6

$$N(0,2k,2n) = \frac{2n}{n+k} = \frac{2n}{n-k}$$

1) Sterling's approximation: $n! \sim \sqrt{2\pi n}(\frac{n}{e})^n$

$$\binom{2n}{n+k} = \frac{2n!}{(n+k)!(n-k)!} = \frac{\sqrt{2\pi 2n}(\frac{2n}{e})^{2n}}{\sqrt{2\pi(n+k)}(\frac{n+k}{e})^{n+k}\sqrt{2\pi(n-k)}(\frac{n-k}{e})^{n-k}}$$

2) Simplify this expression (cancel $\sqrt{2\pi}$ from numerator and denominator):

$$= \frac{\sqrt{2\pi}(2n)^{1/2}(2n)^{(2n)}e^{-(2n)}}{\sqrt{2\pi}(n+k)^{1/2}(n+k)^{(n+k)}e^{-(n+k)}\sqrt{2\pi}(n-k)^{1/2}(n-k)^{(n-k)}e^{-(n-k)}}$$

$$= \frac{(2n)^{1/2}(2n)^{(2n)}e^{-(2n)}}{\sqrt{2\pi}(n+k)^{1/2}(n+k)^{(n+k)}e^{-2n}(n-k)^{1/2}(n-k)^{(n-k)}e^{-(n-k)}}$$

3) Combine and cancel e terms:

$$= \frac{2^{2n+1/2}n^{2n+1/2}e^{-2n}}{\sqrt{2\pi}(n+k)^{1/2+n+k}(n-k)^{n-1/2-k}e^{-2n)}}$$

$$= \frac{2^{2n+1/2}n^{2n+1/2}}{\sqrt{2\pi}(n+k)^{1/2+n+k}(n-k)^{n-1/2-k}}$$

4) rearrange and simplify bold terms to $n^{1/2}$

$$= \frac{2^{2n+1/2}\mathbf{n^{2n+1/2}}}{\sqrt{2\pi}\mathbf{n^{1/2+n+k}}(1+\frac{k}{n})^{1/2+n+k}\mathbf{n^{1/2+n-k}}(1-\frac{k}{n})^{1/2+n-k}}$$

$$= \frac{2^{2n+1/2}}{\sqrt{2\pi}n^{1/2}(1+\frac{k}{n})^{1/2+n+k}(1-\frac{k}{n})^{1/2+n-k}}$$

5) simplify and rearrange

$$= \frac{2^{2n+1/2}}{\sqrt{2\pi}n^{1/2}(1-\frac{k^2}{n^2})^{1/2+n}(1-\frac{k}{n})^{-k}(1+\frac{k}{n})^{k}}$$

6) take the natural log of the bolded

$$= \frac{2^{2n+1/2}}{\sqrt{2\pi}n^{1/2}} * \frac{\mathbf{1}}{\mathbf{(1-\frac{k^2}{n^2})^{1/2+n}(1-\frac{k}{n})^{-k}(1+\frac{k}{n})^{k}}}$$

$$= ln1 - (1/2+n)ln(1-\frac{k^2}{n^2}) - kln(1-\frac{k}{n}) + kln(1+\frac{k}{n})$$

7) apply taylor series expansion $ln(1+z) \simeq z$ as n gets large k goes to infinity, bold expressions cancel

$$= 0 - (1/2+n)\frac{k^2}{n^2} \mathbf{-kln(1-\frac{k}{n}) + kln(1+\frac{k}{n})}$$

$$= -(1/2+n)\frac{k^2}{n^2}$$

8) rearrange and $1/2n$ goes to 0 as n becomes very large, n cancels with n^2 in denominator

$$= -n(1/2n+1)\frac{k^2}{n^2}$$

$$= -\frac{k^2}{n}$$

8) take natural log and substitute back into original expression

$$= \frac{2^{2n+1/2}}{\sqrt{2\pi}n^{1/2}e^{-\frac{k^2}{n}}}$$

9) Given: $n = t/2$ & $k = x/2$. Substitute in for n and k and simplify

$$= \frac{2^{2(t/2)+1/2}}{\sqrt{2\pi}(t/2)^{1/2}e^{-\frac{(x/2)^2}{(t/2)}}}$$

10) Answer

$$Pr(x_t \equiv x) \simeq \frac{1}{(\sqrt{2\pi t})^{e^{-\frac{x^2}{2t}}}}$$

Problem 7:

1) set up problem

- displacement = j - i
- P = # positive steps
- N = # negative steps
- P + N = t
- p - N = displacement

Find....

$$\binom{t}{P} = number\ of\ random\ walks$$

2) solve linear set of equations by addition

$$P + N = t$$

$$P - N = j - i$$

$$2P = t + j - i$$

3) divide both sides by 2

$$P = \frac{t + j - i}{2}$$

4) t choose p is equal to...

$$\binom{t}{\frac{t+j-i}{2}} = number\ of\ random\ walks$$