PoCS Assignment 1

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PoCS Assignment github repo: https://github.com/alexburn17/BurnhamPoCS

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#1)

Assumptions (including the information from assumptions a, b and f):

$D_f \propto V^2 l^2$ (assumption c)

$P \propto D_f V$ (assumption d)

$N \propto l^3$ (assumption e)

$P \propto N$ (assumption g)

Operation:

1) Substitute $D_f \propto V^2 l^2$ into $P \propto D_f V$ then substitute P for l^3 using assumptions g and e:

$$l^3 \propto V^3 l^2$$

2) Divide l^3 by l^2, rearrange assumption e ($l \propto N^{1/3}$) and substitute in after division:

$$l^3/l^2 \propto (V^3 l^2)/l^2$$

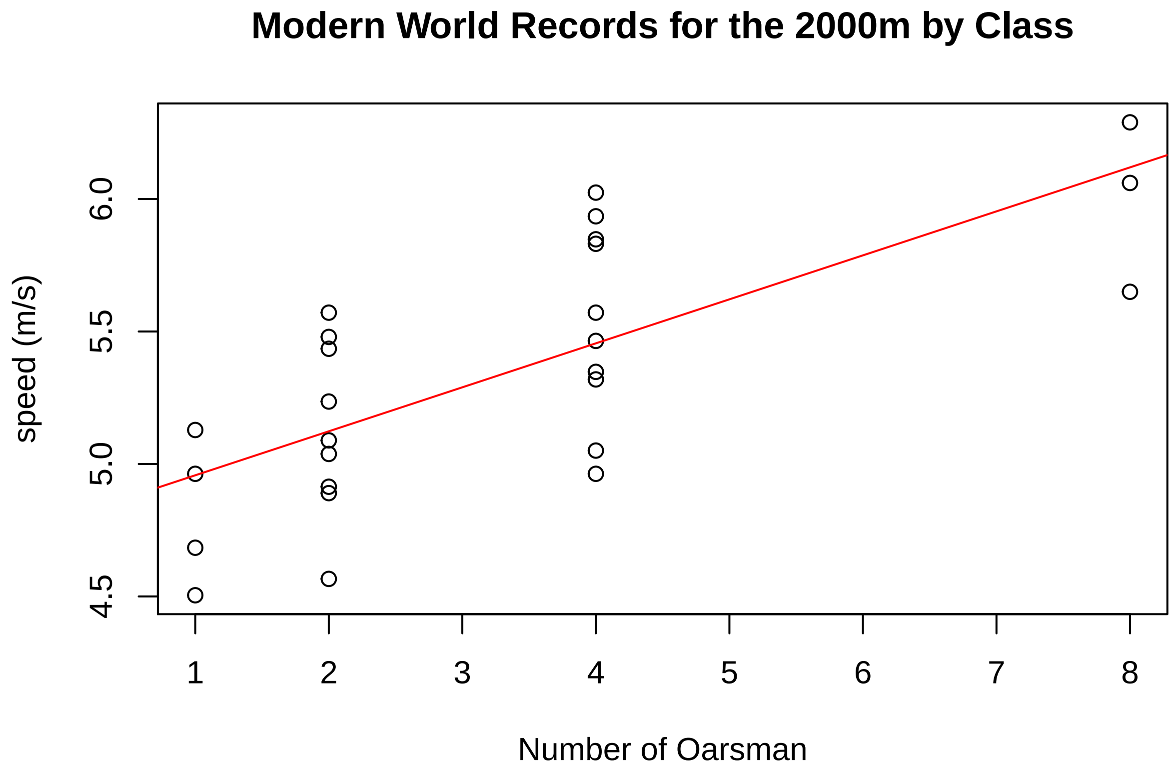
$$N^{1/3} \propto V^3$$

3) Take the cube root of both sides and simplify:

$$\sqrt[3]{N^{1/3}} \propto \sqrt[3]{(V^3}$$

$$V \propto N^{1/9}$$

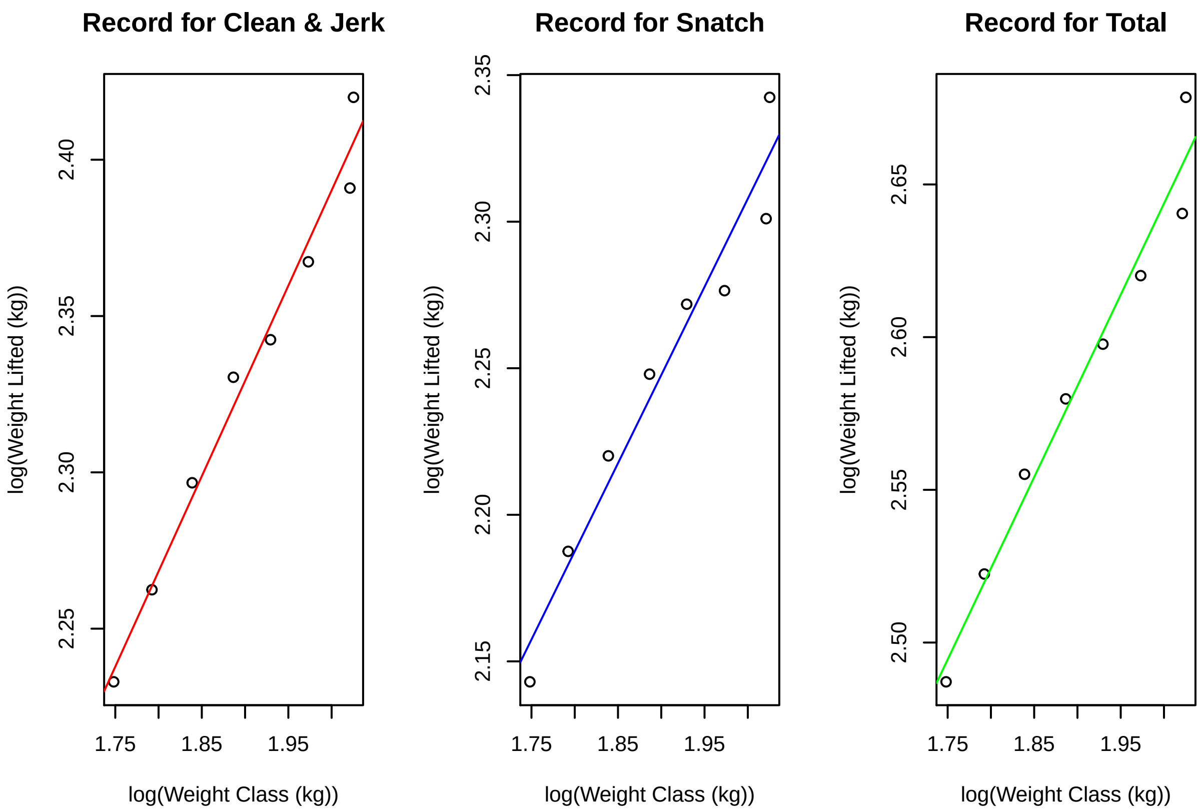
#2) 1/9 roughly equates to a slope of 0.111. After importing the current times for the 2000m (http://www.worldrowing.com/events/statistics/) and fitting a linear model as done in the McMahon and Bonner paper, we see a similar trend with a slope of 0.166. This is a slightly steeper slope but based on the coarseness of the data and the limited range of the variables in terms of orders of magnitude, they can be considered to be very similar. This slight increase, if more than a product of the limited data, might be due to technological advancements that allow for the reduction of friction in larger hulled boats. A premise (at least of sailing) is that larger boats go faster all else being equal so technological advancement might have strengthened this relationship.




```
##   (Intercept) rowing$Class
##      4.791300     0.166003
```

#3)

a) The scale does seem to match the 2/3 scaling hypothesis. The data were taken from (https://www.iwf.net/results/world-records/) body mass and weight lifted were log base 10 transformed and the slopes of the line of best fit for each regression were close to 0.666 (Clean and Jerk = 0.609, Snatch = 0.601 and Total = 0.597).



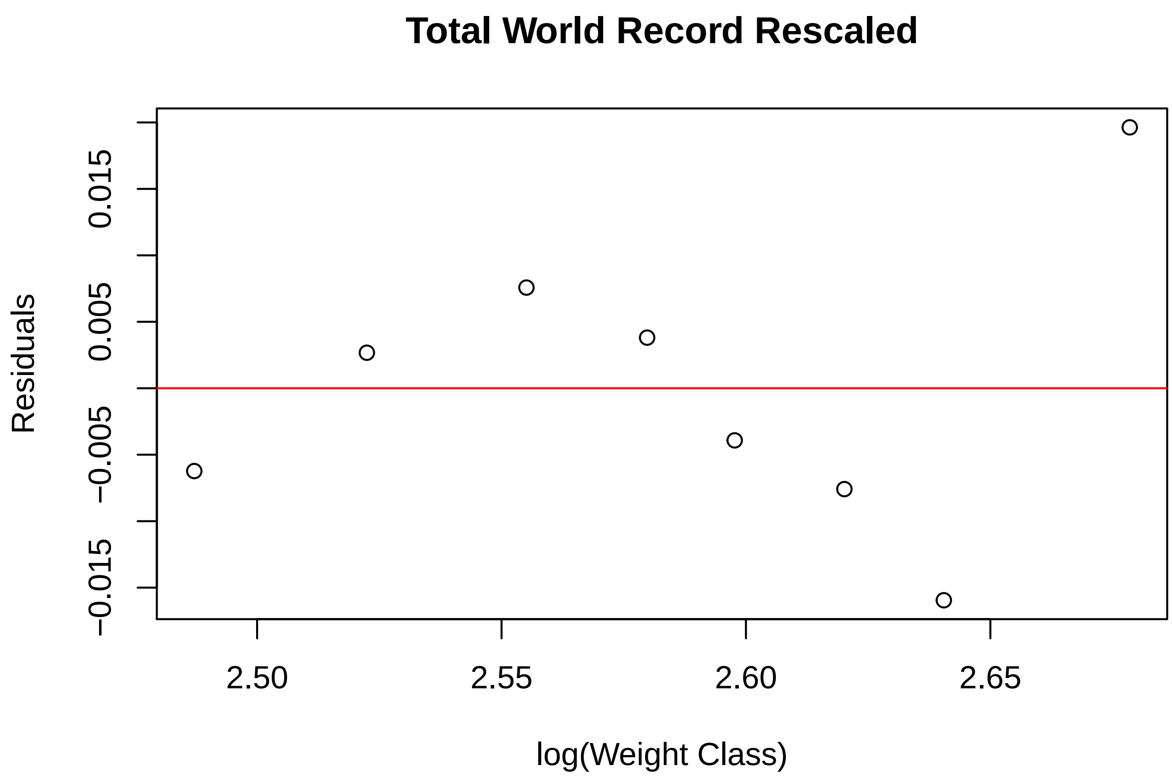
Intercepts and Coefficients for Clean and Jerk, Snatch and Total Respectivly

```
##                 (Intercept) weightssplit$cleanJerk$logMass
##                   1.1714848                      0.6093553
```

```
##              (Intercept) weightssplit$snatch$logMass
##                1.1049376                   0.6014391
```

```
##             (Intercept) weightssplit$Total$logMass
##               1.4491841                  0.5972949
```

b) Using the residuals from the Total catagory, in order to determine overall scaled world champion, we see that the lift fomr the champion in the highest weight class still holds (TALAKHADZE Lasha with 477kg). However, the second place scaled champion is in the 3rd lowest weight class (LIAO Hui with 359 kg).



#4)

Find the Dimensions:

$q_1 : [l] = L$

$q_2 : [m] = M$

$q_3 : [g] = LT^{-2}$

$q_4 : [\tau] = T$

Set up dimensions to find π_1

$\pi_1 = (L)^{x_1}(M)^{x_2}(LT^{-2})^{x_3}(T)^{x_4}$

$L : x_1 + x_3 = 0$

$M : x_2 = 0$

$T : -2x_3 + x_4 = 0$

One dimensionless parameter (4-3=1). Equation is in one space so make assumptions that $x_1 = 1$.

Solve algerbraicially after substitution:

$L : 1 + x_3 = 0$ thus $x_3 = -1$

$M : x_2 = 0$

$T : -2x_3 + x_4 = 0$ substitute -1 in for x_3 to get $x_4 = -2$

$$x_1 = 1, x_2 = 0, x_3 = -1, x_4 = -2$$

$$\pi_1 = l^1 m^0 g^{-1} \tau^{-2}$$

$$\pi_1 = l/(g\tau^2)$$

multiply both sides by g, divide by l and take the square root of both sides

$$\sqrt{\tau^2} \propto \sqrt{l/g}$$

$$\tau \propto \sqrt{l/g}$$

#5)

Find the dimensions:

$q_1 : [b] = L^2/(T^3)$

$q_2 : [\rho] = M/L^3$

$q_3 : [V] = L/T$

$q_4 : [l] = L$

$q_5 : [\sigma] = M/LT^2$

Set up dimensions to find π_1:

$\pi_1 = (L^2/(T^3))^{x_1}(M/L^3)^{x_2}(L/T)^{x_3}(L)^{x_4}(M/LT^2)^{x_5}$

$L : 2x_1 - 3x_2 + x_3 + x_4 - x_5 = 0$

$M : x_2 + x_5 = 0$

$T : -3x_1 - x_3 - 2x_5 = 0$

Two dimensionless parameters (5-3=2). Equation is in two space so make assumptions that $x_1 = 1, x_2 = 1$.

Solve for x_i algebraically:

$M : (1) + x_5 = 0$ thus $x_5 = -1$ and substitute into equation for T

$T : -3(1) - x_3 - 2(-1) = 0$ thus $x_3 = 1$ and substitue into equation for L

$L : 2(1) - 3(1) + (1) + x_4 - (-1) = 0$ thus $x_4 = -1$

Solutions:

$x_1 = 1, x_2 = 1, x_3 = 1, x_4 = -1, x_5 = -1$

Substitute in parameters and exponents:

$$\pi_1 = b^1\rho^1V^1l^{-1}\sigma^{-1}$$

$$\pi_1 = Vb\rho/(l\sigma)$$

multiply both sides by $\sigma/(b\rho)$

$$V/l \propto \sigma/(b\rho)$$

#6) Find the Dimensions:

$q_1 : [E] = ML^2/(T^2)$

$q_2 : [P] = M/L^3$

$q_3 : [R] = L$

$q_4 : [t] = T$

Set up dimensions to find π_1

$\pi_1 = (ML^2/(T^2))^{x_1}(M/L^3)^{x_2}(r)^{x_3}(T)^{x_4}$

$L : 2x_1 - 3x_2 + x_3 = 0$

$M : x_1 + x_2 = 0$

$T : -2x_1 + x_4 = 0$

One dimensionless parameter (4-3=1). Equation is in one space so make assumptions that $x_1 = 1$.

Solve algerbraicially after substitution:

$L : 2 - 3x_2 + x_3 = 0$

$M : 1 + x_2 = 0$ thus $x_2 = -1$

$T : -2 + x_4 = 0$ thus $x_4 = 2$

plug in x_2 into the equation for L in order to get all exponants:

$$x_1 = 1, x_2 = -1, x_3 = -5, x_4 = 2$$

$$\pi_1 = E^1 P^{-1} R^{-5} t^2$$

Multiply to get E to one side then Flip denominators

$$\pi_1 = E^1 t^2/(P^1 R^5)$$

$$E = pR^5/t^2$$

#7)

a)

Find the dimensions:

$q_1 : [G] = L^3/(MT^2)$

$q_2 : [M] = M$

$q_3 : [m] = M$

$q_4 : [\tau] = T$

$q_5 : [r] = L$

b)

Set up dimensions to find π_1:

$\pi_1 = (L^3/(MT^2))^{x_1}(M)^{x_2}(m)^{x_3}(T)^{x_4}(L)^{x_5}$

$L : 3x_1 + x_5 = 0$

$M : -x_1 + x_2 + x_3 = 0$

$T: -2x_1 + x_4 = 0$

Two dimensionless parameters (5-3=2). Equation is in two space so make assumptions that $x_1 = 1, x_2 = 1$.

c)

Solve for x_i algebraically to get:

$x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 2, x_5 = -3$

Substitute in parameters and exponents:

$$\pi_1 = G^1 M^1 m^0 \tau^2 r^{-3}$$

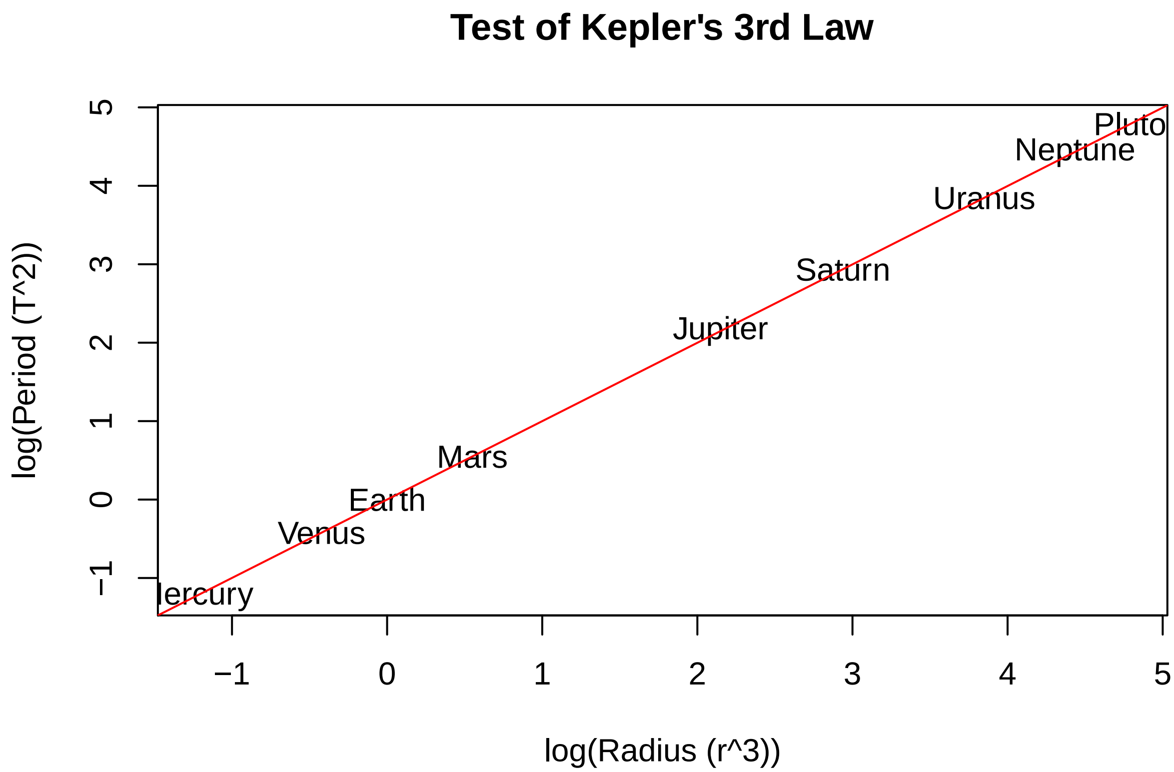
Change sides and remove constants

$$\pi_1 = GM\tau^2/(r^{-3})$$

$$r^3 \propto \tau^2$$

d)

Not supprisingly, Kepler's 3rd law holds up well. Plotting the period squared against the radius cubed creates a near perfect regression (R^2=1 and slope = 0.999). This worked for 8 planets in our solar system as well as one of the thousands of small frozen non-planets. This indicates that Kepler's work in the orbital mechanics of planets generalizes to other things in orbits.




```
##   (Intercept)    log10(r3)
## 0.0001508254 0.9993180577
```

#8)

a)

Show equations in matrix notation:

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} {c_1}^{-1} \\ {c_2}^{-1} \\ {c_3}^{-1} \end{bmatrix} \begin{bmatrix} V^{\gamma_1} \\ V^{\gamma_2} \\ V^{\gamma_3} \end{bmatrix}$$

Substitute:

$$L_1 L_2 L_3 = (c_1 c_2 c_3)^{-1} V^{\gamma_1+\gamma_2+\gamma_3}$$

Contstants and exponants are equal to 1 by definition:

$$L_1 L_2 L_3 = (1) V^1$$

$$L_1 L_2 L_3 = V$$

b)

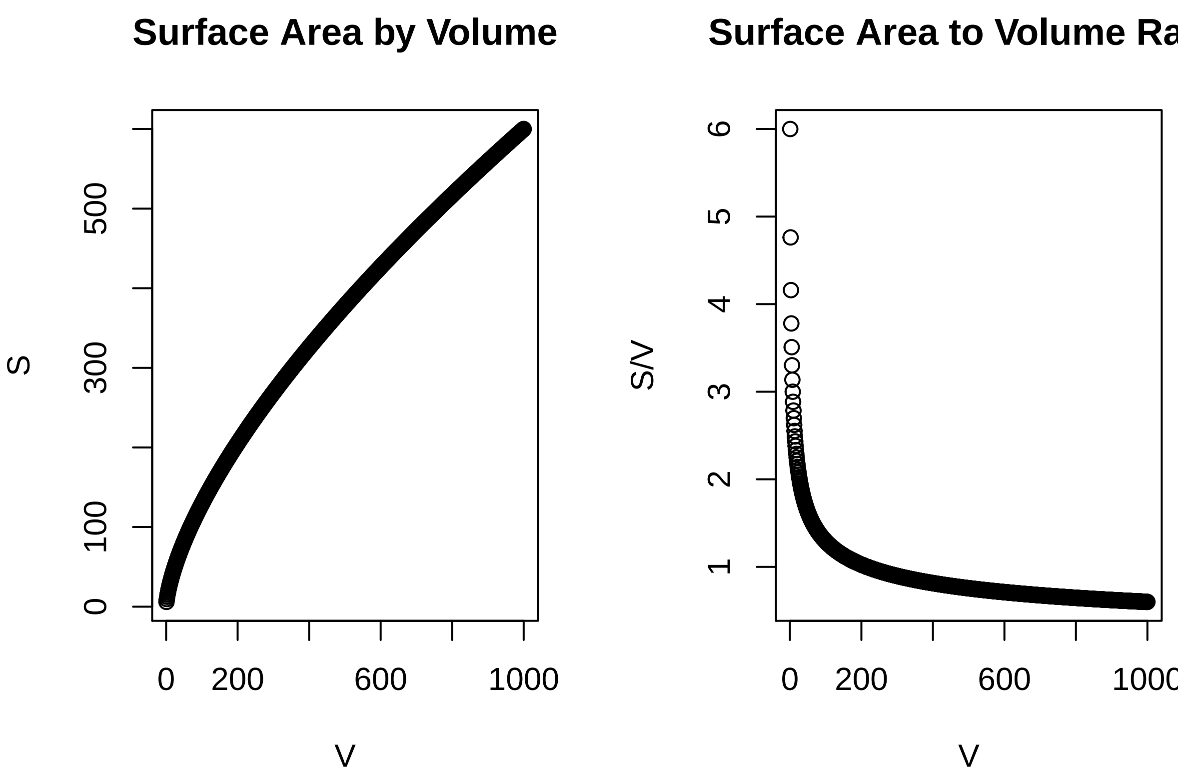
For an isometric scaling $\gamma_i = 1/3$ so all of the 3 dimensions grow at the same rate

c)

$$S = 2(L_1 L_2) + 2(L_2 L_3) + 2(L_1 L_3)$$

d)

Assume, as directed, isometric γ_i and set constants equal to one to see general relationship. We see that as volume increases, surface area increases at a decaying rate and the surface area to volume ration decreases as a negative exponential function, Making volume a dominate variable in this function.



e) The fastest set is when $\gamma_3 = 1$ and the other two are 0 as is permissable by $\gamma_1 + \gamma_2 + \gamma_3 = 1$ & $\gamma_1 \leq \gamma_2 \leq \gamma_3$ and is slowest when the smallest allowable γ_3 value is inputted, 1/3.