PoCS Assignment 5

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PoCS Assignment github repo: https://github.com/alexburn17/BurnhamPoCS

Worked with: Yu Han, Edison & Kewang

Problem 1:

a)

Step 1: sub in $1/D$:

$$S = \sum_{i=1}^{n} p_i^2$$

$$\sum_{i=1}^{n} p_i^2 = \sum_{i=1}^{D} (\frac{1}{D})_i^2$$

Step 2: Do summation and simplify:

$$S = D(\frac{1}{D})^2$$

$$S = \frac{1}{D}$$

$$D = \frac{1}{S}$$

Step 3: Sub in S:

$$D = \frac{1}{\sum_{i=1}^{n} p_i^2}$$

b)

Step 1: sub in from part a ($S = 1/D$)

$$G \equiv 1 - S = 1 - \sum_{i=1}^{n} p_i^2$$

$$G \ = 1 - \frac{1}{D}$$

$$D = \frac{1}{1 - G}$$

Step 2: sub in definition of G

$$D = \frac{1}{1-(1-S)}$$

$$D = \frac{1}{S}$$

Step 3: sub in S

$$D = \frac{1}{\sum_{i=1}^{n} p_i^2}$$

In terms of D, these two metrics are identical.

c)

Part 1: sub in $1/D$

$$H = -\sum_{i=1}^{n} p_i ln(p_i)$$

$$H = -\sum_{i=1}^{D} (\frac{1}{D}) ln(\frac{1}{D})$$

Part 2: cancel

$$H = -D(\frac{1}{D}) ln(\frac{1}{D})$$

$$H = ln(D)$$

$$D = e^H$$

Step 3: sub H back in

$$D = e^{-\sum_{i=1}^{n} p_i ln(p_i)}$$

d)

Step 1: sub in $1/D$

$$H_q^{(R)} = \frac{1}{q-1}(-ln \sum_{i=1}^{n} p_i^q)$$

$$H_q^{(R)} = \frac{1}{q-1}(-ln \sum_{i=1}^{D} (\frac{1}{D})^q)$$

$$H_q^{(R)} = \frac{1}{q-1}(-ln D(\frac{1}{D})^q)$$

Step 2: simplify

$$H_q^{(R)} = \frac{1}{q-1}(-lnD^{(1-q)})$$

$$H_q^{(R)} = \frac{1}{q-1}(lnD^{(q-1)})$$

Step 3: Multiply both sides by $q-1$ and simplify

$$H_q^{(R)}(q-1) = lnD(q-1)$$

$$H_q^{(R)} = lnD$$

$$D = e^{H_q^{(R)}}$$

Step 4: plug in $H_q^{(R)}$

$$D = e^{\frac{1}{q-1}(-ln\sum_{i=1}^{n} p_i^q)}$$

e)

Step 1: plug in $1/D$ and simplify

$$H_q^{(T)} = \frac{1}{q-1}(1-\sum_{i=1}^{n} p_i^q)$$

$$H_q^{(T)} = \frac{1}{q-1}(1-\sum_{i=1}^{D}(\frac{1}{D})^q)$$

$$H_q^{(T)} = \frac{1}{q-1}(1-D(\frac{1}{D})^q)$$

$$H_q^{(T)} = \frac{1}{q-1}(1-D^{(1-q)})$$

Step 2: rearrange and simplify

$$D = ((1-(q-1))H_q^{(T)})^{\frac{1}{(1-q)}}$$

Step 3: sub in $H_q^{(T)}$

$$D = ((2-q)(\frac{1}{q-1}(1-\sum_{i=1}^{n} p_i^q)))^{\frac{1}{(1-q)}}$$

Both of these depend on Shannon's E

f)

$$\lim_{n\to\infty}(1+\frac{H_q^{(T)}}{n})^n = e^{H_q^{(T)}}$$

$$\lim_{q\to 1}(1+\frac{H_q^{(T)}}{\frac{1}{1-q}})^{\frac{1}{1-q}} = e^{H_q^{(T)}}$$

$$D = ((1-(q-1))H_q^{(T)})^{\frac{1}{(1-q)}}$$

$$D = 1+(\frac{H_q^{(T)}}{\frac{1}{1-q}})^{\frac{1}{1-q}}$$

$$\lim_{q\to 1} = 1+(\frac{H_q^{(T)}}{\frac{1}{1-q}})^{\frac{1}{1-q}} = D$$

$$D = e^{H_q^{(T)}}$$

This matches shannon's entropy $D = e^H$

Problem 2: Step 1: Set up equation and sub in

$$\Psi = F + \lambda G$$

$$\Psi = \frac{C}{H} + \lambda G$$

$$= \frac{\sum_{i=1}^n p_i ln(j+a)}{-g\sum_{i=1}^n p_i lnp_i} + \lambda \sum_{i=1}^n p_i - 1$$

Step 2: take partial derivative

$$\frac{\partial \Psi}{\partial p_j} = \frac{ln(j+a)H + Cg(Hlnp_i))}{H^2} + \lambda = 0$$

$$\frac{\partial \Psi}{\partial p_j} = \frac{ln(j+a)(-g\sum_{i=1}^n p_i lnp_i) - (\sum_{i=1}^n p_i h(j+a)(-g(Hlnp_i))}{(-g\sum_{i=1}^n p_i lnp_i)^2} + \lambda$$

Step 3: simplify and rearrange

$$-\lambda H^2 = Hln(j+a) + Cg(Hlnp_i)$$

$$p_i = e^{\frac{-\lambda H^2 - Hln(j+a) - Cg}{Cg}}$$

$$p_i = e^{-\lambda H^2/Cg}(j+a)^{-H/Cg}$$

Step 4: Find expression connecting all variables while also showing that $e^{-\lambda H^2/Cg} = 1$

$$p_i = e^{-1-\frac{\lambda H^2}{Cg}(a+j)^{\frac{-H}{Cg}}}$$

take the natural log:

$$lnp_i = -1 - \frac{\lambda H^2}{Cg} + \frac{-H}{Cg} ln(a+j)$$

$$H = -g \sum_{i=1}^{n} p_i ln(p_i)$$

$$H = -g \sum_{i=1}^{n} p_i \left[\Big(-1 - \frac{\lambda H^2}{Cg} \Big) + \Big(\frac{-H}{Cg} \Big) ln(a+j) \right]$$

$$H = g \left[\sum_{i=1}^{n} p_i \Big(\frac{1+\lambda H^2}{Cg} \Big) + \frac{H}{C} \sum_{i=1}^{n} p_i ln(a+j) \right]$$

$\sum_{i=1}^{n} p_i$ goes to 1 and $\sum_{i=1}^{n} p_i ln(a+j)$ is C

$$H = g + \frac{\lambda H^2}{C} + H$$

$$0 = g + \frac{\lambda H^2}{C}$$

$$-g = \frac{\lambda H^2}{C}$$

Find expression connecting all variables. . . .

$$\lambda = \frac{-gC}{H^2}$$

Step 5: plug $\lambda = \frac{-gC}{H^2}$ into $p_i = e^{-1-\frac{\lambda H^2}{Cg}} (a+j)^{\frac{-H}{Cg}}$

$$p_i = e^0 (a+j)^{\frac{-H}{Cg}}$$

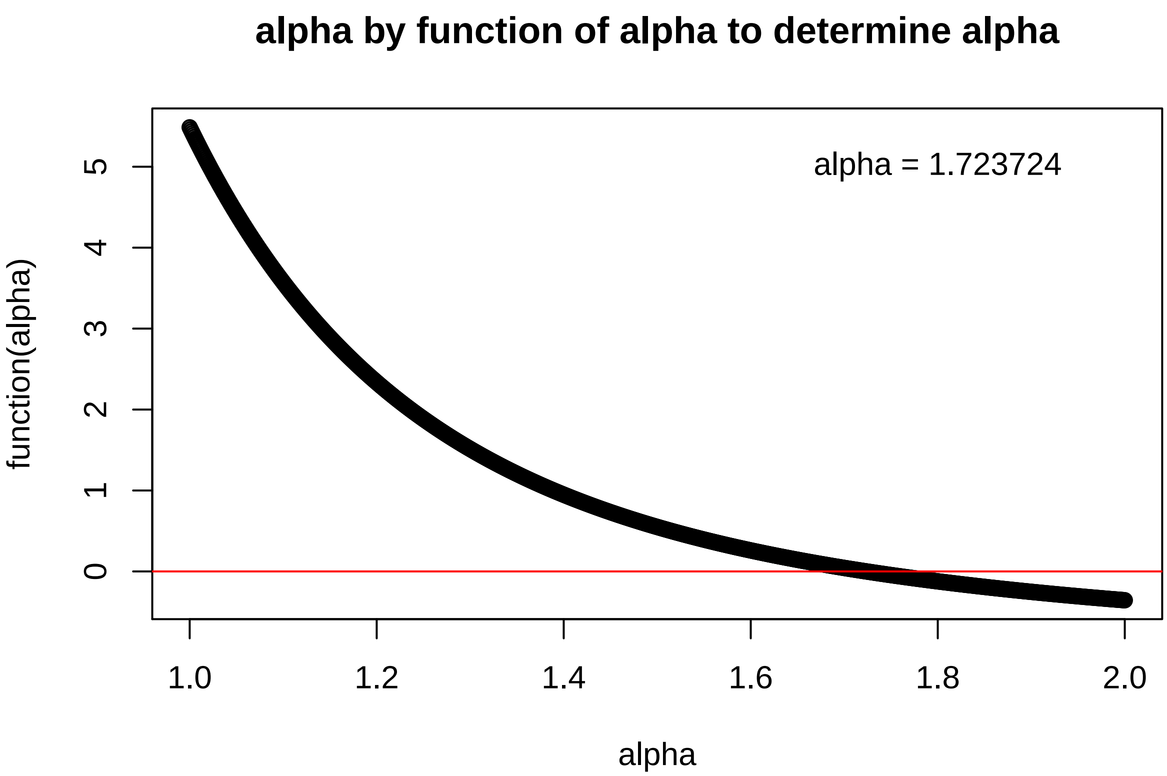
$$p_i = (a+j)^{\frac{-H}{Cg}}$$

Problem 3:

A)

Use this expression for large N for various alphas between 1 and 2. Iterate through the summation:

$$\sum_{j=1}^{N} (j+1)^{-\alpha} - 1 = 0$$



B)

$$\sum_{i=1}^{n}(j+a)^{-1}=1$$

$$\sum_{i=1}^{n}\frac{1}{i+a}=1$$

approximation:

$$\int_{1}^{n}\frac{1}{x+a}dx=1$$

antiderivative:

$$ln(a+x)|_{1}^{n}=1$$

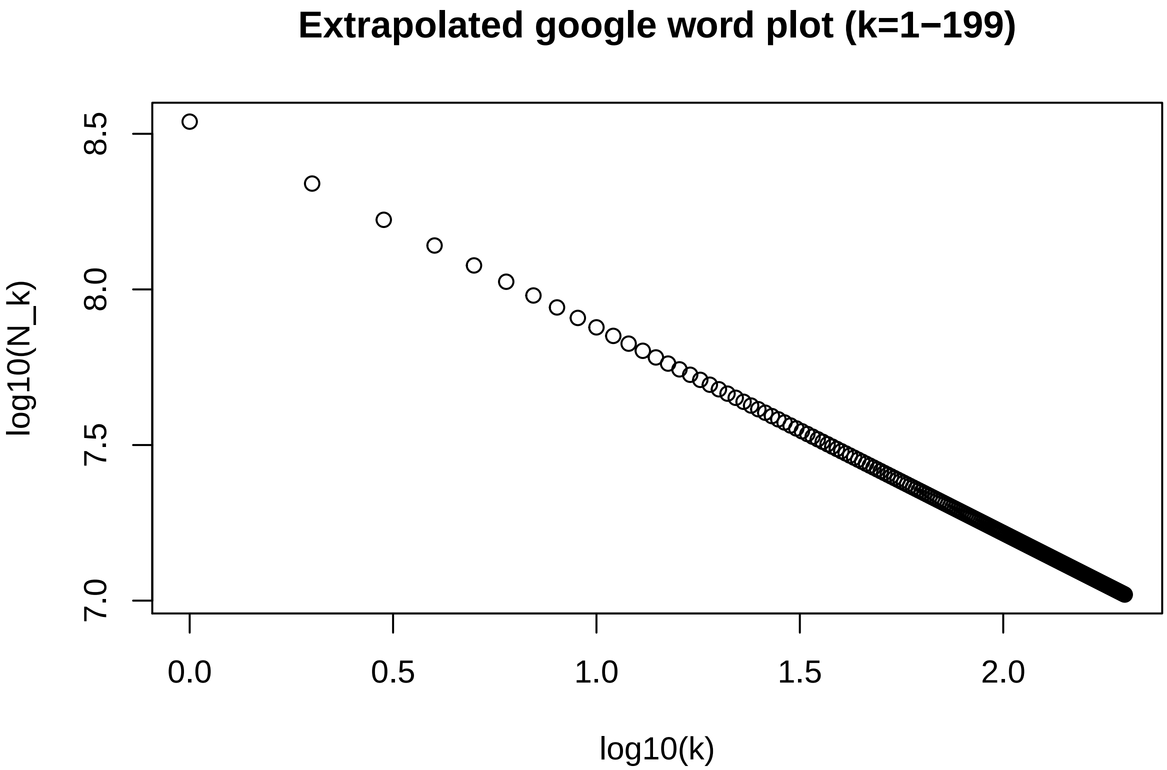
$$ln(a+n)-ln(a+1)=1$$

$$\frac{a+n}{a+1}=e$$

$$a=\frac{e-n}{1-e}$$

as n goes to infinity, a goes to positive infinity

Problem 4 (A, B):

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## [1] "The mean is: 68815.4185654997"
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## [1] "The variance is: 946512455151728"
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C)

$$i = \frac{N_1}{\sum k * N_k(total)}$$

$$ii = \frac{\sum N_k(extrapolated)}{\sum N_k(total)}$$

$$iii = \frac{\sum N_k * k(extrapolated)}{\sum k * N_k(total)}$$

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## [1] "The answer to C part i is: 5.57362869906071e-09"
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## [1] "The answer to C part ii is: 0.00568475335870248"
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## [1] "The answer to C part iii is: 4.99859348109373e-06"
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