Homework 7 - PoCS

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Worked with: Yu, Edison and Kewang

Question 1:

A)

Step 1: frame problem

$p \geq (x) = \int_x^\infty p(x)dx = cx^{-q}$

$P \geq (A) = p \geq (p^{-1}(a^{-\gamma}))$

Step 2: solve for x and simplify

$P(x) = cx^{-(q+1)}$

$x = (\frac{p(x)}{c})^{(\frac{1}{-(q+1)})}$

$(\frac{x}{c})^{(\frac{1}{-q+1})} = p^{-1}q$

Step 3: Sub in $A^{-\gamma(1-1/q)}$ and simplify

$c((\frac{A^{-\gamma}}{c})^{(\frac{-1}{q+1})})^{-q} = x$

$= c\left(\frac{A^{-\gamma(\frac{q}{q+1})}}{c^{(\frac{q}{q+1})}}\right)$

$= c^{1-(\frac{q}{q+1})}A^{-\gamma(\frac{q}{q+1})}$

$= c^{(\frac{1}{q+1})}A^{-\gamma(\frac{q}{q+1})}$

B)

Step 1: solve for x and simplify

$P(x) = ce^{-x}$

$\frac{x}{c} = e^{-p^{-1}}(x)$

$ln(\frac{x}{c}) = -p^{-1}(x)$

Step 2: Find $P \geq (A)$

$P \geq (A) = ce^{-(-ln\frac{A^{-\gamma}}{c})}$

$= ce^{ln\frac{q^{-\gamma}}{c}}$

$= c\frac{A^{\gamma}}{c}$

$= A^{\gamma}$ plug into above for x

Step 3: plug in

$p^{-1}(A^{-\gamma}) = -ln(\frac{A^{-\gamma}}{c})$

C)

Step 1: sub $A^{-\gamma}[ln(A)]^{-1/2}$ for x below and simplify

$P \geq (x) = cx^{-1}e^{-x^2}$

$P \geq (A) = c(\sqrt{\frac{-lnA^{-\gamma}}{c}})^{-1}e^{-(\sqrt{\frac{-lnA^{-\gamma}}{c}})^2}$

$= \frac{c}{\sqrt{\frac{-lnA^{-\gamma}}{c}}}e^{-(-ln\frac{A^{-\gamma}}{c})}$

$= \frac{c}{\sqrt{\frac{-lnA^{-\gamma}}{c}}}\frac{A^{-\gamma}}{c}$

$= A^{-\gamma}(-ln(\frac{A^{-\gamma}}{c}))^{-1/2}$

Question 2:

Step 1: set up equation

$L = \sum p_i a_i + \lambda(\sum a_i\frac{d-1}{d}a_i^{-1} - C)$

Step 2: Take partial derivative with respect to a_i and set equal to 0

$\frac{\partial L}{\partial a_i} = p_i + \lambda(\frac{d-1}{d}a_i^{\frac{d-1}{d}-1}a_i^{-1} + a_i^{\frac{d-1}{d}}(-1)a_i^{-2}) = 0$

Step 2: Factor out $a_i^{\frac{d-1}{d}}$ and a_i^{-2}

$p_i = -\lambda a_i^{\frac{d-1}{d}}(\frac{d-1}{d}a_i^{-2} - a_i^{-2})$

$= -\lambda a_i\frac{d-1}{d}^{-2}(\frac{d-1}{d} - 1)$

Step 3: cancel constants and set proportional

$= \frac{\lambda}{d}a_i^{\frac{-d-1}{d}}$

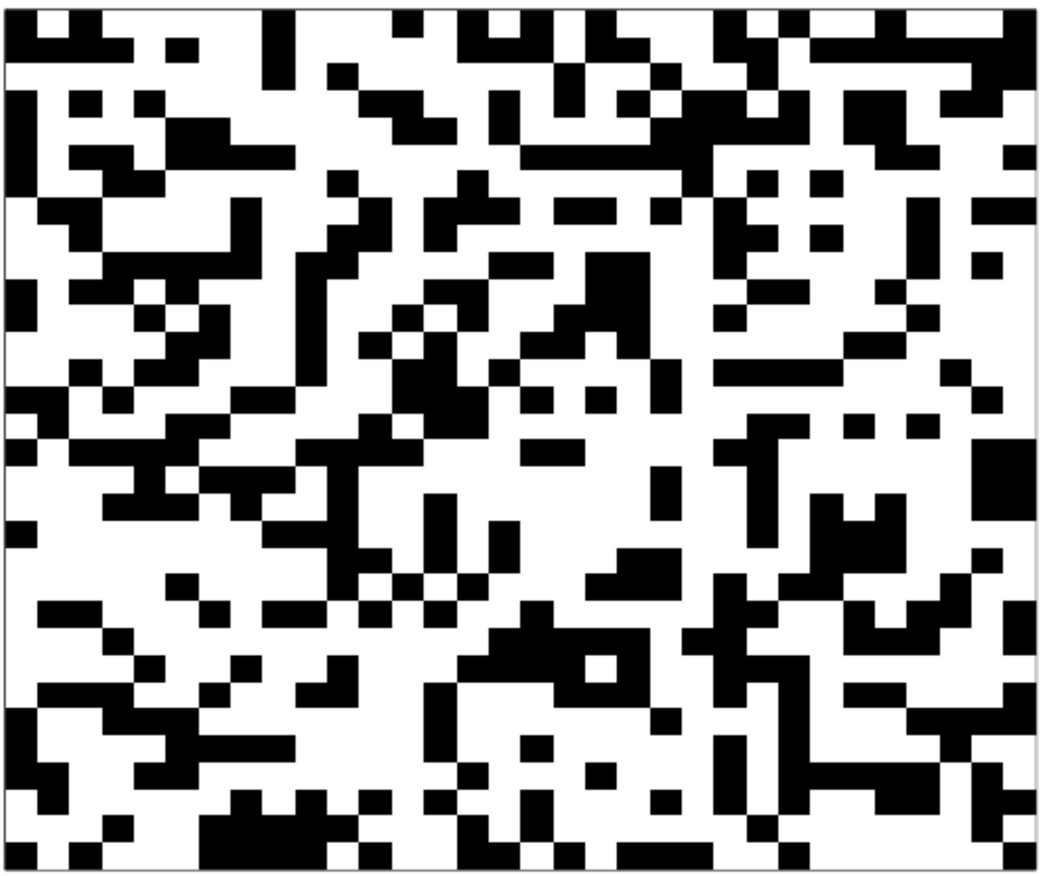
$= \frac{\lambda}{d} a_i^{-1-\frac{1}{d}}$

$p_i \propto a_i^{-1-\frac{1}{d}}$

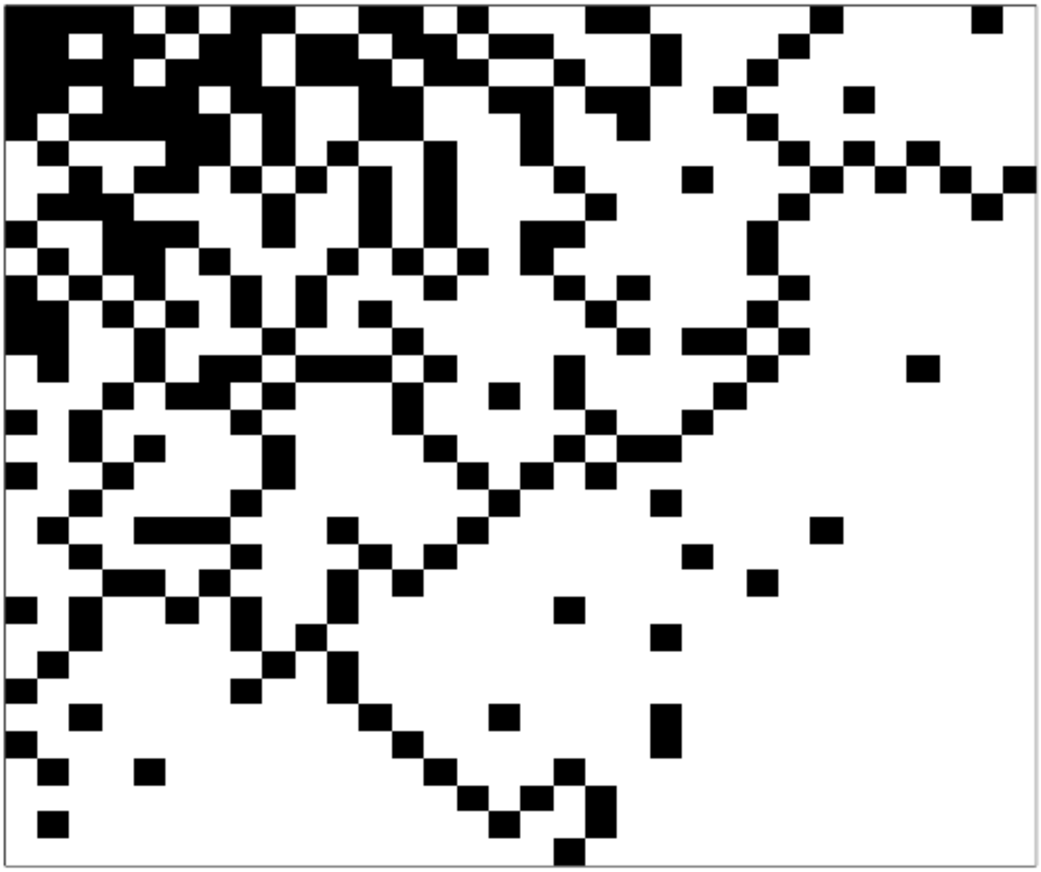
Question 3: Completed with L=32, upgrading to a faster computer soon.

A)

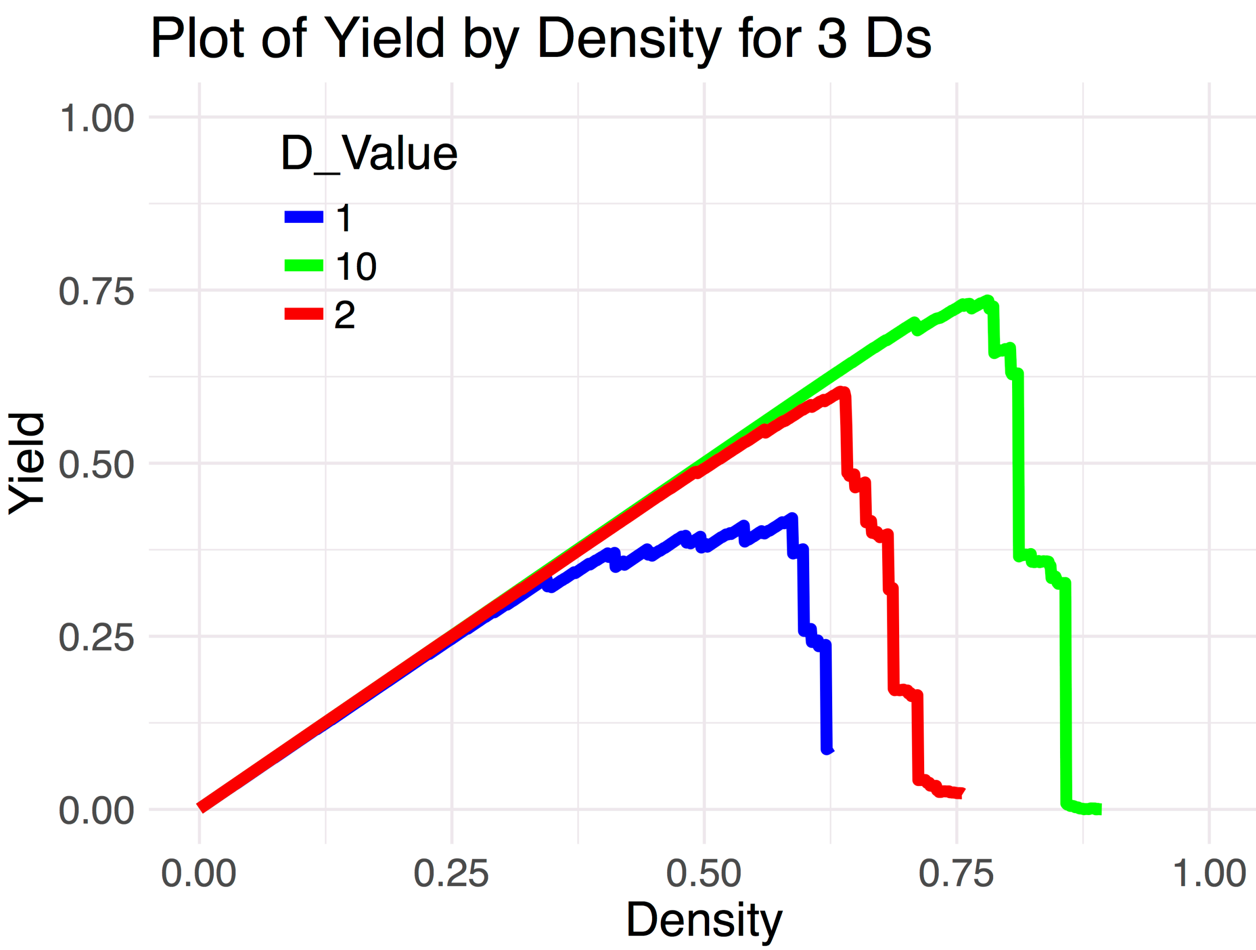
D=1



D=10:

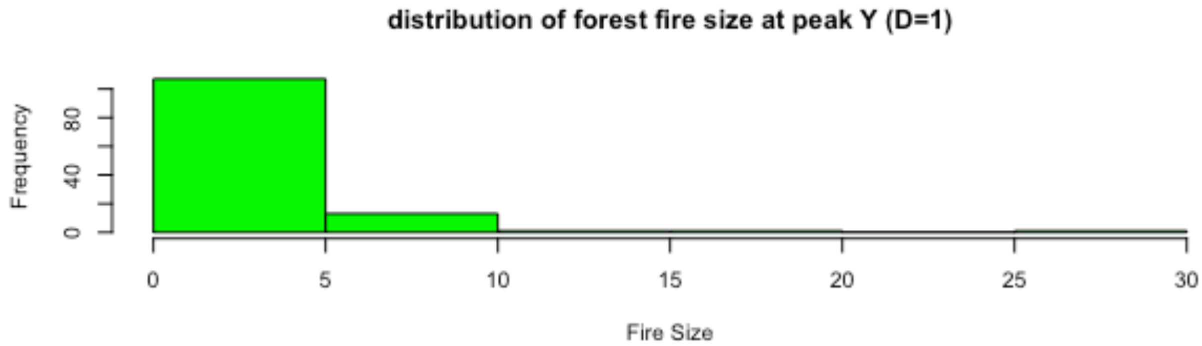


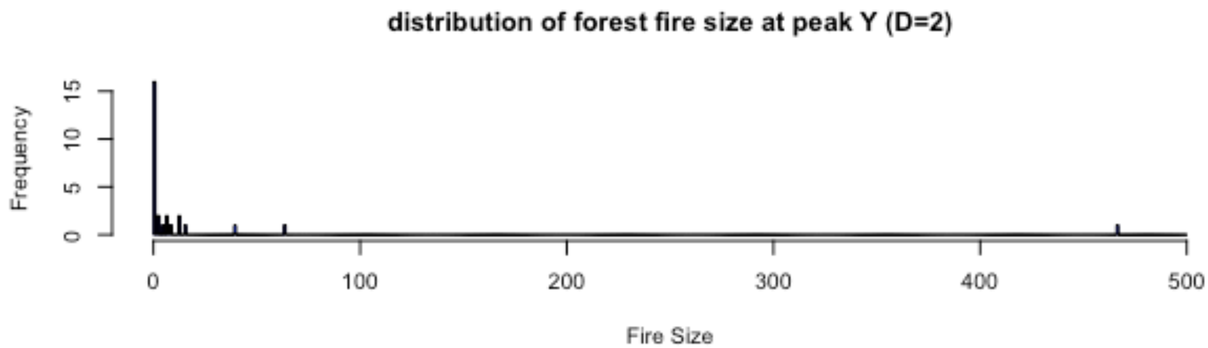
B)

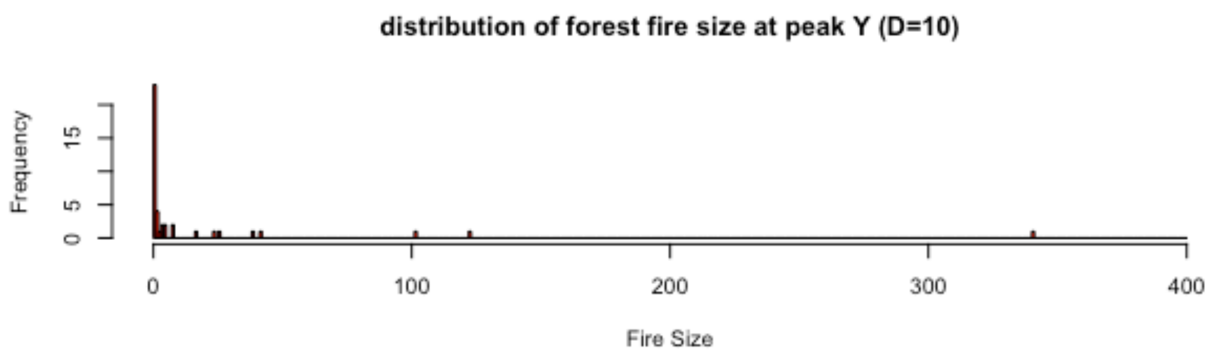


- When $D = 1$, $p_{max} = 0.58$ and $Y_{max} = 0.39$
- When $D = 2$, $p_{max} = 0.625$ and $Y_{max} = 0.625$
- When $D = 10$, $p_{max} = 0.81$ and $Y_{max} = 0.75$

C) Power Law distributions at peak yield







D) Did not attempt as it is optional and I have my comprehensive exam and a NSF proposal decline coming up in 10 days! Sounded interesting though.