AUTOMATIC DIFFERENTIATION

FOR MACHINE LEARNING



ALEX WILTSCHKO RESEARCH ENGINEER TWITTER ADVANCED TECHNOLOGY GROUP

@ AWILTSCH



MATERIAL DEVELOPED WITH HUGO LAROCHELLE RYAN ADAMS



FIRST A BIT ABOUT

NEURAL NETWORKS IN NLP

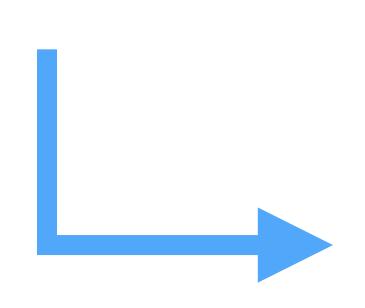
THEN MORE ABOUT

THE POWERFUL ABSTRACTION
THAT POWERS THEM



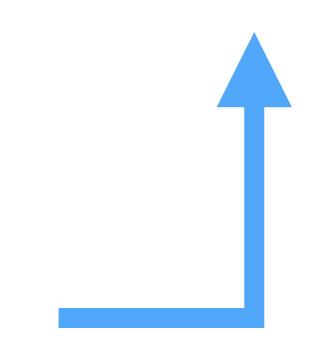
TURNING WORDS INTO NUMBERS

"This was a very good movie"

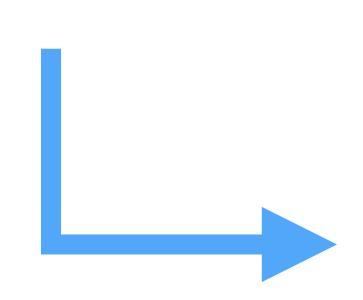


```
array([[-0.52357694, 1.12979217, 1.26719889, ..., 0.73388794, -1.16502713, -1.23081258],
[ 0.90611596, 1.84870625, -0.01891658, ..., -1.2501873, 0.87360637, 1.74786707],
[ 0.17103426, -0.50897518, 0.12341446, ..., -0.02443121, 1.3935029, -0.68816075],
...,
[-0.9147852, 0.80959717, -1.2149677, ..., -0.40054953, -1.12964909, -1.43887948],
[-0.89976191, -1.75732653, 1.29460437, ..., -1.61950746, 0.40906495, 0.36949282],
[-0.37811107, -0.23104273, -1.00433316, ..., 0.60603038, 2.04055542, 0.26861255]])
```



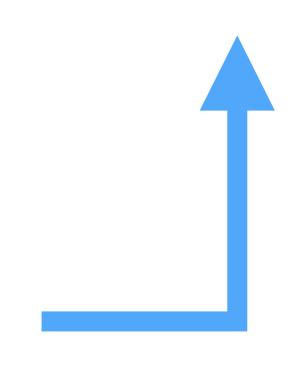


"What a disaster of a movie"



```
1.48081682e-01, ...,
array([[ 1.43408359e+00, -1.14951576e+00,
                                             1.24139221e+00],
                          2.44072829e+00,
         3.15954797e-01,
      [ 4.47487023e-01,
                          3.39139585e+00,
                                            -7.68169864e-02, ...,
        -2.33335039e+00, -1.09712552e+00,
                                             1.30592139e+00],
                                             4.62356615e-01, ...,
      [ -2.27218218e-01,
                          1.15852345e+00,
                           7.53330730e-01,
                                             8.17434705e-01],
         9.38343095e-01,
      [ -7.31496776e-01, -1.53018132e+00,
                                             3.87623099e-01, ...,
                                             1.13800609e+007
                          2.95869429e-01
      [ 1.18416890e+00, -1.59414075e-01,
                                             1.10052558e+00, ....
         -3.26311286e-01, -6.44709655e-01, -2.51885786e-03],
      [ -1.87242824e+00, 1.13216335e-01, -2.48464419e-01, ...,
         -1.09979159e+00, -1.92920004e+00, -2.88726652e-01]])
```



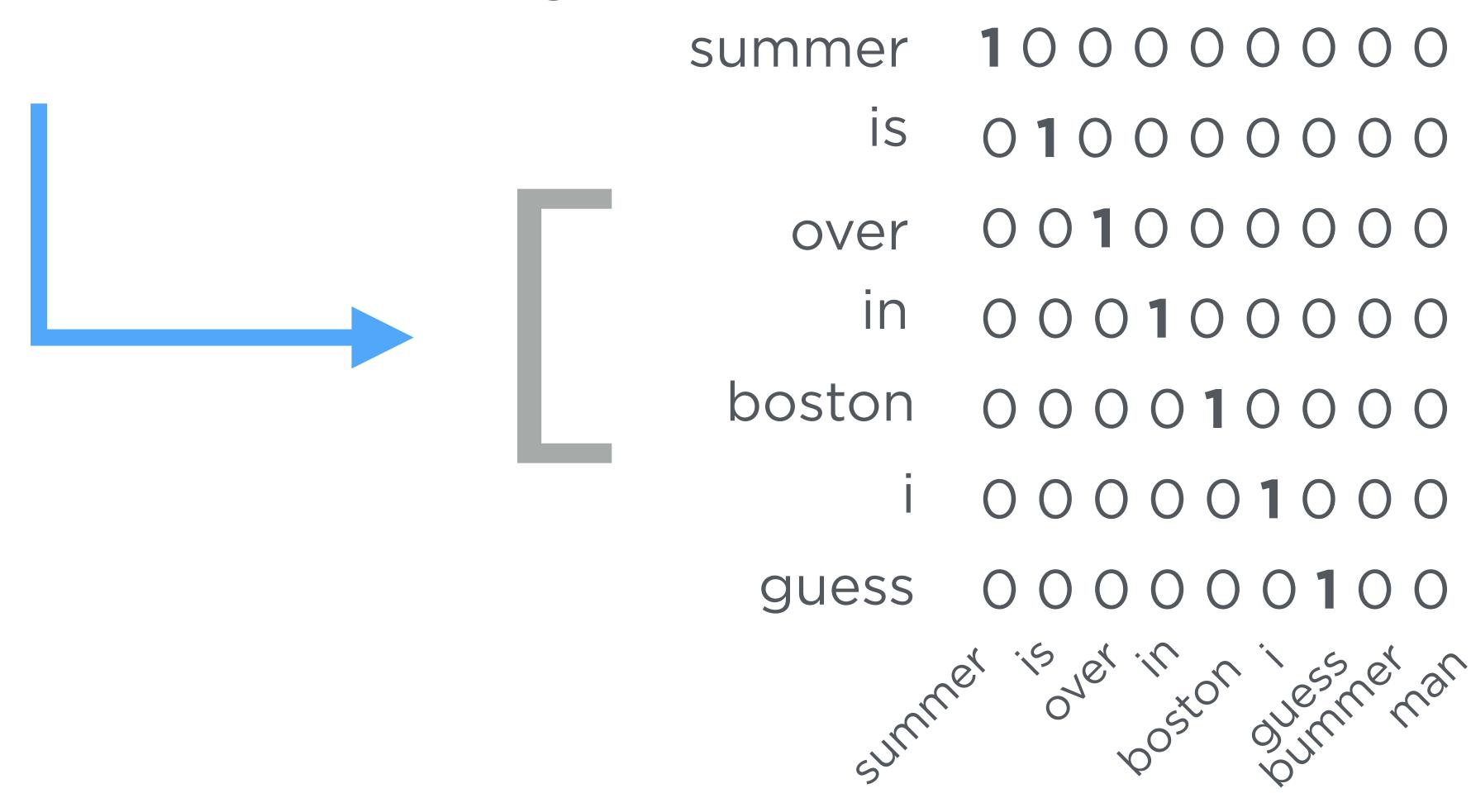




WORDS AS BIT-VECTORS

One-hot word representations are a natural first representation to try

Summer is over in Boston, I guess

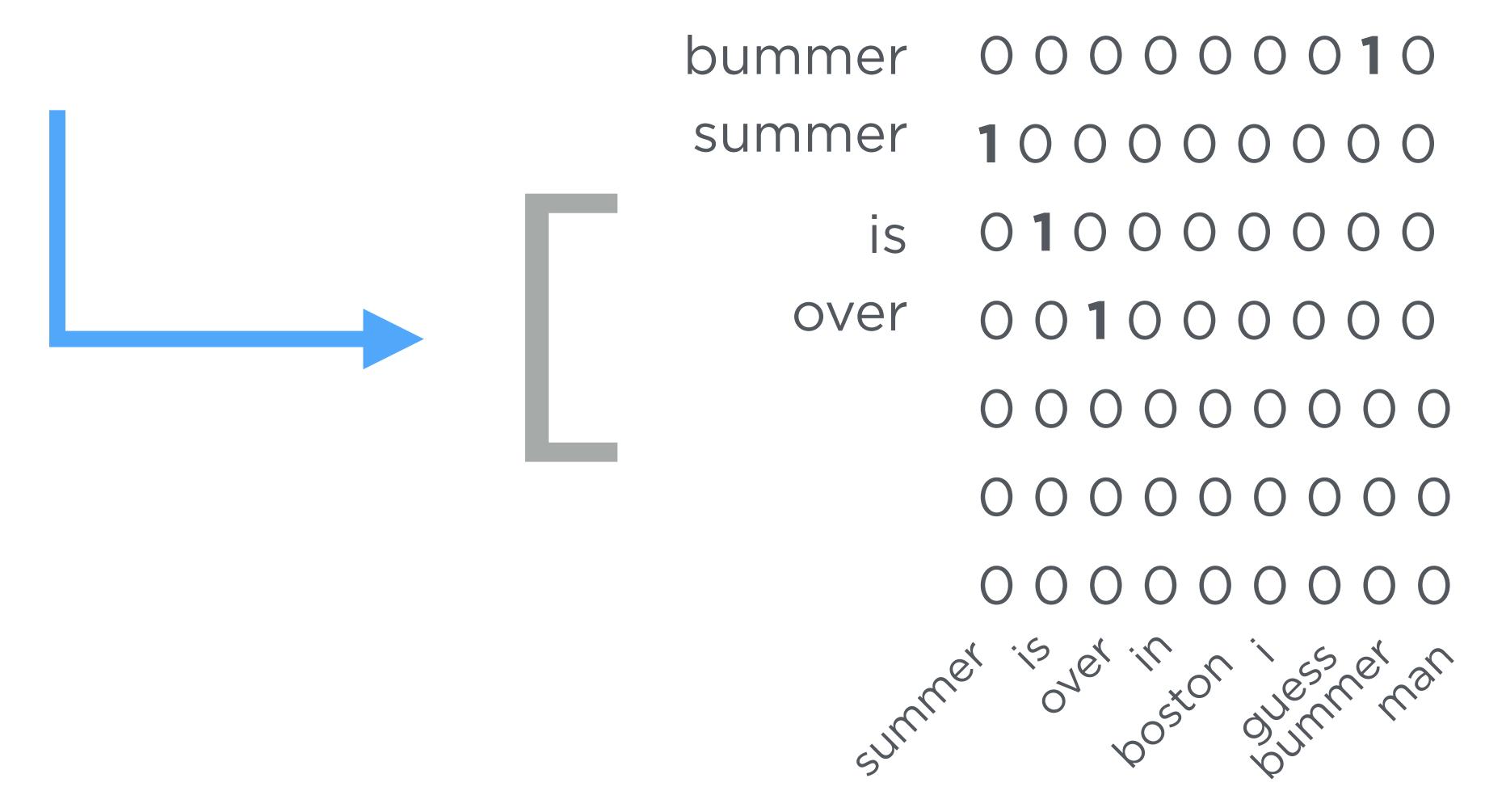




WORDS AS BIT-VECTORS

One-hot word representations are a natural first representation to try

bummer, summer is over





WORD EMBEDDINGS

But!

bummer - summer = over - summer = 2

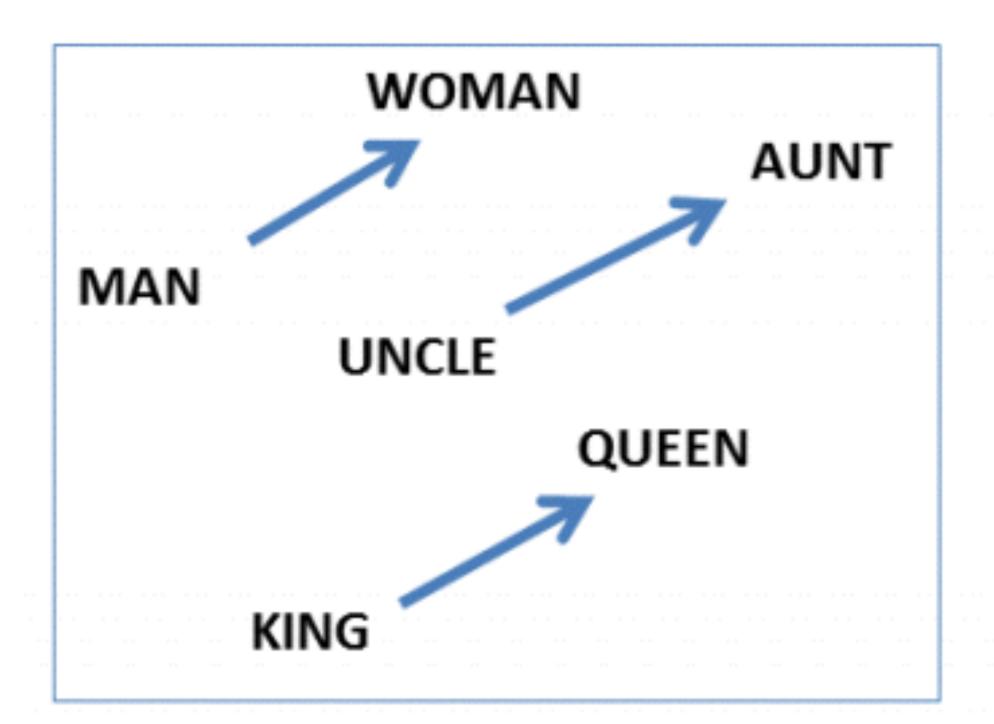
That makes zero sense!

Summer and bummer are not similar to each other.

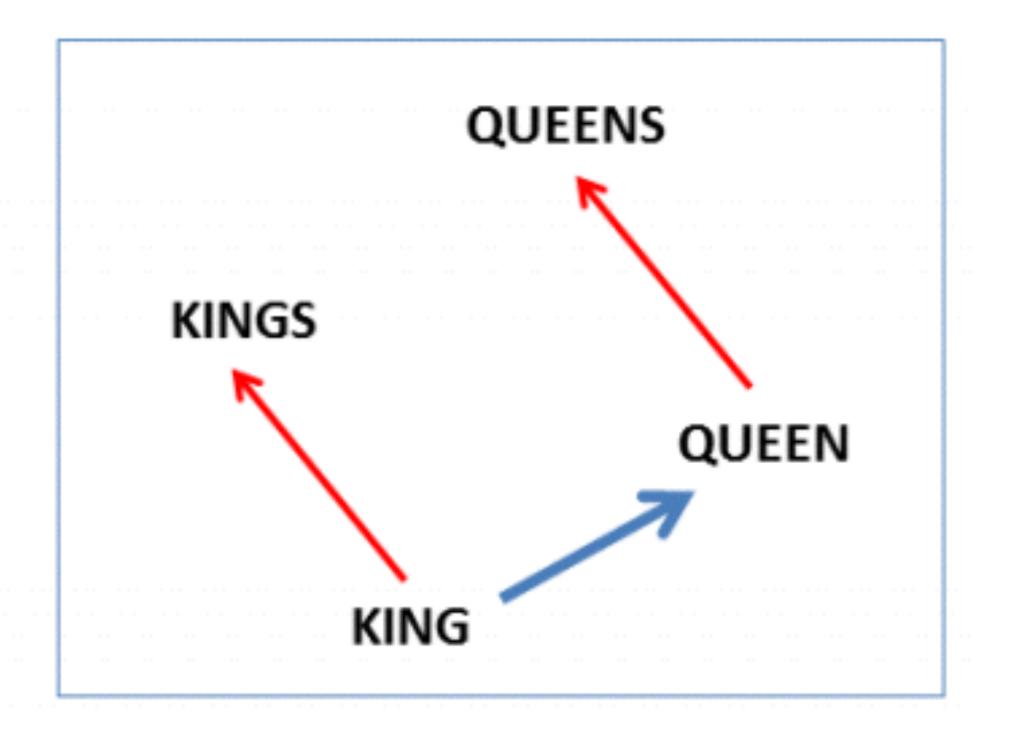


WORD EMBEDDINGS — WORD2VEC

king - queen = woman - man



kings - king + queen = queens

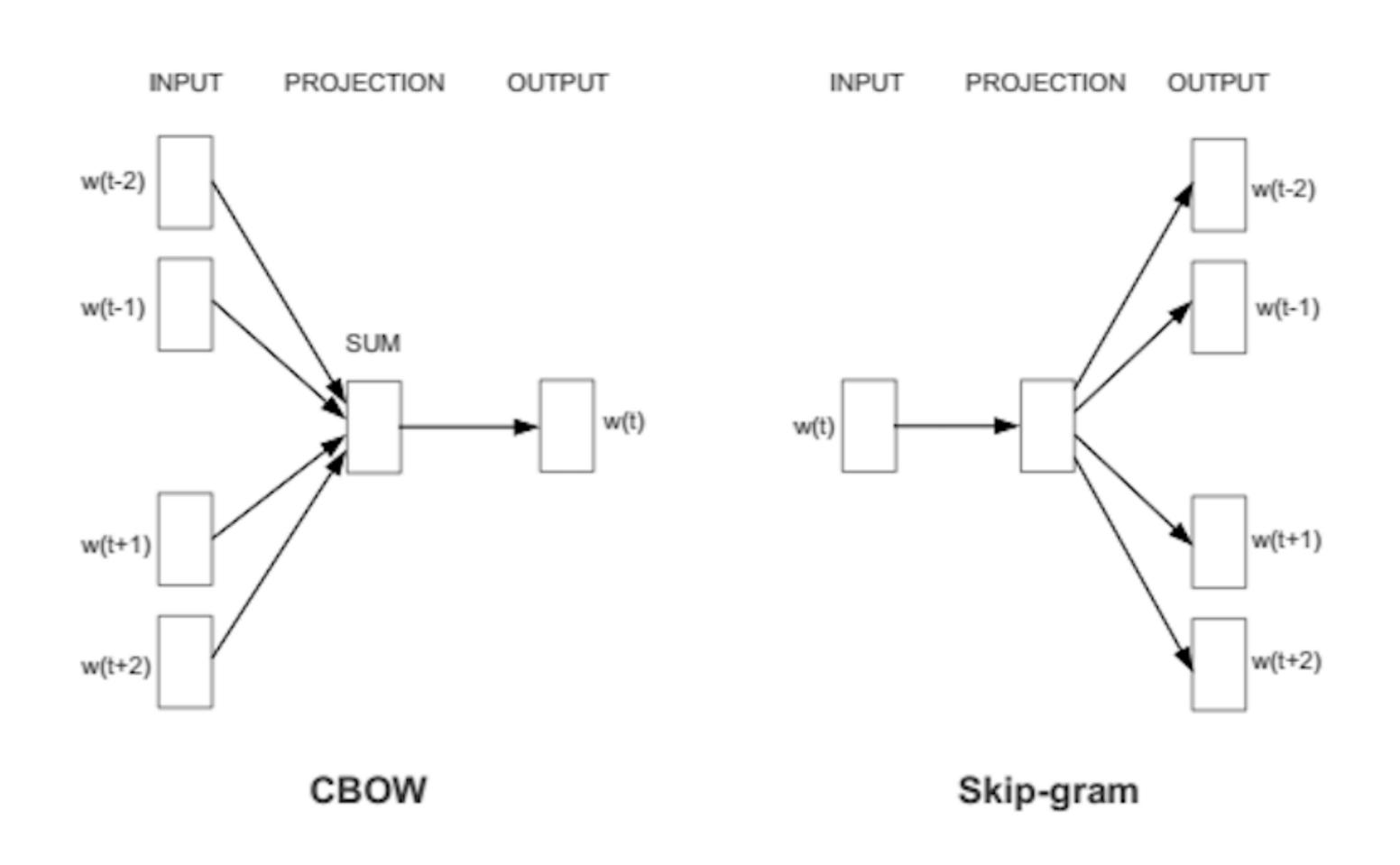


(Mikolov et al., NAACL HLT, 2013)



LEARNING WORD EMBEDDINGS - WORD2VEC

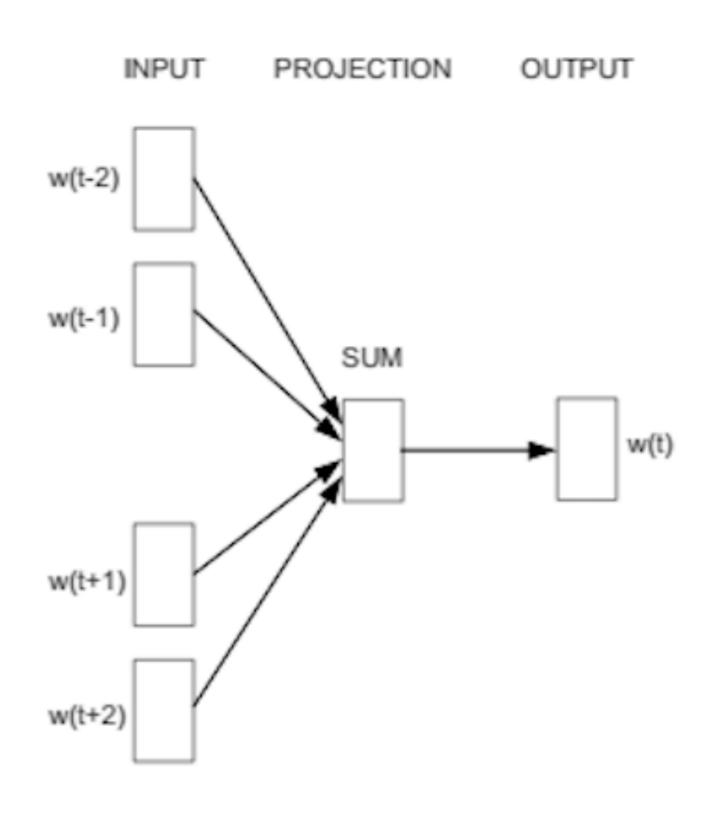
"You shall know a word by the company it keeps" (J. R. Firth, 1957: 11)





LEARNING WORD EMBEDDINGS - CBOW

"You shall know a word by the company it keeps" (J. R. Firth, 1957: 11)

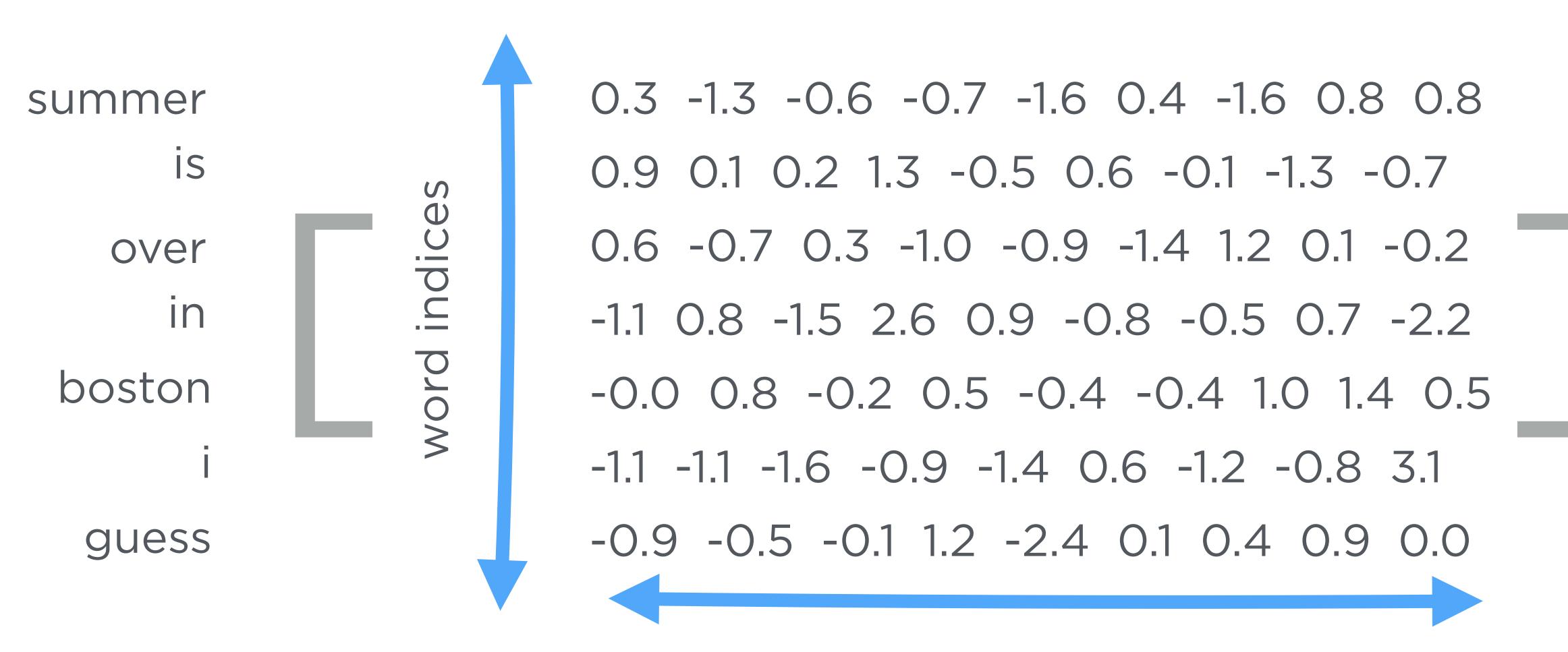


CBOW



WORDS AS BIT-VECTORS

One-hot word representations are a natural first representation to try









LEARNING WORD EMBEDDINGS — CBOW

We want to make this difference small (while having w's not be zero).

$$|| w[i] - (w[i-2] + w[i-1] + w[i+1] + w[i+2]) ||$$



LEARNING WORD EMBEDDINGS — CBOW

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$$|| w[i] - (w[i-2] + w[i-1] + w[i+1] + w[i+2])||$$

How? (also PS, that's not the real CBOW loss, sorry!)

CBOW is like a tiny neural network!

All neural networks are trained using something called

"Backpropagation"

We need derivatives!

Backprop is the chain rule of calculus for computing derivatives.

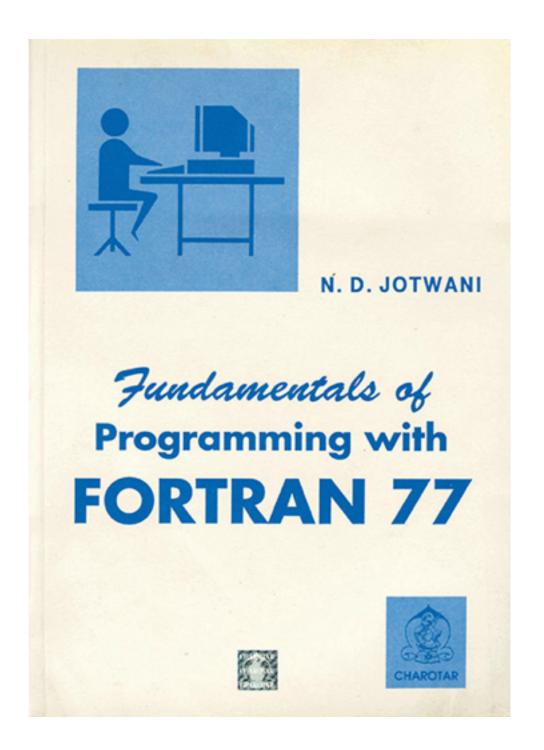
How does the error change if we change our vectors?



WE WORK ON TOP OF STABLE ABSTRACTIONS

We should take these for granted, to stay sane!

Arrays



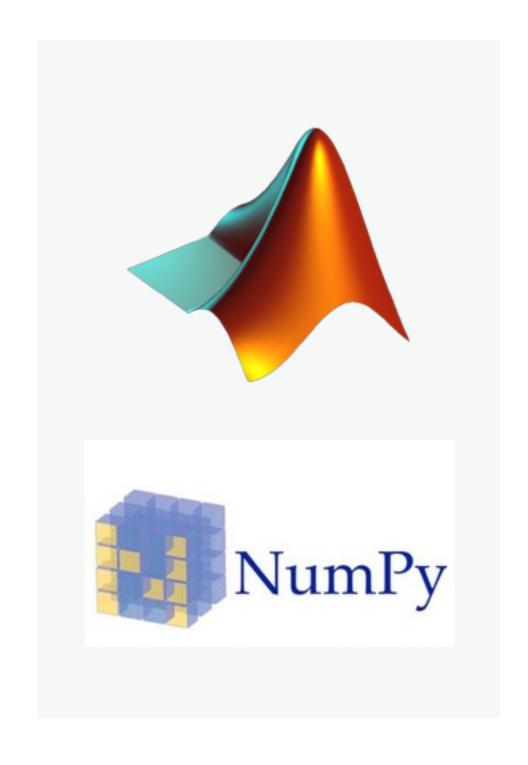
Est: 1957

Linear Algebra



Est: 1979 (now on GitHub!)

Common Subroutines



Est: 1984



MACHINE LEARNING HAS OTHER ABSTRACTIONS

These assume all the other lower-level abstractions in scientific computing

All gradient-based optimization (that includes neural nets) relies on **Automatic Differentiation (AD)**

"Mechanically calculates derivatives as functions expressed as computer programs, at machine precision, and with complexity guarantees." (Barak Pearlmutter).

Not finite differences -- generally bad numeric stability. We still use it as "gradcheck" though.

Not symbolic differentiation -- no complexity guarantee. Symbolic derivatives of heavily nested functions (e.g. all neural nets) can quickly blow up in expression size.



AUTOMATIC DIFFERENTIATION IS THE ABSTRACTION FOR GRADIENT-BASED ML

All gradient-based optimization (that includes neural nets) relies on **Automatic Differentiation** (AD)

- Rediscovered several times (Widrow and Lehr, 1990)
- Described and implemented for FORTRAN by Speelpenning in 1980 (although forward-mode variant that is less useful for ML described in 1964 by Wengert).
- Popularized in connectionist ML as "backpropagation" (Rumelhart et al, 1986)
- In use in nuclear science, computational fluid dynamics and atmospheric sciences (in fact, their AD tools are more sophisticated than ours!)

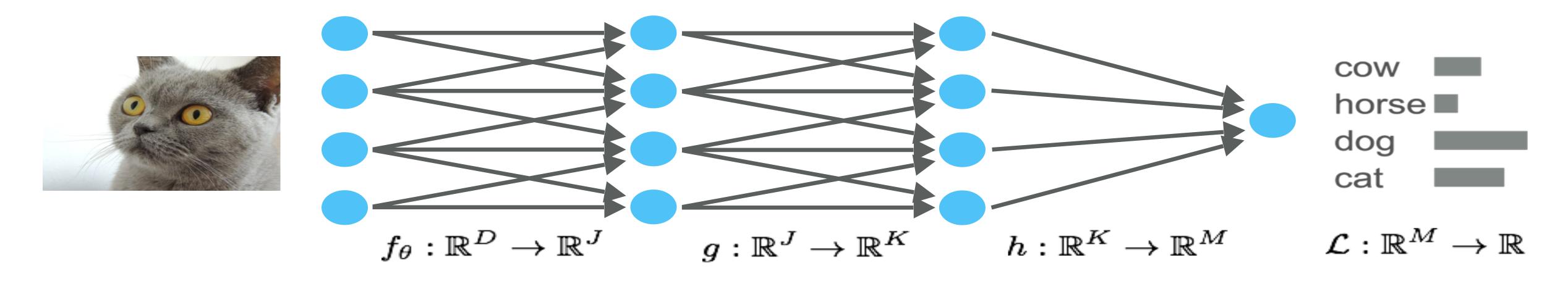


AUTOMATIC DIFFERENTIATION IS THE ABSTRACTION FOR GRADIENT-BASED ML

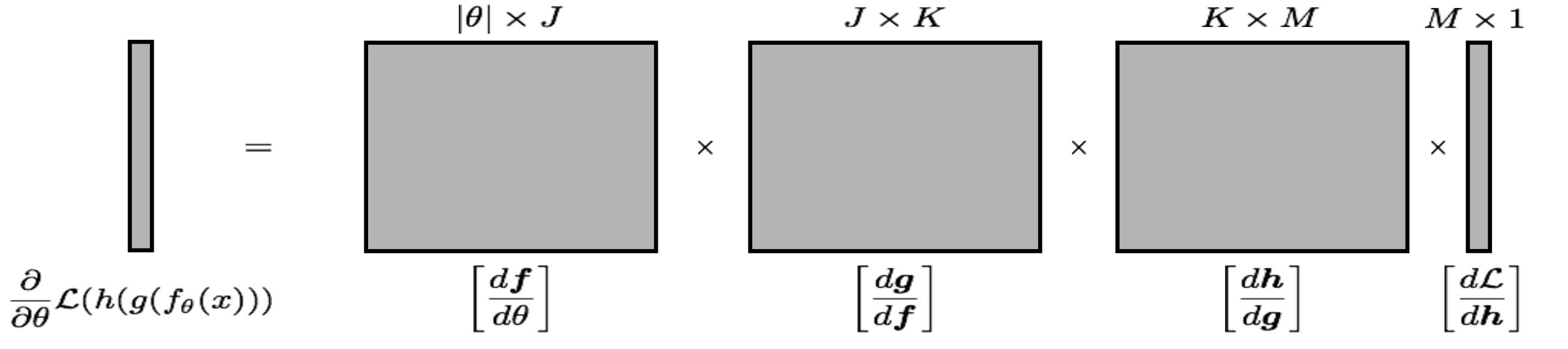
All gradient-based optimization (that includes neural nets) relies on Reverse-Mode Automatic Differentiation (AD)

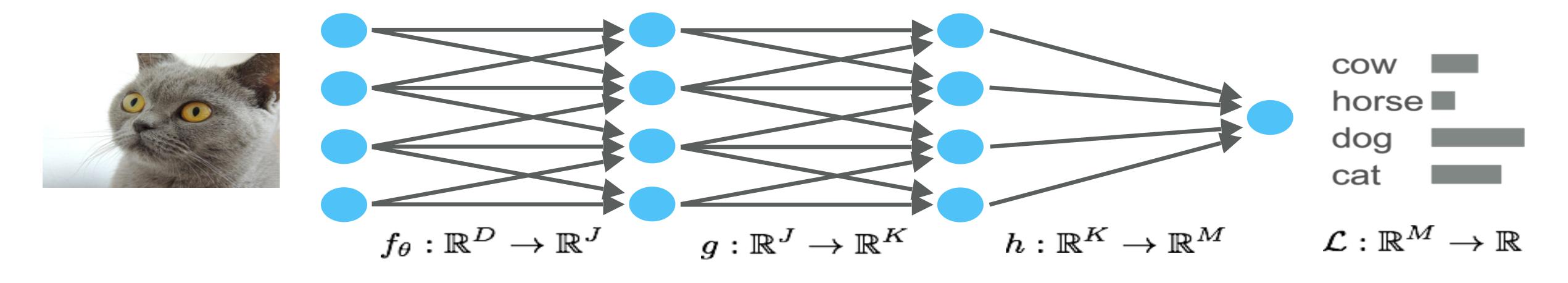
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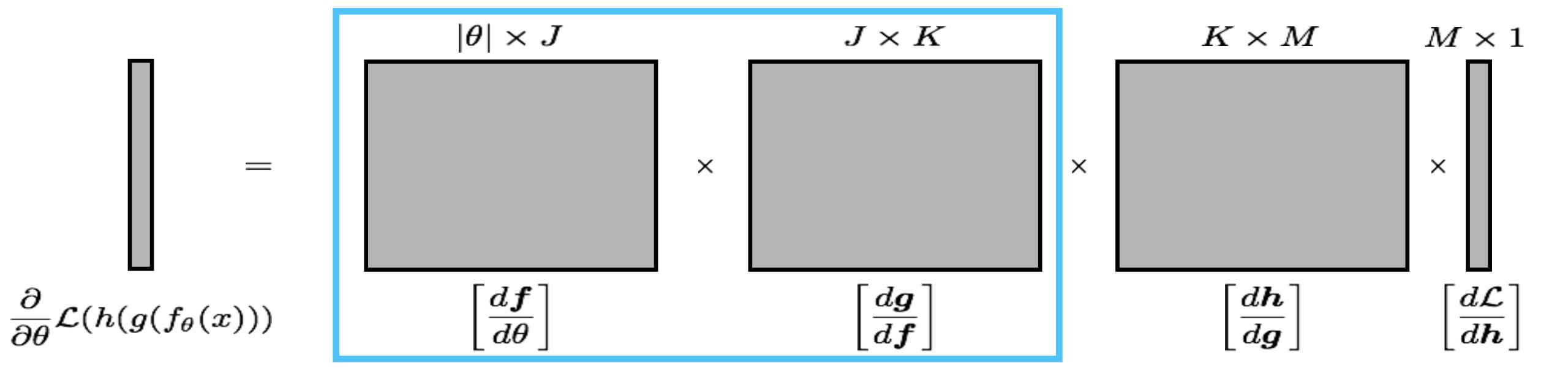


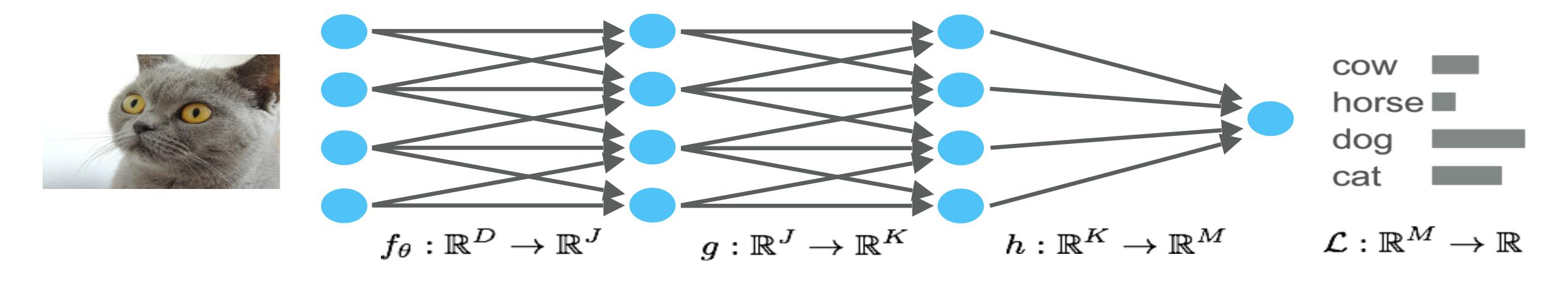
$$\frac{\partial}{\partial \theta} \mathcal{L}(h(g(f_{\theta}(x)))) = \left[\frac{d\mathbf{f}}{d\theta}\right] \left[\frac{d\mathbf{g}}{d\mathbf{f}}\right] \left[\frac{d\mathbf{h}}{d\mathbf{g}}\right] \left[\frac{d\mathbf{L}}{d\mathbf{h}}\right]$$



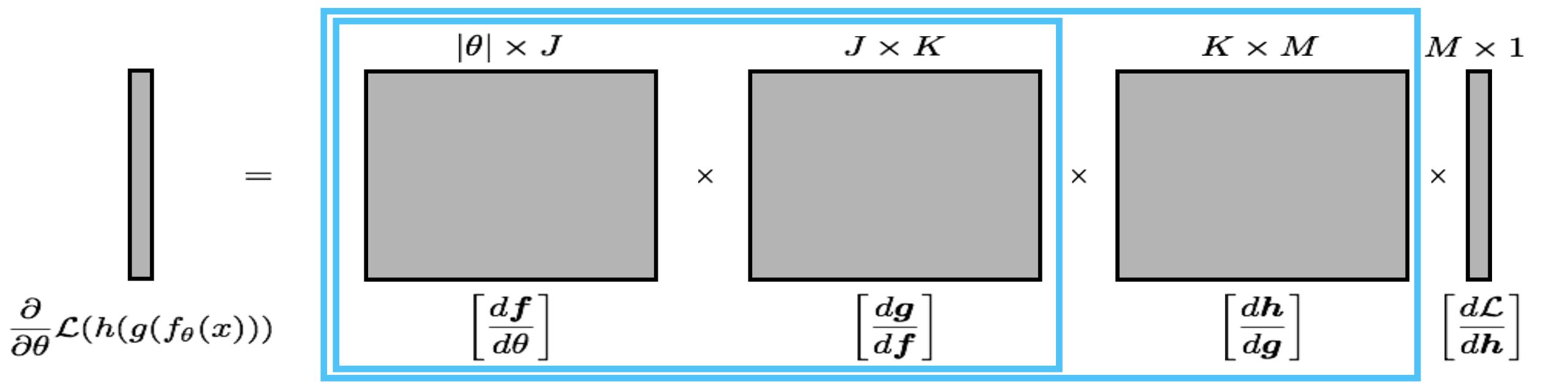


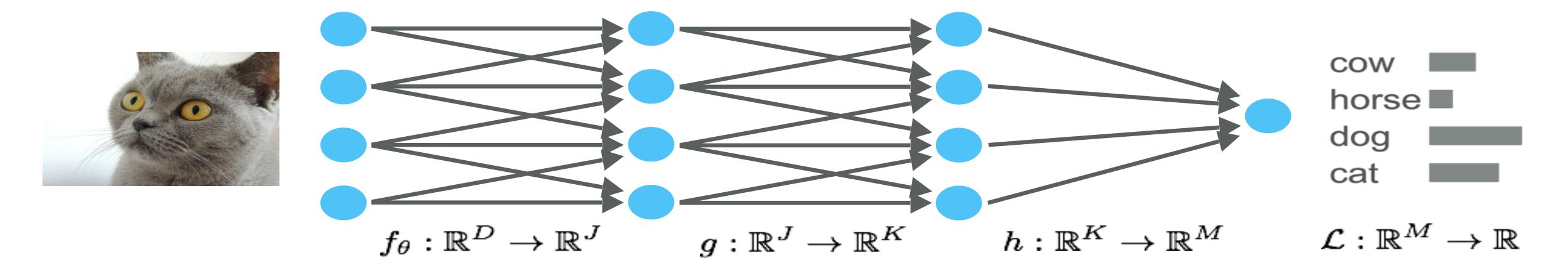
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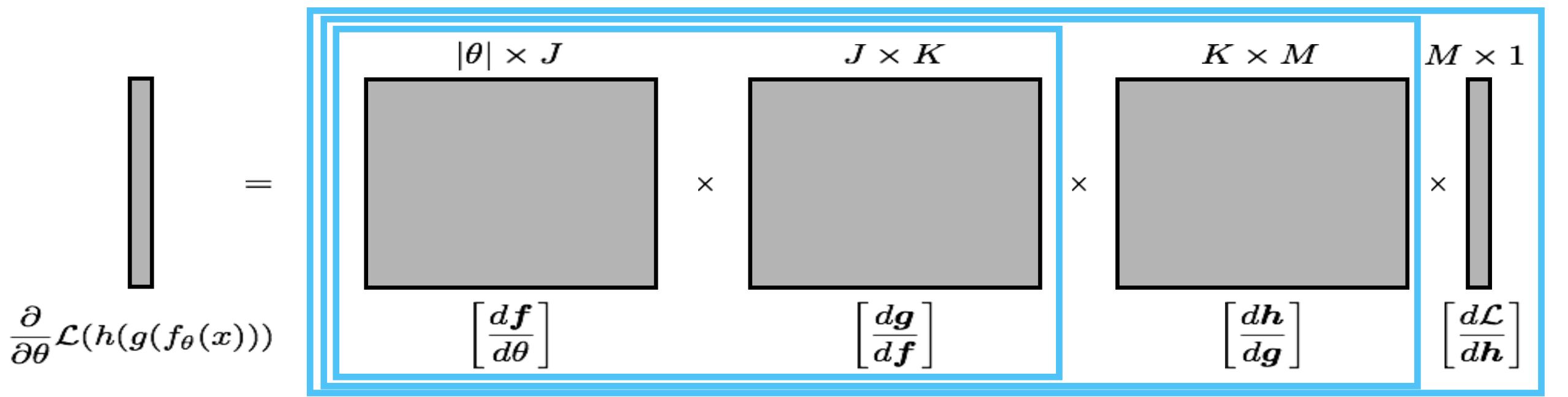


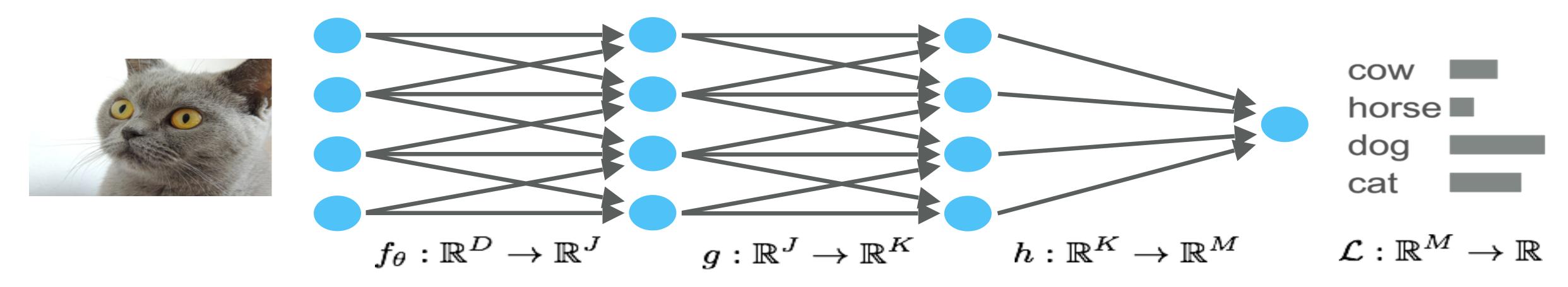
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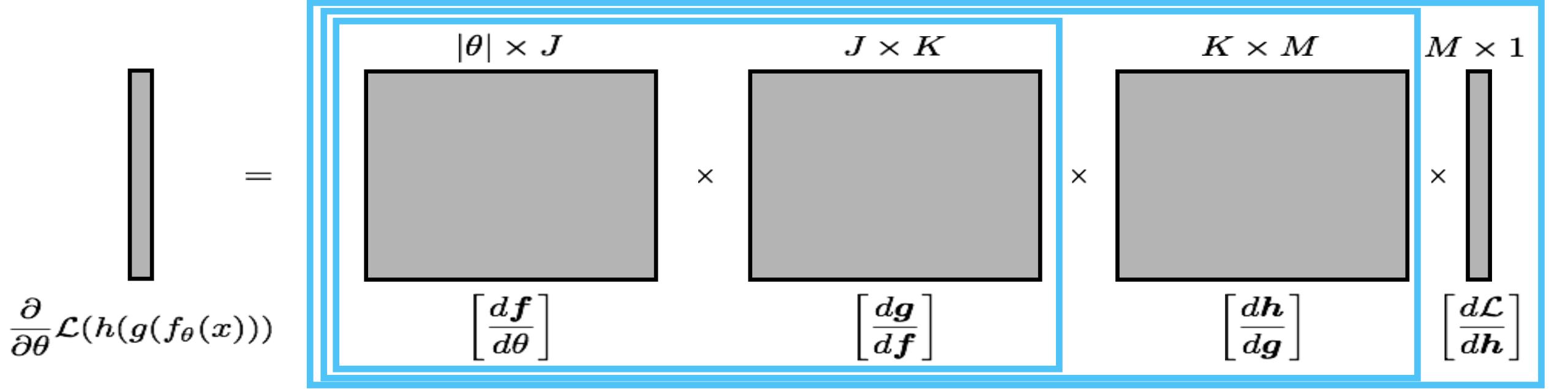


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Left to right: $O(|\theta|JK + |\theta|KM + |\theta|M)$



Left-to-right evaluation of partial derivatives (not so great for optimization)

We can write the evaluation of a program in a sequence of operations, called a "trace", or a "Wengert list"

```
function f(a,b,c)
   if b > c then
      return a * math.sin(b)
   else
      return a + b * c
   end
end
print(f(3,2,1))
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2.727892280477

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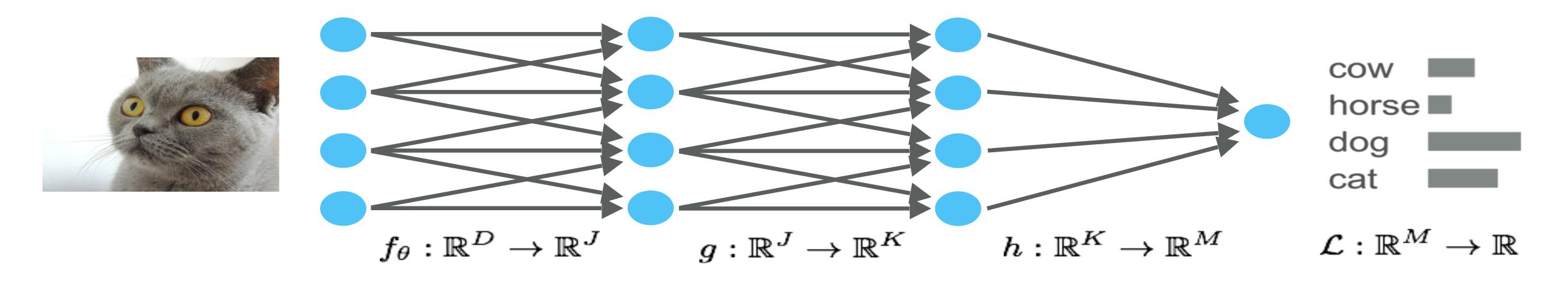
b = 2

We can write the evaluation of a program in a sequence of operations, called a "trace", or a "Wengert list"

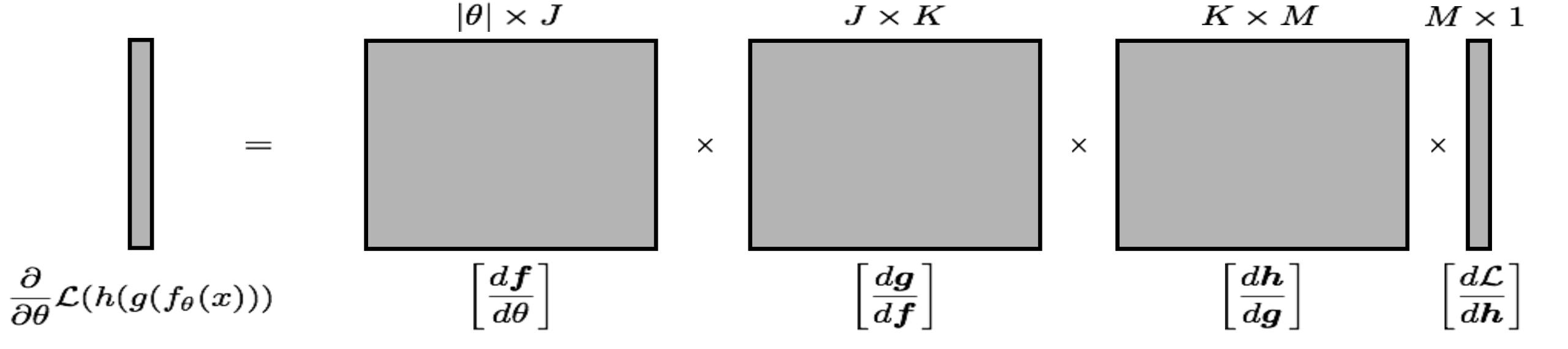
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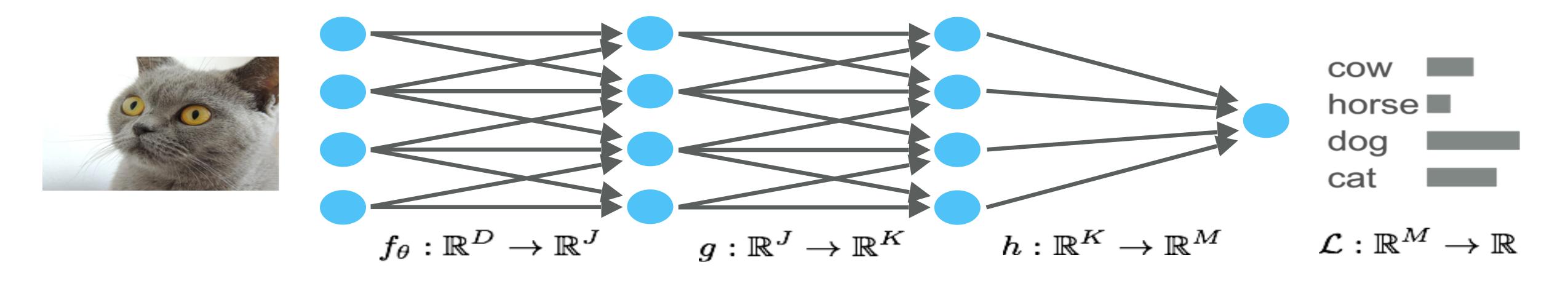
2.727892280477

return 0.909

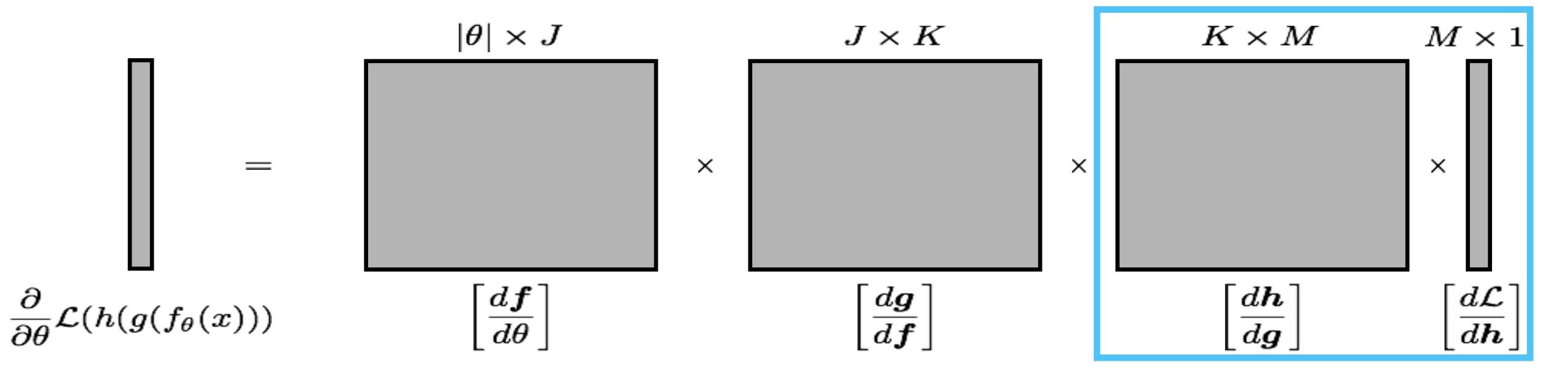


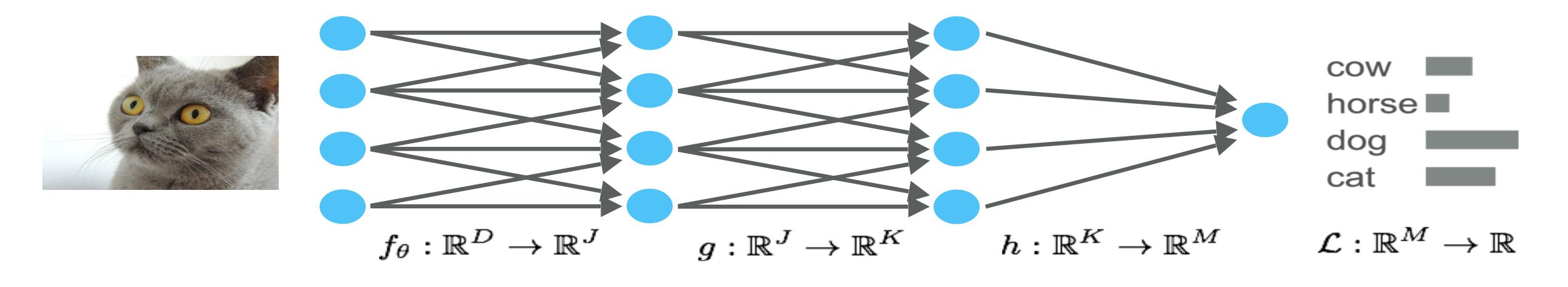
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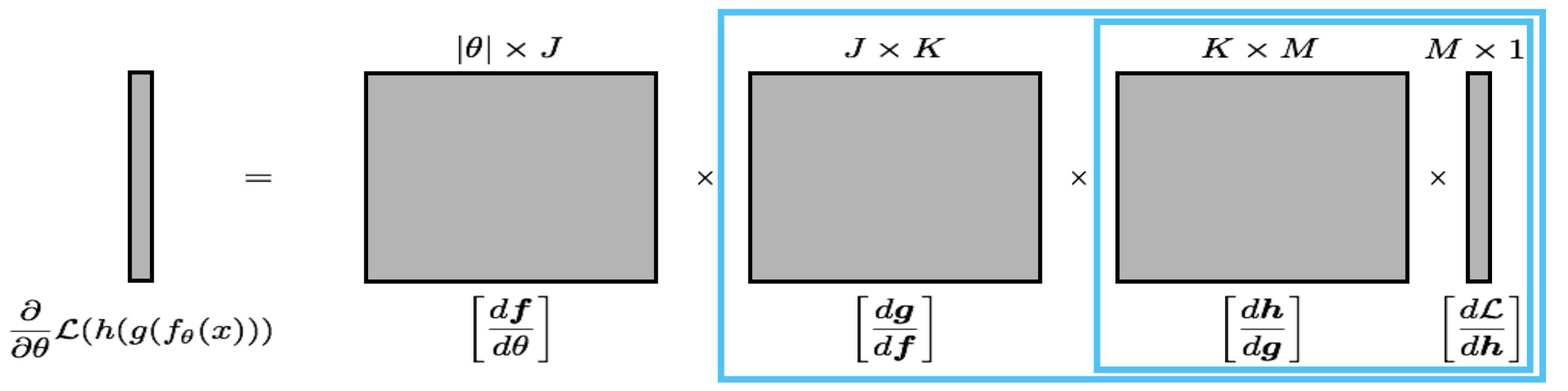


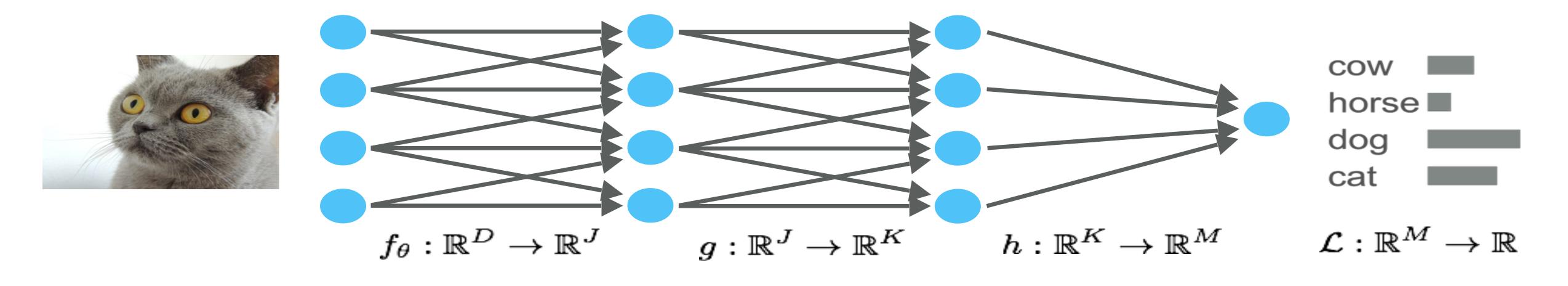
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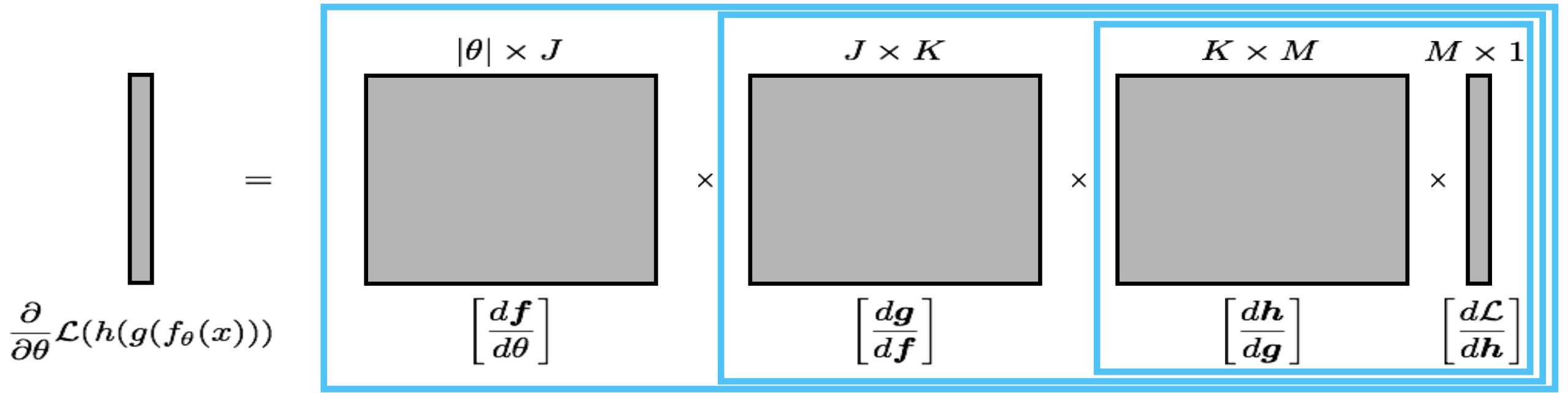


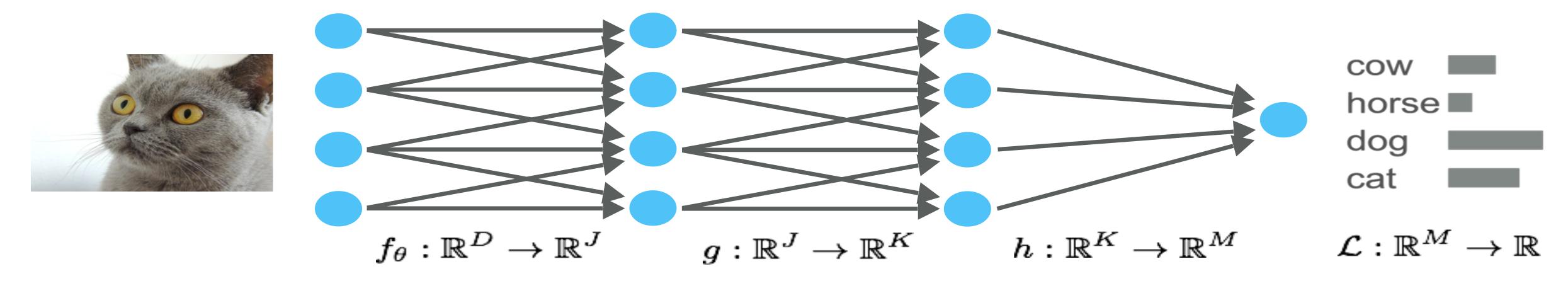
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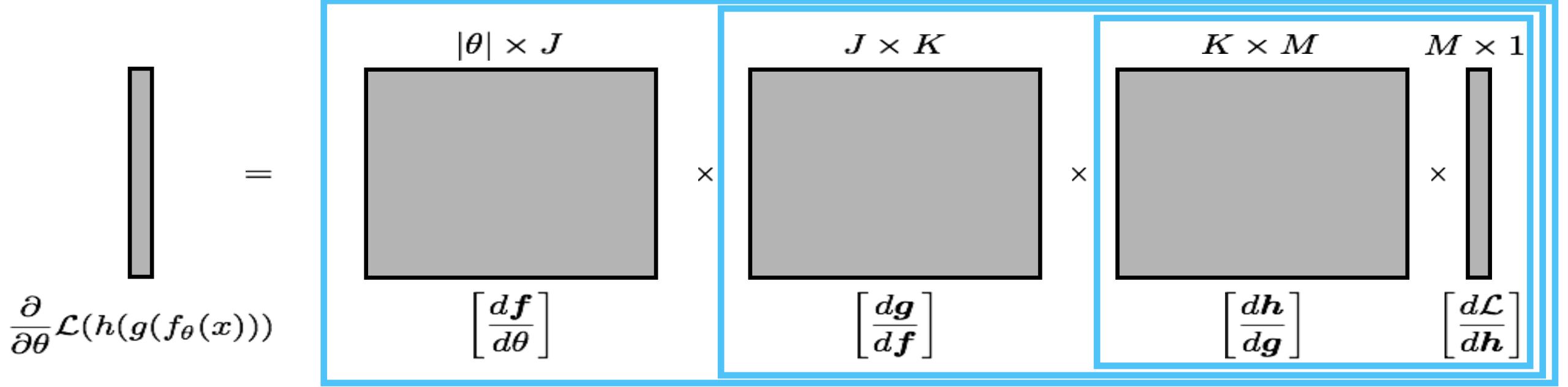
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ight]$$





Right to left:

$$O(KM + JK + \theta J)$$



Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

```
function f(a,b,c)
   if b > c then
       return a * math.sin(b)
   else
       return a + b * c
   end
end
print(f(3,2,1))
```

Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

```
function f(a,b,c)
   if b > c then
      return a * math.sin(b)
   else
      return a + b * c
   end
end
print(f(3,2,1))
```

a = 3

Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

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function f(a,b,c)
   if b > c then
       return a * math.sin(b)
   else
       return a + b * c
   end
end
print(f(3,2,1))
```

a = 3

b = 2

Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

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function f(a,b,c)
   if b > c then
       return a * math.sin(b)
   else
       return a + b * c
   end
end
print(f(3,2,1))
```

a = 3

b = 2

c = 1

Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

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   if b > c then
       return a * math.sin(b)
   else
       return a + b * c
   end
end
print(f(3,2,1))
```

```
a = 3
b = 2
c = 1
d = a * math.sin(b) = 2.728
```

Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

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function f(a,b,c)
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```

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print(f(3,2,1))
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```

$$a = 3$$
 $b = 2$

Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

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   if b > c then
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```

2.727892280477

a = 3
 b = 2
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Right-to-left evaluation of partial derivatives (the right thing to do for optimization)

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   if b > c then
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       return a + b * c
   end
end
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```

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end
print(f(3,2,1))
```

```
a = 3
b = 2
c = 1
d = a * math.sin(b) = 2.728
return 2.728
```

A trainable neural network in torch-autograd

Any numeric function can go here

These two fn's are split only for clarity

This is the API >

This is a how the parameters are updated

```
torch = require 'torch'
     params = {
        W = \{torch.randn(64*64,50), torch.randn(50,4)\},
        b = \{torch.randn(64*64), torch.randn(4)\}
 6
     function neuralNetwork(params, image)
        local h1 = torch.tanh(image*params.W[1] + params.b[1])
10
        local h2 = torch.tanh(h1*params.W[2] + params.b[2])
        return torch.log(torch.sum(torch.exp(h2)))
12
     end
14
     function loss(params, image, trueLabel)
        local prediction = neuralNetwork(params, image)
        return torch.sum(torch.pow(prediction-trueLabel,2))
     end
     grad = require 'autograd'
20
     dloss = grad(loss)
21
     for _,datapoint in dataset() do
        -- Calculate our gradients
23
        local gradients = dloss(params, datapoint.image, datapoint.label)
24
        -- Update parameters
        for i=1,#params.W do
26
           params.W[i] = params.W[i] - 0.01*gradients.W[i]
           params.b[i] = params.b[i] - 0.01*gradients.b[i]
28
29
        end
30
     end
```



A trainable neural network in torch-autograd

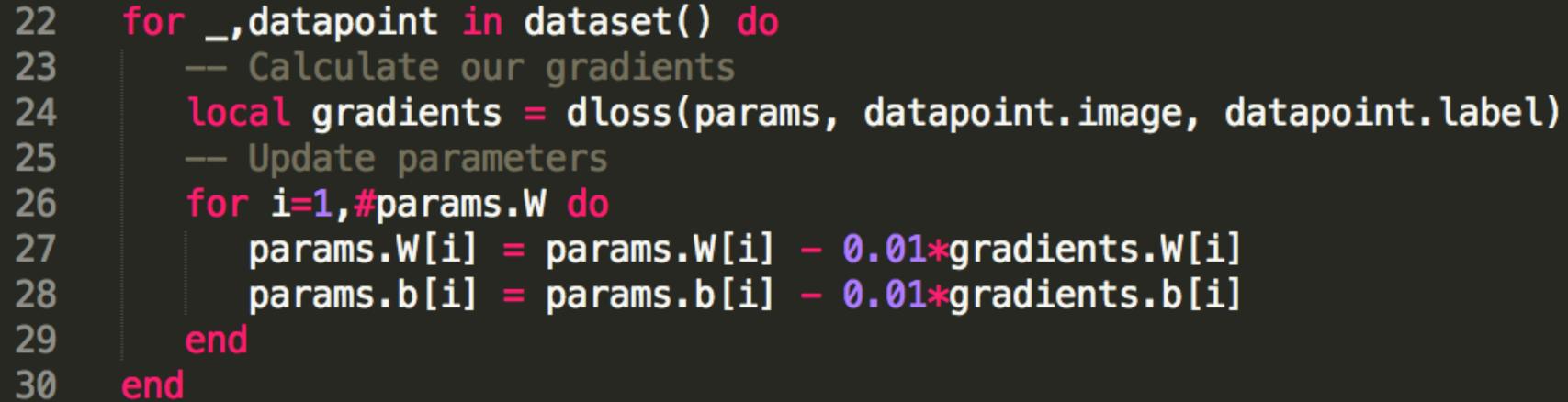
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This is the API >

This is a how the parameters are updated

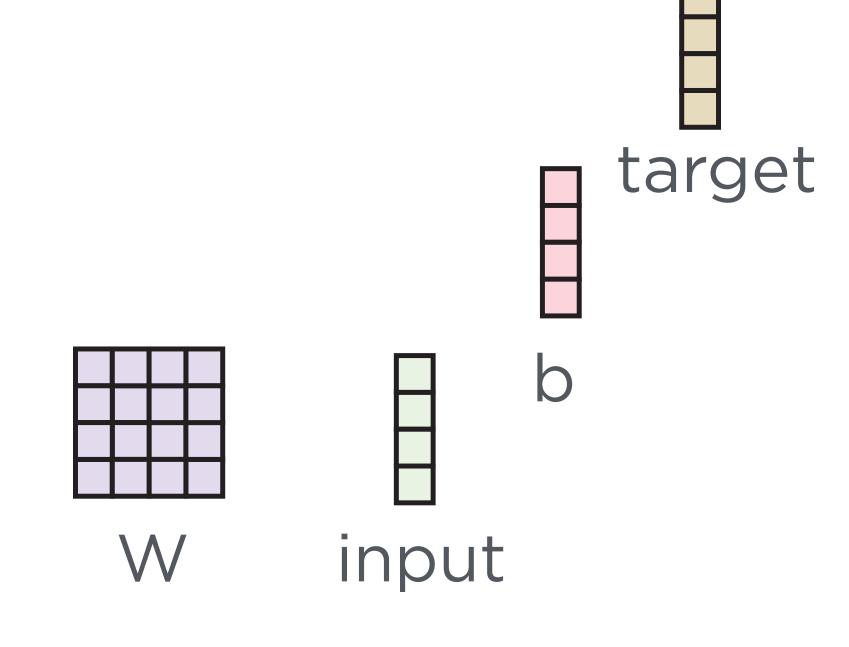
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torch = require 'torch'
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        W = \{torch.randn(64*64,50), torch.randn(50,4)\},
        b = \{torch.randn(64*64), torch.randn(4)\}
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        local h1 = torch.tanh(image*params.W[1] + params.b[1])
        local h2 = torch.tanh(h1*params.W[2] + params.b[2])
10
        return torch.log(torch.sum(torch.exp(h2)))
12
     end
14
     function loss(params, image, trueLabel)
        local prediction = neuralNetwork(params, image)
        return torch.sum(torch.pow(prediction-trueLabel,2))
16
     end
     grad = require 'autograd'
     dloss = grad(loss)
20
22
     for _,datapoint in dataset() do
23
        -- Calculate our gradients
24
        -- Update parameters
```





```
params = {W=torch.randn(4,4),b=torch.randn(4)}
input = torch.randn(4)
target = torch.randn(4)

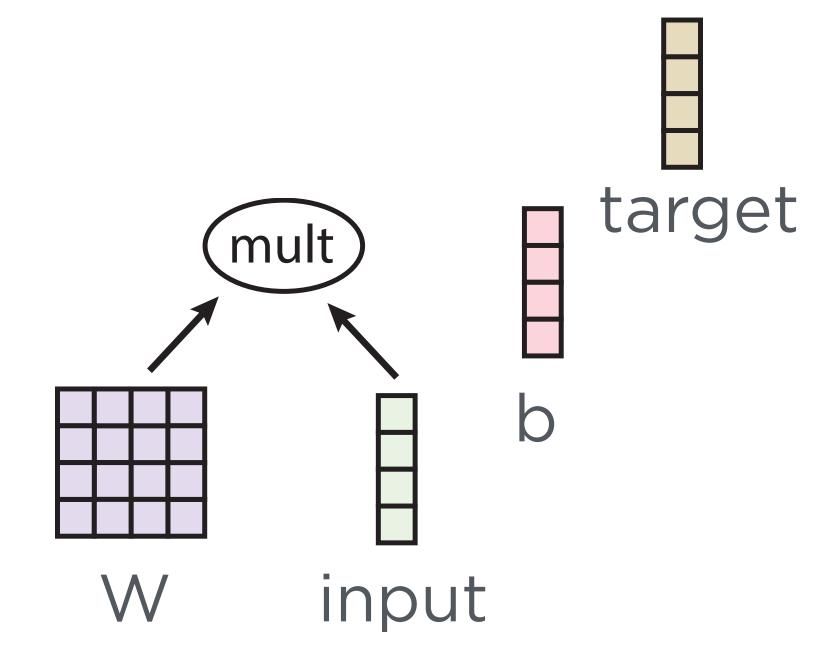
/ function simpleFn(params, input, target)
local h1 = params.W*input
local h2 = h1 + params.b
local h3 = h2 - target
local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
return h5
end
```





```
params = {W=torch.randn(4,4),b=torch.randn(4)}
input = torch.randn(4)
target = torch.randn(4)

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local h1 = params.W*input
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local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
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end
```





```
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target = torch.randn(4)

function simpleFn(params, input, target)

local h1 = params.W*input

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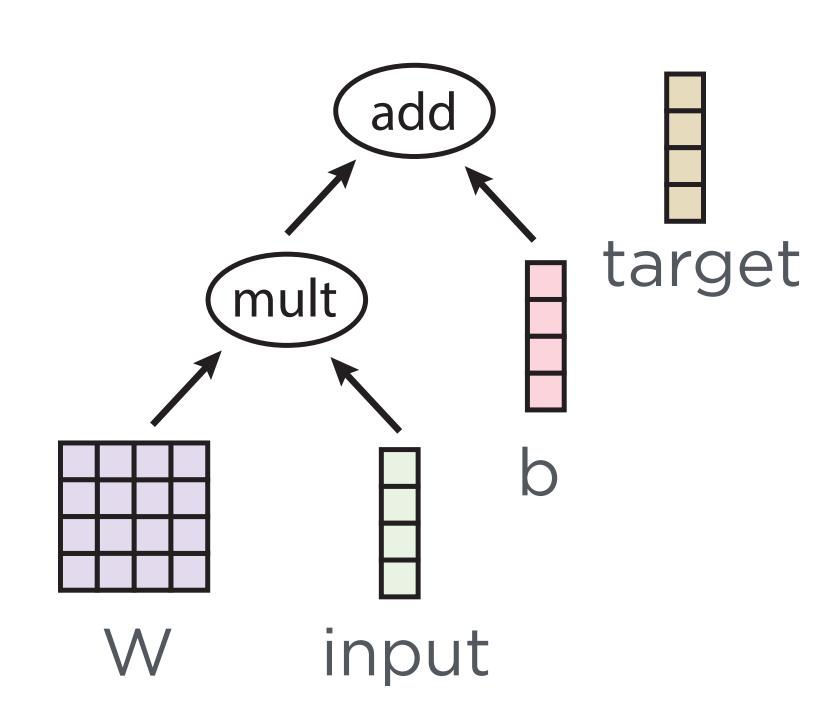
local h3 = h2 - target

local h4 = torch.pow(h3,2)

local h5 = torch.sum(h4)

return h5

end
```





```
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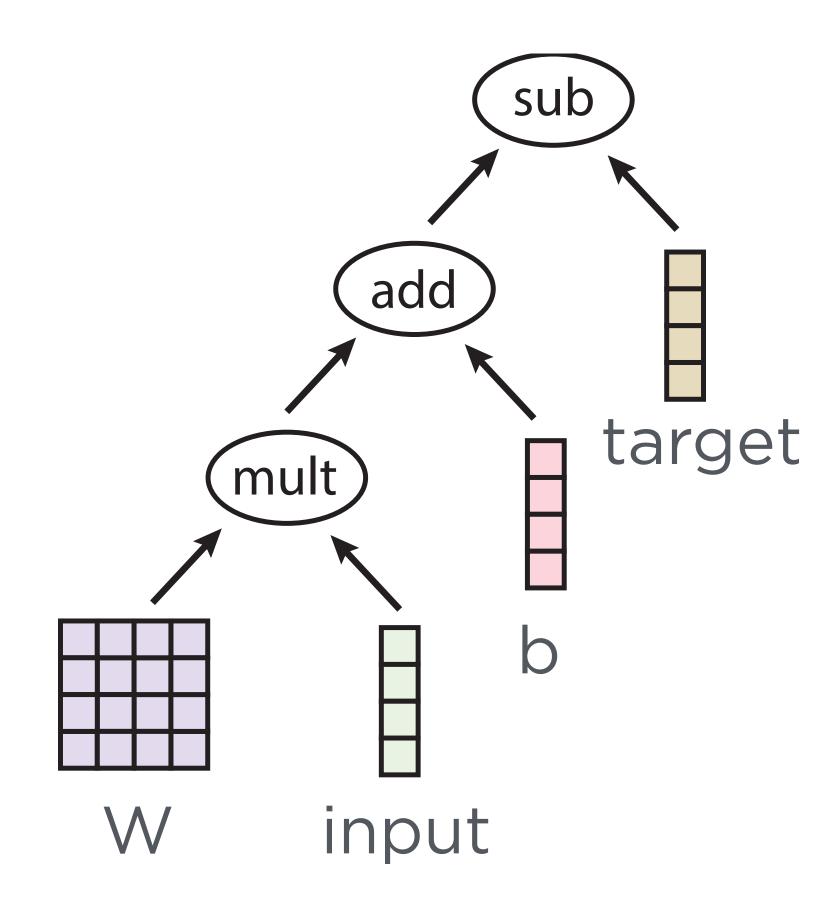
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end
```





```
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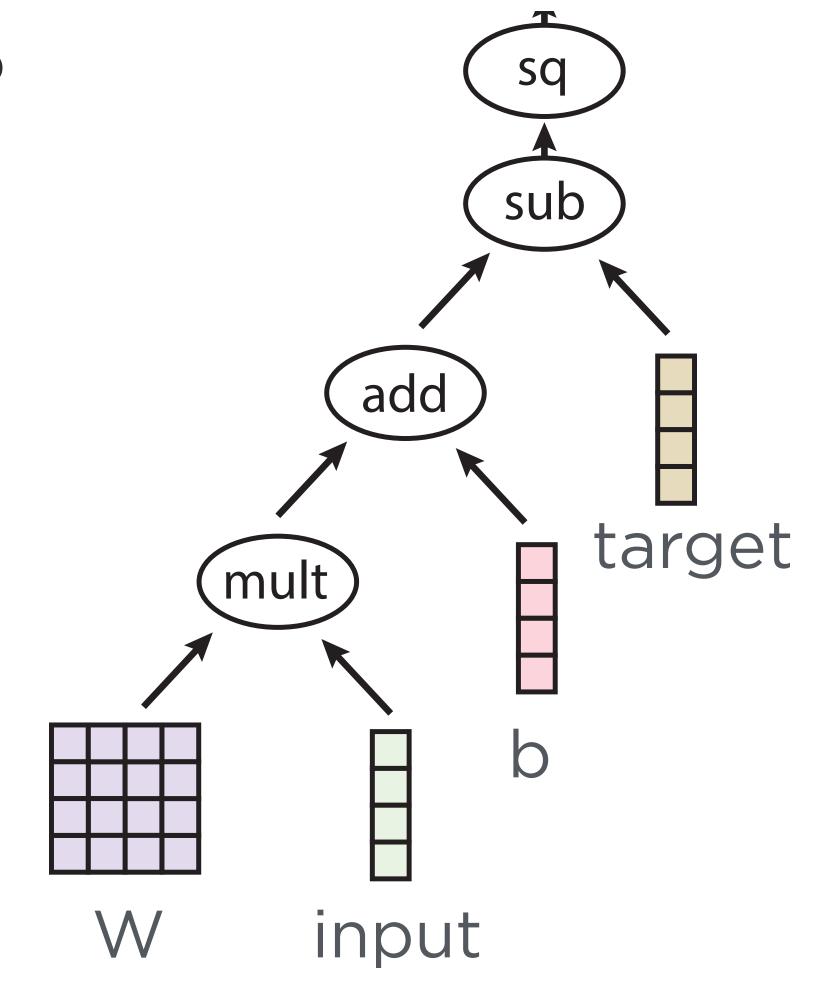
local h3 = h2 - target

local h4 = torch.pow(h3,2)

local h5 = torch.sum(h4)

return h5

end
```





```
params = {W=torch.randn(4,4),b=torch.randn(4)}
input = torch.randn(4)
target = torch.randn(4)

function simpleFn(params, input, target)

local h1 = params.W*input

local h2 = h1 + params.b

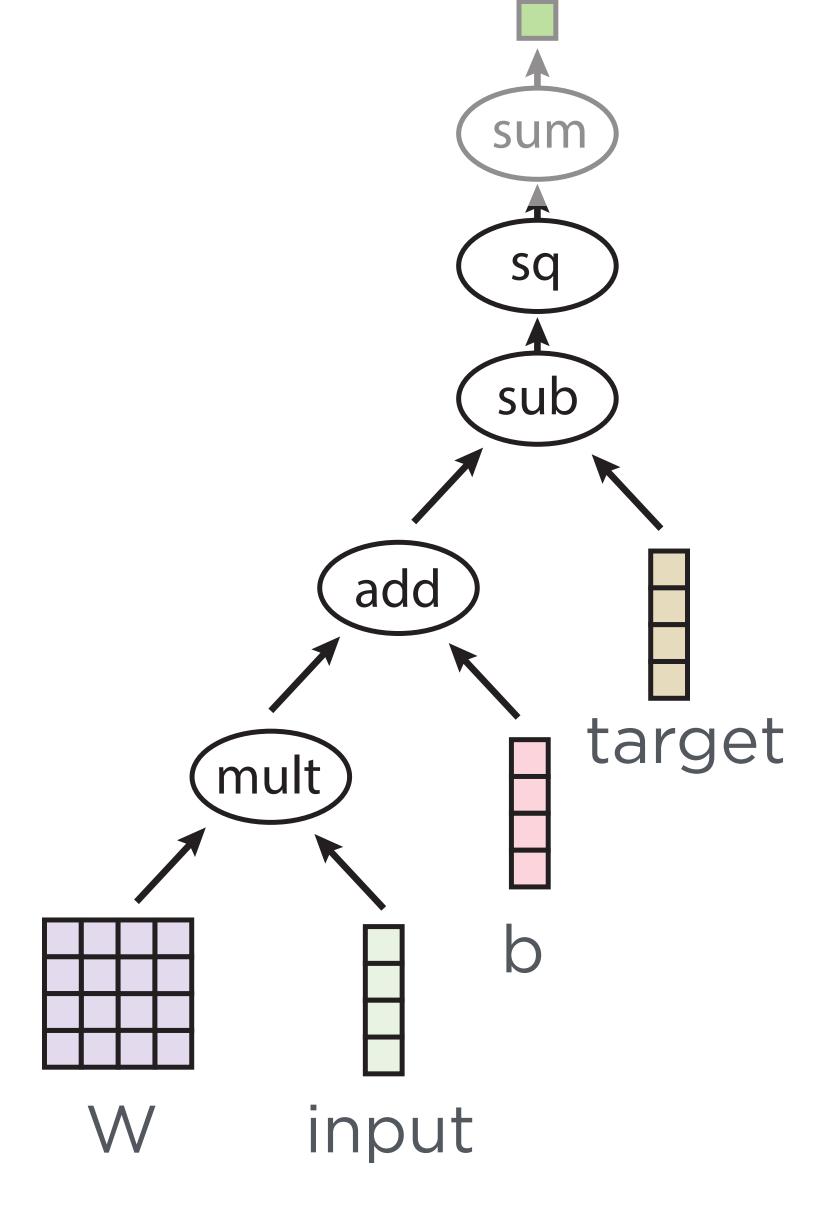
local h3 = h2 - target

local h4 = torch.pow(h3,2)

local h5 = torch.sum(h4)

return h5

end
```





WE TRACK COMPUTATION VIA OPERATOR OVERLOADING

Linked list of computation forms a "tape" of computation

```
local origSum = torch.sum
    torch.sum = function(arg)
       — Check if the argument has been used before in an overloaded function
       if not isNodeType(arg) then
           return origSum(arg)
6
       else
7~
          — Run the function
           local outputVal = origSum(unpackNode(arg))
10
          — Build a data structure that will track computaiton via linked list
11
           local outputNode = {fn=origSum,parent=arg,val=outputVal}
13
       end
14
    end
15
    -- Now overload all other numeric functions...
16
17
    -- sin,cos,tan,sinh,cosh,tanh,add,sub,mul,div,pow
    -- select,narrow,size,new,zeros, ...
```



When it comes time to evaluate partial derivatives, we just have to look up the partial derivatives from a table

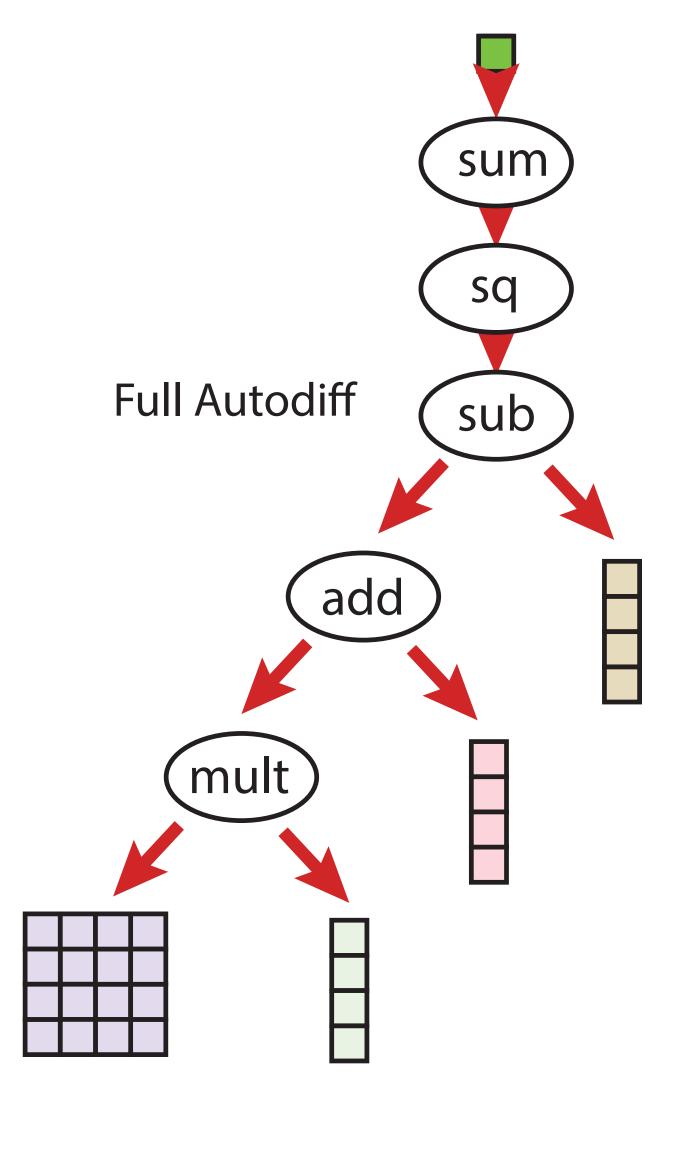
```
gradients[torch.sqrt] = {
    function(g, ans, x) return torch.cmul(torch.cmul(g,0.5), torch.pow(x,-0.5)) end
}

gradients[torch.sin] = {
    function(g, ans, x) return torch.cmul(g, torch.cos(x)) end
})

gradients[torch.cos] = {
    function(g, ans, x) return torch.cmul(g, -torch.sin(x)) end
})

gradients[torch.tan] = {
    function(g, ans, x) return torch.cdiv(g, torch.pow(torch.cos(x), 2.0)) end
})

gradients[torch.log] = {
    function(g, ans, x) return torch.cdiv(g,x) end
})
```



We can then calculate the derivative of the loss w.r.t. inputs via the chain rule!



Autograd gives you derivatives of numeric code, without a special mini-language

```
-- Arithmetic is no problem
grad = require 'autograd'
function f(a,b,c)
    return a + b * c
end
df = grad(f)
da, val = df(3.5, 2.1, 1.1)
print("Value: "..val)
print("Gradient: "..da)
```

Value: 5.81 Gradient: 1



Control flow, like if-statements, are handled seamlessly

```
-- If statements are no problem
grad = require 'autograd'
function f(a,b,c)
    if b > c then
        return a * math.sin(b)
    else
        return a + b * c
    end
end
g = grad(f)
da, val = g(3.5, 2.1, 1.1)
print("Value: "..val)
print("Gradient: "..da)
```

Value: 3.0212327832711

Gradient: 0.86320936664887



Scalars are good for demonstration, but autograd is most often used with tensor types

```
-- Of course, works with tensors
grad = require 'autograd'
function f(a,b,c)
    if torch.sum(b) > torch.sum(c) then
        return torch.sum(torch.cmul(a,torch.sin(b)))
    else
        return torch.sum(a + torch.cmul(b,c))
    end
end
g = grad(f)
a = torch.randn(3,3)
b = torch.eye(3,3)
c = torch.randn(3,3)
da, val = g(a,b,c)
print("Value: "..val)
print("Gradient: ")
print(da)
```

```
Value: 0.40072414956087

Gradient:
    0.8415    0.0000    0.0000
    0.0000    0.8415    0.0000
    0.0000    0.8415

[torch.DoubleTensor of size 3x3]
```



Autograd shines if you have dynamic compute graphs

```
-- Autograd for loop
function f(a,b)
    for i=1,b do
        a = a*a
    end
    return a
end
g = grad(f)
da, val = g(3,2)
print("Value: "..val)
print("Gradient: "..da)
```

Value: 81

Gradient: 108



Recursion is no problem.

Write numeric code as you ordinarily would, autograd handles the gradients

```
-- Autograd recursive function
function f(a,b)
    if b == 0 then
        return a
    else
        return f(a*a,b-1)
    end
end
g = grad(f)
da, val = g(3,2)
print("Value: "..val)
print("Gradient: "..da)
```

Value: 81

Gradient: 108



Need new or tweaked partial derivatives? Not a problem.

```
-- New ops aren't a problem
function f(a)
    return torch.sum(torch.floor(torch.pow(a,3)))
end
g = grad(f)
da, val = g(torch.eye(3))
print("Value: "..val)
print("Gradient:")
print(da)
Value: 3
Gradient:
 0 0 0
 0 0 0
 0 0 0
[torch.DoubleTensor of size 3x3]
```



Need new or tweaked partial derivatives? Not a problem.

```
-- New ops aren't a problem
grad = require 'autograd'
special = {}
special.floor = function(x) return torch.floor(x) end
-- Overload our new mini-module, called "special"
grad.overload.module("special", special, function(module)
    -- Define a gradient for the member function "floor"
   module.gradient("floor", {
                -- Here's our new partial derivative
                -- (if we had two arguments,
                -- we'd define two functions)
                function(g,ans,x)
                    return g
                end
            })
    end)
```



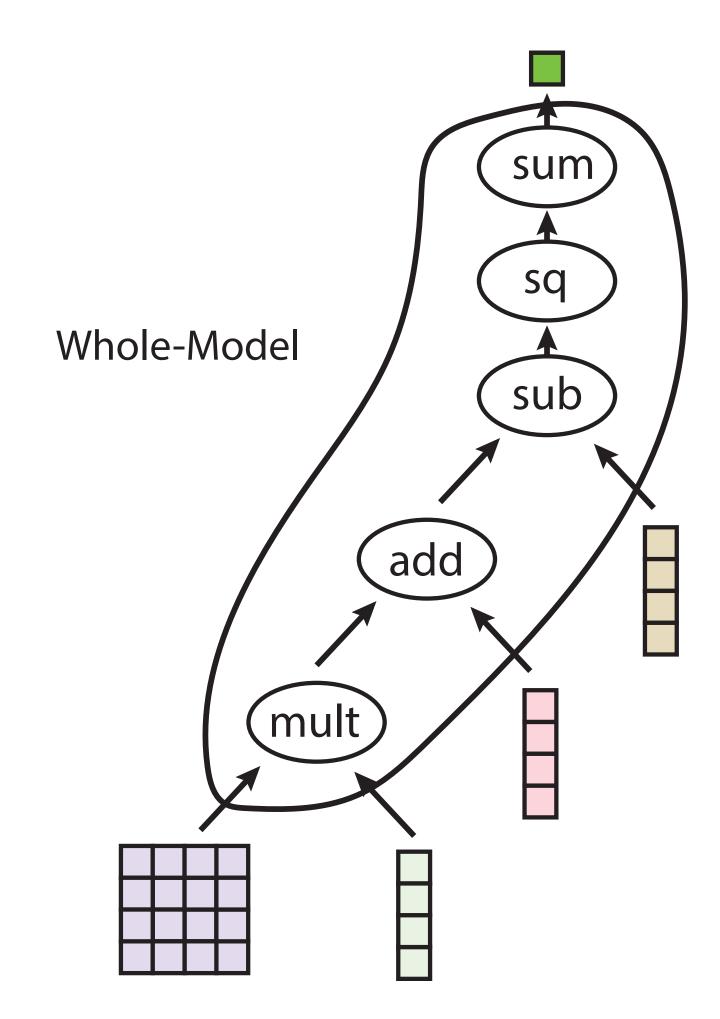
Need new or tweaked partial derivatives? Not a problem.

```
function f(a)
    return torch.sum(special.floor(torch.pow(a,3)))
end
g = grad(f)
da, val = g(torch.eye(3))
print("Value: "..val)
print("Gradient:")
print(da)
```

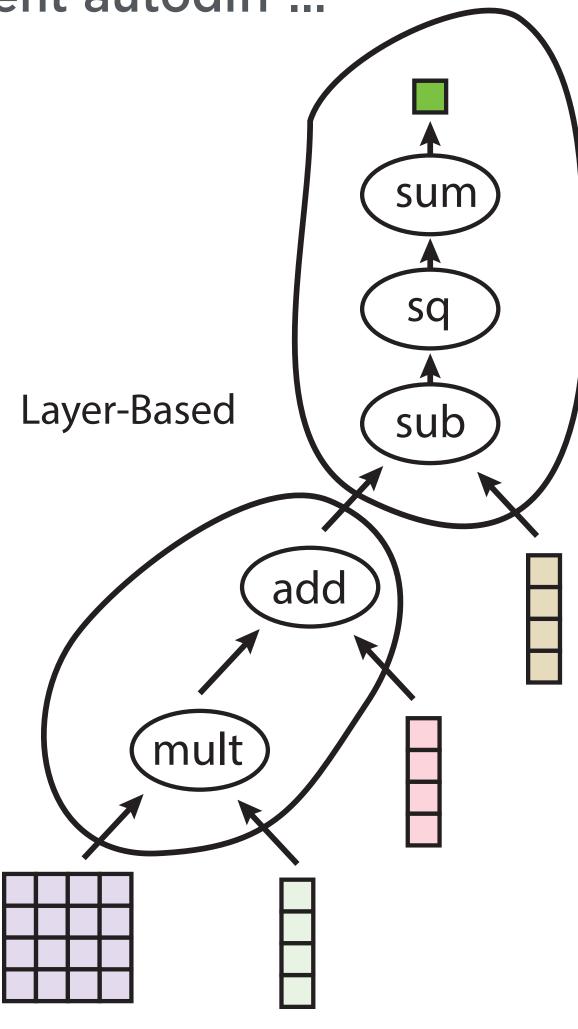
```
Value: 3
Gradient:
3 0 0
0 3 0
0 0 3
[torch.DoubleTensor of size 3x3]
```



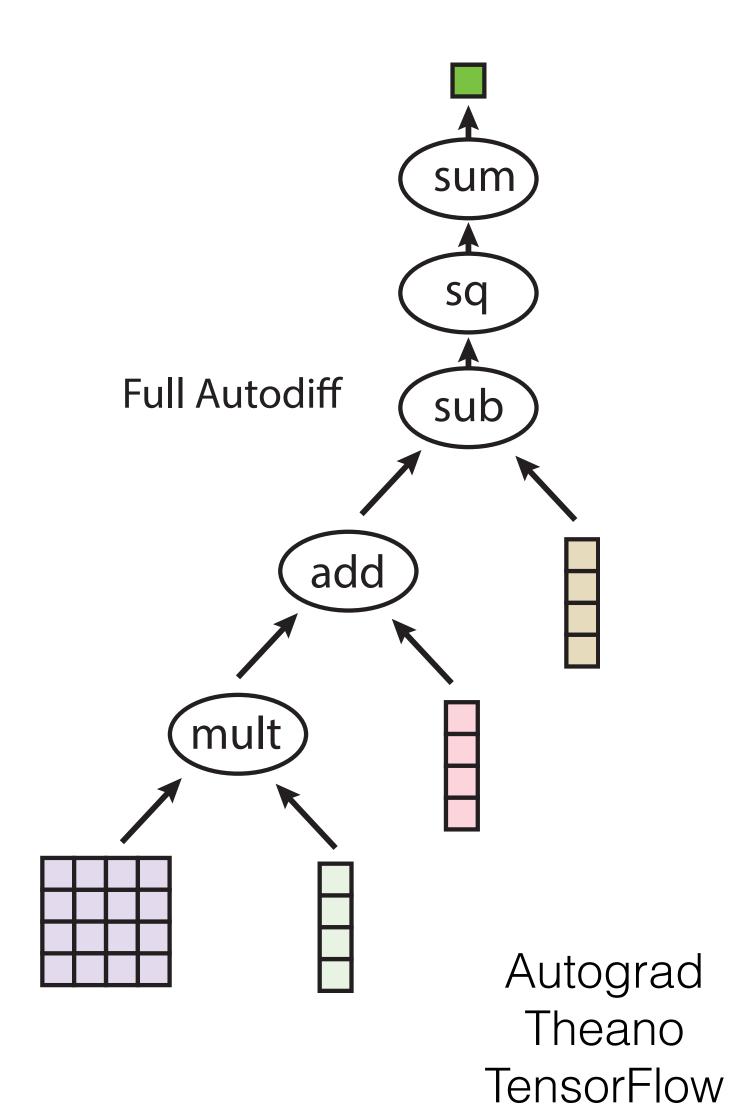
The granularity at which they implement autodiff ...





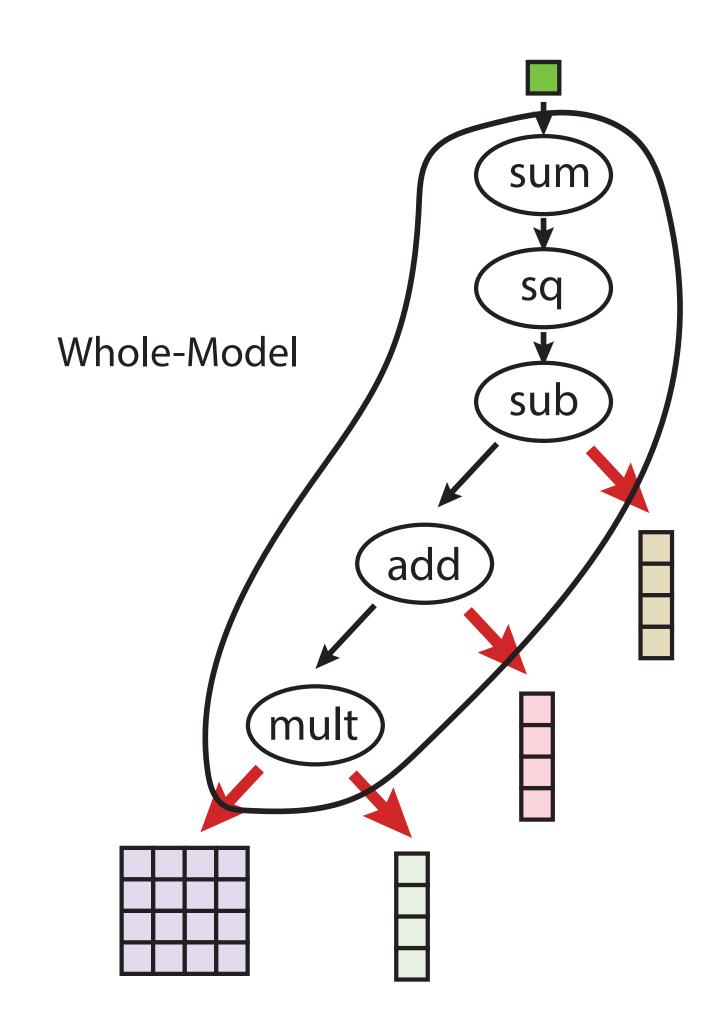


Torch NN Keras Lasagne

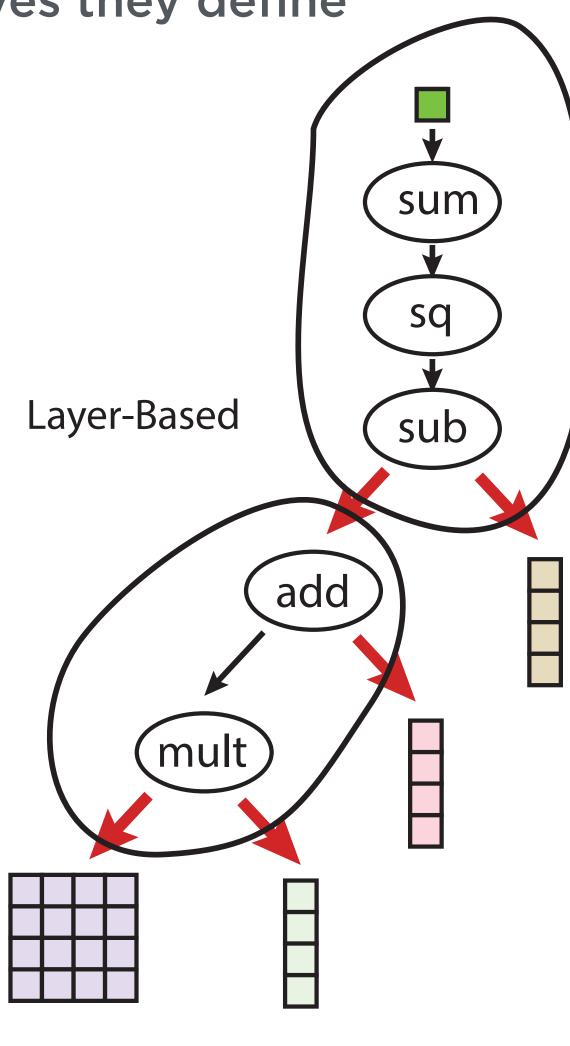




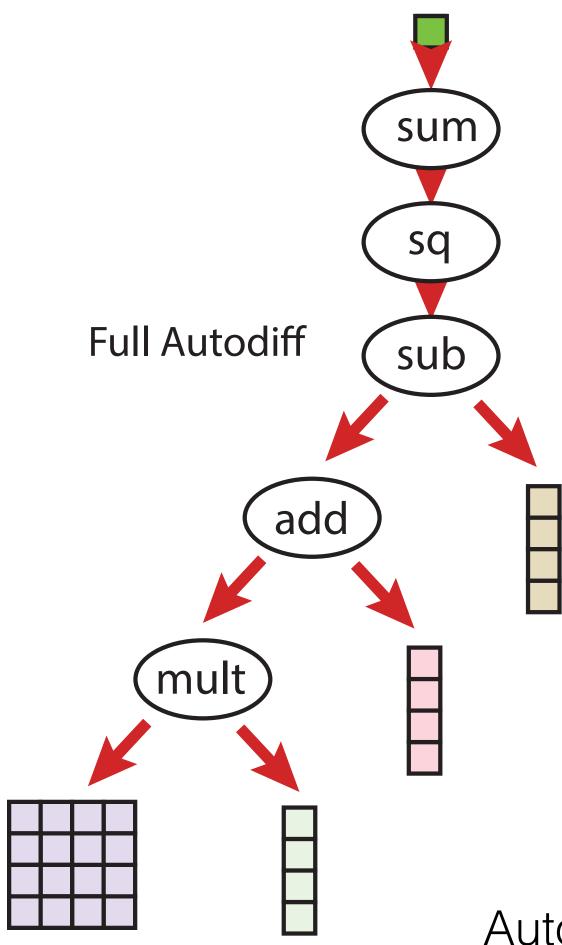
... which is set by the partial derivatives they define







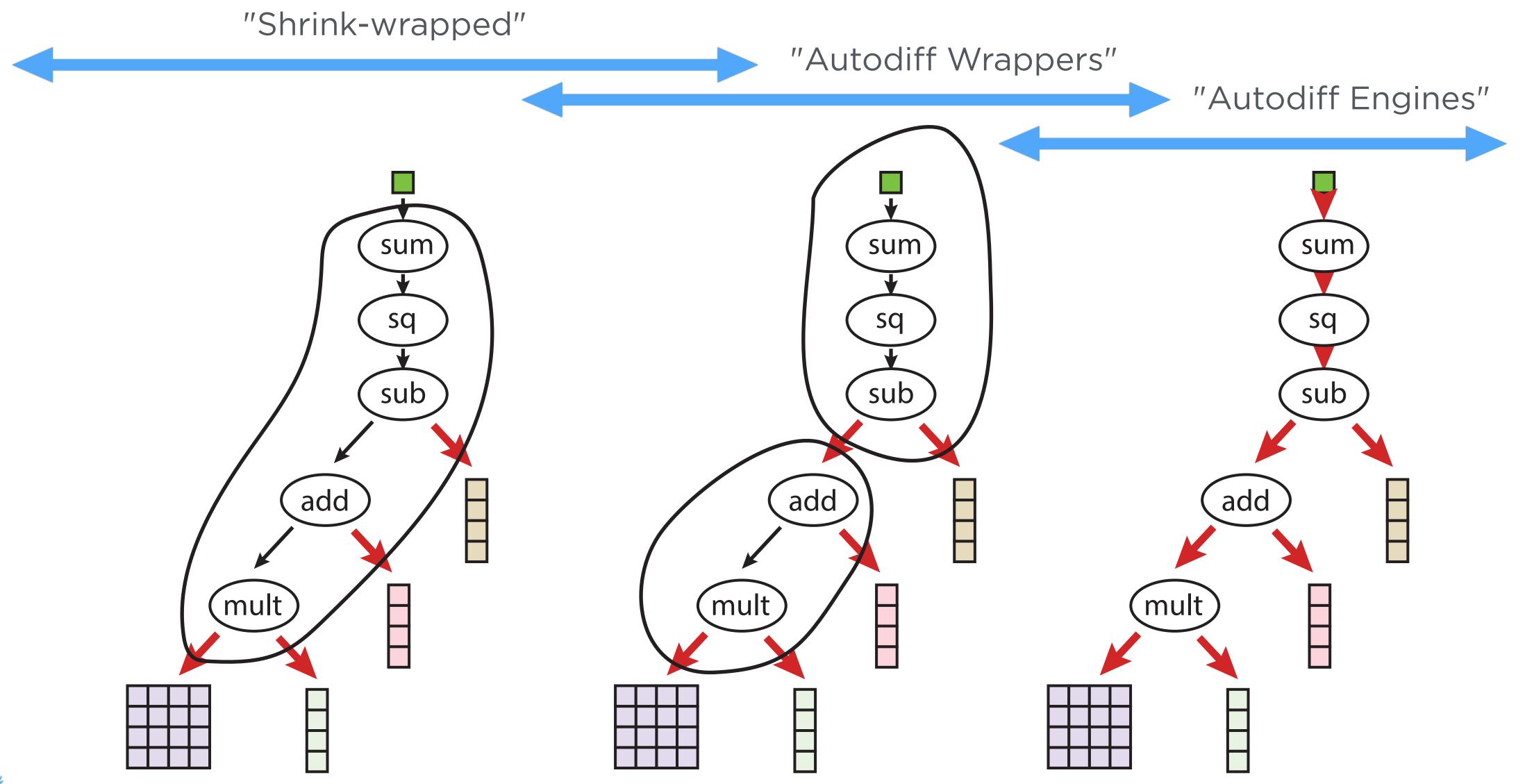
Torch NN Keras Lasagne



Autograd
Theano
TensorFlow



Do they actually implement autodiff, or do they wrap a separate autodiff library?



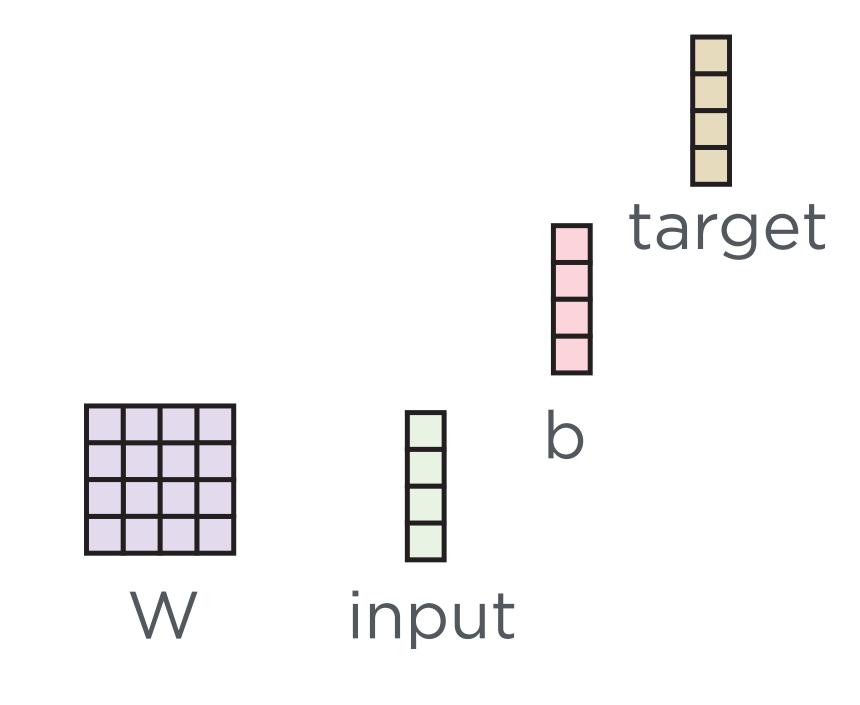


```
params = {W=torch.randn(4,4),b=torch.randn(4)}
input = torch.randn(4)
target = torch.randn(4)

function simpleFn(params, input, target)

local h1 = params.W*input
local h2 = h1 + params.b

local h3 = h2 - target
local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
return h5
end
```

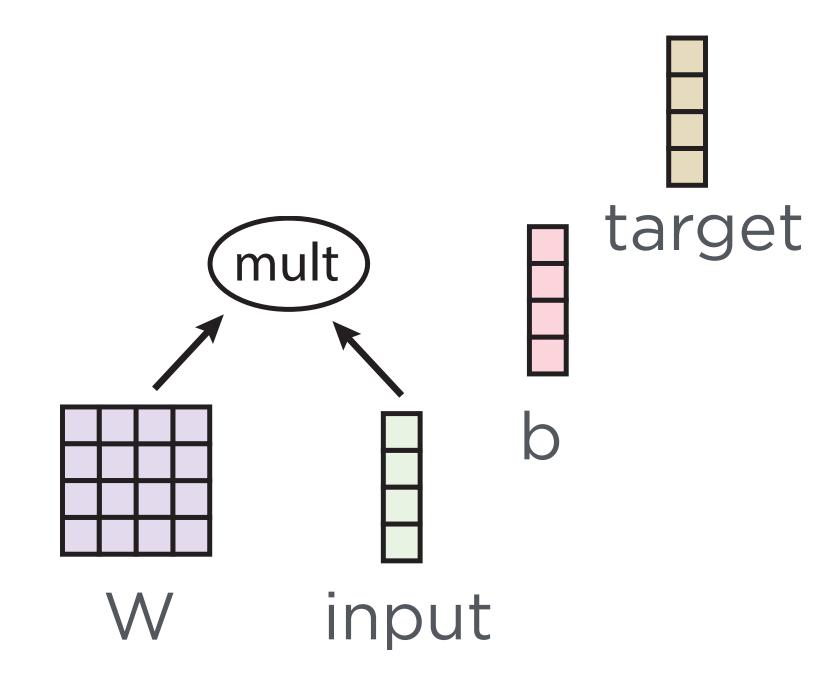




```
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local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
return h5
end
```

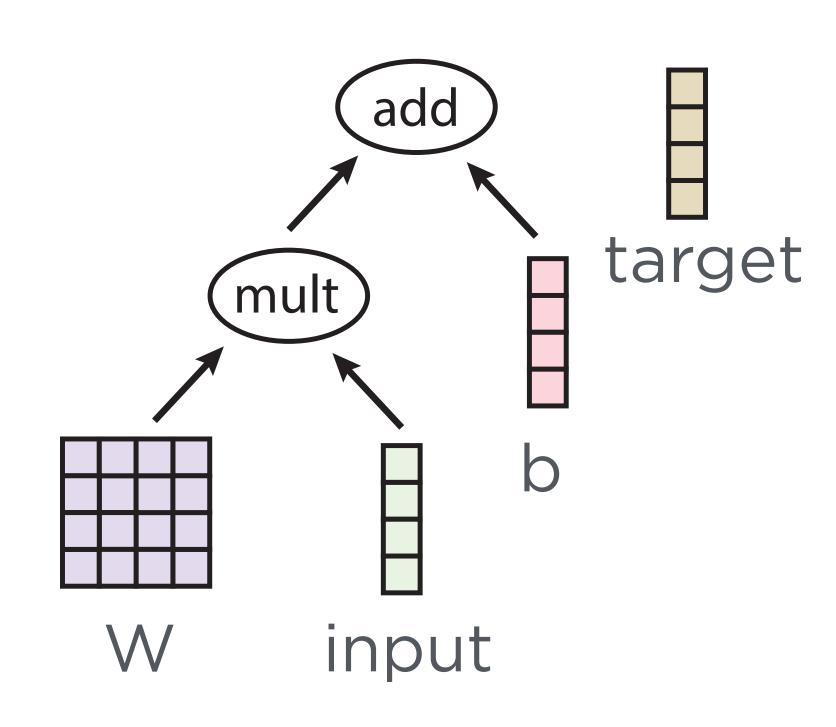




```
params = {W=torch.randn(4,4),b=torch.randn(4)}
input = torch.randn(4)
target = torch.randn(4)

function simpleFn(params, input, target)

local h1 = params.W*input
local h2 = h1 + params.b
local h3 = h2 - target
local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
return h5
end
```



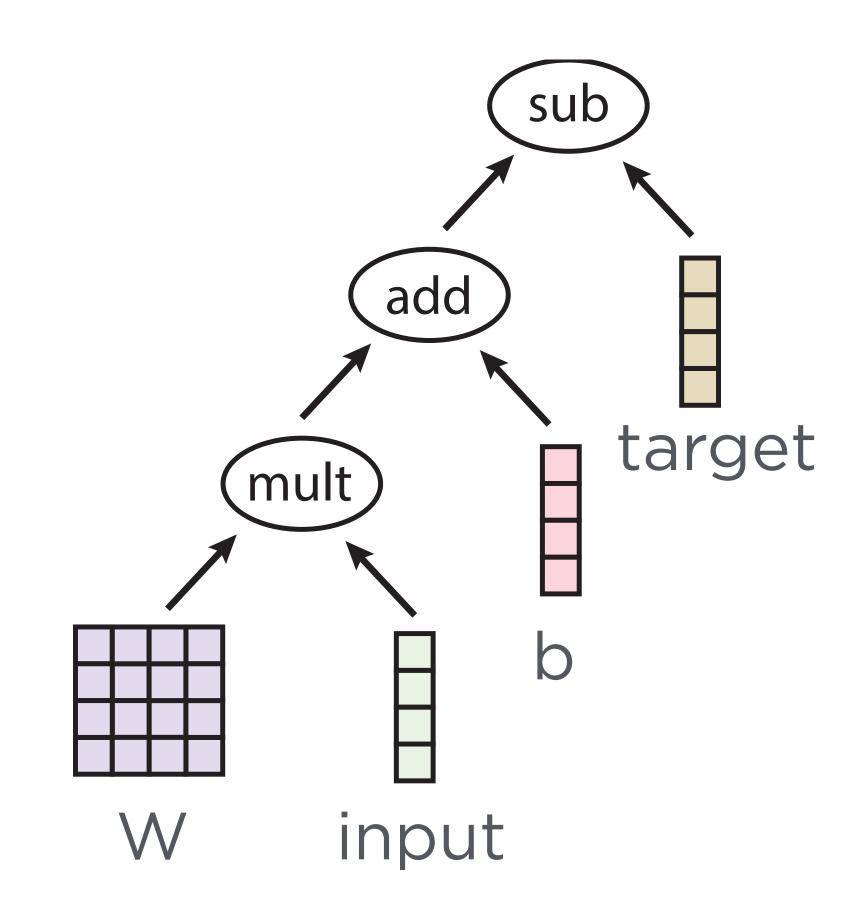


```
params = {W=torch.randn(4,4),b=torch.randn(4)}
input = torch.randn(4)
target = torch.randn(4)

// function simpleFn(params, input, target)

local h1 = params.W*input
local h2 = h1 + params.b

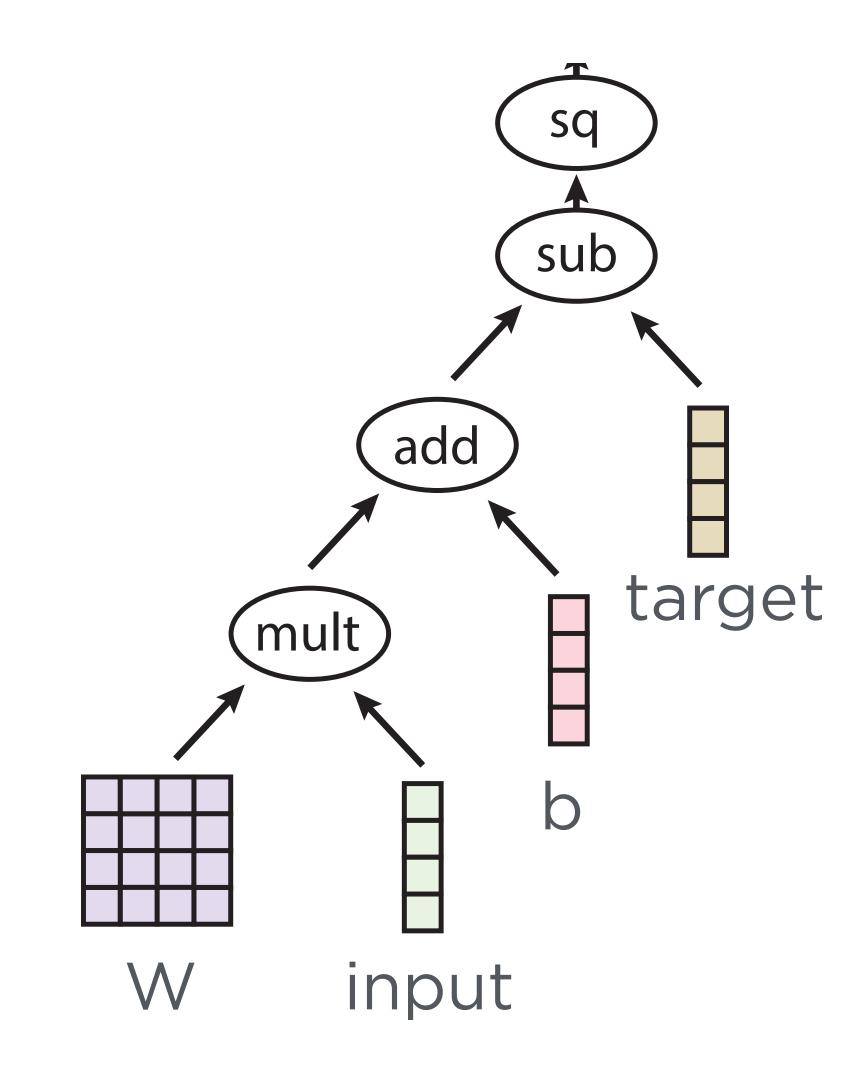
local h3 = h2 - target
local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
return h5
end
```





```
params = {W=torch.randn(4,4),b=torch.randn(4)}
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local h2 = h1 + params.b
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local h4 = torch.pow(h3,2)
local h5 = torch.sum(h4)
return h5
end
```





```
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input = torch.randn(4)
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function simpleFn(params, input, target)

local h1 = params.W*input

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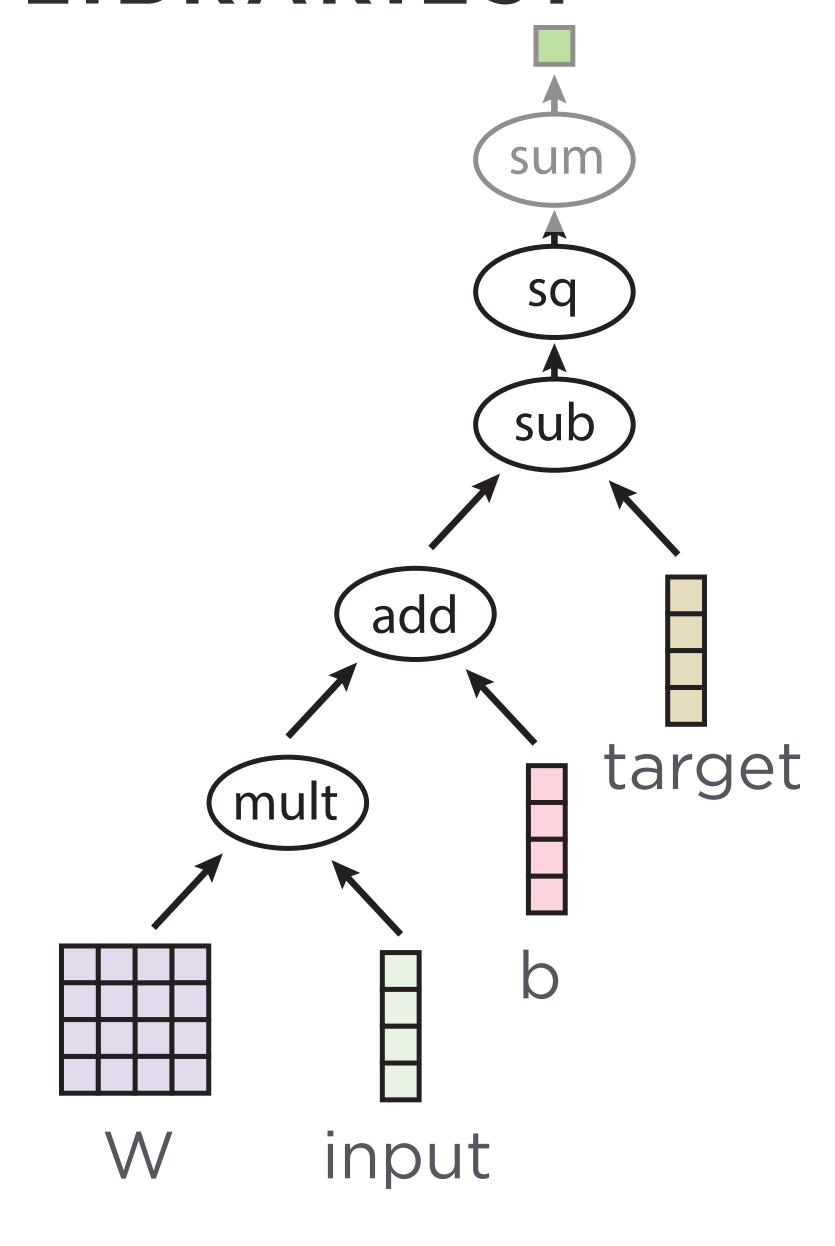
local h3 = h2 - target

local h4 = torch.pow(h3,2)

local h5 = torch.sum(h4)

return h5

end
```





Explicit	Ahead-of-Time	Just-in-Time
NN	TensorFlow	Autograd
Caffe	Theano	Chainer
No compiler optimizations, no dynamic graphs	Dynamic graphs can be awkward to work with	No compiler optimizations



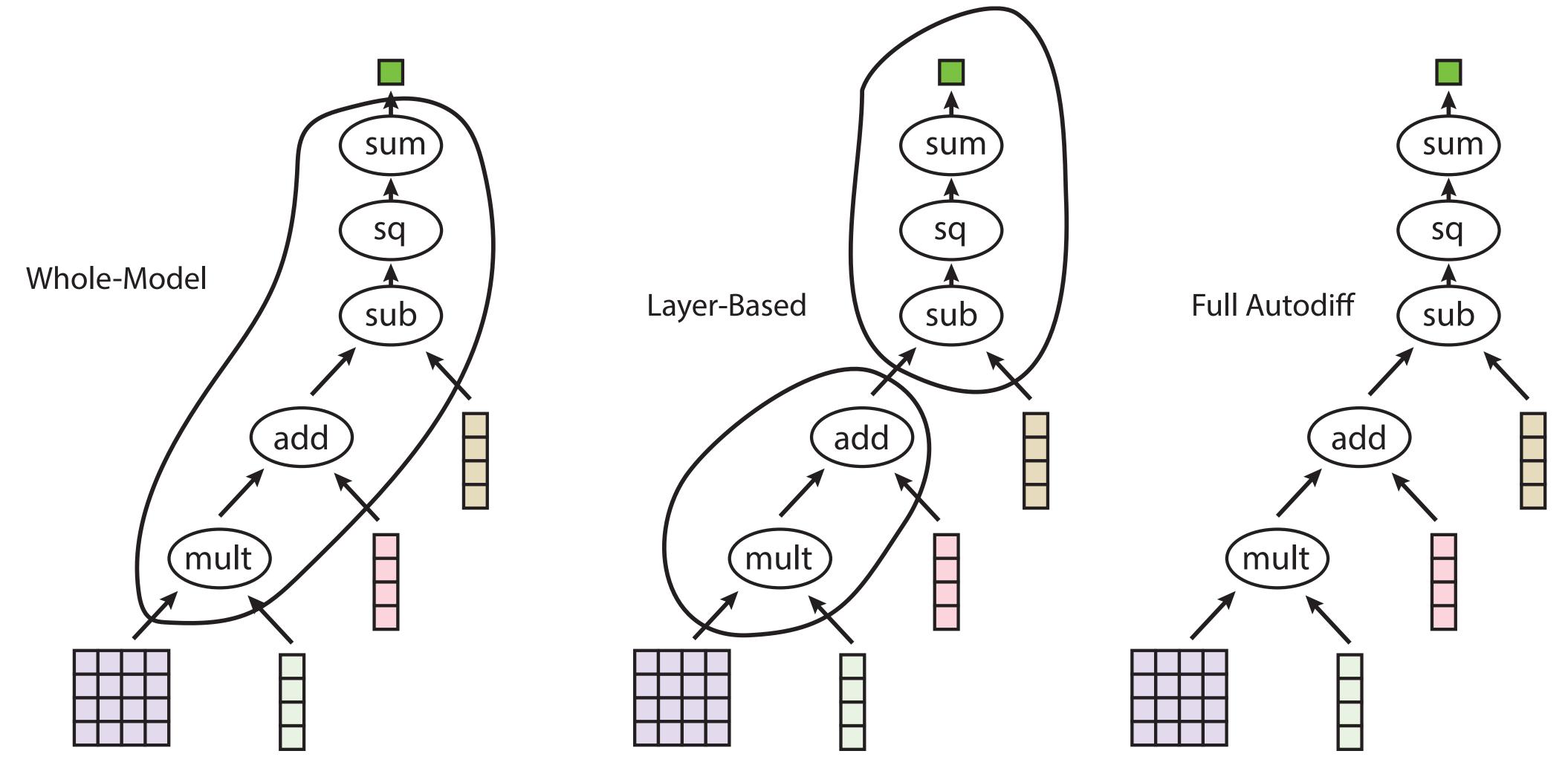
What is the graph?

Static Dataflow	Hybrid	JIT Dataflow	Sea of Nodes (Click & Paleczny, 1995)
NN	TensorFlow	Autograd	?
Caffe	Theano	Chainer	
Ops are nodes, edges are data, graph can't change	Ops are nodes, edges data, but the runtime has to work hard for control flow	Ops are nodes, edges are data, graph can change freely	Control flow and dataflow merged in this AST representation



WE WANT NO LIMITS ON THE MODELS WE WRITE

Why can't we mix different levels of granularity?



These divisions are usually the function of wrapper libraries



NEURAL NET THREE WAYS

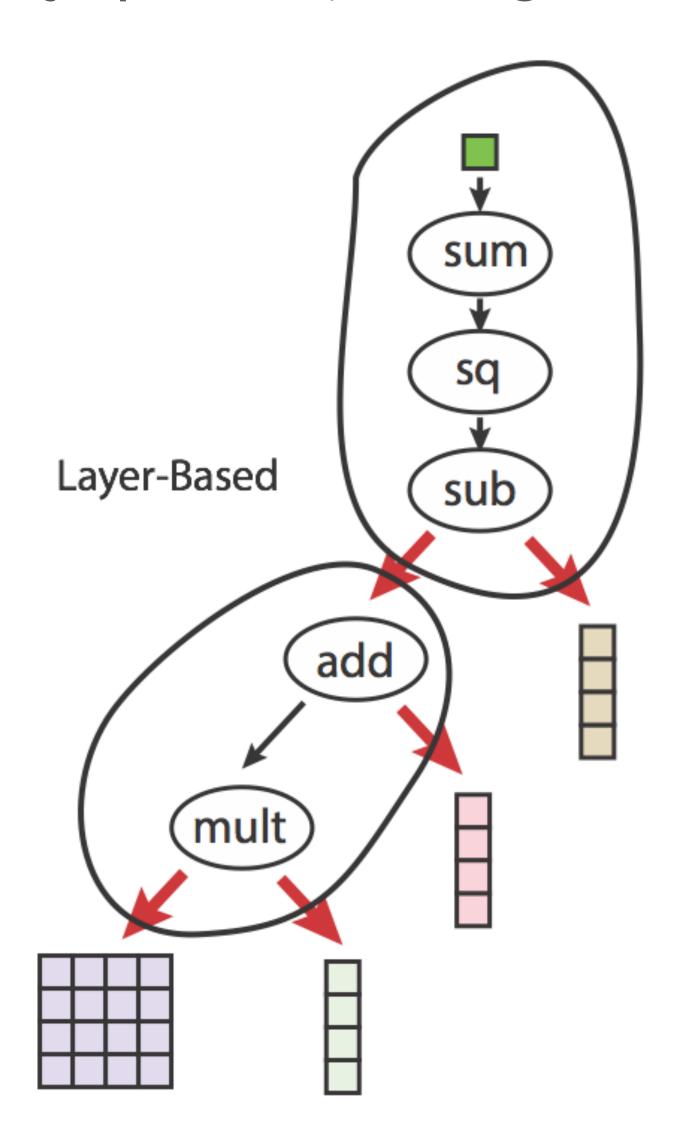
The most granular — using individual Torch functions

```
-- Define our parameters
local W1 = torch.FloatTensor(784,50):uniform(-1/math.sqrt(50),1/math.sqrt(50))
                                                                                                   sum
local B1 = torch.FloatTensor(50):fill(0)
local W2 = torch.FloatTensor(50,50):uniform(-1/math.sqrt(50),1/math.sqrt(50))
local B2 = torch.FloatTensor(50):fill(0)
                                                                                                    sq
local W3 = torch.FloatTensor(50,#classes):uniform(-1/math.sqrt(#classes),1/math.sqrt(#classes)
local B3 = torch.FloatTensor(#classes):fill(0)
                                                                                   Full Autodiff
local params = {
   W = \{W1, W2, W3\},\
   B = \{B1, B2, B3\},\
-- Define our neural net
local function mlp(params, input, target)
   local h1 = torch.tanh(input * params.W[1] + params.B[1])
   local h2 = torch.tanh(h1 * params.W[2] + params.B[2])
   local h3 = h2 * params.W[3] + params.B[3]
   local prediction = autograd.util.logSoftMax(h3)
   local loss = autograd.loss.logMultinomialLoss(prediction, target)
   return loss, prediction
end
```

NEURAL NET THREE WAYS

Composing pre-existing NN layers. If we need layers that have been highly optimized, this is good

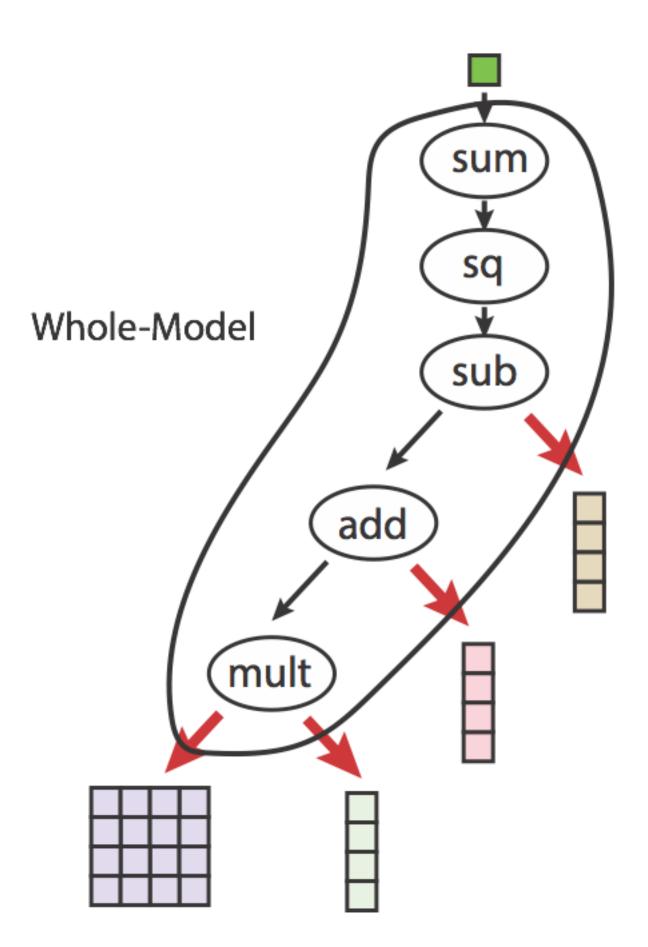
```
— Define our layers and their parameters
local params = {}
local linear1, linear2, linear3, acts1, acts2, lsm, lossf
linear1, params.linear1 = autograd.nn.Linear(784, 50)
acts1 = autograd.nn.Tanh()
linear2,params.linear2 = autograd.nn.Linear(50, 50)
acts2 = autograd.nn.Tanh()
linear3, params.linear3 = autograd.nn.Linear(50,#classes)
lsm = autograd.nn.LogSoftMax()
lossf = autograd.nn.ClassNLLCriterion()
-- Tie it all together
local function mlp(params)
   local h1 = acts1(linear1(params.linear1, params.x))
   local h2 = acts2(linear2(params.linear2, h1))
   local h3 = linear3(params.linear3, h2)
   local prediction = lsm(h3)
   local loss = lossf(prediction, target)
   return loss, prediction
end
```



NEURAL NET THREE WAYS

We can also compose entire networks together (e.g. image captioning, GANs)

```
-- Grab the neural network all at once
local f,params = autograd.model.NeuralNetwork({
   inputFeatures = 784,
   hiddenFeatures = {50,#classes},
   classifier = true,
lsm = autograd.nn.LogSoftMax()
lossf = autograd.nn.ClassNLLCriterion()
-- Link the model and the loss
local loss = function(params, input, target)
   local prediction = lsm(f(params, input))
   local loss = lossf(prediction, target)
   return loss, prediction
end
```

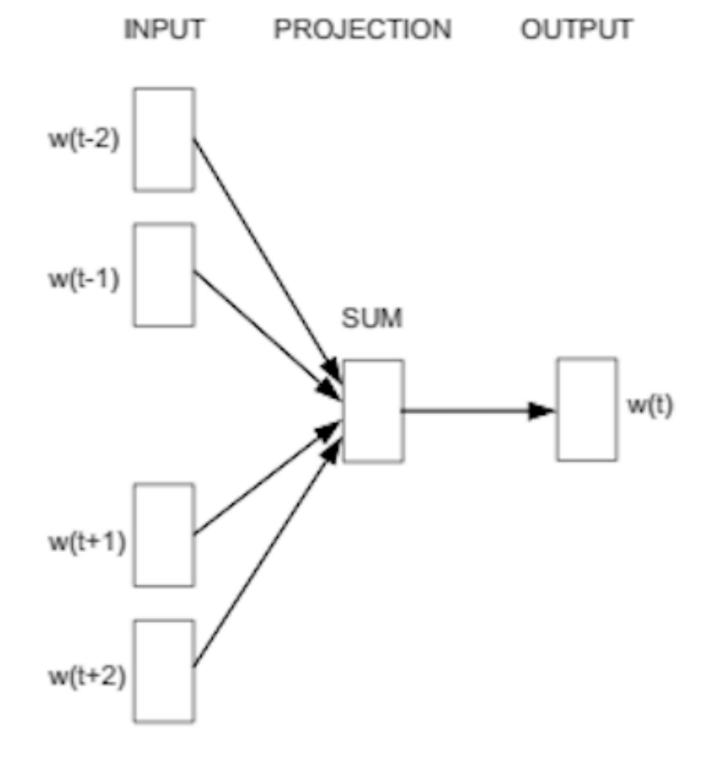


AUTODIFF & NLP

A trainable implementation of something like CBOW (just an example! CBOW loss is more complicated)

def cbow(w, i):

return abs(sum(
$$w[i] - (w[i-2] + w[i-1] + w[i+1] + w[i+2]))$$





IMPACT AT TWITTER

Prototyping without fear

- We try crazier, potentially high-payoff ideas more often, because autograd makes it essentially free to do so (can write "regular" numeric code, and automagically pass gradients through it)
- We use weird losses in production: large classification model uses a loss computed over a tree of class taxonomies
- Models trained with autograd running on large amounts of media at Twitter
- Often "fast enough", no penalty at test time
- · "Optimized mode" is nearly a compiler, but still a work in progress



OTHER AUTODIFF IDEAS

That haven't fully landed in machine learning yet

- **Checkpointing** don't save all of the intermediate values. Recompute them when you need them (memory savings, potentially speedup if compute is faster than load/store, possibly good with pointwise functions like ReLU). MXNet I *think* is the first to implement this generally for neural nets.
- **Mixing forward and reverse mode** called "cross-country elimination". No need to evaluate partial derivatives only in one direction! For diamond or hour-glass shaped compute graphs, perhaps dynamic programming can find the right order of partial derivative folding.
- **Stencils** image processing (convolutions) and element-wise ufuncs can be phrased as stencil operations. More efficient general-purpose implementations of differentiable stencils needed (compute graphics does this, Guenter 2007, extending with DeVito et al 2016).
- **Source-to-source** All neural net autodiff packages are either AOT or JIT graph construction, with operator overloading. The original autodiff (in FORTRAN, in the 80s) was source transformation. Considered gold-standard for performance in autodiff field. Challenge is control flow.
- **Higher-order gradients** hessian = grad(grad(f)). Not many efficient implementations, need to take advantage of sparsity. Fully closed versions in e.g. autograd, DiffSharp, Hype.



YOU SHOULD BE USING IT

It's easy to try

```
# Install anaconda if you don't have it (instructions here for OS X)
wget http://repo.continuum.io/miniconda/Miniconda-latest-MacOSX-x86_64.sh
sh Miniconda-latest-MacOSX-x86_64.sh -b -p $HOME/anaconda

# Add anaconda to your $PATH
export PATH=$HOME/anaconda/bin:$PATH

# Install Lua & Torch
conda install lua=5.2 lua-science -c alexbw

# Available versions of Lua: 2.0, 5.1, 5.2, 5.3
# 2.0 is LuaJIT
```



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- Anaconda is the de-facto distribution for scientific Python.
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- https://github.com/alexbw/conda-lua-recipes

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QUESTIONS?

Happy to help. Find me at:

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awiltschko@twitter.com
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