Universidad de Carabobo - Facultad de Ingenieria - Direccion de Postgrado
Programa: Maestria Matematica y Computacion - **Asignatura**: Introduccion al Calculo
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Titulo: Asignacion - **Parte 3)** Integrales

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> restart: with(Student[Calculus1]):
```

Ejercicio 3.1 $\int \frac{x^2}{\sqrt{x^2-2}}\,dx$

```
> Int(((x)^(2))/(sqrt((x)^(2)-2)),x); ShowSolution(%);
```

$$\int \frac{x^2}{\sqrt{x^2-2}}\,dx$$

$$\begin{aligned}
&\int \frac{x^2}{\sqrt{x^2-2}}\,dx \\
&= \int\left(-\frac{u}{4}+\frac{-4u^2-4}{4u^3}\right)du && \left[change,\ u=\sqrt{x^2-2}-x,\ u\right] \\
&= \int\left(-\frac{u}{4}-\frac{u^2+1}{u^3}\right)du && \left[rewrite,\ -\frac{u}{4}+\frac{-4u^2-4}{4u^3}=-\frac{u}{4}-\frac{u^2+1}{u^3}\right] \\
&= \int -\frac{u}{4}\,du+\int -\frac{u^2+1}{u^3}\,du && [sum] \\
&= -\frac{\left(\int u\,du\right)}{4}+\int -\frac{u^2+1}{u^3}\,du && [constantmultiple] \\
&= -\frac{u^2}{8}+\int -\frac{u^2+1}{u^3}\,du && [power] \\
&= -\frac{u^2}{8}-\left(\int \frac{u^2+1}{u^3}\,du\right) && [constantmultiple] \\
&= -\frac{u^2}{8}-\left(\int\left(\frac{1}{u}+\frac{1}{u^3}\right)du\right) && \left[rewrite,\ \frac{u^2+1}{u^3}=\frac{1}{u}+\frac{1}{u^3}\right] \\
&= -\frac{u^2}{8}-\left(\int \frac{1}{u}\,du\right)-\left(\int \frac{1}{u^3}\,du\right) && [sum] \\
&= -\frac{u^2}{8}-\ln(u)-\left(\int \frac{1}{u^3}\,du\right) && [power] \\
&= -\frac{u^2}{8}-\ln(u)+\frac{1}{2u^2} && [power] \\
&= -\frac{x^2}{4}+\frac{1}{4}+\frac{x\sqrt{x^2-2}}{4}-\ln\left(\sqrt{x^2-2}-x\right)+\frac{1}{2\left(\sqrt{x^2-2}-x\right)^2} && [revert]
\end{aligned} \tag{1.1}$$

Se agrega constante de integracion

```
> Int(((x)^(2))/(sqrt((x)^(2)-2)),x) = int(((x)^(2))/(sqrt((x)^(2)-2)),x) + C;
```

$$\int \frac{x^2}{\sqrt{x^2-2}}\,dx=\frac{1}{2}x\sqrt{x^2-2}+\ln\left(x+\sqrt{x^2-2}\right)+C \tag{1.2}$$

Nota: con la funcion int(f(x),x), expresa el resultado de la integral es una expresion equivalente al resultado del paso a paso.

Ejercicio 3.2 $\int\sqrt{(x^2+4)^3}\,dx$

```
> Int(sqrt(((x)^(2) + 4)^(3)),x); ShowSolution(%);
```

$$\int\sqrt{(x^2+4)^3}\,dx$$

$$\int \sqrt{(x^2+4)^3}\, dx$$

$$= \int (x^2+4)^{3/2}\, dx$$

$$= \int \left(-\frac{u^3}{16} - u + \frac{-96\,u^4 - 256\,u^2 - 256}{16\,u^5} \right) du$$

$$= \int \left(-\frac{u^3}{16} - u - \frac{2\,(3\,u^4 + 8\,u^2 + 8)}{u^5} \right) du$$

$$= \int -\frac{u^3}{16}\, du + \int -u\, du + \int -\frac{2\,(3\,u^4 + 8\,u^2 + 8)}{u^5}\, du$$

$$= -\frac{\left(\int u^3\, du\right)}{16} + \int -u\, du + \int -\frac{2\,(3\,u^4 + 8\,u^2 + 8)}{u^5}\, du$$

$$= -\frac{u^4}{64} + \int -u\, du + \int -\frac{2\,(3\,u^4 + 8\,u^2 + 8)}{u^5}\, du$$

$$= -\frac{u^4}{64} - \left(\int u\, du\right) + \int -\frac{2\,(3\,u^4 + 8\,u^2 + 8)}{u^5}\, du$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} + \int -\frac{2\,(3\,u^4 + 8\,u^2 + 8)}{u^5}\, du$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 2\left(\int \frac{3\,u^4 + 8\,u^2 + 8}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 2\left(\int \left(\frac{3}{u} + \frac{8}{u^3} + \frac{8}{u^5}\right) du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 2\left(\int \frac{3}{u}\, du\right) - 2\left(\int \frac{8}{u^3}\, du\right) - 2\left(\int \frac{8}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 6\left(\int \frac{1}{u}\, du\right) - 2\left(\int \frac{8}{u^3}\, du\right) - 2\left(\int \frac{8}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 6\ln(u) - 2\left(\int \frac{8}{u^3}\, du\right) - 2\left(\int \frac{8}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 6\ln(u) - 16\left(\int \frac{1}{u^3}\, du\right) - 2\left(\int \frac{8}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 6\ln(u) + \frac{8}{u^2} - 2\left(\int \frac{8}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 6\ln(u) + \frac{8}{u^2} - 16\left(\int \frac{1}{u^5}\, du\right)$$

$$= -\frac{u^4}{64} - \frac{u^2}{2} - 6\ln(u) + \frac{8}{u^2} + \frac{4}{u^4}$$

$$= -\frac{(x^2+4)^2}{64} + \frac{(x^2+4)^{3/2}\,x}{16} - \frac{3\,(x^2+4)\,x^2}{32} + \frac{x^3\sqrt{x^2+4}}{16} - \frac{x^4}{64} - x^2 - 2 + \sqrt{x^2+4}\,x - 6\ln\left(\sqrt{x^2+4} - x\right) + \frac{8}{\left(\sqrt{x^2+4} - x\right)^2} + \frac{4}{\left(\sqrt{x^2+4} - x\right)^4}$$

Se agrega constante de integracion.

```
> Int(sqrt(((x)^(2) + 4)^(3)),x) = int(sqrt(((x)^(2) + 4)^(3)),x) + C;
```

$$\int \sqrt{(x^2+4)^3}\, dx = \frac{1}{4}\, \frac{\sqrt{(x^2+4)^3}\left((x^2+4)^{3/2}\,x + 6\sqrt{x^2+4}\,x + 24\,\mathrm{arcsinh}\left(\frac{1}{2}\,x\right)\right)}{(x^2+4)^{3/2}} + C \qquad \textbf{(2.2)}$$

Nota: con la funcion int(f(x),x), agrupa los terminos del resultado del paso a paso de la integral, y los simplifica en una expresion equivalente de arcoseno hiperbolico.

Ejercicio 3.3 $\int \frac{1}{t^2\sqrt{3\,t^2+5}}\, dt$

```
> Int((1)/((t)^(2)*sqrt(3*(t)^(2)+5)),t); ShowSolution(%);
```

$$\int \frac{1}{t^2\sqrt{3\,t^2+5}}\, dt$$

$$\int \frac{1}{t^2\sqrt{3t^2+5}}\,dt$$

$$\begin{aligned} &= \int -\frac{1}{5}\,du && \left[change,\ 3+\frac{5}{t^2}=u^2,\ u\right] \\ &= -\frac{u}{5} && [constant] \\ &= -\frac{\sqrt{3t^2+5}}{5t} && [revert] \end{aligned} \tag{3.1}$$

Se agrega constante de integracion

```
> Int((1)/((t)^(2)*sqrt(3*(t)^(2)+5)),t) = int((1)/((t)^(2)*sqrt(3*(t)^(2)+5)),t) + C;
```

$$\int \frac{1}{t^2\sqrt{3t^2+5}}\,dt = -\frac{1}{5}\,\frac{\sqrt{3t^2+5}}{t} + C \tag{3.2}$$

Ejercicio 3.4 $\int \frac{1}{t\sqrt{3t-5}}\,dt$

```
> Int((1)/((t)*sqrt(3*(t)-5)),t); ShowSolution(%):
```

$$\int \frac{1}{t\sqrt{3t-5}}\,dt$$

$$\int \frac{1}{t\sqrt{3t-5}}\,dt$$

$$\begin{aligned} &= \int \frac{2}{u^2+5}\,du && [change,\ 3t-5=u^2,\ u] \\ &= 2\left(\int \frac{1}{u^2+5}\,du\right) && [constantmultiple] \\ &= 2\left(\int \frac{\sqrt{5}}{5}\,du1\right) && [change,\ u=\sqrt{5}\tan(u1),\ u1] \\ &= \frac{2\sqrt{5}\,u1}{5} && [constant] \\ &= \frac{2\sqrt{5}\arctan\left(\frac{u\sqrt{5}}{5}\right)}{5} && [revert] \\ &= \frac{2\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{3t-5}}{5}\right)}{5} && [revert] \end{aligned} \tag{4.1}$$

Se agrega constante de integracion

```
> Int((1)/((t)*sqrt(3*(t)-5)),t) = int((1)/((t)*sqrt(3*(t)-5)),t) + C;
```

$$\int \frac{1}{t\sqrt{3t-5}}\,dt = \frac{2}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\sqrt{3t-5}\right) + C \tag{4.2}$$

Ejercicio 3.5 $\int x^4 \ln(x)^4\,dx$

```
> Int(((x))^(4)*(ln(x))^(4),x); ShowSolution(%):
```

$$\int x^4 \ln(x)^4\,dx$$

$$\int x^4 \ln(x)^4 \, dx$$

$$= \frac{x^5 \ln(x)^4}{5} - \left(\int \frac{4 x^4 \ln(x)^3}{5} \, dx \right) \qquad \left[parts, \ln(x)^4, \frac{x^5}{5} \right]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 \left(\int x^4 \ln(x)^3 \, dx \right)}{5} \qquad [constantmultiple]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{4 \left(\int \frac{3 x^4 \ln(x)^2}{5} \, dx \right)}{5} \qquad \left[parts, \ln(x)^3, \frac{x^5}{5} \right]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{12 \left(\int x^4 \ln(x)^2 \, dx \right)}{25} \qquad [constantmultiple]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{12 x^5 \ln(x)^2}{125} - \frac{12 \left(\int \frac{2 x^4 \ln(x)}{5} \, dx \right)}{25} \qquad \left[parts, \ln(x)^2, \frac{x^5}{5} \right] \tag{5.1}$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{12 x^5 \ln(x)^2}{125} - \frac{24 \left(\int x^4 \ln(x) \, dx \right)}{125} \qquad [constantmultiple]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{12 x^5 \ln(x)^2}{125} - \frac{24 x^5 \ln(x)}{625} + \frac{24 \left(\int \frac{x^4}{5} \, dx \right)}{125} \qquad \left[parts, \ln(x), \frac{x^5}{5} \right]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{12 x^5 \ln(x)^2}{125} - \frac{24 x^5 \ln(x)}{625} + \frac{24 \left(\int x^4 \, dx \right)}{625} \qquad [constantmultiple]$$

$$= \frac{x^5 \ln(x)^4}{5} - \frac{4 x^5 \ln(x)^3}{25} + \frac{12 x^5 \ln(x)^2}{125} - \frac{24 x^5 \ln(x)}{625} + \frac{24 x^5}{3125} \qquad [power]$$

Se agrega constante de integracion

```
> Int(((x))^(4)*(ln(x))^(4),x) = int(((x))^(4)*(ln(x))^(4),x) + C;
```

$$\int x^4 \ln(x)^4 \, dx = \frac{1}{5} x^5 \ln(x)^4 - \frac{4}{25} x^5 \ln(x)^3 + \frac{12}{125} x^5 \ln(x)^2 - \frac{24}{625} x^5 \ln(x) + \frac{24}{3125} x^5 + C \tag{5.2}$$

Ejercicio 3.6 $\int x^7 e^{x^2} \, dx$

```
> Int(((x))^(7)*(exp((x)^(2))),x); ShowSolution(%):
```

$$\int x^7 e^{x^2} \, dx$$

$$\tag{6.1}$$

$$\int x^7 e^{x^2}\,dx$$

$$= \frac{x^6 e^{x^2}}{2} - \left(\int 3 e^{x^2} x^5\,dx\right) \qquad \left[parts, x^6, \frac{e^{x^2}}{2}\right]$$

$$= \frac{x^6 e^{x^2}}{2} - 3\left(\int e^{x^2} x^5\,dx\right) \qquad [constantmultiple]$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 3\left(\int 2 e^{x^2} x^3\,dx\right) \qquad \left[parts, x^4, \frac{e^{x^2}}{2}\right]$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 6\left(\int e^{x^2} x^3\,dx\right) \qquad [constantmultiple]$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 3x^2 e^{x^2} - 6\left(\int x e^{x^2}\,dx\right) \qquad \left[parts, x^2, \frac{e^{x^2}}{2}\right] \tag{6.1}$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 3x^2 e^{x^2} - 6\left(\int \frac{e^u}{2}\,du\right) \qquad [change, u = x^2, u]$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 3x^2 e^{x^2} - 3\left(\int e^u\,du\right) \qquad [constantmultiple]$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 3x^2 e^{x^2} - 3e^u \qquad [\exp]$$

$$= \frac{x^6 e^{x^2}}{2} - \frac{3x^4 e^{x^2}}{2} + 3x^2 e^{x^2} - 3e^{x^2} \qquad [revert]$$

Se agrega constante de integracion

```
> Int(((x))^(7)*(exp((x)^(2))),x) = int(((x))^(7)*(exp((x)^(2))),x) + C
```

$$\int x^7 e^{x^2}\,dx = \frac{1}{2}\left(x^6 - 3x^4 + 6x^2 - 6\right) e^{x^2} + C \tag{6.2}$$

Ejercicio 3.7 $\int x\sqrt{-x^2+2x}\,dx$

```
> Int((x)*(sqrt(2*x-(x)^(2))),x); ShowSolution(%):
```

$$\int x\sqrt{-x^2+2x}\,dx$$

(7.1)

$$\int x\sqrt{-x^2+2x}\,dx$$

$$= \int \sqrt{-u^2+1}\,(u+1)\,du \qquad [change, u = x-1, u]$$

$$= \int \left(-\sin(u1)^3 - \sin(u1)^2 + \sin(u1) + 1\right) du1 \qquad [change, u = \sin(u1), u1]$$

$$= \int -\sin(u1)^3\,du1 + \int -\sin(u1)^2\,du1 + \int \sin(u1)\,du1 + \int 1\,du1 \qquad [sum]$$

$$= -\left(\int \sin(u1)^3\,du1\right) + \int -\sin(u1)^2\,du1 + \int \sin(u1)\,du1 + \int 1\,du1 \qquad [constantmultiple]$$

$$= -\left(\int \left(1-\cos(u1)^2\right)\sin(u1)\,du1\right) + \int -\sin(u1)^2\,du1 + \int \sin(u1)\,du1 + \int 1\,du1 \qquad [rewrite, \sin(u1)^2 = 1-\cos(u1)^2]$$

$$= -\left(\int \left(-\sin(u1)\cos(u1)^2 + \sin(u1)\right) du1\right) + \int -\sin(u1)^2\,du1 + \int \sin(u1)\,du1 + \int 1\,du1 \qquad [rewrite, (1-\cos(u1)^2)\sin(u1) = -\sin(u1)\cos(u1)^2 + \sin(u1)]$$

$$= -\left(\int -\sin(u1)\cos(u1)^2\,du1\right) + \int -\sin(u1)^2\,du1 + \int 1\,du1 \qquad [sum]$$

$$= \int \sin(u1)\cos(u1)^2\,du1 + \int -\sin(u1)^2\,du1 + \int 1\,du1 \qquad [constantmultiple]$$

$$= \int -u2^2\,du2 + \int -\sin(u1)^2\,du1 + \int 1\,du1 \qquad [change, u2 = \cos(u1), u2]$$

$$= -\left(\int u2^2\,du2\right) + \int -\sin(u1)^2\,du1 + \int 1\,du1 \qquad [constantmultiple]$$

$$= -\frac{u2^3}{3} + \int -\sin(u1)^2\,du1 + \int 1\,du1 \qquad [power]$$

$$= -\frac{\cos(u1)^3}{3} + \int -\sin(u1)^2\,du1 + \int 1\,du1 \qquad [revert]$$

$$= -\frac{\cos(u1)^3}{3} - \left(\int \sin(u1)^2\,du1\right) + \int 1\,du1 \qquad [constantmultiple]$$

$$= -\frac{\cos(u1)^3}{3} - \left(\int \left(\frac{1}{2} - \frac{\cos(2\,u1)}{2}\right) du1\right) + \int 1\,du1 \qquad \left[rewrite, \sin(u1)^2 = \frac{1}{2} - \frac{\cos(2\,u1)}{2}\right] \tag{7.1}$$

$$= -\frac{\cos(u1)^3}{3} - \left(\int \frac{1}{2}\,du1\right) - \left(\int -\frac{\cos(2\,u1)}{2}\,du1\right) + \int 1\,du1 \qquad [sum]$$

$$= -\frac{\cos(u1)^3}{3} - \frac{u1}{2} - \left(\int -\frac{\cos(2\,u1)}{2}\,du1\right) + \int 1\,du1 \qquad [constant]$$

$$= -\frac{\cos(u1)^3}{3} - \frac{u1}{2} + \frac{\left(\int \cos(2\,u1)\,du1\right)}{2} + \int 1\,du1 \qquad [constantmultiple]$$

$$= -\frac{\cos(u1)^3}{3} - \frac{u1}{2} + \frac{\left(\int \frac{\cos(u2)}{2}\,du2\right)}{2} + \int 1\,du1 \qquad [change, u2 = 2\,u1, u2]$$

$$= -\frac{\cos(u1)^3}{3} - \frac{u1}{2} + \frac{\left(\int \cos(u2)\,du2\right)}{4} + \int 1\,du1 \qquad [constantmultiple]$$

$$= -\frac{\cos(u1)^3}{3} - \frac{u1}{2} + \frac{\sin(u2)}{4} + \int 1\,du1 \qquad [\cos]$$

$$= -\frac{\cos(u1)^3}{3} - \frac{u1}{2} + \frac{\sin(2\,u1)}{4} + \int 1\,du1 \qquad [revert]$$

$$= -\frac{\cos(u1)^3}{3} + \frac{u1}{2} + \frac{\sin(2\,u1)}{4} \qquad [constant]$$

$$= -\frac{(-u^2+1)^{3\backslash 2}}{3} + \frac{\arcsin(u)}{2} + \frac{u\sqrt{-u^2+1}}{2} \qquad [revert]$$

$$= -\frac{(-x^2+2x)^{3\backslash 2}}{3} + \frac{\arcsin(x-1)}{2} + \frac{x\sqrt{-x^2+2x}}{2} - \frac{\sqrt{-x^2+2x}}{2} \qquad [revert]$$

Se agrega constante de integracion

```
> Int((x)*(sqrt(2*x-(x)^(2))),x) = int((x)*(sqrt(2*x-(x)^(2))),x) + C;
```

$$\int x\sqrt{-x^2+2x}\,dx = -\frac{1}{3}\left(-x^2+2x\right)^{3/2} - \frac{1}{4}\left(-2x+2\right)\sqrt{-x^2+2x} + \frac{1}{2}\arcsin(x-1) + C \tag{7.2}$$

Ejercicio 3.8 $\int \frac{\sqrt{-x^2+2x}}{x^2}\,dx$

```
> Int(((sqrt(2*x-(x)^(2)))/((x))^(2)),x); ShowSolution(%):
```

$$\int \frac{\sqrt{-x^2+2x}}{x^2}\,dx$$

$$\int \frac{\sqrt{-x^2+2x}}{x^2}\,dx$$

$$\begin{aligned}
&= \int\left(\frac{2}{u^2+1}-2\right)du && \left[change, \frac{2}{x}-1=u^2, u\right]\\
&= \int \frac{2}{u^2+1}\,du + \int(-2)\,du && [sum]\\
&= 2\left(\int \frac{1}{u^2+1}\,du\right) + \int(-2)\,du && [constantmultiple]\\
&= 2\left(\int 1\,du1\right) + \int(-2)\,du && [change, u=\tan(u1), u1]\\
&= 2\,u1 + \int(-2)\,du && [constant]\\
&= 2\arctan(u) + \int(-2)\,du && [revert]\\
&= 2\arctan(u) - 2u && [constant]\\
&= -\frac{2\left(-\arctan\left(\frac{\sqrt{-x(x-2)}}{x}\right)x+\sqrt{-x(x-2)}\right)}{x} && [revert]
\end{aligned} \tag{8.1}$$

Se agrega constante de integracion

```
> Int(((sqrt(2*x-(x)^(2)))/((x))^(2)),x)= int(((sqrt(2*x-(x)^(2)))/((x))^(2)),
  x) + C;
```

$$\int \frac{\sqrt{-x^2+2x}}{x^2}\,dx = -\frac{(-x^2+2x)^{3/2}}{x^2} - \sqrt{-x^2+2x} - \arcsin(x-1) + C \tag{8.2}$$

Nota: con la funcion int(f(x),x), expresa el resultado de la integral en una expresion equivalente al resultado del paso a paso.

Ejercicio 3.9 $\int \frac{1}{(-x^2+4x)^{3/2}}\,dx$

```
> Int(((1))/(sqrt((4*x-(x)^(2)))^(3)),x); ShowSolution(%):
```

$$\int \frac{1}{(-x^2+4x)^{3/2}}\,dx$$

$$\int \frac{1}{(-x^2+4x)^{3/2}}\,dx$$

$$\begin{aligned}
&= \int\left(\frac{1}{8}+\frac{1}{8u^2}\right)du && \left[change, \frac{4}{x}-1=u^2, u\right]\\
&= \int \frac{1}{8}\,du + \int \frac{1}{8u^2}\,du && [sum]\\
&= \frac{u}{8} + \int \frac{1}{8u^2}\,du && [constant]\\
&= \frac{u}{8} + \frac{\left(\int \frac{1}{u^2}\,du\right)}{8} && [constantmultiple]\\
&= \frac{u}{8} - \frac{1}{8u} && [power]\\
&= \frac{\sqrt{-x(x-4)}}{8x} - \frac{x}{8\sqrt{-x(x-4)}} && [revert]
\end{aligned} \tag{9.1}$$

Se agrega constante de integracion

```
> Int(((1))/(sqrt((4*x-(x)^(2)))^(3)),x)= int(((1))/(sqrt((4*x-(x)^(2)))^(3)),
  x) + C;
```

$$\int \frac{1}{(-x^2+4x)^{3/2}}\,dx = -\frac{1}{4}\,\frac{x\,(x-4)\,(x-2)}{(-x^2+4x)^{3/2}} + C \tag{9.2}$$

Nota: con la funcion int(f(x),x), expresa el resultado de la integral en una expresion equivalente al resultado del paso a paso, donde simplifica y racionaliza la expresion algebraica.

Ejercicio 3.10 $\int \frac{1}{(-x^2+4x)^2}\,dx$

```
> Int(((1))/(sqrt((4*x-(x)^(2)))^(4)),x); ShowSolution(%):
```

$$\int \frac{1}{(-x^2+4x)^2}\,dx$$

$$\begin{aligned}
&\int \frac{1}{(-x^2+4x)^2}\,dx \\
&= \int\left(-\frac{1}{32\,(x-4)}+\frac{1}{16\,(x-4)^2}+\frac{1}{32\,x}+\frac{1}{16\,x^2}\right)dx && [partialfractions] \\
&= \int -\frac{1}{32\,(x-4)}\,dx+\int\frac{1}{16\,(x-4)^2}\,dx+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [sum] \\
&= -\frac{\left(\int\frac{1}{x-4}\,dx\right)}{32}+\int\frac{1}{16\,(x-4)^2}\,dx+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [constantmultiple] \\
&= -\frac{\left(\int\frac{1}{u}\,du\right)}{32}+\int\frac{1}{16\,(x-4)^2}\,dx+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [change, u=x-4, u] \\
&= -\frac{\ln(u)}{32}+\int\frac{1}{16\,(x-4)^2}\,dx+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [power] \\
&= -\frac{\ln(x-4)}{32}+\int\frac{1}{16\,(x-4)^2}\,dx+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [revert] \\
&= -\frac{\ln(x-4)}{32}+\frac{\left(\int\frac{1}{(x-4)^2}\,dx\right)}{16}+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [constantmultiple] \\
&= -\frac{\ln(x-4)}{32}+\frac{\left(\int\frac{1}{u^2}\,du\right)}{16}+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [change, u=x-4, u] \\
&= -\frac{\ln(x-4)}{32}-\frac{1}{16\,u}+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [power] \\
&= -\frac{\ln(x-4)}{32}-\frac{1}{16\,(x-4)}+\int\frac{1}{32\,x}\,dx+\int\frac{1}{16\,x^2}\,dx && [revert] \\
&= -\frac{\ln(x-4)}{32}-\frac{1}{16\,(x-4)}+\frac{\left(\int\frac{1}{x}\,dx\right)}{32}+\int\frac{1}{16\,x^2}\,dx && [constantmultiple] \\
&= -\frac{\ln(x-4)}{32}-\frac{1}{16\,(x-4)}+\frac{\ln(x)}{32}+\int\frac{1}{16\,x^2}\,dx && [power] \\
&= -\frac{\ln(x-4)}{32}-\frac{1}{16\,(x-4)}+\frac{\ln(x)}{32}+\frac{\left(\int\frac{1}{x^2}\,dx\right)}{16} && [constantmultiple] \\
&= -\frac{\ln(x-4)}{32}-\frac{1}{16\,(x-4)}+\frac{\ln(x)}{32}-\frac{1}{16\,x} && [power]
\end{aligned} \tag{10.1}$$

Se agrega constante de integracion

```
> Int(((1))/(sqrt((4*x-(x)^(2)))^(4)),x)= int(((1))/(sqrt((4*x-(x)^(2)))^(4)),
  x) + C;
```

(10.2)

$$\int \frac{1}{(-x^2+4x)^2}\,dx = -\frac{1}{32}\ln(x-4) - \frac{1}{16(x-4)} + \frac{1}{32}\ln(x) - \frac{1}{16x} + C \qquad \textbf{(10.2)}$$