Calculation of Covariance and Correlation

Given data: $(1, 5), (4, 18), (3, 17), (8, 35), (1, 6), (3, 20)$

Step 1: Calculate sample means

$$\bar{X} = \frac{1 + 4 + 3 + 8 + 1 + 3}{6} = \frac{20}{6} = \frac{10}{3}$$

$$\bar{Y} = \frac{5 + 18 + 17 + 35 + 6 + 20}{6} = \frac{101}{6}$$

Step 2: Calculate deviations and required sums

$$\sum_{i=1}^{6}(X_i - \bar{X})(Y_i - \bar{Y}) = (1 - \frac{10}{3})(5 - \frac{101}{6}) + (4 - \frac{10}{3})(18 - \frac{101}{6}) \tag{1}$$

$$+ (3 - \frac{10}{3})(17 - \frac{101}{6}) + (8 - \frac{10}{3})(35 - \frac{101}{6}) \tag{2}$$

$$+ (1 - \frac{10}{3})(6 - \frac{101}{6}) + (3 - \frac{10}{3})(20 - \frac{101}{6}) \tag{3}$$

Converting to common denominators:

$$X_i - \bar{X} : -\frac{7}{3}, \frac{2}{3}, -\frac{1}{3}, \frac{14}{3}, -\frac{7}{3}, -\frac{1}{3}$$

$$Y_i - \bar{Y} : -\frac{71}{6}, \frac{7}{6}, \frac{1}{6}, \frac{109}{6}, -\frac{65}{6}, \frac{19}{6}$$

$$\sum_{i=1}^{6}(X_i - \bar{X})(Y_i - \bar{Y}) = \frac{497}{18} + \frac{14}{18} - \frac{1}{18} + \frac{1526}{18} + \frac{455}{18} - \frac{19}{18} = \frac{2472}{18} = \frac{412}{3}$$

$$\sum_{i=1}^{6}(X_i - \bar{X})^2 = \frac{49}{9} + \frac{4}{9} + \frac{1}{9} + \frac{196}{9} + \frac{49}{9} + \frac{1}{9} = \frac{300}{9} = \frac{100}{3}$$

$$\sum_{i=1}^{6}(Y_i - \bar{Y})^2 = \frac{5041}{36} + \frac{49}{36} + \frac{1}{36} + \frac{11881}{36} + \frac{4225}{36} + \frac{361}{36} = \frac{21558}{36} = \frac{3593}{6}$$

a. Sample Covariance

$$\text{Cov}(X, Y) = \frac{1}{n-1}\sum_{i=1}^{6}(X_i - \bar{X})(Y_i - \bar{Y}) = \frac{1}{5} \cdot \frac{412}{3} = \frac{412}{15}$$

b. Sample Correlation Coefficient

$$s_X = \sqrt{\frac{1}{5} \cdot \frac{100}{3}} = \sqrt{\frac{20}{3}} = \frac{2\sqrt{15}}{3}$$

$$s_Y = \sqrt{\frac{1}{5} \cdot \frac{3593}{6}} = \sqrt{\frac{3593}{30}}$$

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{s_X \cdot s_Y} = \frac{\frac{412}{15}}{\frac{2\sqrt{15}}{3} \cdot \sqrt{\frac{3593}{30}}} = \frac{412}{15} \cdot \frac{3}{2\sqrt{15}} \cdot \frac{\sqrt{30}}{\sqrt{3593}}$$

$$\rho(X, Y) = \frac{412\sqrt{2}}{\sqrt{15 \cdot 3593}} \approx 0.891$$

Final Answers:

$$\text{Cov}(X, Y) = \frac{412}{15} \approx 27.47$$

$$\rho(X, Y) \approx 0.891$$