1. Expectation for Uniform Distribution on [a,b]

For a uniform distribution on $[a, b]$, the probability density function is:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

The expectation is:

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx = \int_a^b x \cdot \frac{1}{b-a}\, dx$$

$$E[X] = \frac{1}{b-a} \int_a^b x\, dx = \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$

$$E[X] = \frac{1}{b-a} \cdot \frac{1}{2}(b^2 - a^2) = \frac{1}{b-a} \cdot \frac{1}{2}(b-a)(b+a)$$

$$E[X] = \frac{b+a}{2}$$

2. Distribution Function for Exponential Distribution with Parameter λ

For an exponential distribution with parameter $\lambda > 0$, the probability density function is:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

The cumulative distribution function (CDF) is:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$$

For $x < 0$: $F(x) = 0$

For $x \geq 0$:

$$F(x) = \int_0^x \lambda e^{-\lambda t}\, dt$$

$$F(x) = \lambda \left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^x = \lambda \cdot \frac{1}{-\lambda}(e^{-\lambda x} - e^0)$$

$$F(x) = -(e^{-\lambda x} - 1) = 1 - e^{-\lambda x}$$

Therefore, the distribution function is:

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$