1. Sample Space of X

$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

2. Probabilities P(X = x)

$$P(X = 2) = \frac{1}{36} \tag{1}$$
$$P(X = 3) = \frac{2}{36} = \frac{1}{18} \tag{2}$$
$$P(X = 4) = \frac{3}{36} = \frac{1}{12} \tag{3}$$
$$P(X = 5) = \frac{4}{36} = \frac{1}{9} \tag{4}$$
$$P(X = 6) = \frac{5}{36} \tag{5}$$
$$P(X = 7) = \frac{6}{36} = \frac{1}{6} \tag{6}$$
$$P(X = 8) = \frac{5}{36} \tag{7}$$
$$P(X = 9) = \frac{4}{36} = \frac{1}{9} \tag{8}$$
$$P(X = 10) = \frac{3}{36} = \frac{1}{12} \tag{9}$$
$$P(X = 11) = \frac{2}{36} = \frac{1}{18} \tag{10}$$
$$P(X = 12) = \frac{1}{36} \tag{11}$$

3. PMF and CDF

PMF:

$$f(x) = P(X = x) = \begin{cases} \frac{1}{36} & \text{if } x = 2, 12 \\ \frac{2}{36} & \text{if } x = 3, 11 \\ \frac{3}{36} & \text{if } x = 4, 10 \\ \frac{4}{36} & \text{if } x = 5, 9 \\ \frac{5}{36} & \text{if } x = 6, 8 \\ \frac{6}{36} & \text{if } x = 7 \\ 0 & \text{otherwise} \end{cases}$$

CDF:

$$F(x) = P(X \leq x) = \begin{cases} 0 & \text{if } x < 2 \\ \frac{1}{36} & \text{if } 2 \leq x < 3 \\ \frac{3}{36} & \text{if } 3 \leq x < 4 \\ \frac{6}{36} & \text{if } 4 \leq x < 5 \\ \frac{10}{36} & \text{if } 5 \leq x < 6 \\ \frac{15}{36} & \text{if } 6 \leq x < 7 \\ \frac{21}{36} & \text{if } 7 \leq x < 8 \\ \frac{26}{36} & \text{if } 8 \leq x < 9 \\ \frac{30}{36} & \text{if } 9 \leq x < 10 \\ \frac{33}{36} & \text{if } 10 \leq x < 11 \\ \frac{35}{36} & \text{if } 11 \leq x < 12 \\ 1 & \text{if } x \geq 12 \end{cases}$$

4. Bernoulli Distribution

$$\text{a. } E(X) = p \tag{12}$$
$$\text{b. } E(X^2) = p \tag{13}$$
$$\text{c. } \text{Var}(X) = p(1-p) \tag{14}$$

5. Poisson Distribution

The smallest integer value of λ is 7.

Working:

$$P(X \geq 2) = 1 - P(X=0) - P(X=1) \tag{15}$$
$$= 1 - e^{-\lambda} - \lambda e^{-\lambda} \tag{16}$$
$$= 1 - e^{-\lambda}(1+\lambda) > 0.99 \tag{17}$$
$$\therefore e^{-\lambda}(1+\lambda) < 0.01 \tag{18}$$
$$\text{For } \lambda = 7 : e^{-7}(1+7) = 8e^{-7} \approx 0.0073 < 0.01 \checkmark \tag{19}$$