Logic Questions

Question 1: Translate the following sentences into propositional logic

a) The flat is vacant.

Let V = "The flat is vacant"

Translation: V

This is straightforward - a simple atomic proposition.

b) The flat can be let only if it is vacant and has been cleaned.

Let V = "The flat is vacant"

Let C = "The flat has been cleaned"

Let L = "The flat can be let"

The phrase "can be let only if" indicates a conditional relationship. "A only if B" means "if A then B", or A → B.

Translation: L → (V ∧ C)

c) Don't drink and drive!

Let D = "You drink"

Let R = "You drive"

This imperative sentence is commanding someone not to do both actions simultaneously.

Translation: ¬(D ∧ R)

Question 2 [6]: Translate the following sentences into predicate logic

Your formalizations should be as detailed as possible.

a) Some teachers are strict and others are not.

This sentence asserts two things:

Some teachers are strict

Some teachers are not strict

There exists someone who is a teacher and is strict, and there exists someone who is a teacher and is not strict.

Let Domain: All people

Let T(x) = "x is a teacher"

Let S(x) = "x is strict"

Translation: $\exists x(T(x) \land S(x)) \land \exists y(T(y) \land \neg S(y))$

b) Some dogs' owners do not collect their dog's litter

There exists a person x and a dog y and x owns y and x does not collect y's litter.

Let Domain: All people and dogs Let P(x) = "x is a person" Let D(x) = "x is a dog" Let O(x,y) = "x owns y" Let C(x,y) = "x collects the litter of y"

Translation: $\exists x \exists y(P(x) \land D(y) \land O(x,y) \land \neg C(x,y))$

c) All animals are equal.

We interpret this as a universal statement about all pairs of animals.

For all x and y, if x is an animal and y is an animal, then x is equal to y.

Let Domain: All entities Let A(x) = "x is an animal" Let E(x,y) = "x is equal to y"

Translation: $\forall x \forall y((A(x) \land A(y)) \rightarrow E(x,y))$

Question 3 [6]: Consider the following formula of propositional logic

$P = ((A \lor B) \land C \rightarrow (A \land B) \lor C)$

a) Assume that A is true, and B,C are false. Is P true or false? Briefly explain why.

Given: A = T, B = F, C = F

$A \lor B = T \lor F = T$

$(A \lor B) \land C = T \land F = F$

$A \land B = T \land F = F$

$(A \land B) \lor C = F \lor F = F$

$P = ((A \lor B) \land C) \rightarrow ((A \land B) \lor C) = F \rightarrow F = T$

P is true.

This is because a conditional statement (→) is true whenever the antecedent is false, regardless of the truth value of the consequent.

b) Which truth values for A,B,C will result in P being false?

For P to be false, we need the antecedent $((A \lor B) \land C)$ to be true and the consequent $((A \land B) \lor C)$ to be false.

For the antecedent to be true:

(A ∨ B) must be true, so at least one of A or B must be true

C must be true

For the consequent to be false:

(A ∧ B) must be false, so at least one of A or B must be false

C must be false

However, this creates a contradiction: C cannot be both true (for the antecedent) and false (for the consequent) simultaneously.

Therefore, **P is always true (it's a tautology)** - there are no truth value assignments that make P false.