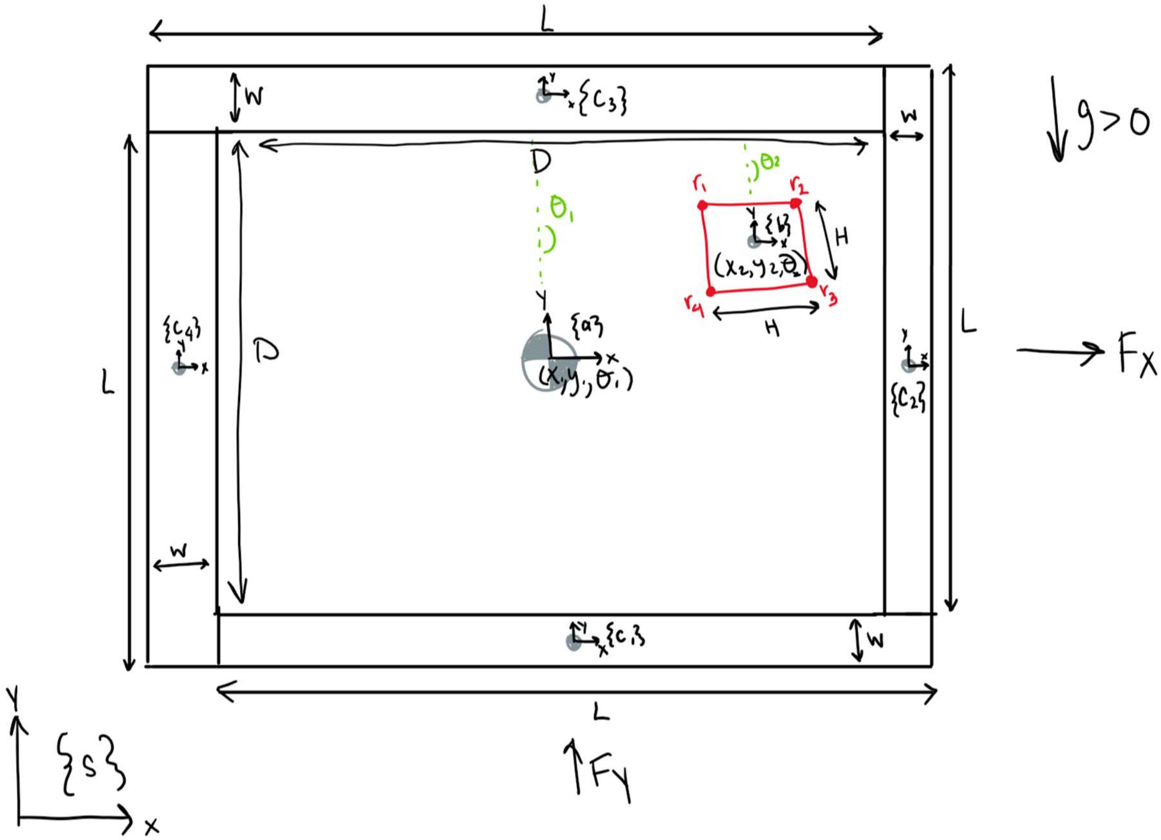
The project completed as the final assessment was the default jacks game.

<u>System Model:</u>



Configuration: $q = (x_1, y_1, \theta_1, x_2, y_2, \theta_2)$, where q is in the s frame and all terms are with respect to time.

Parameters:

$H = 0.5\text{m}$, $L = 5\text{m}$, $W = 0.25\text{m}$, $m_{\text{cube side}} = 2\text{g}$, $m_{\text{jack}} = 0.5\text{g}$, $g = 9.8\text{m/s}^2$

<u>Rigid body transformations:</u>

g_{sa}: $R = \theta_1$, $p = (x_1, y_1, 0)$

g_{sb}: $R = \theta_2$, $p = (x_2, y_2, 0)$

g_{ac1}: $R=0$, $p = (W/2, -L/2, 0)$

g_{ac2}: $R=0$, $p = (L/2, W/2, 0)$

g_{ac3}: $R=0$, $p = (-W/2, L/2, 0)$

g_{ac4}: $R=0$, $p = (-L/2, -W/2, 0)$

g_{sci}: $g_{sa} * g_{aci}$ (for all i = [1, 4])

g_{cib}: $g_{sci}^{-1} * g_{sb}$ (for all i = [1, 4])

Euler-Lagrange equations:

$\frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) - \frac{dL}{dq} = F$, where the forcing term is an applied external force vector F(q).

$F(q) = \begin{bmatrix} \cos\left(\frac{t}{2}\right) & m_{cube} g y_1 + \cos\left(\frac{t}{2}\right) & 0 & 0 & 0 & 0 \end{bmatrix}$

The forcing term was generated to "shake" the cube while offsetting the force of gravity.

To generate the Lagrangian of the system, the kinetic and potential energies of the cup and jack were assessed separately.

Cup:

Rotational inertia of rectangle = $\frac{1}{12} m L^2 W^2$

To find the inertia of the cup, I_{cup}, it is treated as subtracting the inertia of a rectangle of the inner dimensions from a rectangle of the outer dimensions. Both the rectangles were treated as having a mass equal to the sum of the masses of the four walls, m_{cup}.

Mass-inertia matrix:

$$\mathfrak{I}_{cup} = \begin{vmatrix} m_{cup} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{cup} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{cup} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{cup} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{cup} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{cup} \end{vmatrix}$$

Energies:

$$KE_{cup} = \frac{1}{2} \mathcal{V}_{b,cup}^{T} \mathfrak{I}_{cup} \mathcal{V}_{b,cup}$$

$$PE_{cup} = m_{cup} g y_1$$

Jack:

Rotational inertia of the jack was found using the rectangle formula and the jack dimensions.

Mass-inertia matrix:

$$\mathfrak{I}_{jack} = \begin{vmatrix} m_{jack} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{jack} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{jack} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{jack} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{jack} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{jack} \end{vmatrix}$$

Energies:

$$KE_{jack} = \frac{1}{2}\mathcal{V}_{b,jack}^{T}\Im_{jack}\mathcal{V}_{b,jack}$$

$$PE_{jack} = m_{jack}gy_2$$

L = (KE_{jack} + KE_{cup}) – (PE_{jack} + PE_{cup})

Impact Update:

Given an impact condition Φ(q)=0, the impact update equations are:

$$\frac{dL}{d\dot{q}}|_{\tau-}^{\tau+} = \lambda\frac{d\Phi}{dq}$$

$$\frac{dL}{d\dot{q}}\cdot\dot{q} - L(q,\dot{q})|_{\tau-}^{\tau+} = 0$$

To determine the Φ(q) at each impact, each corner of the jack was tracked with respect to each wall. If a corner was found to have impacted a wall, the limiting condition of that wall and the given corner was used as Φ(q). An impact was defined as the position of a corner being within W/2 m of a wall origin on the axis entering into the free space of the cup (ex. C_1:+y axis).

The two equations were then solved for λ and $\dot{q}(\tau^+)$, and these values were substituted into the previous state equation, keeping the same q(τ^-), as position does not have discontinuities with impact.

Brief dynamics functions overview:

integrate: runge kutta integration scheme

impact_conditions: using a distance threshold of 0.1m, checks the position of each corner of the jack with respect to the coordinate frame of each wall, outputs Φ(q) if impact condition is met

impactFuncts: takes in $\frac{d\Phi}{dq}$ and returns it with dummy variables without time dependance

impact_update_dice: takes in state and $\frac{d\Phi}{dq}$ and returns an updated state following an application of the impact update

Code Assessment:

Generally, the code functions as I would expect. Since the mass of the cup is significantly greater than the mass of the jack, as well as the differences in the rotational inertia between the two shapes, impacts yield a visible velocity change in the opposite direction for the jack and no visible change for the cup. The jack hits all four walls of the cup and “bounces off” of them and back into the free space. Since the jack it initialized with zero velocity, it falls straight down until making contact with the cup, as there are no external forces applied to it. The cup has a gravity-offset force and cosine force functions applied in the x and y directions. This can be seen by the cup not leaving the animation visual frame (does not free fall in space) as well as it changing angle and position over time independent of contact with the jack.

The code breaks down at some points in time, with the corner of the jack sometimes visibly going through the box. To fix this, I attempted to reduce the timestep and add a collision detection threshold, but neither fully fixed the error. The threshold I settled on was 0.1m, as a larger threshold (ex.0.25m) lead to the jack "impacting" the wall without visibly hitting it as well as eventually going through the wall and into the outside space. The timestep settled upon was 0.005s, which was an adjustment from the original of 0.01s. With 0.01s, there was a point where the impact update led to the jack spinning rapidly in a side wall before returning to the open space and behaving as expected again. Reducing the timesteps appears to have fully removed this problem.