

Continuous time incomplete market codes

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1 Introduction

This repository provides some basic but clean codes for the solution of heterogeneous agent incomplete markets models à la [Bewley \(1986\)](#), [Imrohoroglu \(1989\)](#), [Huggett \(1993\)](#), and [Aiyagari \(1994\)](#) in continuous time. The codes are applications of the [Achdou et al. \(2022\)](#) methods, and details of the methods can be found in that paper and its online appendix.

The codes set out a generic consumer problem which can be applied to all models. A switch then allows you to control whether we solve the model in partial equilibrium or in the general equilibrium of either [Aiyagari \(1994\)](#) (i.e. with capital) or [Huggett \(1993\)](#) (i.e. with bonds). The problem is set and solved in steady state without aggregate shocks.

2 A generic consumer problem

2.1 Agent's problem

A unit continuum of infinitely lived agents faces both idiosyncratic and aggregate productivity risk. Each agent saves using asset holdings $a \geq \underline{a}$, subject to a borrowing constraint $\underline{a} \leq 0$. Agents inelastically provide one unit of labour, and have idiosyncratic productivity shocks z , which follow a Poisson jump process with discrete values z_j for $j = 1, \dots, N_z$. A new value is drawn at rate α_z from a Markov distribution $\gamma_{j,j'}$. They take the wage w and interest rate r as given.

Agents choose consumption c to maximize expected lifetime utility, discounted at rate ρ . The value function $v(a, z)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho v(a, z_j) = \max_c u(c) + v_a(a, z_j)(wz_j + ra - c) + \alpha_z \left(\sum_{j'} \pi_{j,j'} v(a, z_{j'}) - v(a, z_j) \right) \quad (1)$$

2.2 Kolmogorov Forward Equation

$\mu(a, z)$ is the equilibrium distribution of agents across asset and productivity levels. The distribution evolves over time according to the Kolmogorov Forward (KF, or Fokker-Planck) equation.

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Letting $\dot{\mu}(a, z)$ denote the time derivative, we have

$$\dot{\mu}(a, z_j) = -\frac{\partial}{\partial a} [(wz_j + ra - c(a, z_j)) \mu(a, z_j)] + \alpha_z \sum_{j'} \pi_{j,j'} \mu(a, z_{j'}) - \alpha_z \mu(a, z_j) \quad (2)$$

In steady state $\dot{\mu}(a, z_j) = 0$.

3 Market clearing and calibration

One unit of time is taken as one year. We take standard values for most parameters. We fix the borrowing constraint to be equal to the quarterly average wage: $\underline{a} = -1/4wE[z]$. The code provides options for several income processes.

3.1 Partial equilibrium

The worker takes some fixed w and r as given and we solve the value function and distribution.

3.2 Huggett (1993) style model

There is no capital and production is only through labour, giving a fixed wage which we set to $w = 1$. Workers save in a bond in zero net supply, so the interest rate r adjusts so that $\int a d\mu(a, z) = 0$. We use a loop to adjust r to clear the market.

3.3 Aiyagari (1994) style model

Firms operate a constant returns production function and rent capital and labor from households. The aggregate wage and interest rate satisfy:

$$w = ZF_L(K, L), \quad r = ZF_K(K, L) - \delta \quad (3)$$

where aggregate output Y is produced from the production function $Y = ZF(K, L)$. K and L denote aggregate capital and efficiency units of labor, and δ is the depreciation rate. Given the assumed idiosyncratic risk process, total labour is fixed at, $L = \int z d\mu(a, z) = \bar{L}$. Market clearing requires:

$$K = \int a d\mu(a, z), \quad C = \int c(a, z) d\mu(a, z) \quad (4)$$

Here $c(a, z)$ is the consumption policy function in equilibrium. These conditions equivalently imply the aggregate resource constraint $Y = C + I$ where $I = \int \dot{a}(a, z) d\mu(a, z) - \delta K$ is aggregate investment.

We choose Z to target $w = 1$. We use a loop to jointly find the equilibrium K and calibrate Z .

References

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