Continuous time incomplete market codes

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1 Introduction

This repository provides some basic but clean codes for the solution of heterogeneous agent incomplete markets models à la Bewley (1986), Imrohoroğlu (1989), Huggett (1993), and Aiyagari (1994) in continuous time. The codes are applications of the Achdou et al. (2022) methods, and details of the methods can be found in that paper and its online appendix.

The codes set out a consumer problem which can be applied to both models. A switch then allows you to control whether we solve the model in partial equilibrium or in the general equilibrium of either Aiyagari (1994) (i.e. with capital) or Huggett (1993) (i.e. with bonds). The problem is set and solved in steady state without aggregate shocks.

2 A generic consumer problem

2.1 Agent's problem

A unit continuum of infinitely lived agents faces both idiosyncratic and aggregate productivity risk. Each agent saves using asset holdings $a \geq \underline{a}$, subject to a borrowing constraint $\underline{a} \leq 0$. Agents inelastically provide one unit of labour, and have idiosyncratic productivity shocks z, which follow a Poisson jump process with discrete values z_j for $j = 1, ..., N_z$. A new value is drawn at rate α_z from a Markov distribution $\gamma_{j,j'}$. They take the wage w and interest rate r as given.

Agents choose consumption c to maximize expected lifetime utility, discounted at rate ρ. The value function $v(a, z)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho v(a, z_j) = \max_c \; u(c) + v_a(a, z_j)\big(wz_j + ra - c\big) + \alpha_z \left(\sum_{j'} \pi_{j,j'} v(a, z_{j'}) - v(a, z_j) \right) \tag{1}$$

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2.2 Kolmogorov Forward Equation

$\mu(a,z)$ is the equilibrium distribution of agents across asset and productivity levels. The distribution evolves over time according to the Kolmogorov Forward (KF, or Fokker-Planck) equation. Letting $\dot{\mu}(a,z)$ denote the time derivative, we have

$$\dot{\mu}(a,z_j) = -\frac{\partial}{\partial a}\left[\left(wz_j + ra - c(a,z_j)\right)\mu(a,z_j)\right] + \alpha_z \sum_{j'} \pi_{j,j'}\mu(a,z_{j'}) - \alpha_z \mu(a,z_j) \tag{2}$$

In steady state $\dot{\mu}(a,z_j) = 0$.

3 Market clearing and calibration

One unit of time is taken as one year. We take standard values for most parameters. We fix the borrowing constraint to be equal to the quarterly average wage: $\underline{a} = -1/4wE[z]$.

3.1 Partial equilibrium

The worker takes some fixed w and r as given and we solve the value function and distribution.

3.2 Aiyagari (1994) style model

Firms operate a constant returns production function and rent capital and labor from households. The aggregate wage and interest rate satisfy:

$$w = ZF_L(K,L), \qquad r = ZF_K(K,L) - \delta \tag{3}$$

where aggregate output Y is produced from the production function $Y = ZF(K,L)$. K and L denote aggregate capital and efficiency units of labor, and δ is the depreciation rate. Given the assumed idiosyncratic risk process, total labour is fixed at, $L = \int z\, d\mu(a,z) = \bar{L}$. Market clearing requires:

$$K = \int a\, d\mu(a,z), \qquad C = \int c(a,z,\Omega)\, d\mu(a,z) \tag{4}$$

Here $c(a,z,\Omega)$ is the consumption policy function in equilibrium. These conditions equivalently imply the aggregate resource constraint $Y = C + I$ where $I = \int \dot{a}(a,z,\Omega)\, d\mu(a,z) - \delta K$ is aggregate investment.

We choose Z to target $w = 1$. We use a single loop to find the equilibrium K and calibrate Z.

3.3 Huggett (1993) style model

There is no capital and production is only through labour, giving a fixed wage which we set to $w = 1$. Workers save in a bond in zero net supply, so the interest rate r adjusts so that $\int a\, d\mu(a,z) = 0$. We use a loop to adjust r to clear the market.

References

Achdou, Y., J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll (2022): "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," *The Review of Economic Studies*, 89, 45–86.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109, 659–684.

Bewley, T. F. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," in *Contributions to Mathematical Economics in Honor of Gérard Debreu*, ed. by W. Hildenbrand and A. Mas-Colell, Amsterdam: North-Holland, 79–102.

Huggett, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953–969.

Imrohoroğlu, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 97, 1364–1383.