

Data Science Fundamentals Final

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Introduction

I created a report based on Class 8 Heavy-Duty vehicle fleet ($\geq 33,000 \text{ lbs}$) data to evaluate the relationship between oil degradation and engine wear. The data is contained in two .xlsx tables. Table one has 111 observations ($n=111$), and is organized into identifiers, condition indicators (soot, viscosity, acidity, base), chemical degradation indicators (oxidation and nitration), and wear metals (iron, lead, copper, chromium, aluminum), and contaminants (silicon, sodium, potassium).

Table two has 24 observations ($n=24$), and gives the fuel efficiency for each fleet, which was not used in final evaluations.

Research Questions & Hypotheses

- Does an increase in degradation indicators (oxidation/nitration) effect engine wear metals?
- Does soot increasing correspond to more engine wear metals
- Do wear metals increase linearly or non-linearly

Hypotheses

Research Question 1:

H_0 for Oxidation: The vector of regression coefficients will be zero for all six wear metals, meaning Oxidation has no effect on the presence of wear metals. H_0 for Nitration: The vector of regression coefficients will be zero for all six wear metals, meaning Nitration has no effect on the presence of wear metals.

H_1 : At least one of the six coefficients for a given predictor is not zero for either Nitration, Oxidation, or both.

Research Question 2:

H_0 : Soot has no effect on wear metals, and the coefficients will be 0 for all wear metals.

H_1 : Soot has an effect on wear metals, and at least one coefficient will not be 0.

Research Question 3:

H_0 : $\beta_{\text{quadratic}} = 0$, the relationship between time and log-metal concentration is linear.

H_1 : $\beta_{\text{quadratic}} \neq 0$, the relationship between time and log-metal concentration is non-linear.

Analyses

Research Question 1: Does an increase in degradation indicators effect wear metals

This research question has two predictors and five response variables. We are unable to use linear regression due to more than one response variable, otherwise we would inflate Type I Error rates and erroneously ignore covariance. Two-way MANOVA is applicable in this situation as we have two predictors and more than one response variable.

Multivariate linear model

	Value	Num DF	Den DF	F Value	Pr > F
<hr/>					
Intercept					
Wilks' lambda	0.0937	6.0000	96.0000	154.7053	0.0000
Pillai's trace	0.9063	6.0000	96.0000	154.7053	0.0000
Hotelling-Lawley trace	9.6691	6.0000	96.0000	154.7053	0.0000
Roy's greatest root	9.6691	6.0000	96.0000	154.7053	0.0000
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Oxidation	Value	Num DF	Den DF	F Value	Pr > F
Wilks' lambda	0.5338	6.0000	96.0000	13.9723	0.0000
Pillai's trace	0.4662	6.0000	96.0000	13.9723	0.0000
Hotelling-Lawley trace	0.8733	6.0000	96.0000	13.9723	0.0000
Roy's greatest root	0.8733	6.0000	96.0000	13.9723	0.0000
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Nitration	Value	Num DF	Den DF	F Value	Pr > F
Wilks' lambda	0.6959	6.0000	96.0000	6.9916	0.0000
Hotelling-Lawley trace	0.4370	6.0000	96.0000	6.9916	0.0000
Roy's greatest root	0.4370	6.0000	96.0000	6.9916	0.0000
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Test Statistics

The MANOVA produces several multivariate test statistics, each yielding identical F and p values. For Oxidation we observed an F value of \$13.9\$ with a p-value of \$<.0001\$. For nitration we observe an F value of \$6.99\$ and a p-value of \$<.0001\$.

After confirming there exists a statistically significant predictor via MANOVA, we performed OLS regression for each individual

Iron:

Oxidation p = 0.7796
Nitration p = 0.1469

Lead:

Oxidation p = 0.0000
Nitration p = 0.0062

Copper:

Oxidation p = 0.1673
Nitration p = 0.0597

Chro:

Oxidation p = 0.6530
Nitration p = 0.1926

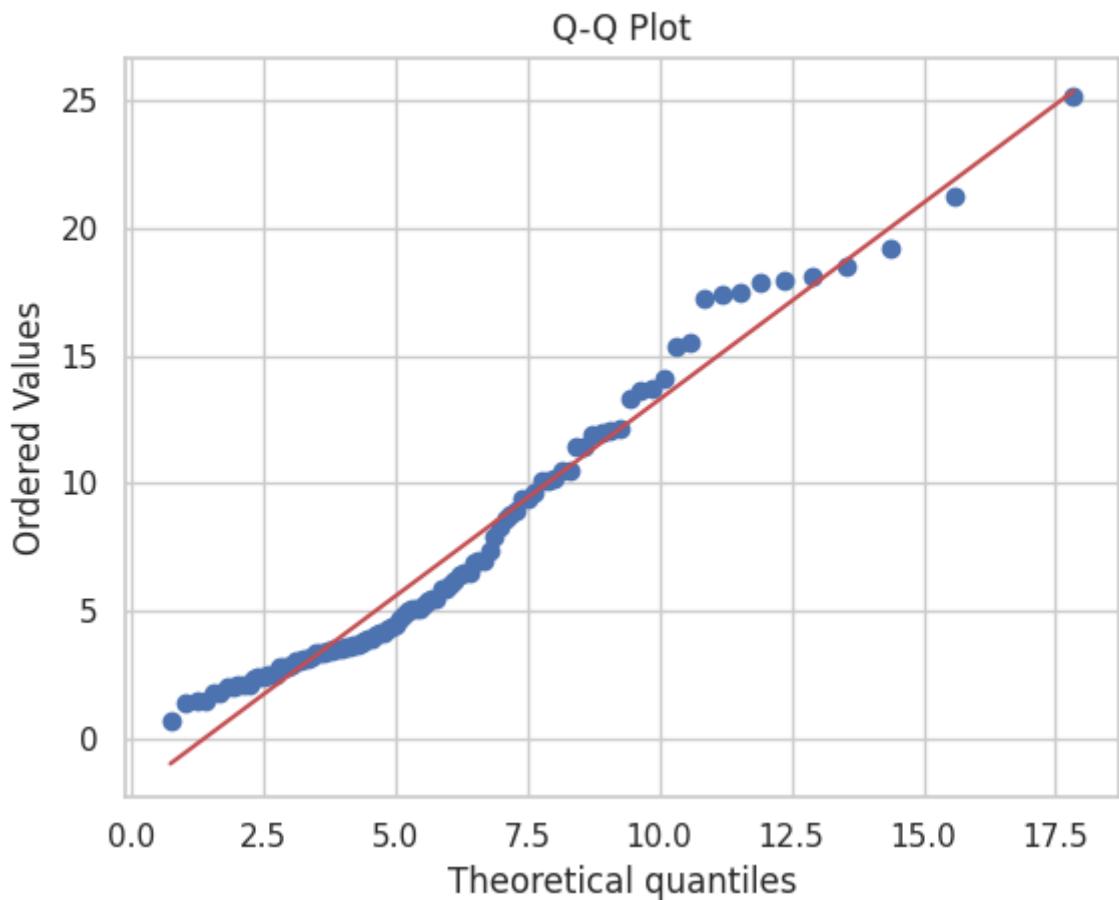
Aluminum:

Oxidation p = 0.2193
Nitration p = 0.2182

Silicon:

Oxidation p = 0.6611
Nitration p = 0.6443

Assumptions



1. Multivariate Normality

Based on the Q-Q graph we can see that the response variables are multivariate normally distributed within each group after the log transformation

2. Independence

Can be assumed based on the context of the dataset.

3. Equal Variance

The test statistic for Pillai's Trace is significant and large enough for unequal variance to be unimportant, as the test statistic is resilient to unequal variance.

4. No outliers

We address this using SciKit Learn's Elliptic Envelope function to remove substantial outliers.

Research Question 2: Does soot increasing correspond to more engine wear metals:

Multivariate linear model

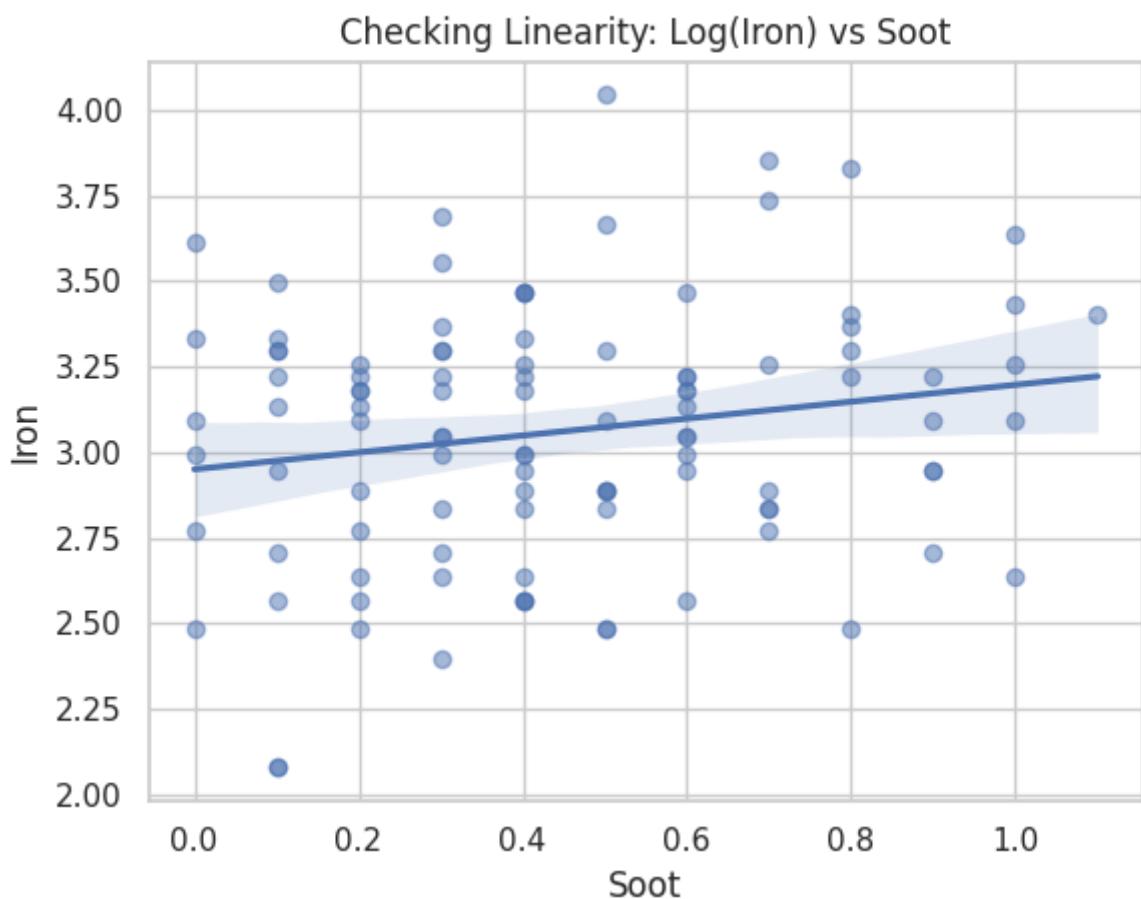
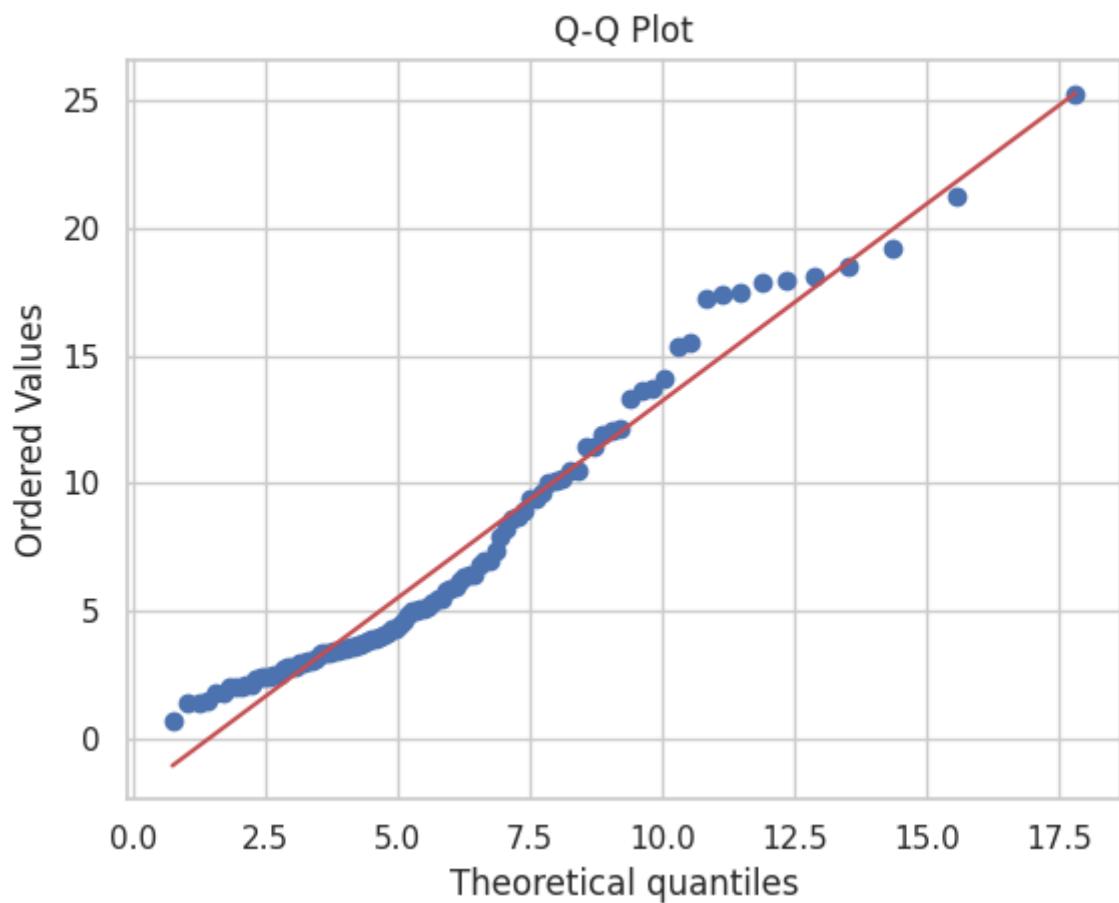
	Intercept	Value	Num DF	Den DF	F	Value	Pr > F
	Wilks' lambda	0.0254	6.0000	96.0000	614.7153	0.0000	
	Pillai's trace	0.9746	6.0000	96.0000	614.7153	0.0000	
	Hotelling-Lawley trace	38.4197	6.0000	96.0000	614.7153	0.0000	
	Roy's greatest root	38.4197	6.0000	96.0000	614.7153	0.0000	

	Soot	Value	Num DF	Den DF	F	Value	Pr > F
	Wilks' lambda	0.8393	6.0000	96.0000	3.0636	0.0087	
	Pillai's trace	0.1607	6.0000	96.0000	3.0636	0.0087	
	Hotelling-Lawley trace	0.1915	6.0000	96.0000	3.0636	0.0087	
	Roy's greatest root	0.1915	6.0000	96.0000	3.0636	0.0087	

Test Statistics

A one way MANOVA was performed comparing Soot to wear metals. We observed an F value of \$3.06\$ and a p of \$.008\$. We observed a Pillai's trace of \$.16\$, indicating a moderate multivariate effect of soot on wear metals.

Assumptions:



1. Multivariate Normality

Based on the Q-Q graph we can see that the response variables are multivariate normally distributed within each group after the log transformation

2. Independence

Can be assumed based on the context of the dataset.

3. Equal Variance

The test statistic for Pillai's Trace is significant and large enough for unequal variance to be unimportant, as the test statistic is resilient to unequal variance.

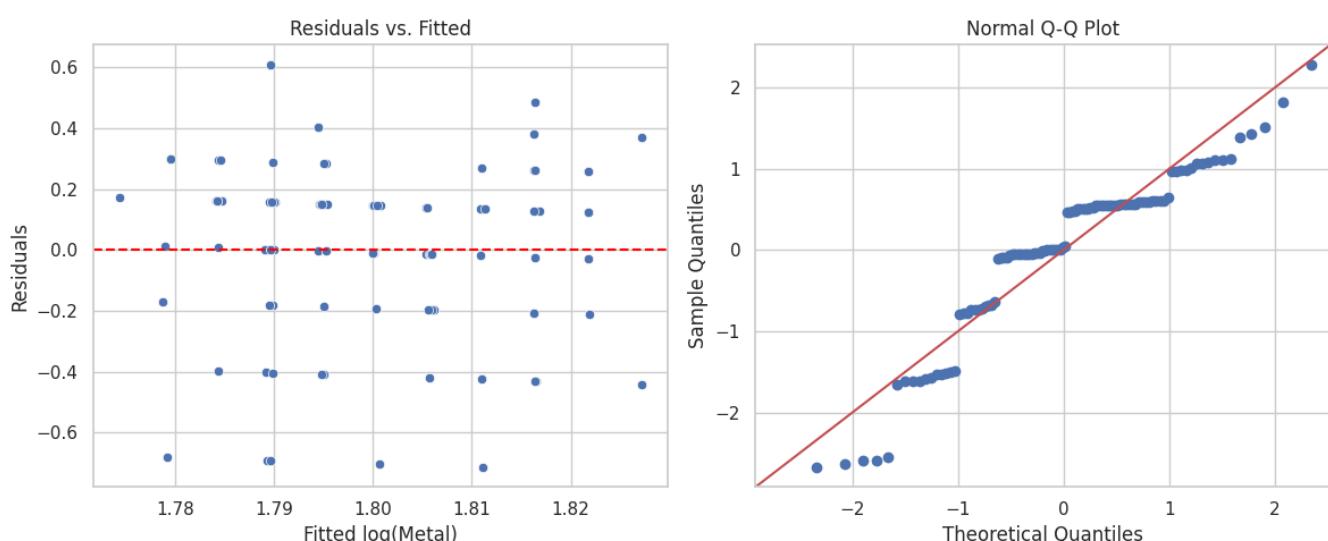
4. No outliers

We address this using SciKit Learn's Elliptic Envelope function to remove substantial outliers.

Research Question 3: Do wear metals increase linearly or non-linearly:

Metal	Result	P_Linear	P_Quadratic	R_squared
Iron	Quadratic	0	0	0
Lead	Quadratic	0.0001	0.0209	0.2561
Copper	No Significant Trend	0.6468	0	0.2561
Chro	Quadratic	0	0.2508	0
Aluminum	Quadratic	0.0181	0.0247	0.0163
Silicon	Quadratic	0.0181	0.0247	0.0163

Assumptions:



1. Linearity While this model does contain a quadratic term, the linearity assumption only cares about linearity of coefficients. The residual plot shows no patterns, is roughly symmetrical around 0, and has no U shape or bow. This assumption is met.
2. Independence Assumed from dataset context.

3. Normality While the Q-Q plot has a quirk of being step-like, this is likely due to the data being not truly continuous. This is not violated under log-normality.
4. Equal Variance of Residuals In the above graph, we can see that there is no clear pattern in the residuals. This is met.

Conclusions

Research Question 1:

We reject both null hypotheses as both our p values are below our alpha significance level. We are not able to isolate which of the wear metals are associated with oxidation and nitration, so we will follow up this test with individual univariate ANOVAs with the Bonferroni Correction.

Using an updated α of $.05/6=.0083$ as per our Bonferroni Correction, we can observe that the only significant interactions are between Lead-Oxidation and Lead-Nitration, for which we will reject the null hypothesis as we have strong evidence of an effect. For all other results we fail to reject the null hypothesis under the stricter significance value.

This has the strongest significance and effect. Using this model we could use Oxidation and Nitration as an early warning indicator for proactive maintenance. Soot has a weaker but detectable relationship.

Research Question 2:

At a significance level of $\alpha = .05$, we reject the null hypothesis, as we have sufficient evidence at the 95% confidence level that soot has an effect on the log concentration of wear metals.

Research Question 3:

For Iron, Lead, Chromium, Aluminum, and Silicon, we observe p values $< .05$, therefore we reject the null hypothesis for these metals, and the relationship is significantly non-linear. For copper we fail to reject the null as we do not have sufficient evidence to suggest a non-linear trend in the data. R^2 values were low across the board however, meaning while there is a significant effect, there is still a large amount of noise, which is to be expected on fleet data where daily operating conditions can change dramatically.

Appendix

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from statsmodels.multivariate.manova import MANOVA
import statsmodels.api as sm
import scipy.stats as stats
from sklearn.covariance import EllipticEnvelope
import statsmodels.formula.api as smf
from statsmodels.stats.stattools import durbin_watson
from statsmodels.stats.diagnostic import het_breuschpagan
```

```
fleet = pd.read_excel('10_1_Class 8 Fleet Data.xlsx', header=0)
efficiency = pd.read_excel('10_2_Class 8 Fleet Fuel Economy.xlsx')
fleet.drop(columns=['Unnamed: 0'], inplace=True)
fleet.drop(index=fleet.index[0], inplace=True)
fleet.columns = fleet.columns.str.strip()

fleet.head()

efficiency.tail()

fleet.describe()

# convert non-numeric values to NaN and compute correlations
numeric = fleet.apply(pd.to_numeric, errors='coerce')

# correlation matrix (drop all-empty columns first)
corr = numeric.dropna(axis=1, how='all').corr()

# heatmap of correlation matrix
plt.figure(figsize=(10, 8))
plt.imshow(corr, cmap='coolwarm', vmin=-1, vmax=1)
plt.colorbar(label='Pearson correlation')
cols = corr.columns
plt.xticks(range(len(cols)), cols, rotation=90)
plt.yticks(range(len(cols)), cols)
plt.title('Correlation matrix heatmap')
plt.tight_layout()
plt.savefig(fname='Heatmap.png')
plt.show()

# pairwise scatter matrix for numeric columns
pd.plotting.scatter_matrix(numeric.dropna(axis=1, how='all'), figsize=(12, 12), diagonal='kde')
plt.suptitle('Scatter matrix (pairwise plots)', y=1.02)
plt.tight_layout()
plt.savefig(fname='pairplot.png')
plt.show()

print('Observations in fleet data: ' + str(len(fleet.index)))
print('Observations in efficiency data: ' + str(len(efficiency.index)))

wear_metals = ['Iron', 'Lead', 'Copper', 'Chro', 'Aluminum']
wear_metals_corr = corr.loc[wear_metals, wear_metals]
with pd.option_context('display.float_format', '{:.3f}'.format,
                      'display.width', None):
    print(wear_metals_corr)

df_long = fleet.melt(
    value_vars=wear_metals,
    var_name="Wear Metal",
    value_name="Concentration"
)
sns.set_theme(style="whitegrid")
```

```
g = sns.catplot(
    data=df_long,
    x="Wear Metal",
    y="Concentration",
    col="Wear Metal",
    kind="box",
    col_wrap=3,
    sharey=False,
    height=4,
    aspect=0.9
)

g.set_titles("{col_name}")
g.set_axis_labels("", "Concentration (ppm)")
g.fig.suptitle("Distribution of Wear Metals Across Fleets", y=1.05)
g.savefig(fname="boxplot.png")
plt.show()

# %%

# Filter to soot < 2%
fleet_filtered = fleet[fleet["Soot"] < 2]

# Wear metal columns
wear_metals = [
    "Iron", "Lead", "Copper", "Chro", "Aluminum", "Silicon"
]

plt.figure(figsize=(8, 6))

for metal in wear_metals:
    plt.scatter(fleet_filtered["Soot"], fleet_filtered[metal], label=metal)

plt.xlabel("Soot (%)")
plt.ylabel("Wear Metal Concentration")
plt.title("Soot (<2%) vs Wear Metals")
plt.legend()
plt.grid(True)
plt.savefig(fname="soot.png")
plt.show()

formula = 'Iron + Lead + Copper + Chro + Aluminum + Silicon ~ Oxidation + Nitration'
predictors = ['Oxidation', 'Nitration']

manova_df = fleet[predictors + wear_metals].copy().apply(pd.to_numeric,
errors='coerce').dropna()
manova_df[wear_metals] = np.log1p(manova_df[wear_metals])
detector = EllipticEnvelope(contamination=0.06, random_state=42) # Removes significant outliers
```

```
manova_df['is_outlier'] = detector.fit_predict(manova_df[wear_metals])

manova_df = manova_df[manova_df['is_outlier'] == 1]

maov = MANOVA.from_formula(formula, data=manova_df)
print(maov.mv_test())

mahal_distances = detector.mahalanobis(manova_df[wear_metals])
stats.probplot(mahal_distances, dist="chi2", sparams=(len(wear_metals),),
plot=plt)
plt.title("Q-Q Plot")
plt.savefig("Q1_qq.png")
plt.show()

X = sm.add_constant(manova_df[predictors])

results = {}
for metal in wear_metals:
    model = sm.OLS(manova_df[metal], X).fit()
    results[metal] = model

for metal in wear_metals:
    print(f"\n{metal}:")
    print(f" Oxidation p = {results[metal].pvalues['Oxidation']:.4f}")
    print(f" Nitration p = {results[metal].pvalues['Nitration']:.4f}")

one_way_df = fleet[['Soot'] + wear_metals].copy()
one_way_df[wear_metals] = one_way_df[wear_metals].astype(float)
one_way_df['Soot'] = pd.to_numeric(one_way_df['Soot'], errors='coerce')
one_way_df[wear_metals] = one_way_df[wear_metals].apply(pd.to_numeric,
errors='coerce')
one_way_df[wear_metals] = np.log1p(one_way_df[wear_metals])
soot_threshold = one_way_df['Soot'].quantile(0.99)
one_way_df = one_way_df[one_way_df['Soot'] <= soot_threshold]

detector = EllipticEnvelope(contamination=0.05, random_state=42) # Removes
significant outliers
one_way_df['is_outlier'] = detector.fit_predict(one_way_df[wear_metals])

one_way_df = one_way_df[one_way_df['is_outlier'] == 1]

formula = 'Iron + Lead + Copper + Chro + Aluminum + Silicon ~ Soot'
maov = MANOVA.from_formula(formula, data=one_way_df)
print(maov.mv_test())

sns.regplot(data=one_way_df, x='Soot', y='Iron', scatter_kws={'alpha':0.5})
plt.title("Checking Linearity: Log(Iron) vs Soot")
plt.savefig("Q2-linearity")
plt.show()

mahal_distances = detector.mahalanobis(one_way_df[wear_metals])
```

```
stats.probplot(mahal_distances, dist="chi2", sparams=(len(wear_metals),),
plot=plt)
plt.title("Q-Q Plot")
plt.savefig("Q2-qq.png")
plt.show()

def test_wear_trends(df):
    results = []
    working_df = df.copy()

    for metal in wear_metals:
        working_df[metal] = pd.to_numeric(working_df[metal],
errors='coerce')
        temp_df = working_df.dropna(subset=[metal, 'Samp. #']).copy()
        temp_df['log_metal'] = np.log1p(temp_df[metal])

        temp_df['time_step'] = temp_df['Samp. #']
        temp_df['time_step_sq'] = temp_df['time_step']**2

    try:

        formula = 'log_metal ~ time_step + time_step_sq'
        model = smf.ols(formula=formula, data=temp_df).fit()

        p_val_linear = model.pvalues['time_step']
        p_val_quad = model.pvalues['time_step_sq']
        coeff_quad = model.params['time_step_sq']

        if p_val_quad < 0.05:
            trend = "Quadratic" if coeff_quad > 0 else "Curved"
        else:
            trend = "Linear" if p_val_linear < 0.05 else "No
Significant Trend"

        results.append({
            'Metal': metal,
            'Result': trend,
            'P_Linear': round(p_val_linear, 4),
            'P_Quadratic': round(p_val_quad, 4),
            'R_squared': round(model.rsquared, 4)
        })
    except Exception as e:
        results.append({'Metal': metal, 'Result': f"Error: {str(e)}"})

    return pd.DataFrame(results)

test_wear_trends(fleet)

residuals = model.resid
fitted = model.fittedvalues

fig, ax = plt.subplots(1, 2, figsize=(12, 5))
```

```
sns.scatterplot(x=fitted, y=residuals, ax=ax[0])
ax[0].axhline(0, color='red', linestyle='--')
ax[0].set_title('Residuals vs. Fitted')
ax[0].set_xlabel('Fitted log(Metal)')
ax[0].set_ylabel('Residuals')

sm.qqplot(residuals, line='45', fit=True, ax=ax[1])
ax[1].set_title('Normal Q-Q Plot')

plt.tight_layout()
plt.savefig('Q3-diagnostics.png')
plt.show()
```