Last Chances: Finding Love in a Hopeless Place

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1 Abstract

In the modern world, statistics show that younger people are getting married less frequently, and more marriages are ending in divorce. For our project, we try to solve a double optimization problem. We model how many people in a finite population will get married given a threshold attractiveness. Using this model, we solved the optimization problem for an individual to maximize their threshold attraction for marrying a partner and their likelihood of getting married.

We did this by setting a threshold attraction and pairing members from a random population. Those who met the threshold attraction were counted as married and removed from the population. We played multiple games with a set population size and number of rounds and through an iterative process found that the optimum attraction threshold for individuals (which optimizes the balance between getting married and being attracted to one's partner) is 0.75, meaning one should be willing to marry the upper 25% of individuals one finds attractive.

2 Introduction

Love is a dangerous game.

According to Pew Research Center, in 2020 44% of Millennials (ages 23 to 38) were unmarried, compared to 81% of members of the silent generation. For those lucky enough to tie the knot, there was about a 37% chance their marriage would end in divorce in 2019. For second marriages, the rate was a staggering 60-67%. Over the past 20 years, people have also been waiting longer to put on the ring as the average age at first marriage has increased by nearly 4 years for both men and women. For many people marriage, especially to the right individual, has many benefits and is deeply fulfilling. Therefore, the aforementioned statistics are disconcerting, as they suggest it is becoming increasingly difficult for people to find partners with whom they can enjoy a happy marriage.

For people who do want to get married, there is naturally a balancing act of choosing to marry someone, and wondering whether or not you are sufficiently compatible with the person to have a successful marriage. In our project, we took advantage of concepts from game theory to model love in a population. Specifically, how we could use game theory to solve a double optimization problem in order to maximize a person's chances of getting married while ensuring that they are attracted to the person whom they are marrying, understanding that these two desires conflict: If you are willing to marry anyone the odds you get married are quite high, but there is a sizeable chance that you are not sufficiently attracted to your love interest such that the marriage does not go well. Conversely, if you are only willing to marry the top 1% of people you are compatible with you will almost inevitably have a successful marriage with that person, but the odds of you meeting them are low enough that having such high standards may reduce your chances of getting married. In light of this conflict, we ask: what percentage of the population should I be willing to marry in order to maximize my chances of getting married to someone I am attracted to?

We modeled this by creating a game that analyzed the amount of people that would get married in a given population if they all had the same threshold for attraction (e.g. if each individual was willing to date 90%, 60%, or 30% of the population). Clearly, if someone was willing to date a majority of the population their odds of getting married are high but such a low selection threshold would likely result in incompatibility. Conversely, if an individual had a high threshold for attraction, they would likely be compatible, but they may be less likely to meet someone who meets their standards. Our game used an iterative process that cycled through different threshold attractions in the population and analyzed what percentage of the population ended up married at different thresholds. We found that a threshold of 25% optimized having a high attraction threshold and the odds of getting married.

3 Literature Review

Gale and Shapely (1962) write a seminal paper on the idea of matching theory, in "College Admissions and the Stability of Marriage". In it, they propose a model over many different scenarios, including colleges trying to get an optimal level of students by admitting more, and a group of men and women trying to end up in stable marriages. They propose (no pun intended) what is now called the Gale Shapely algorithm. This algorithm involves one group proposing to their best match and that match holding to accept until other proposals are sent. Although this takes exponential time, it always results in so-called "stable" marriages, where no two people could be better off by divorcing their relative partners and remarrying each other. In the context of our work, this shows that matching all partners in an optimal manner is possible, but not in a way necessarily conducive to actual human dating behavior.

The next important paper, particularly useful because it can model dating in a way that is more applicable to our work, is Corbin's "The secretary problem as a Model of Choice" (1980). The secretary problem proposes a boss with the goal being to hire the best secretary from a pool of n applicants. By interviewing, she may rank the applicants she views, but after accepting or rejecting one, she cannot change her choice. As well, she cannot view the features of unseen applicants.

The optimization issue lies with rejecting applicants in order to get more information about the population risks rejecting optimal choices that may not come back up. This paper applies heuristics to the model and offers strategies to improve estimates for what the best applicant could be. By reading this as a romantic partner instead of a secretary, we get similar results. Finding a good partner is an optimal stopping problem and though human behavior is not such that recommending one reject $\frac{n}{e}$ partners before marriage, it still can give us insights.

Another paper that provides valuable insight into matching theory and its applications to relationships is "A Marriage Matching Mechanism Menagerie" by Boudreau and Knoblauch. In this paper, Boudreau and Knoblauch acknowledge the stability of the Gale Shapely method but also propose a series of other algorithms that, rather than focusing on stability, prioritize meeting wide ranges–or even combinations–of criteria for the formation of a match. By reducing the graph representing the matching problem to a collection of subgraphs, each satisfying a specified set of criteria, and then allowing individuals to choose partners based on a predefined ordering, the individuals are able to find matches that are Pareto-optimal among all potential matchings that achieve the desired combination(s) of criteria. This process takes polynomial time, and in the context of our work, shows that it is possible to find all partners a match such that all of their criteria are met to some extent, although the Pareto-optimal matching means that not every individual's match will be the best possible match for him or her specifically.

4 Methods

4.1 Theory

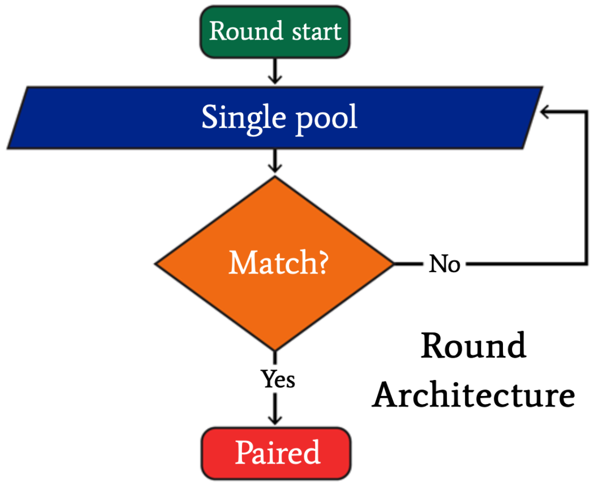
The setup of this model is solving a double optimization problem. On one hand, people should be very picky, because the pickier they are, the more likely it is that they end up with a more attractive partner. On the other hand, being very picky leads to a longer time dating and a greater chance of ending up single. To model this behavior, we decided on different ways to represent populations and romantic interactions.

The first thing we decided is that the population should be homogeneous and unisexual. The selection from two groups among each other doesn't really change the fundamental question of modeling selection and attraction so we decided to simplify the model that way. As well, adding sexual preferences would complicate the model but not contribute to novel results, so we omitted that.

We then sought to model attraction, which is a bit of a vague and personal concept. We considered different methods but realized that the most simple solution was the one people use regularly: just assigning a number and comparing people ordinally. Instead of ranking people 1-10, we used a scale from 0-1, going by 0.01. The way people are attracted to each other is not expected to be perfectly transitive or complete or logical, but we considered this to be the closest to the real thing.

Once we had a model population and a method for measuring attraction, we thought of a way to model dating behavior. Obviously, this is an entirely personal process for everyone, so some major simplifications were necessary. First, we decided to break it into rounds. Second, we only allowed two people to pair at a time. These two rules are not truly capturing behavior, but are good enough proxies that also allow us to work with the model.

In short, we are trying to answer a double optimization problem to see what percentage of the population an individual should be willing to marry in order to maximize their chances of getting married while marrying the most attractive person they can. In order to do this we create a mathematical representation of a population of individuals who all have different degrees of attraction to one another and pair them together. If two people are sufficiently attracted to each other, then when they are paired they are a match for marriage and can be removed from the "single" population. In our model we discretize the process of looking for a partner by creating rounds in which individuals are paired, quantify the relative attraction individuals have for each other, and set a numerical threshold for which paired individuals have sufficient attraction to be married.



4.2 Algorithm

We set up the problem with three variables: population size, rounds, and threshold attraction. For our model, we made the population size 100. In each round members of the population are randomly paired together. Each individual has an array of their relative attraction to all the other individuals in the population on a scale from 0 to 1, with 1 being the person an individual is most attracted to and 0 being the person they are least attracted to. We begin with a threshold value of attraction. If two individuals are paired and their attraction values to each other exceed the threshold then we count those two individuals as married and remove them from the single population for the next round. All the single individuals remain in the population for the next round. We conducted the experiment with uniform, normal, and exponential distributions of attraction. In the uniform distribution unattractive, neutral, and attractive are all equally likely so the probability of someone having a 0.9 attraction value (which, again, is high for our game given that 1 is maximum attraction) is the same as someone having a 0.1 attraction and someone having a 0.5 attraction value–all attraction values are equally likely. For a normal distribution attraction values are distributed normally such that values around 0.5 are most likely and they become increasingly rare as you move out to the edges of the curve with 0.9 and 0.1 being the least likely. Finally, in an exponential distribution, being relatively unattractive is the most common with the likelihood decreasing more and more as you approach higher attractiveness values.

4.3 Test

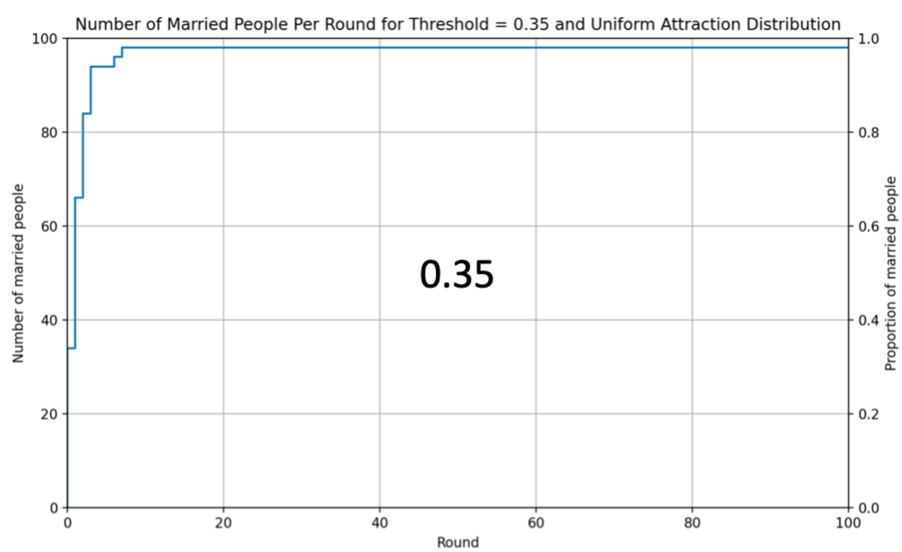
For each of the distributions, we used a population of 100 individuals and cycled through 100 rounds. We looped through the game for different thresholds increasing the thresholds incrementally by 0.01 from 0 to 1. For each threshold value, we ran 100 simulations and recorded the average number of people that ended up being matched at that threshold. We took the average number of people matched at each threshold and plotted it against the average threshold value. Additionally, we made a plot of a total error function. To do this, we computed the error as

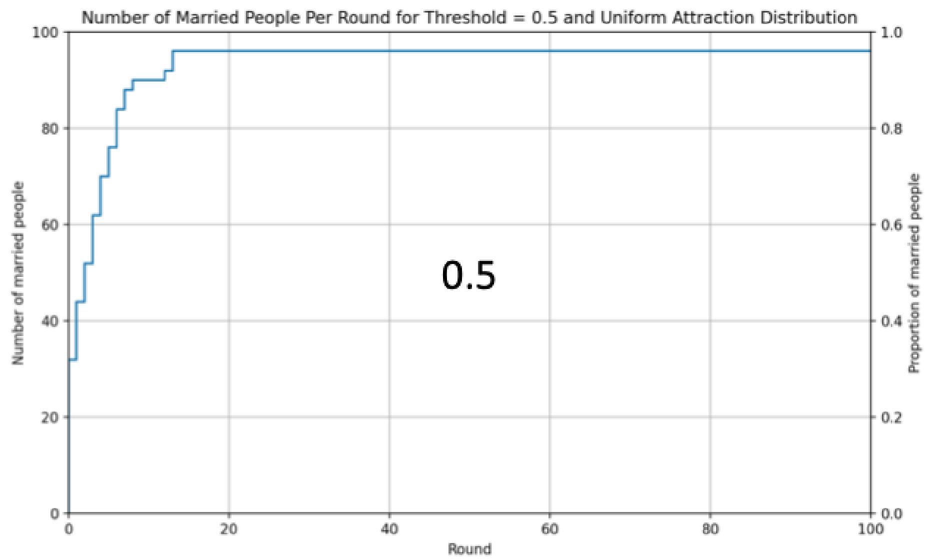
$$(1 - \text{percentage of people matched}) + (1 - \text{threshold})$$

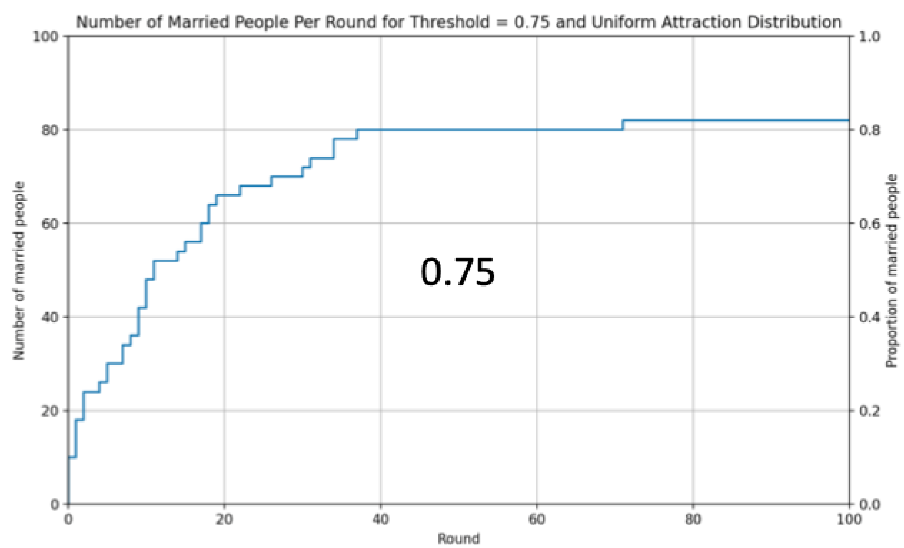
and plotted these values against the threshold value. This modeled our double optimization because in an ideal set up the whole population would be matched such that (1-percentage of people matched) would equal 0 and the threshold would be 1 such that people were only matching with individuals with whom they were the most compatible (and so in the ideal setting the y-value would be zero). Creating this plot allows us to find the threshold at which (1-percentage of people matched)+(1-threshold) is minimized. In our testing, we ran the test to produce these plots for the uniform, exponential, and normalized attractive distributions, and optimized each one accordingly.

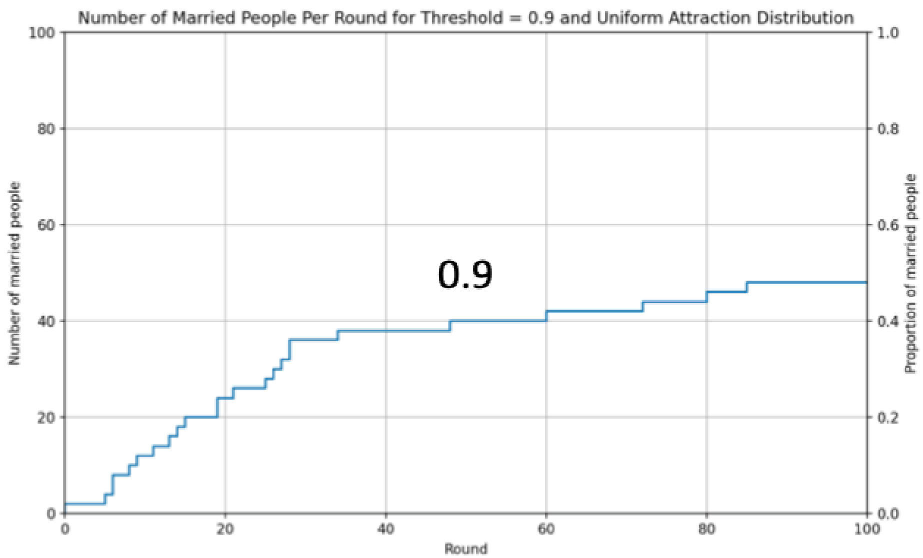
4.4 Results

Below are different outcomes depending on the threshold indicated in the middle (0.35, 0.5, -.75, and 0.9). Accordingly, the 0.9 indicates a willingness to marry 10% of the population, while the 0.35 indicates a willingness to marry 65% of the population. You can see that the pattern through which the population pairs remains the same across the different thresholds. However, as expected, the higher the threshold, the longer it takes for people to pair and the lower the amount of paired people at the end of the simulation.

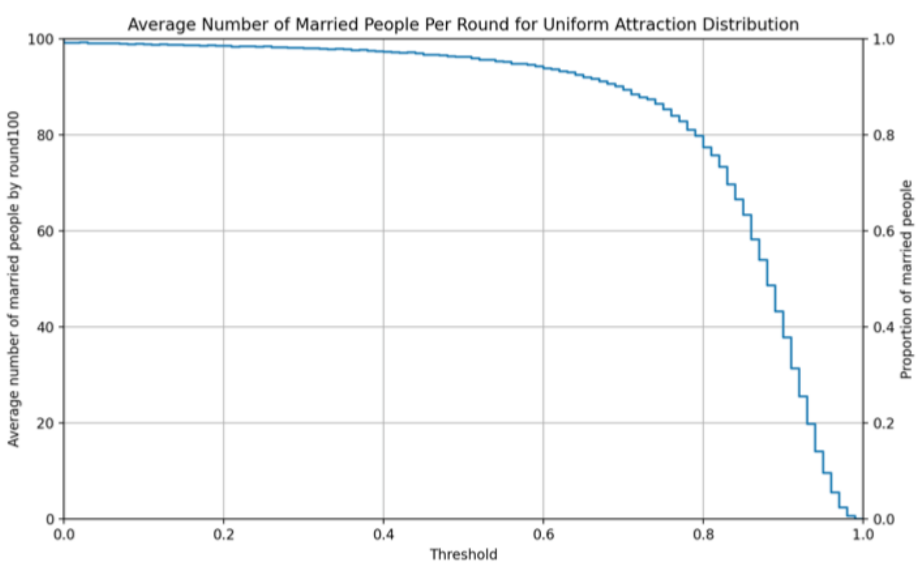



Number of Married People Per Round for Threshold = 0.5 and Uniform Attraction Distribution
Number of married people
Proportion of married people
Round
0.5
0
20
40
60
80
100
0.0
0.2
0.4
0.6
0.8
1.0


Number of Married People Per Round for Threshold = 0.75 and Uniform Attraction Distribution
Number of married people
Proportion of married people
Round
0.75
0
20
40
60
80
100
0.0
0.2
0.4
0.6
0.8
1.0



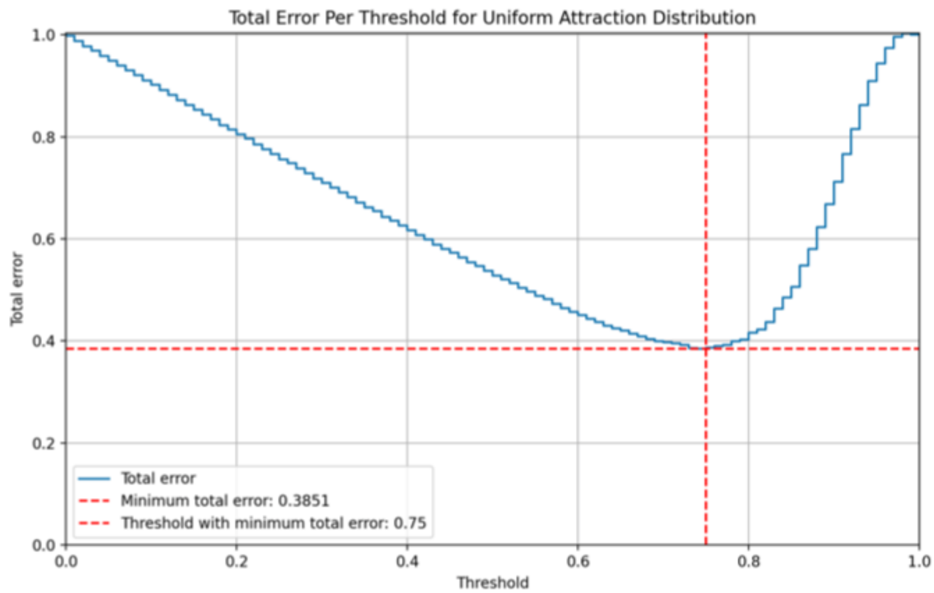
Next, we aggregated all of the different simulations, running the simulation 100 times and raising the threshold values from 0.01-1 throughout the aggregation.
Below you can see the average number of married people per round for uniform attraction distribution.



Using the above graph, we calculated the total error (ROC-like analysis).

$$TE = w_t(1 - t) + w_m(1 - m)$$

This provided us with the following figure.



From this graph, we can see that the optimized threshold to achieve

1. Highest proportion of married individuals, and
2. Highest attraction between any given individual pair

was 0.75. This means that the best outcomes for the population at large occur when people are willing to marry about 25% of the population.

Interestingly, when the population and number of rounds were increased to 1,000, the attractiveness threshold increased to 0.85, meaning one should only be willing to date the top 15% of the eligible population. Intuitively, this makes sense: a greater number of rounds corresponds to a longer period of time in which a person can choose a partner, and with more time and more options for potential partners one can afford to be pickier (hence the lower percentage of people one should be willing to marry).

5 Discussion

5.1 Problem Set-Up Limitations

When interpreting these results it is important to consider certain limitations of our problem set-up. A significant limitation in our setup is the use of rounds. In the real world, circumstances and chance determine when and how someone might meet a romantic partner. Furthermore, even once two individuals meet they often date for a long period of time during which outside factors, including changes in location, occupation, health, and other circumstances beyond compatibility may prevent two individuals from ultimately getting married. In our model, we simplify this process as individuals are paired in iterative rounds. This is a discretization of a continuous process of dating and also removes the chance of outside factors preventing two individuals from meeting or dating successfully. Likewise, after two individuals "meet" in our program compatibility is the only factor in determining whether or not two individuals get married, again removing external factors that exist in the real world.

Another limitation is the fact that we assume every individual has roughly the same attractiveness as the mean attractiveness is randomly distributed with a mean of 0.5. This is likely not an accurate representation of the real world as there are some people who are considered to have high attractiveness to the majority of the population while others have low attractiveness to the majority of the population. We are able to account for this somewhat using the normal distribution of attractiveness. It also should be noted that the terms attraction and compatibility are somewhat conflated in our setup.

5.2 Next Steps: A Better Model

There are a number of possible improvements that are available to us. In some cases, getting closer to realism would improve the performance while allowing results to be more directly applicable. In others, simplification allows us to make faster decisions without losing important information. We will share some of these next steps for a better model now.

The first option is to improve the temporal element of our model. We've discretized a continuous process, but going back to continuous is very doable. The Poisson distribution calculates the probability of certain numbers of discrete events happenings over a continuous period, like the number of goals in a hockey game. If we repurpose it for the number of dates one experiences during a period, we can begin to model behavior in a way much better than simple rounds. This would also allow us to add actual "dating periods" in which a certain number of successful dates need to be completed before the long-term relationship, for example.

We also think another method to improve the model could be the implementation of a moving threshold. Similar to the methods recommended by an optimal stopping problem, rejecting an initial amount of suitors, or at least being picky is very sensible. This allows someone to get a better sense of their dating pool and also minimizes the risk of settling early. However, as the rounds go on, we presume one's desperation leads them to reduce their threshold of attractiveness. This could be easily implemented and instead of looking at ideal set thresholds, we could look at ideal rates of threshold decay.

Another potential idea for improvement that we have is the use of mapping the algorithm over a social network, maybe even an evolving one. At the edges, or "chance encounters," we could implement probabilities that work similarly in theory to our thresholds and threshold distributions – this could present a case where the distributions have relevance, unlike our current model. Another idea is the use of stochastic probabilities to govern these edges. With both of these ideas in use, the model would more accurately represent real-life social encounters.

The last possible extension is one to increase the realism of our model. Our implementation relies on a random distribution of all players over all other players. In reality, this is not even close to the case - some people are inherently more attractive than others, and that changes their optimal behavior for dating. By adding a base attractiveness score, we can see what it is like for someone who is lucky and born attractive versus the opposite. Although this would make results a lot more difficult to interpret, depending on our implementation, this would also be an important step to ensuring the validity of our results.

6 Conclusion

In the digital age, online dating has provided individuals with relatively easy access to compatible partners. Despite, this young people seem to be less likely to get married and more likely not to stay married than members of previous generations. If an algorithm can find someone a compatible partner, then perhaps an algorithm could help people decide on who they should be willing to marry in order to maximize their chances of getting and staying married. Our algorithm attempted to solve this double optimization problem and produced interesting results.

Regardless of how we distribute attractiveness in our unisex and homogenous population of 100, whether uniform, normal, or exponential, our results indicated that one should have an attractiveness threshold of 0.75, meaning that one should be willing to date the upper 25% of individuals they're attracted to in the eligible dating population. When the population and number of rounds were increased to 1,000, however, the ideal threshold rose to 0.85, indicating that higher standards are reasonable when individuals are given more options for potential partners and more time to match with a suitable partner.

In interpreting these results, it is important to recognize this model and analysis are preliminary. We sought to solve the double optimization problem of maximizing one's odds of getting married while ensuring the partner was sufficiently attractive. In order to do this we made a simplified model that used many parameters that do not accurately reflect the real world or real-world dating in an accurate way. At most, the results should give individuals looking for a spouse a very rough sense of how picky they should be in their partners relative to the rest of the eligible dating population, but the primary of goal of this project was to take a first step in using game theory and optimum stopping problem to study the marriage problem faced by the modern world. We hope that our model can be improved and developed into an accurate simulation of real world dating to help individuals looking for a spouse answer important questions like: who should I be willing to date? At what point should I propose to my partner? How compatible should I be with my partner in order for marriage to be successful? Obviously, these are deeply personal questions, and to perfectly model these complex relationships and feelings of humans would be a virtually impossible task. Through this model and project, however, we hope to help people find love and find fulfilling relationships because when we do this it is a "win-win" and we make the world a more loving and happy place.

7 Bibliography

Barroso, Amanda and Parker Kim. "As millenials near 40, they're approaching family life differently than previous generations." Pew Research Center, May, 2020.

Boudreau, J. W., and V. Knoblauch. "A marriage matching mechanism menagerie." Operations Research Letters, vol. 45, no. 1, 2017, pp. 68-71. ScienceDirect, https://www.sciencedirect.com/science/article/pii/S0167-637716302449. Accessed 24 May 2023.

Corbin, R. M. "The secretary problem as a model of choice." The Journal of Mathematical Psychology, vol. 21, no. 1, 1980, pp. 1-29. ScienceDirect, https://www.sciencedirect.com/science/article/pii/0022249680900255. Accessed 24 May 2023.

Gale, D., and L. S. Shapley. "College Admissions and the Stability of Marriage." The American Mathematical Monthly, vol. 69, no. 1, 1962, pp. 9–15. JSTOR, https://doi.org/10.2307/2312726. Accessed 26 May 2023.