M50 Homework 1

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Exercise 1.

Part A.

Suppose that Y $Bernoulli(q)$ and let $Z = \frac{1}{1+Y} + Y$. What is the probability function of Z?

Solution

We can start with the probability mass function of Y, defined at the probability mass function of any Bernoulli random variable with parameter q:

$$f_Y(y) = Pr(Y = y) = \begin{cases} q & \text{if } y = 1 \\ 1 - q & \text{if } y = 0 \\ 0 & \text{o.w.} \end{cases}$$

We can then use this to find the probability mass function of Z by defining the two cases of when $Y = 1$ and $Y = 0$. When $Y = 1$, we now that $Z = \frac{1}{1+1} + 1 = \frac{3}{2}$. When $Y = 0$, we know that $Z = \frac{1}{1+0} + 0 = 1$. Thus, we can define the probability mass function of Z as:

$$f_Z(z) = P(Z = z) = \begin{cases} q & \text{if } z = \frac{3}{2} \\ 1 - q & \text{if } z = 1 \\ 0 & \text{o.w.} \end{cases}$$

Part B.

Suppose a coin is flipped. If the coin is heads, we write down 0. If the coin is tails, we roll a dice and write down the number. Write down the probability distribution for Y, the number that we write down.

Solution

Let us start by defining random variables X and Z. Let X be the random variable that represents the result of the coin flip (let 0 denote heads, and 1 denote tails), and let Z be the random variable that represents the dice roll. We know that $X \sim bernoulli(\frac{1}{2})$, and we know that Z is a discrete uniform random variable that can take on values in the set $\{1, 2, 3, 4, 5, 6\}$, each with probability $\frac{1}{6}$ given that we flip tails on the initial coin flip. We can therefore define the probability mass function of X amd Z as:

$$f_X(x) = Pr(X = x) = \begin{cases} \frac{1}{2} & \text{if } x = 0 \\ \frac{1}{2} & \text{if } x = 1 \\ 0 & \text{o.w.} \end{cases}$$

$$f_Z(z) = Pr(Z = z) = \begin{cases} \frac{1}{6} & \text{if } z \in \{1, 2, 3, 4, 5, 6\} \wedge X = 1 \\ 0 & \text{o.w.} \end{cases}$$

Let us now define our sample space S for the outcomes of Y:

$$S = \{0, 1, 2, 3, 4, 5, 6\}$$

$Y = 0$ in the case where we flip heads on the initial coin flip. Therefore:

$$f_Y(0) = Pr(Y = 0) = Pr(X = 0) = \frac{1}{2}$$

For all other values of Y in the sample space, we know that we must have flipped tails on the initial coin flip, and then rolled that value on the dice. We can therefore define the probability of Y being these values as the probability of the intersection of flipping tails on the initial coin flip and flipping that value on the dice roll:

$$f_Y(z) = Pr(Y = z \cap X = 1) = Pr(Z = z \cap X = 1) = Pr(Z = z \mid X = 1) \times Pr(X = 1)$$

$$= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} : z \in \{1, 2, 3, 4, 5, 6\}$$

We can therefore define the probability mass function of Y as:

$$f_Y(y) = Pr(Y = y) = \begin{cases} \frac{1}{2} & \text{if } y = 0 \\ \frac{1}{12} & \text{if } y \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{o.w.} \end{cases}$$

Part C.

For the previous problem, conditioned on the dice rolling a 4, what is the probability we write down 0? Conditioned on the coin being tails, what is the probability the dice rolls a 3?

Solution

Given that the dice rolled a 4, we know that $Y = 4$ because in all the cases where we roll a dice, we write down the value that we roll. Therefore, the probability that we write down 0 is 0.

Given that the coin is tails, we know that $X = 1$. We may still define the probability that the dice rolls a 3 as the probability of the intersection of the dice rolling a 3 and the coin being tails, but in this case $Pr(X = 1) = 1$:

$$Pr(Z = 3 \cap X = 1) = Pr(Z = 3 \mid X = 1) \times Pr(X = 1) = Pr(Z = 3) \times Pr(X = 1) = \frac{1}{6} \times 1 = \frac{1}{6}$$

Part D.

Consider the geometric distribution discussed in lecture. What are 3 examples of variables in the real world for which this might be a good model and what are some limitations of these models.

Solution

Examples:

1. Shooting basketball free throws until you make one.
2. Playing lottery scratch tickets until you win.
3. Flying on an airplane until you get into a fatal crash.

Limitations:

1. The probability of success must be constant for each trial, so changes in the probability of success over time are not accounted for.
2. The trials must be independent, so the outcome of one trial cannot affect the outcome of another trial.

Exercise 2.

Consider the following code:

```python
for i in range(5):
    for j in range(i + 1):
        print(i, end = "")
    print("")
```

```
0
11
222
3333
44444
```

Modify the code so that it prints out:

```
0
01
012
0123
01234
012345
```

Solution

```python
for i in range(6):
    for j in range(i + 1):
        print(j, end = "")
    print("")
```

```
0
01
012
0123
01234
012345
```

Exercise 3.

(Washington post data): Below I load some data on homocide victims in US from the washington post. Don't worry about how I process it, all you need to work with is the DataFrame "data" on the very last line.

```python
import pandas as pd

data = pd.read_csv("https://raw.githubusercontent.com/washingtonpost/data-homicides/master/homicide-data.csv",

data["victim age"] = pd.to_numeric(data["victim_age"],errors="coerce")
```

Part A.

For each age $a = 1, ..., 100$ determine the number of victims $n(a)$ with an age $< a$. You can ignore the effects of those entries with missing ages. Make a plot of $n(a)$ vs. a.

Solution

Part B.

What do you notice about the plot. Does it have the expected behavior?

Solution

Part C.

Break the data up into white and non-white victims. Then, for each group make the plot from part **(a)**. Comment on what you find.

Solution