M50 Homework 2

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Exercise 1.

(Computing conditional averages): Consider a random variable $Y = (Y_1, Y_2)$ which takes values in the sample space:

$$S = \mathbb{N} \times \mathbb{N} = (i, j), i, j \in \mathbb{N}$$

That is, the sample space consists of all possible pairs of numbers (i, j). Now suppose we have some data:

$$(1, 2), (1, 2), (3, 1), (1, 4), (3, 3), (2, 2), (1, 5)$$

Give you best estimates of the following (either by hand, with Python, or a calculator)

$$E[Y_1], \quad E[Y_1 \mid Y_2 = 2], \quad E[Y_2 \mid Y_1 = 1], \quad E[Y_2 \mid Y_1 > 1]$$

Solution

$$E[Y_1] \approx \frac{1 + 1 + 3 + 1 + 3 + 2 + 1}{7} = \frac{12}{7}$$

$$E[Y_1 \mid Y_2 = 2] \approx \frac{1 + 1 + 2}{3} = \frac{4}{3}$$

$$E[Y_2 \mid Y_1 = 1] \approx \frac{2 + 2 + 4 + 5}{4} = \frac{13}{4}$$

$$E[Y_2 \mid Y_1 > 1] \approx \frac{1 + 3 + 2}{3} = 2$$

Exercise 2.

(Independence and conditional expectation): Let X and Y be two random variables with sample spaces S_X and S_Y.

Part A

Prove that if X and Y are independent $E[X \mid Y = y] = E[X]$ and $E[Y \mid X = x] = E[Y]$ for all $x \in S_X$ and $y \in S_Y$.

Solution

Part B

Prove the tower property of expectation that is stated in the class notes.

Solution