

MIDTERM QUIZ – PRACTICE

Instructions

- You have 1 hour to complete the quiz.
- You may use your written notes, but NO electronics (computer, calculator, etc.).
- Each problem is worth 3 points.
- Show all your work but **circle your final answer**.

Exercise 1 (Conditional probability): Suppose

$$X \sim \text{Bernoulli}(1/2)$$

$$Y|X \sim \text{Bernoulli}(X/4 + 1/2)$$

Compute

- (a) $E[Y]$
- (b) $\text{var}(Y)$

Exercise 2 (Python code): Consider the following code which generates samples of variables X and Y .

```
> import numpy as np
> n = 1000
> x = np.random.choice([0,1],n,p=[0.5,0.5])
> y = np.random.normal(x,2,n)
```

- (a) Write down a probability model that is simulated by this code.
- (b) Write the code to compute $P(X = 1|Y > 0.3)$.

Exercise 3 (Estimators): Suppose you are designing a study to investigate the effect of a supplement on performance in a 5k. Let Y denote the random variable representing someone's time (in minutes) and X represent whether they took the supplement ($X = 1$ if they took it). If your probability model is

$$Y|X \sim \text{Normal}(\mu_0 + \Delta X, \sigma^2)$$

write down unbiased estimators of μ_0 , ΔX and σ^2 in terms of N samples $(X_1, Y_1), \dots, (X_N, Y_N)$.

Exercise 4 (Computations with data): Given the data in the table below, compute

- (a) $E[X]$
- (b) $E[X|Y = 0]$.

i	X_i	Y_i
1	0	0
2	0	0
3	0	0
4	1	0
5	1	0
6	0	1
7	0	1
8	1	1
9	1	1

Exercise 5 (Understanding continuous distributions): Suppose
 $Y \sim \text{Uniform}(0, 1)$

On the following diagram, labeling the probabilities/conditional probabilities which are given by the area of the green and blue rectangles.

