Math 70 Homework 2

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Part 1.

Instructions: Generate z_1 and z_2 as above and graph the scatter plot. Compute and show the regression lines z_1 on z_2, z_2 on z_1, and the major principle axis. Print out the four slopes and explain the results.

Part 2.

Instructions: Prove that $\mathbf{A}^{1/2}$, $\mathbf{A}^{-1}$, and $\mathbf{A}^{-1/2}$ derived through the matrix function meet their definitions.

If $\mathbf{A}$ is a symmetric matrix with spectral decomposition $\mathbf{A} = \mathbf{P\Lambda P}^{-1}$, then

$$f(\mathbf{A}) = \mathbf{P}f(\mathbf{\Lambda})\mathbf{P}^T$$

Keep in mind that matrix $\mathbf{P}$ is orthogonal, so $\mathbf{P}^T = \mathbf{P}^{-1}$, so $\mathbf{P}^T\mathbf{P} = \mathbf{PP}^T = \mathbf{I}$.

2.1 Proving $\mathbf{A}^{1/2}$ Meets Definition

By definition, $\mathbf{A}^{1/2}\mathbf{A}^{1/2} = \mathbf{A}$. Lets prove this identity through the matrix function.

$$\mathbf{A}^{1/2}\mathbf{A}^{1/2} = \mathbf{P\Lambda^{1/2}P}^T\mathbf{P\Lambda^{1/2}P}^T = \mathbf{P\Lambda^{1/2}I\Lambda^{1/2}P}^T$$

$$= \mathbf{P\Lambda P}^T = \mathbf{A}$$

2.2 Proving $\mathbf{A}^{-1}$ Meets Definition

By definition, $\mathbf{AA}^{-1} = \mathbf{I}$. Lets prove this identity through the matrix function.

$$\mathbf{AA}^{-1} = \mathbf{P\Lambda P}^T\mathbf{P\Lambda^{-1}P}^T = \mathbf{P\Lambda I\Lambda^{-1}P}^T = \mathbf{PIP}^T = \mathbf{I}$$

$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$ is also true.

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{P\Lambda^{-1}P}^T\mathbf{P\Lambda P}^T = \mathbf{P\Lambda^{-1}I\Lambda P}^T = \mathbf{PIP}^T = \mathbf{I}$$

2.3 Proving $\mathbf{A}^{-1/2}$ Meets Definition

By definition, $\mathbf{A}^{-1/2}\mathbf{A}^{-1/2} = \mathbf{A}^{-1}$. Lets prove this identity through the matrix function.

$$\mathbf{A}^{-1/2}\mathbf{A}^{-1/2} = \mathbf{P\Lambda^{-1/2}P}^T\mathbf{P\Lambda^{-1/2}P}^T = \mathbf{P\Lambda^{-1/2}I\Lambda^{-1/2}P}^T = \mathbf{P\Lambda^{-1}P}^T = \mathbf{A}^{-1}$$

Part 3.

Instructions: Prove that $\frac{\partial ||x||}{\partial x} = \frac{x}{||x||}$.

$||x||$ is the norm of x and can be defined as the square root of the scalar product of x with itself. Thus

$$||x|| = \sqrt{x^T x}$$

We can then differentiate this expression with respect to x.

$$\frac{\partial ||x||}{\partial x} = \frac{\partial \sqrt{x^T x}}{\partial x}$$

Apply chain rule

$$\frac{\partial\sqrt{x^T x}}{\partial x} = \frac{1}{2\sqrt{x^T x}}\frac{\partial(x^T x)}{\partial x}$$

Apply product rule

$$\frac{\partial(x^T x)}{\partial x} = \frac{\partial x^T}{\partial x}x + x^T\frac{\partial x}{\partial x} = Ix + x^T I = 2x$$

$$\Rightarrow \frac{1}{2\sqrt{x^T x}}\frac{\partial(x^T x)}{\partial x} = \frac{1}{2\sqrt{x^T x}}2x = \frac{x}{\sqrt{x^T x}} = \frac{x}{||x||}$$

$$\Rightarrow \frac{\partial||x||}{\partial x} = \frac{x}{||x||}$$