M86 Homework 1

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Exercise 1

Wilmott Ch. 1 Questions

Question 1.1 A company makes a three-for-one stock split. What effect does this have on the share price?

Answer The share price will decrease by a factor of three. So if the share price was \$90 before the split, it will be \$30 after the split. The total value of the shares will remain the same.

Question 1.2 A company whose stock price is currently S pays out a dividend DS, where $0 \leq D \leq 1$. What is the price of the stock just after the dividend date?

Answer If the stock price is \$100 and the dividend is 0.01, the price of the stock after the dividend date will be $S - DS = \$100 - \$100 \times 0.01 = \$99$.

Question 1.3 The dollar sterling exchange rate (colloquially known as 'cable') is 1.83, £1 = \$1.83. The sterling euro exchange rate is 1.41, £1 = €1.41. The dollar euro exchange rate is 0.77, \$1 = €0.77. Is there an arbitrage, and if so, how does it work?

Answer Let us test if there is an arbitrage opportunity. Let's start with \$1.

$\$1 \rightarrow £\frac{1}{1.83} \rightarrow €\frac{1.41}{1.83} \rightarrow \$\frac{1.41}{1.83 \times 0.77} = \$\frac{1.41}{1.4091} = \$1.0006387$

So we have made a profit of \$0.0006387. Very small, but still an arbitrage opportunity. The exchange fees would probably eat up the profit, but if we had a large amount of money, we might make a profit.

Another way to represent the conversion is:

$\$1 \times \frac{£1}{\$1.83} \times \frac{€1.41}{£1} \times \frac{\$1}{€0.77} = \$1.0006387$

Question 1.5 A spot exchange rate is currently 2.350. The one-month forward is 2.362. What is the one-month interest rate assuming there is no arbitrage?

Answer The one-month interest rate is $\frac{2.362-2.350}{2.350} = 0.005106383$ or 0.5106383%.

Question 1.6 A particular forward contract costs nothing to enter into at time t and obliges the holder to buy the asset for an amount F at expiry T. The asset pays a dividend DS at time t_d, where $0 \leq D \leq 1$ and $t \leq t_d \leq T$. Use an arbitrage argument to find the forward price $F(t)$.

Hint: Consider the point of view of the writer of the contract when the dividend is reinvested immediately in the asset.

Answer The value of the dividend including interest rate can be represented as $DS(t_d) \times e^{r(T-t_d)}$. For no-arbitrage, the party entering the forward must be compensated for this opportunity cost, so the forward price will be the interest-adjusted price of the underlying stock minus the interest-adjusted price of the dividend, or $F(t) = S(t) \times e^{r(T-t)} - DS(t_d) \times e^{r(T-t_d)}$.

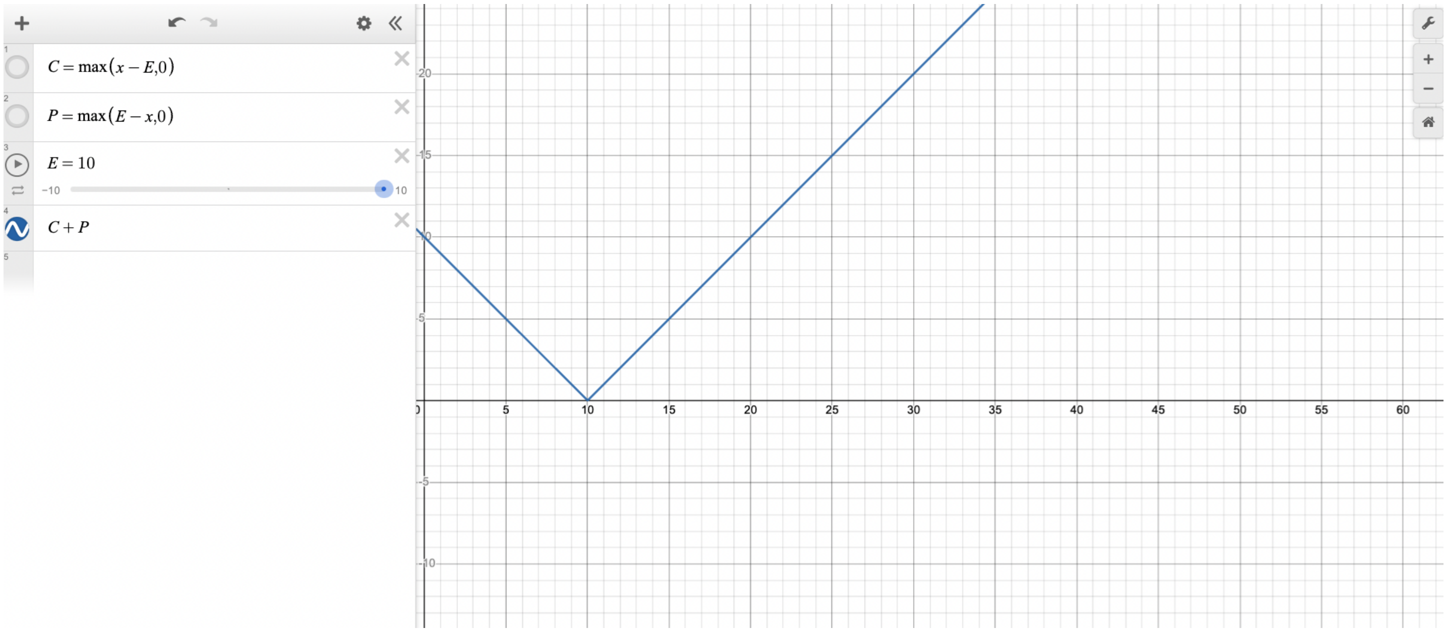
Exercise 2

Wilmott Ch. 2 Questions

Question 2.1 Find the value of the following portfolios of options at expiry, as a function of the share price:

2.1b Long one call and one put, both with exercise price E.

Answer This is a straddle. The value of the call is $C = \max(S - E, 0)$ and the value of the put is $P = \max(E - S, 0)$. The value of the portfolio is $C + P = \max(S - E, 0) + \max(E - S, 0) = \max(S - E, E - S, 0) = |S - E|$.



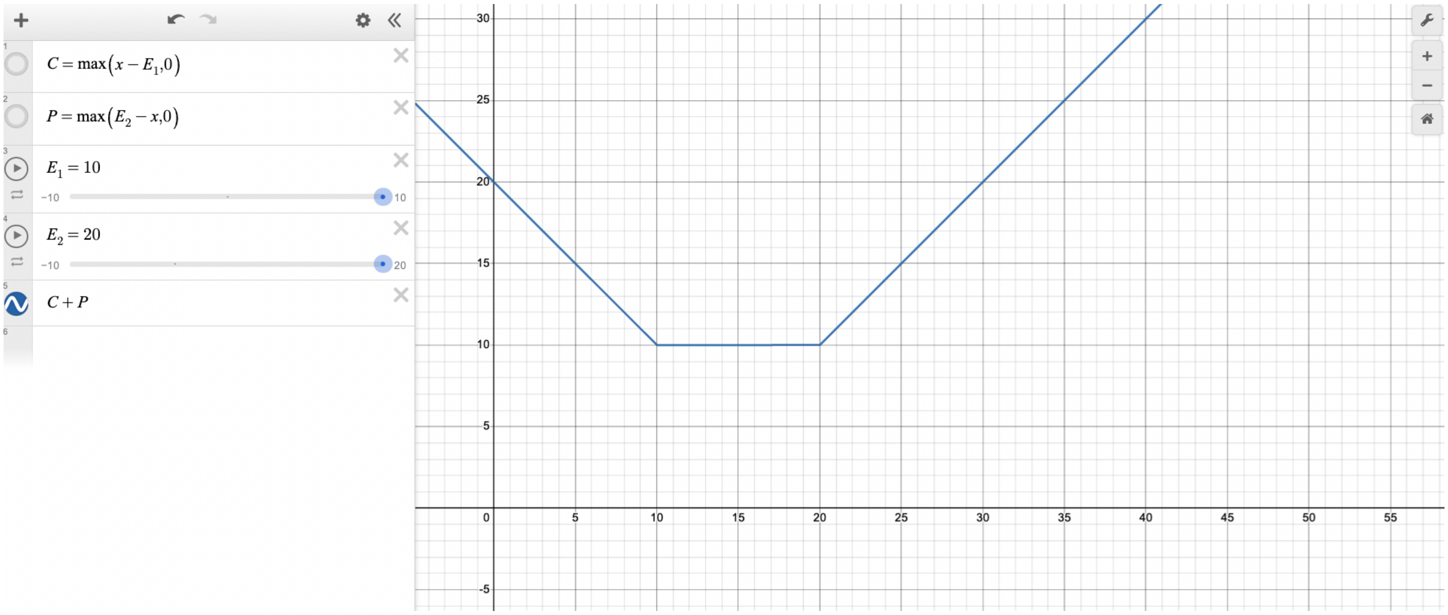
2.1c Long one call, exercise price E_1, short one call, exercise price E_2, where $E_1 < E_2$.

Answer This is a bull spread. The value of the portfolio is $C_1 - C_2 = \max(S - E_1, 0) - \max(S - E_2, 0)$.

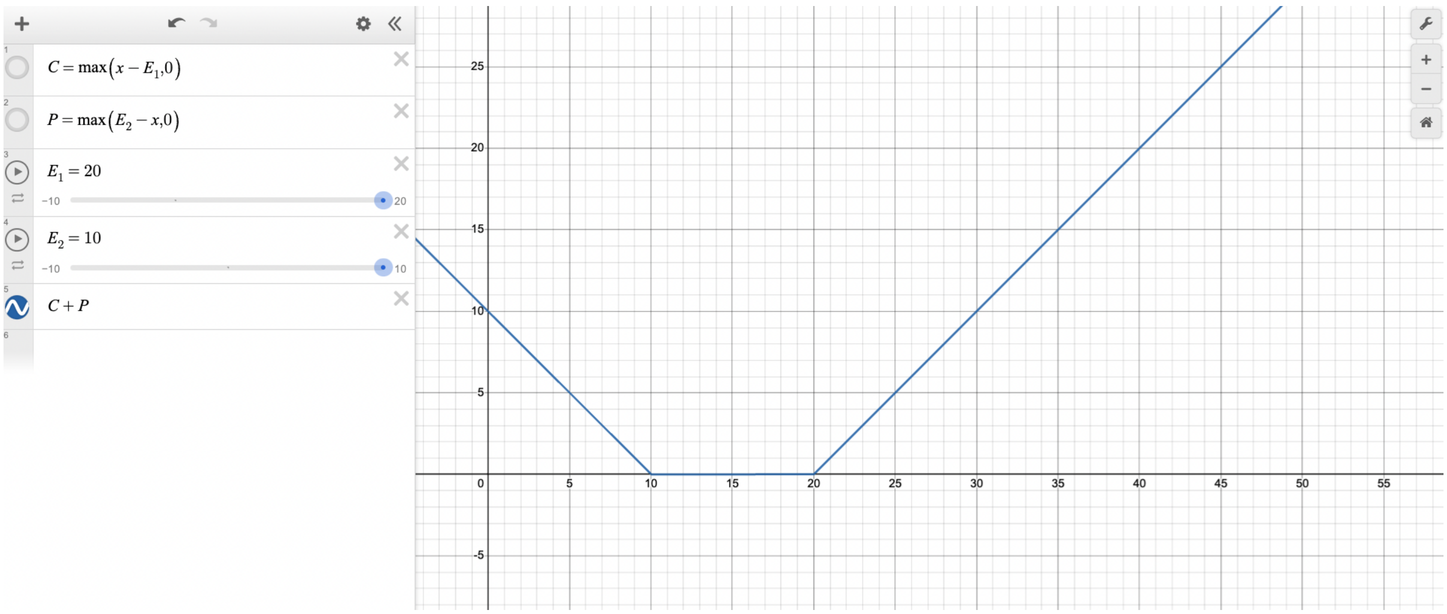
2.1d Long one call at exercise price E_1, long one put at exercise price E_2 . There are three cases to consider.

Answer In all three cases, the value of the portfolio is $C + P = \max(S - E_1, 0) + \max(E_2 - S, 0)$.

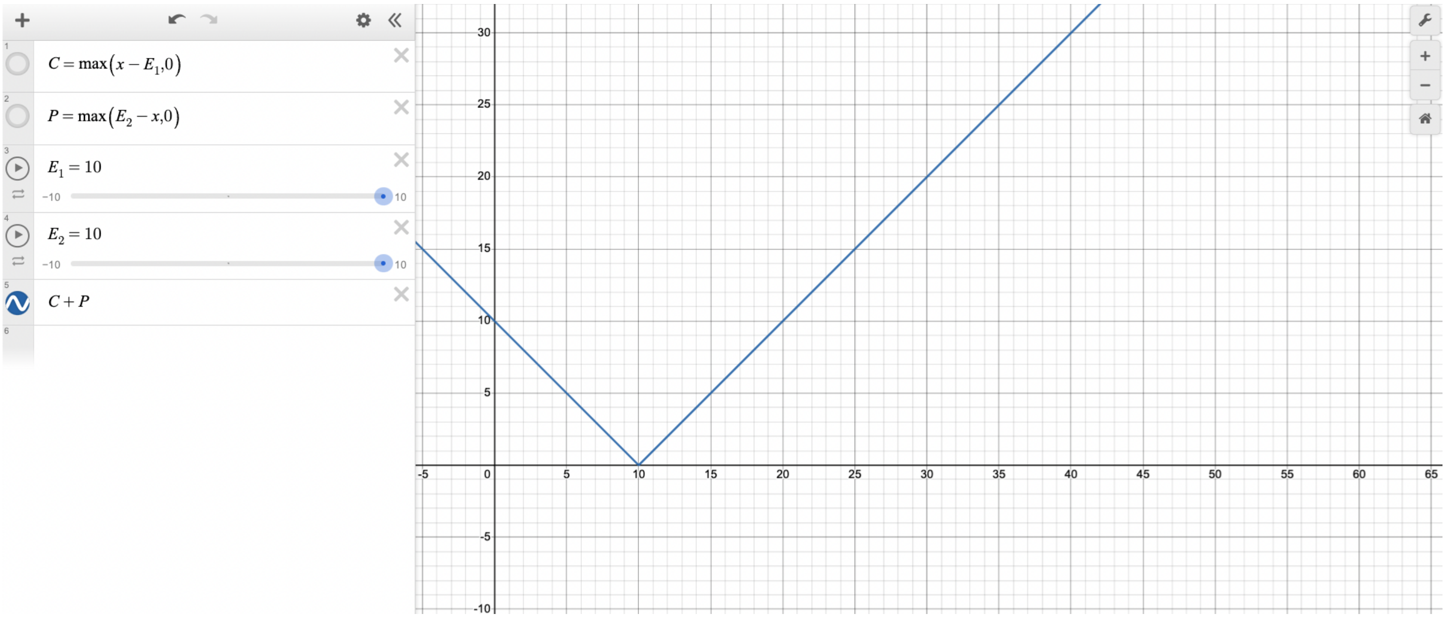
Case 1: $E_1 < E_2$



Case 2: $E_1 > E_2$



Case 3: $E_1 = E_2$


$C = \max(x - E_1, 0)$
$P = \max(E_2 - x, 0)$
$E_1 = 10$
$E_2 = 10$
$C + P$