ACS336/6336
Take-Home Quiz Briefing

Payam Soulatiantork

January 18, 2019

Introduction

This document contains a set of questions designed to assess your knowledge of the modelling, simulation and control aspects of this course. This assignment is worth 35% of the overall module mark.

Submission Details

Students are requested to prepare a short lab report that answers each of the questions below, and submit a hard copy to ACSE reception no later than 4pm on Friday May 10th. **Please remember to attach a coursework coversheet[1] to the front of your report**. Students are encouraged to answer the questions in a concise and precise fashion. Late submissions will be docked 5% for each day late, up to a maximum of 5 working days. In the event of extenuating circumstances, students must submit an extenuating circumstances form if they have any medical or special circumstances that may have affected their performance on the assignment. **You may not collaborate with other students, past or present, and instances of unfair means will be dealt with severely.**

Feedback

This is the final component of assessment for this module. As a consequence of this the marks need to be checked and verified by the Department before they are returned to you. This means that you will receive your overall mark after the exam board and at the same time as the final marks for the rest of your semester 2 modules. Feedback on class performance will be made available to all students as part of the module review. This is sufficiently detailed to allow students to understand the common mistakes made in each question. This feedback will be made available shortly after the Summer Exam Board Meeting. Please note, it is not our policy at this time to release marked scripts back to students for this particular assignment. In the event that you are unsatisfied with the provided feedback, individual scripts are viewable in line with the Department's procedures for viewing exam papers:
http://www.sheffield.ac.uk/acse/current/examviewing

[1] Available from http://www.cpe-electronics.group.shef.ac.uk/bcstudent/

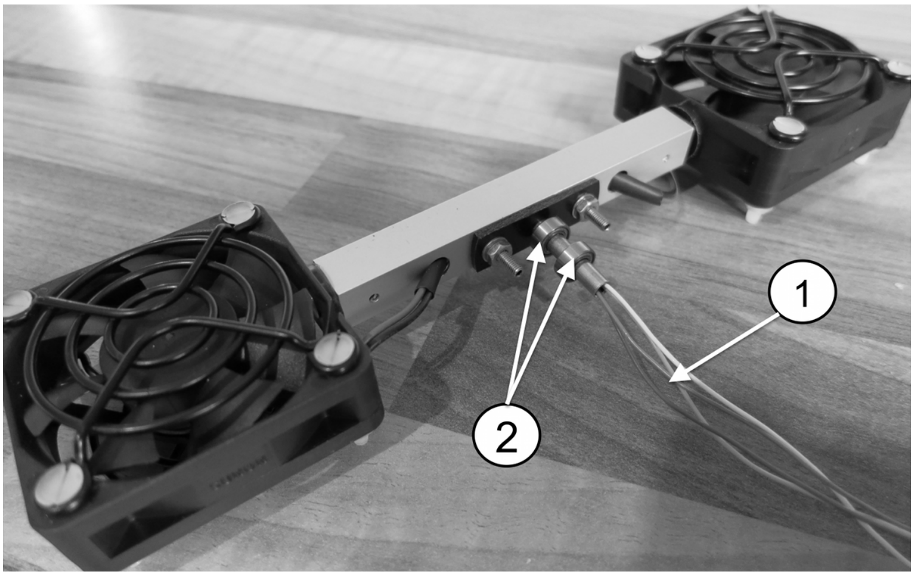

Figure 1: Photo of the pitch-beam sub-assembly, showing (1) the fan wires emerging from a hollow brass insert, and (2) miniature ball-bearings.

Questions

1. The motion about the pitch axis on most of the helicopters displays a damped oscillatory response when released from rest and from a nonzero initial condition of pitch axis angle. Briefly explain the physical source of this response. You may wish to refer to Figure 1. (10 marks)

2. List five ways in which the dynamics of your SIMULINK model of the helicopter hardware differs from the dynamics of the actual hardware. (20 marks)

   Please be precise with your answers. For example, an answer along the lines *dynamic friction about the elevation axis exists in the hardware, but has not been included in the simulation model* is acceptable[2], whilst *the hardware is more complicated than the simulation model* is not.

3. Imperfect modelling results in the output response (in terms of the elevation, pitch and travel angles) to differ between the SIMULINK model and the actual hardware. With respect to your answers to the previous question, select the modelling discrepancy that you believe creates the largest error in output response between the SIMULINK model and the actual hardware. *Carefully* describe how you would improve the SIMULINK model with a view towards reducing this modelling discrepancy. (10 marks)

   An example of a good answer is as follows[3]. *The dynamic friction about the elevation axis could be approximated by a linear damping term (τ_{e}, say) that resists angular velocity $\dot{E}$, and which would appear as an extra term on the right hand side of the governing equation for motion about the elevation axis. This could be modelled as $\tau_{\mathrm{e}}(t) = -k_{\mathrm{e}}\dot{E}(t)$, with the value for the damping coefficient k_{e} determined from an experiment that involved measuring the time taken for the helicopter to drop to its lower angular limit when released from rest from its upper angular limit. The* SIMULINK *model for the elevation dynamics block would then be modified for the inclusion of this extra damping term.*

   An example of a bad answer, lacking in sufficient detail, is as follows. *I would include a model of the friction.*

[2]NB, students are not permitted to re-use this, or any variant of this, as their answer
[3]NB, students are not permitted to re-use this, or any variant of this, as their answer

4. A major source of plant/model mismatch is the lack of a model that accounts for the dynamic friction that exists about the elevation axis. As mentioned above, this could be approximated by a linear damping term of the form $\tau_{\rm e}(t) = -k_{\rm e}\dot{E}(t)$, where $k_{\rm e} \in \mathbb{R}_+$ is a linear damping coefficient. The inclusion of this term would result in the following modified governing equation of motion about the elevation axis:

$$J_E\ddot{E}(t) = -J_E\dot{\Theta}^2(t)\sin E(t)\cos E(t) + g(m_2l_3 - 2m_1l_1)\cos E(t) + l_1(F_a(t)+F_b(t))\cos\Psi(t) - k_{\rm e}\dot{E}(t), \tag{1}$$

where all terms in (1) are defined as above and in the lecture notes.

- An incomplete SIMULINK block diagram of (1) is shown in Figure 2. Sketch on the additional block(s), together with the necessary connections between all blocks, in order to complete the block diagram representation of (1). Scan, or photo the completed diagram and append to the back of your lab report. (5 marks)
- Your existing helicopter controller (prior to state augmentation) was synthesised from the following linear state-space model:

$$\frac{d}{dt}\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\\ \dot{\epsilon}(t)\\ \dot{\psi}(t)\\ \dot{\theta}(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -\frac{k_s}{J_\Psi} & 0 & 0 & -\frac{k_d}{J_\Psi} & 0\\ 0 & -\frac{2l_1(\alpha k_a U_e+\beta)}{J_\Theta} & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\\ \dot{\epsilon}(t)\\ \dot{\psi}(t)\\ \dot{\theta}(t)\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{l_1\alpha k_a}{J_E} & \frac{l_1\alpha k_a}{J_E}\\ \frac{l_2\alpha k_a}{J_\Psi} & -\frac{l_2\alpha k_a}{J_\Psi}\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_a(t)\\ u_b(t)\end{bmatrix},$$

$$\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\\ \dot{\epsilon}(t)\\ \dot{\psi}(t)\\ \dot{\theta}(t)\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_a(t)\\ u_b(t)\end{bmatrix},$$

  where all terms are defined as in the lecture notes. A copy of this state-space model is included in the back of this handout. The linear damping term on the far right of (1) can be included in the controller design by appropriate modification of the state-space matrices above. On the copy in the back of this handout, circle any element(s) of the state-space matrices that you would change in order to accommodate the linear damping term, and state the new value of the circled element(s). Scan, or photo the completed diagram and append to the back of your lab report. (5 marks)

5. As part of your lab exercises, you were asked to note the value of the constant input voltage U_e required to achieve level hover. The modelled value of U_e is unlikely to have been equal to the actual value of U_e. One consequence of this is that the helicopter will not achieve hover in the horizontal position under the action of an open-loop controller. Carefully explain how you would design a feedback controller to achieve horizontal hover assuming your modelled value of U_e is different from the actual value of U_e. (10 marks)

6. If a pitch angle sensor was not available to measure the pitch angle, would you still be able to use a full state feedback controller? Carefully explain your answer, using mathematical treatment, if necessary. (10 marks)

7. Write down your final design values for Q_x and Q_u, the state and control penalty matrices used in the design of your LQR controller[4]. Briefly explain your reasons for selecting these values. (10 marks)

[4] If you have employed different controllers for your hardware and simulation model, then use the values of Q_x and Q_u from your simulated model

8. Provide plots of your simulated closed-loop controller response. Specifically, provide the following:

   - A plot of reference and output elevation angles against time,
   - A plot of reference and output travel angles against time,
   - A plot of the pitch angle against time,
   - A plot of the control signal $U_{a,b}$ against time.

   You may wish to refer to the plots at the end of Lecture 8 to see examples of these plots. Now imagine you are presenting your controller to the manufacturers of the helicopter hardware, with a view to licensing it, and making a small fortune in the process. The manufacturers obviously need some convincing that your controller is worthy of investment. Referring to your plotted responses and in fewer than 100 words, explain the performance of your helicopter in terms of how it responds to changes in reference inputs. (20 marks)

Answer sheet 1 for Question 4 - append to back of lab report

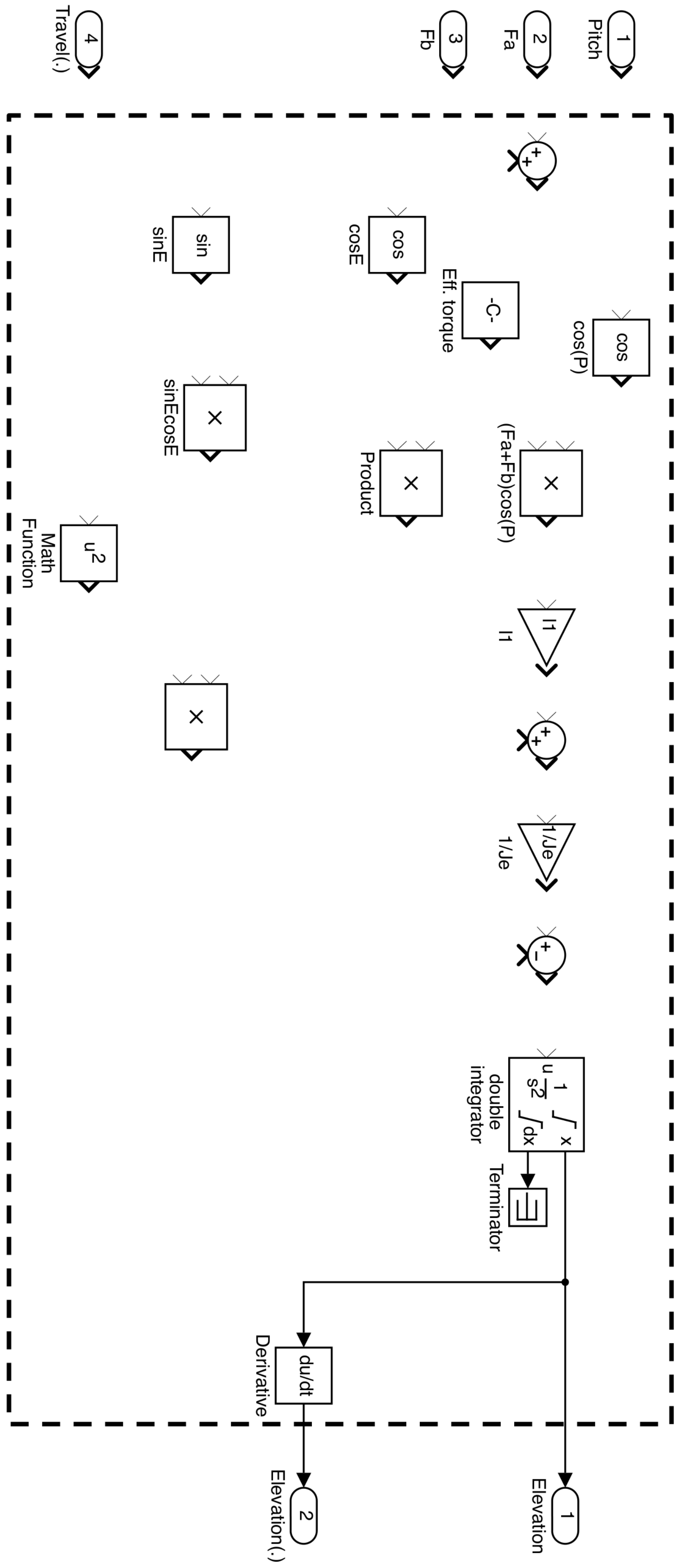

Figure 2: Partially completed elevation dynamics block diagram.

Answer sheet 2 for Question 4 - append to back of lab report

Circle any element(s) of the following state-space matrices that would change in order to accommodate the linear damping term in (1), and state what the new value of the circled element(s) would be.

$$
\frac{d}{dt}\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\\ \dot{\epsilon}(t)\\ \dot{\psi}(t)\\ \dot{\theta}(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -\frac{k_s}{J_\Psi} & 0 & 0 & -\frac{k_d}{J_\Psi} & 0\\ 0 & -\frac{2l_1(\alpha k_a U_e+\beta)}{J_\Theta} & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\\ \dot{\epsilon}(t)\\ \dot{\psi}(t)\\ \dot{\theta}(t)\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{l_1\alpha k_a}{J_E} & \frac{l_1\alpha k_a}{J_E}\\ \frac{l_2\alpha k_a}{J_\Psi} & -\frac{l_2\alpha k_a}{J_\Psi}\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_a(t)\\ u_b(t)\end{bmatrix},
$$

$$
\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\epsilon(t)\\ \psi(t)\\ \theta(t)\\ \dot{\epsilon}(t)\\ \dot{\psi}(t)\\ \dot{\theta}(t)\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_a(t)\\ u_b(t)\end{bmatrix}.
$$