

Interim Report

Dynamic Modelling, Simulation and Control of
Asymmetrical VTOL Multi-Rotor UAVs

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Project Background

Unmanned Aerial Vehicle (UAV) are aerial systems which are not directly controlled by a human onboard the vehicle. There are many types of UAV platforms chiefly defined by the characteristics of their mechanical constructions, such as the on frame position, number and type of their actuators. One such type of UAVs are multi-rotors which are defined as such due to their multiple rotor wing actuators [5],[6].

These types of UAVs have a wide range of applications due to their ability to perform vertical take-off and landing (VTOL), stationary and low speed flight [5], coupled with their relatively simple mechanical designs when compared to single rotor constructions, such as more traditional helicopters [2]. These features enable them to be utilised in many wide ranging applications, where it would not be possible to use other platforms, such as; large scale directed administration of substances to plants in precision agriculture, frequent inspection of standing structures, wide area surveillance, amongst many others [10],[11].

The major challenge in dealing with these types of UAVs is their inherent instability in flight, save for the intervention of complex control systems [1],[2],[3],[4],[5]. For this reason, control schemes must be implemented in order for multi-rotor UAVs to be utilised. Therefore in order to ensure control schemes meet performance specifications in an energy efficient manner, it is necessary to understand the physical characteristics and response to stimuli of the system which is done through mathematical modelling. This level of insight is also required in the development of many advanced control schemes. [11],[14].

Aims and Objectives

The framework of this project is set around the design and development of control software for Vertical/Short Take-off and Landing (VSTOL) model aircraft. From this theme, the aims of modelling and simulating the non-linear dynamics of an asymmetrical VTOL multi-rotor platform and the development of multiple control schemes were derived. These control schemes will then be implemented on the multi-rotor platform via an embedded microprocessor unit.

These aims breaks down in to multiple objective milestones which must each be achieved to fulfil the full scope of that the aim outlines. These Objectives can be Classified into 2 categories:

Basic Objectives

1. Develop a mathematical model representing the dynamics of the multi-rotor aircraft.
2. Develop a dynamic simulation of the crafts behaviour.
3. Develop feedback control laws: Proportional Integral Derivative(PID) and Linear Quadratic Gaussian(LQG).
4. Investigate feedback control laws in simulation with the mathematical model to achieve behavioural targets.
5. Develop flight control software to interface with sensors and implement control laws.
6. Implement the flight control software on an embedded microprocessor unit.
7. Implement controller unto multi-rotor platform.
8. Discuss the results of performance comparisons between the simulation and hardware implementations.

Advanced Objectives

1. Investigate the application of Model Predictive Control(MPC) schemes in simulation.
2. Incorporate and implement MPC schemes unto the flight control software.
3. Discuss the differences between the PID, LQG and MPC implementations.

Review of Literature and Work Till Date

The acquired literature take the form of articles published in scientific journals, Research publications, exerts from university Lectures, Masters and PhD thesis publications and published books, which present 3 major themes that are relevant to the project's investigation:

- Mathematical Modelling and Systems Identification.
- Control Design.
- Simulation and Implementation.

UAV



Figure 1: Y6 Hexarotor CAD Graphic

The selected UAV under consideration is one of my own design, which is an asymmetrical Y6 Hexarotor, meaning that it has 6 actuators which are arranged in such a configuration where it resembles the shape of a "Y". This kind of configuration has its actuators in 3 coaxial pairs, meaning two motors one atop the other back to back, which has the advantage of minimising the size of the frame while enabling higher thrust and stability. However there are added power demands and inefficiency due to the increased number of actuators and the aerodynamic interaction of coaxial actuator combinations [6].

The UAV is actuated by 6 1000Kv brushless direct current (BLDC) motors attached to 6 10x45inch propellers and is powered by a 4-cell lithium-polymer battery with an operational voltage range of 16.8 volts to 12.8 volts. The UAV has dimensions of, 45 cm in length from rear motor centres to front motor centres, 44 cm in length from left motor centres to right motor centres, and a mass of 2.1 kilograms. control systems will be implemented on a BeagleBone Blue linux based single board computer (SBC).

The exact details involved with the design, development and construction of UAVs is beyond the scope of the project investigation but are fully explored in [19].

Mathematical Modelling and System Identification

Rigid-Body Dynamics

In modelling, it is necessary to state assumptions made about its characteristics. Multi-rotors can be defined as a rigid body free to move within 3D space. Therefore it moves with 6 degrees of freedom (6-DOF). With this modelling assumption, the motion of the vehicle is subject to the laws of rigid body Kinematics and Kinetics. Vehicle motion is defined in terms of 2 coordinate frames moving relative to one another where the physical quantities that change with time, its states, change with respect to one frame or the other. They are; Earth Frame: F_e , Attached to the earth, Body Frame: F_b , Attached to the vehicle body. Each frame consists of 3 orthogonal axes, $x_e y_e z_e$ and $x_b y_b z_b$ respectively about which rotational motion can occur, and along which translational motion can occur [1],[2],[3],[4].

The sources obtained explore 2 ways for representing rotational motion. Euler angles are used to describe arbitrary orientation in the 3-dimensional Euclidean space, three parameters are required. Euler angles represent a sequence of three elemental rotations about each axes of a coordinate system. Any orientation in 3D space can be achieved by composing these 3 elemental rotations [2],[4]. Another representation is through Quaternions which solve issues present with computing Euler angles such as computational expense and rotational singularities which occur when during certain orientations, as it does not require the calculation of sin and cos when certain angles go to 0 or 90 degrees and their multiples. Quaternions represent orientation using a single angle about an axis [4],[14].

The rigid body assumptions also allows for the utilisation of several Mechanical Modelling techniques and conventions which are used to derive the non-linear dynamics of mechanical systems taking into account both kinematics and Kinetics:

- The Newton-Euler. [1],[2],[3],[4],[5],[10],[13],[14],[15],[16]
- The Euler-Lagrange. [4],[16]
- The Newton-Hamiltonian.
- Hardware-in-Loop system identification. [11]

This investigation utilises both the Euler-angle representation and the Newton-Euler convention due to its intuitively familiar representation of systems dynamics from the first princi-

ples of newtons 2nd law showing the effects of forces applied to the rigid body by its actuators. Due to the UAVs Layout, it is asymmetric, this creates a system where inertia is not represented by a diagonal matrix but instead one with off diagonal elements and transposed reference frames [7],[8]. However, for the purpose of this investigation, it is assumed that the system is symmetrical as the off diagonal terms in the inertia matrix are far lower in magnitude than the diagonal terms, thus can be reasonably neglected. Otherwise, the entire coordinates system would be adjusted and transformed to produce a true diagonal matrix as presented in [8].

In order to design the controllers, the non-linear model obtained via the Newton-Euler convention needs to be linearised. This is accomplished through the application of Jacobi's lineari- sation as presented in [12]. This process produces a full linear state space representation of systems dynamics around stable operating points.

Actuator Dynamics

As stated in the *UAV* section, the multi-rotor is actuated by BLDC motors. To fully develop a system model, these must be modelled. This can either be done via first principles calculations as in [2],[3],[16] or via experimental systems identifications as presented in [4],[6],[9],[10],[11],[14],[17]. This approaches involves the derivation of a lumped parameter linear input output model, encompassing the electronic speed control, motor and propeller dynamics, between input Pulse width modulated (PWM) signal duty cycles or pulse width and the output angular velocities torque and thrust forces. This has the advantage of reducing complex dynamics enabling the utilisation of a minimum viable model and as such is being implemented in this investigation. Once the actuator dynamics are identified they will substitute the dummy place holder values taken from data presented in [14].

Results

Presentation of the system's non-linear dynamics.

In the Body frame:

$$I\mathbf{v}' = \boldsymbol{\tau} - \mathbf{v}X(I\mathbf{v})$$

$$I \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} (X)I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Where p, q, r, p', q', r' are angular velocities and accelerations, I is the diagonal inertia matrix and τ is a vector of the torques generated from the actuators and UAV body.

In the Earth frame:

$$M\ddot{\zeta}'' = G + RT_b - D\dot{\zeta}'$$

$$M \begin{bmatrix} \ddot{x}'' \\ \ddot{y}'' \\ \ddot{z}'' \end{bmatrix} = -Mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + T_b \begin{bmatrix} C\phi S\theta C\psi + S\psi S\phi \\ C\phi S\theta S\psi - S\phi C\psi \\ C\phi C\theta \end{bmatrix} - D \begin{bmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{bmatrix}$$

Where M is mass, $x, y, z, x', y', z', x'', y'', z''$ are linear position, velocity and acceleration, ϕ, θ, ψ are Euler angles, D is the diagonal matrix of air resistance and T_b is a vector of the Forces generated from the actuators. This was then turned into a Simulink Function block:

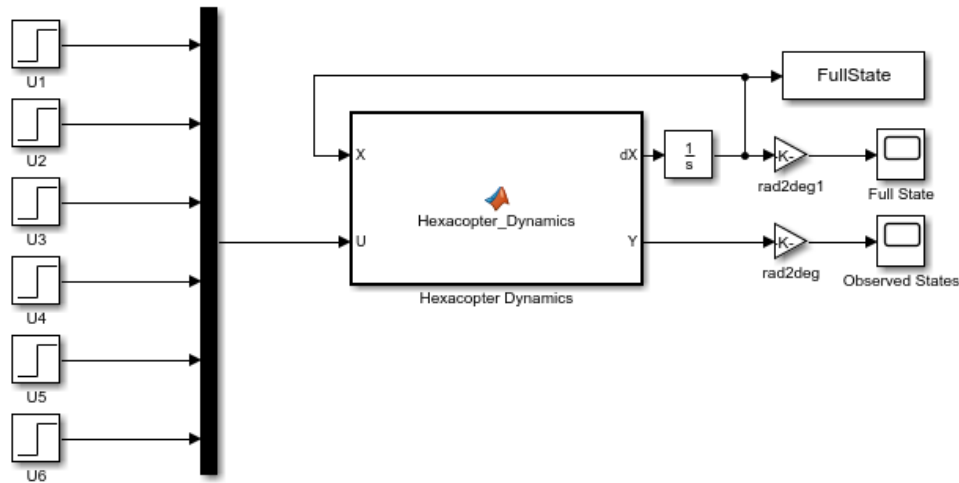


Figure 2: Non Linear Dynamics in Simulink

Control Design simulation and Implementation

The three control schemes under investigation are:

PID

PID controllers consist of a component proportional to the error between a desired value and the observed value, a component proportional to the derivative of the error, and on which is

proportional to the integral of the error. Adding an integral term causes any remaining steady-state error to build up and enact a change on the input signal to the system, so a PID controller should be able to track target trajectories with small to zero steady-state error.[3] In practical implementations, pure PID controllers come with drawbacks like integral wind-up and derivative noise sensitivity which must be dealt with during the implementation stage[21]. Another issue is, PID controllers are designed for Single input and output systems thus any implementation for a multiple input and output system like a UAV will not take into account dynamic coupling between states. [14] presents a novel way to overcome this by designing the PID controller using LQG techniques.

LQG

The Linear Quadratic Gaussian makes use of the full state of a system through the application of a Gaussian estimator, in this case a Kalman filter, to obtain the full state. It is a form of optimal control where its objective is to minimize a cost or performance function to bring the system's state to a desired set of values while minimizing the use of the control inputs[2][14][20][21]. As of the creation of this report, significant work has been done ahead of schedule in beginning the development an LQG controller. Below is the MATLAB Simulink simulation showing the an LQG control structure in series with the multi-rotor dynamics.

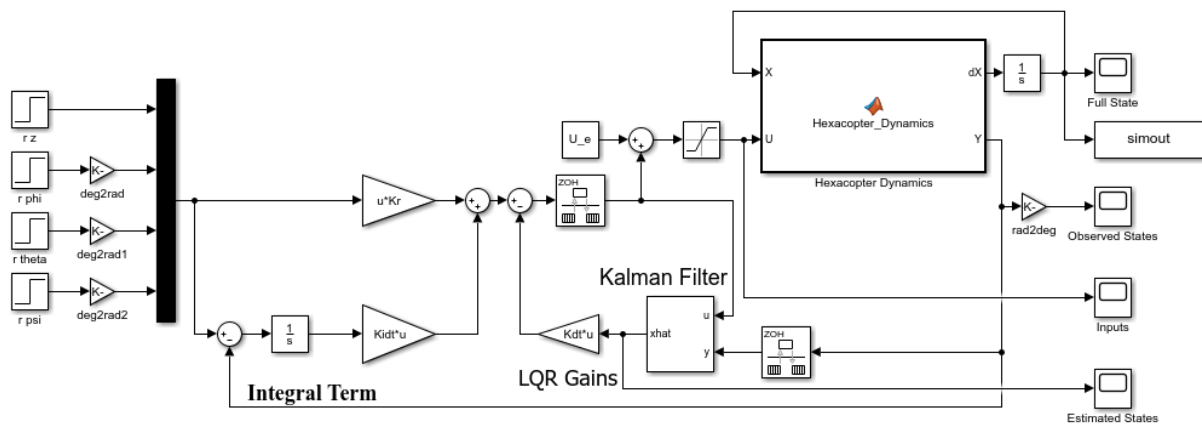


Figure 3: Dynamics and LQG controller in Simulink

MPC

Like LQG, MPC is also a kind of optimal control, however, this technique's cost function includes the values of future states over some fixed finite horizon and control signals can be

determined so that the system can meet physical constraints. MPC is also known as receding horizon control(RHC) as when it only acts on the first step in the horizon of states before recalculating and performing this action until the target is achieved[11]. The LQG can be referred to as an infinite horizon controller as it does not recede the horizon[18].

Study [11] also discusses various implementations methods including utilising machine learning techniques to estimate certain optimisation parameters, as well as proposing and implementing such a controller.

Summary

Thus far, the assembly of the multi-rotor platform, displayed in the *UAV* section, has been completed with all the parts of the frame including the actuators and propellers present, access to pertinent literature concerning; the mathematical modelling of rigid bodies, the system identification of actuator dynamics, the design and implementation of a myriad of linear and non-linear state feedback techniques, as well as to those beginning investigations into the more advanced topic of model based predictive control, have been obtained.

In working towards the objectives layed out in the *Aims and Objectives* section, the full non-linear mathematical model of the rigid body dynamics of the multi-rotor platform have been developed and implemented as a simulation in MATLAB/Simulink, as well as Linear State Space representations of the system for the purposes of control design, with work having commenced on the design of an LQG controller

Project Management

In accordance with the project *Aims and Objectives*, significant progress has been made particularly towards the timeous completions of *Basic Objectives* points 1, 2, 3 and 4 which will be due for completion and review in accordance with the project plan. Thus far, the project has proceeded on schedule, therefore no changes have been made to the plan submitted on week 6. Purchases have been made acquiring key hardware such as the main flight computer, the radio control system, which will serve as the main communication unit sending control commands to the multi-rotor. Orders have also been put in for the more motors, electronics speed controllers, and load cells which will be used to develop the lab-bench platforms that will serve to identify the actuator dynamics specified in the *Actuator Dynamics* section. Future completion dates for further objectives are displayed in the project plan expressed in the table below with the in the full project Gantt chart available in the *Appendix*.

		Name	Duration	Start	Finish	Predecessors
1		Development of V/STOL MultiRotor	189 days	21/10/19 08:00	27/04/20 08:00	
2		S1 Week 4 Aims and Objectives	0 days	21/10/19 08:00	21/10/19 08:00	
3		S1 Week 6 Progress Review	0 days	04/11/19 09:00	04/11/19 09:00	
4		S1 Week 10 Second Reviewer	0 days	02/12/19 09:00	02/12/19 09:00	
5		S1 Week 11 Interim Report	0 days	09/12/19 09:00	09/12/19 09:00	
6		S1 Week 12 Second Reader Meeting	0 days	16/12/19 09:00	16/12/19 09:00	
7		S2 Week 1 Intereme Report Feedback	0 days	03/02/20 09:00	03/02/20 09:00	
8		S2 Week 6 Progress Review	0 days	02/03/20 09:00	02/03/20 09:00	
9		S2 Week 11 Final Report	0 days	20/04/20 08:00	20/04/20 08:00	
10		S2 Week 12 Presentation	0 days	27/04/20 08:00	27/04/20 08:00	
11		Modelling	68 days	21/10/19 08:00	28/12/19 08:00	
12		Obtain Parts	14 days	21/10/19 08:00	04/11/19 08:00	
13		Complete Multirotor Platform	14 days	04/11/19 08:00	18/11/19 08:00	12
14		Investigate Literature	14 days	21/10/19 08:00	04/11/19 08:00	
15		Develop Dynamical Model	40 days	04/11/19 08:00	14/12/19 08:00	14
16		Develop Model Simulation	14 days	14/12/19 08:00	28/12/19 08:00	15
17		Control Systems Design	85 days	04/11/19 08:00	28/01/20 08:00	
18		Investigate Literature	30 days	04/11/19 08:00	04/12/19 08:00	14
19		PID Control Design	14 days	28/12/19 08:00	11/01/20 08:00	16;18
20		LQG Control Design	21 days	28/12/19 08:00	18/01/20 08:00	16;18
21		Obtain results in Simulation	10 days	18/01/20 08:00	28/01/20 08:00	20
22		Flight Software Development	150 days	21/10/19 08:00	19/03/20 08:00	
23		Obtain key Libraries	7 days	21/10/19 08:00	28/10/19 08:00	
24		Discretize Control Laws	7 days	28/01/20 08:00	04/02/20 08:00	20;19;21
25		Modify/Develop Existing Software	30 days	04/02/20 08:00	05/03/20 08:00	23;24
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33		Modify Flight Software	21 days	03/03/20 08:00	24/03/20 08:00	32
34		Implement/test on Hadware	5 days	09/04/20 08:00	14/04/20 08:00	33;29

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Figure 4: Table of Project Objectives and Dates

Self Review

In dealing with project operations, there have been several challenges most prominently those involved with obtaining and simulating the multi-rotor's dynamics. The work done in understanding the complex mechanics, which is one of my weakest subjects, has been very helpful in gaining insight into the workings of dynamical systems at large. Proving to be a task of decent difficulty is learning how to utilise the full range of capabilities provided by Matlab/Simulink. This is displayed by the fact that the rigid body simulation the presented in the *Results* section breaks down due to singularities forming, the issue raised in the *Rigid-Body Dynamics* section. I am yet to fully diagnose the issue, most likely this is due still no fully understanding how to represent a simulation of such complex dynamics.

I have had to balance work on this projects with other modules which has been challenging. Though, at the time of creating this report, work has progressed on time and on-schedule towards achieving the first 4 objectives listed in the *Basic Objectives* section. This has been a great exercise in developing the ability to manage a project not only in the scope of the project across time with setting achievable objectives, as the scope of the project has been revised down from designing control systems for a full Quad-Tilt multi-rotor, to where it is now. but also in managing individual activities such as parts purchasing.

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Gantt Chart

Figure 5: Gantt Table of Project Objectives and Dates

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Figure 6: Gantt Chart Page 1

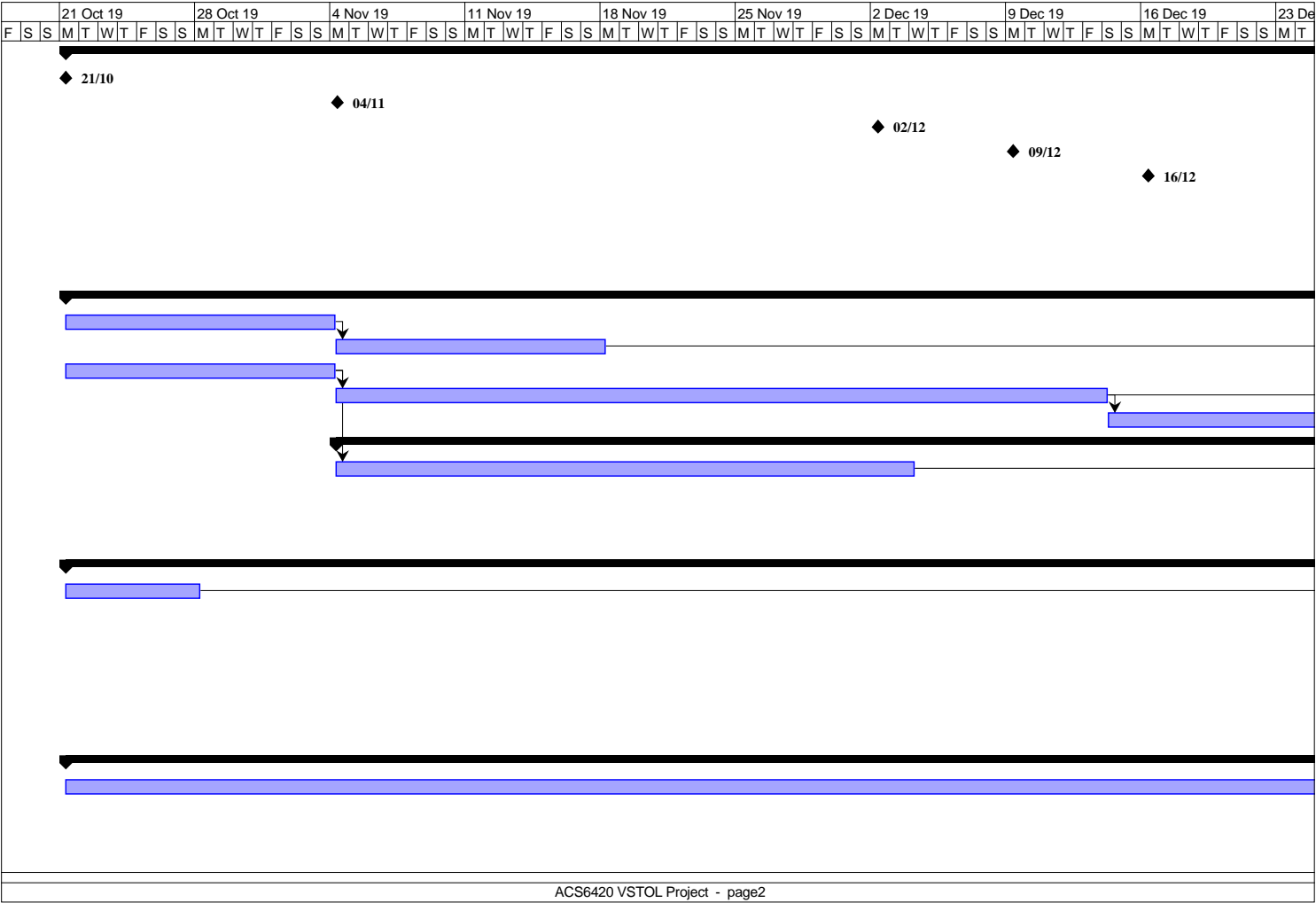


Figure 7: Gantt Chart Page 2

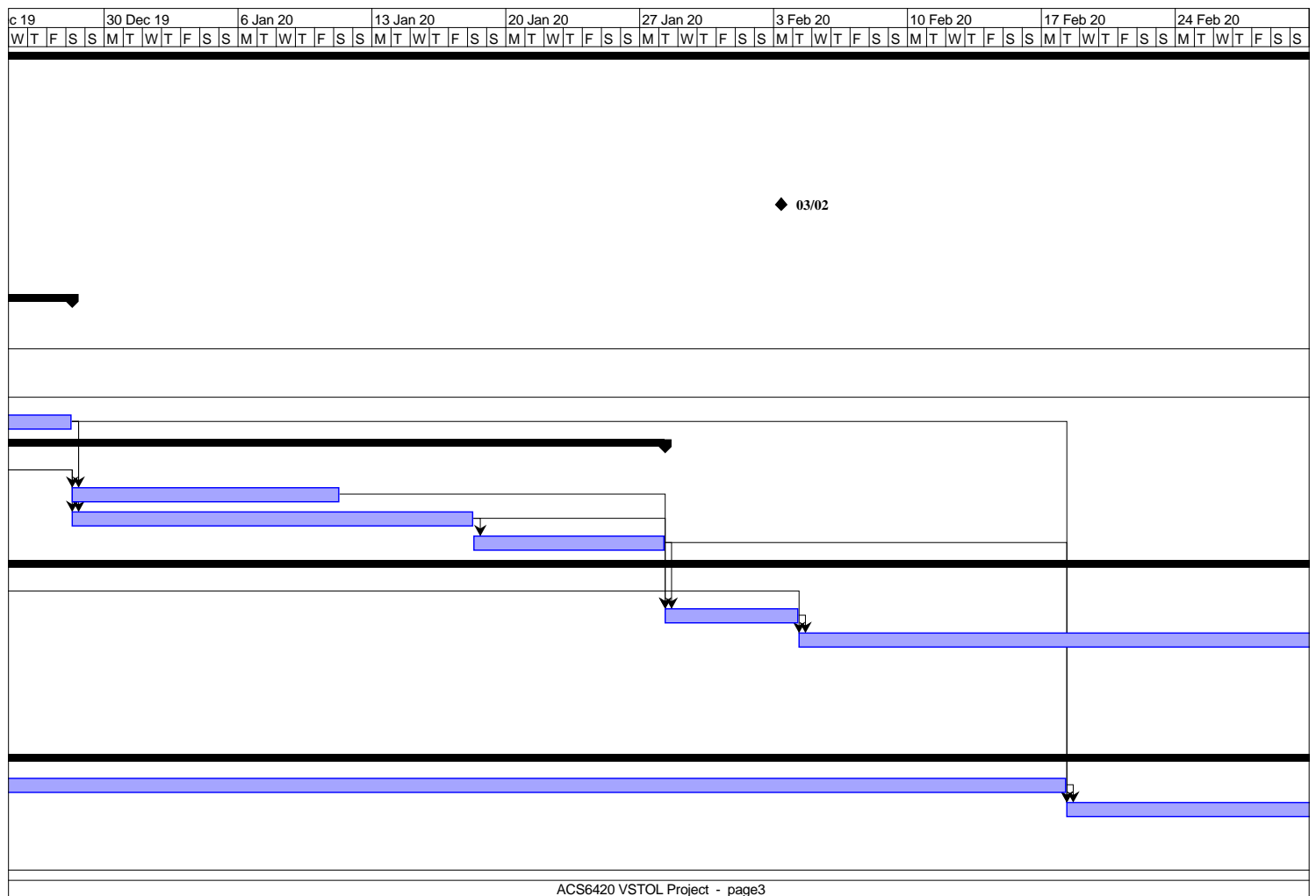


Figure 8: Gantt Chart Page 3

