Sparsity Misspecification and Robust Covariate Effect Estimation for Sparse Social Networks

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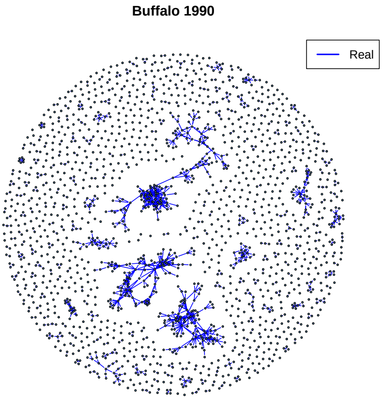
Network Link Generation Problem



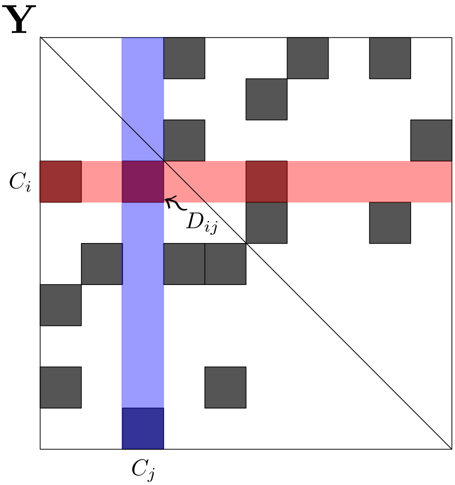
Sample of n actors.

Explain or predict pairwise outcomes Y using pairwise covariates X.

X may be observed or latent.

Network Sample Representation

Generalized Random Graph



Y entries in **arbitrary sample space** with some element $\mathbf{0}$.

Binarized graph A with

$$A_{ij} \equiv \mathbf{1}_{Y_{ij} \neq \mathbf{0}}.$$

Covariates X combine observed, latent attributes,

$$X_{ij} = f(C_i, C_j, D_{ij}).$$

Generative Network Models

Tradeoffs

Local structure: Homophily, Heterophily, Transitivity, etc.

Global structure: Sparsity, Percolation, etc.

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Local structure dominates generative network modeling.

Generative Network Models

Common local approaches

Conditionally independent dyads (regression):

$$P(Y \mid X) = \prod_{i<j<n} P(Y_{ij} \mid X_{ij}).$$

Infinitely exchangeable dyads (Aldous-Hoover):

$$P(Y \mid X) = \int_{\mathcal{C}} \prod_{i<j<n} P(Y_{ij} \mid X_{ij}(C_i, C_j)) dF(C).$$

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Do not capture global features, e.g., sparsity. Does this matter?

Inference Paradigms

Finite/Fixed Population

For a fixed set of actors

- Impute unmeasured links.
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For differing sets of actors

- Compare networks.
- Pool information or predict across networks.
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Extrapolating

Network Superpopulation Inference

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Definition 1 (Generalized Random Graph Process).

A generalized random graph process $Y_{\mathbb{V}}$ is a stochastic process indexed by a countably infinite vertex set $\mathbb{V}$ whose finite-dimensional distribution for any finite subset $V \subset \mathbb{V}$ defines a generalized random graph Y_V with vertex set V.

Network Superpopulation Inference

Inferential procedure and assumptions

Likelihood inference procedure

1. Propose a model family $\mathcal{P}$ of models $P_{\beta,\gamma}$.
2. $\mathcal{P}$ implies a log-likelihood $L_{\beta,\gamma,n}$ on the sampled index set. Compute

$$\hat{\beta}_n, \hat{\gamma}_n = \arg\max_{B,\Gamma} L_{\beta,\gamma,n}(Y_n). \tag{1}$$

3. Interpret $\hat{\beta}_n$ as a population parameter estimate.

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Step 3 requires **coherence** between inferences from different samples drawn from same population.

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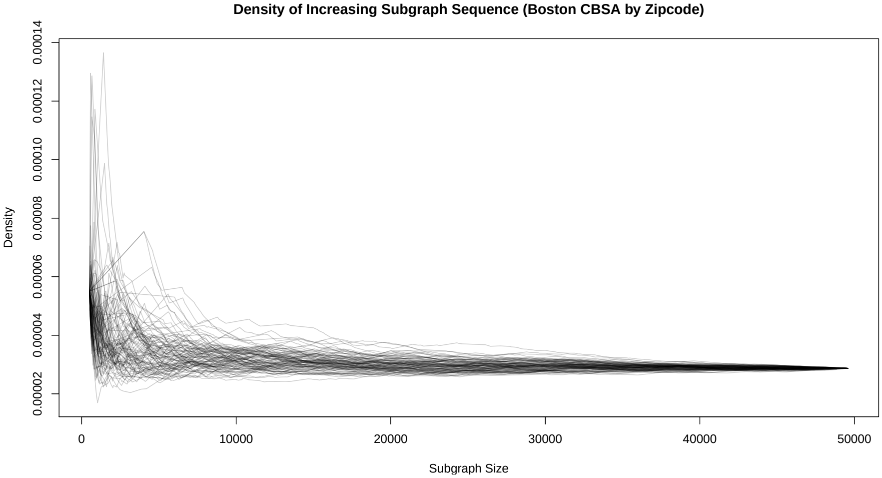
Under misspecified global structure?

Sparsity

Illustration



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Definition 1 (Sparse Generalized Random Graph Process).

Let $Y_{\mathbb{V}}$ be a generalized random graph process on $\mathbb{V}$. $Y_{\mathbb{V}}$ is *sparse* if and only if for any $\epsilon > 0$, there exists an n such that for any subset of vertices $V \in \mathbb{V}$ with $|V| > n$ the corresponding finite dimensional generalized random graph Y_V has the property $\mathbb{E}(D(Y_V)) < \epsilon$.

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Also, **sparsity rate** $\epsilon(n)$.

Sparsity Misspecification

Definition

A model family $\mathcal{P}_{\beta,\gamma}$ is **sparsity misspecfied** iff

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For example,

- For CID (under regularity) and exchangeable models, population extension is **dense** or **empty** (e.g., Orbanz and Roy, 2013).
- For process models, most lock in a given form for $\epsilon(n)$ (e.g., power law for preferential attachment).

Sparsity Misspecification

Consequences

Theorem 2 (Moving target theorem).

Suppose that the following hold:

1. *The inferential family $\mathcal{P}$ is sparsity misspecified for the true population process P_0.*
2. *The marginal distribution of the binarized data A identifies β in the presenece of nuisance parameters γ in $\mathcal{P}$.*
3. *The inferential model is* **responsive** *to the sample density $D(Y_n)$ under the true population process and*

$$|\mathbb{E}_{\bar{\beta}_n,\bar{\gamma}_n}(D(Y_n)) - \mathbb{E}_0(D(Y_n))| \in O(\epsilon_0(n)). \tag{2}$$

Then, for any n, there exists an $n' > n$ such that $\bar{\beta}_n \neq \bar{\beta}_{n'}$.

Example: Real Model Output

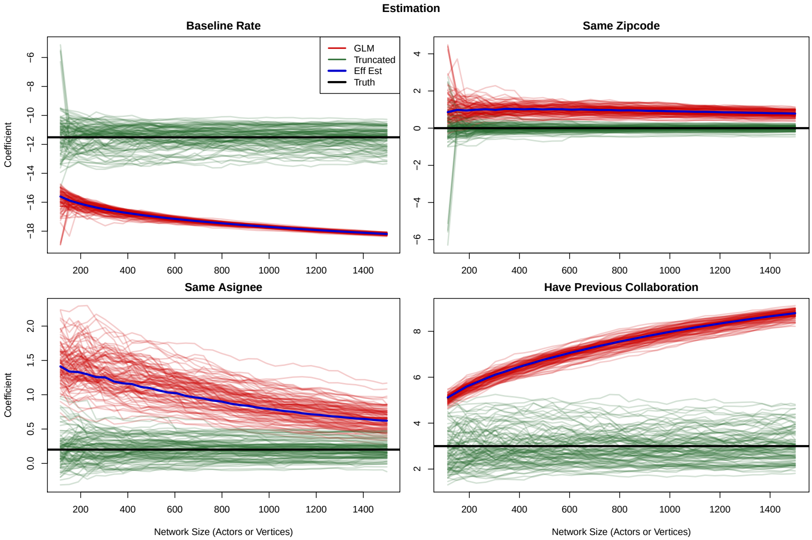
Cox PH regression. (Perry and Wolfe, 2013)
Inventor coauthorships in Michigan's motor industry 1982-1988.

Covariates (Coefs are log-ratios):

- `post85`: After 1985.
- `asgnum`: Work for same firm.
- `Ng0`: Have worked together before.

	lower	est	upper
post85	15.49	15.84	16.20
asgnum	4.65	4.83	5.02
Ng0	11.36	11.73	12.10
post85:asgnum	-4.77	-4.40	-4.03
post85:Ng0	-14.57	-14.00	-13.44
asgnum:Ng0	-5.56	-5.16	-4.76
post85:asgnum:Ng0	3.91	4.52	5.13

Example: Simulation



Partial Resolution

Salvaging conditional independence

Conditional independence assumptions are desirable.

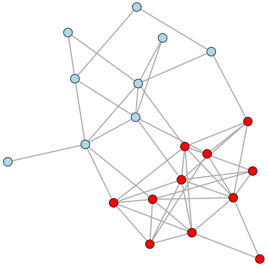
Can propose more complex latent structures R that induce conditional independence on conditional process.

Changes question:

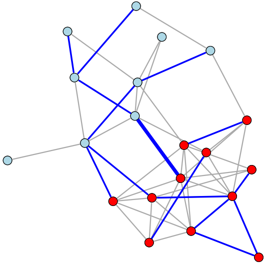
$$P_{\beta,\gamma}(Y \mid X) \to P_{\beta}(Y \mid R, X).$$

Partial Resolution

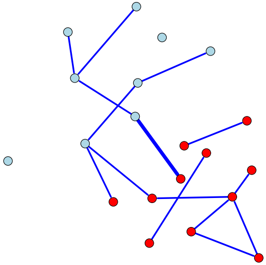
Conditionally Independent Relationship model



Unobservable Relationship/Risk



Both



Observable Interactions

Partial Resolution

Partial likelihood inference

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Exploit conditional distribution $P_\beta(Y \mid A, X)$, or **zero-truncated likelihood**.

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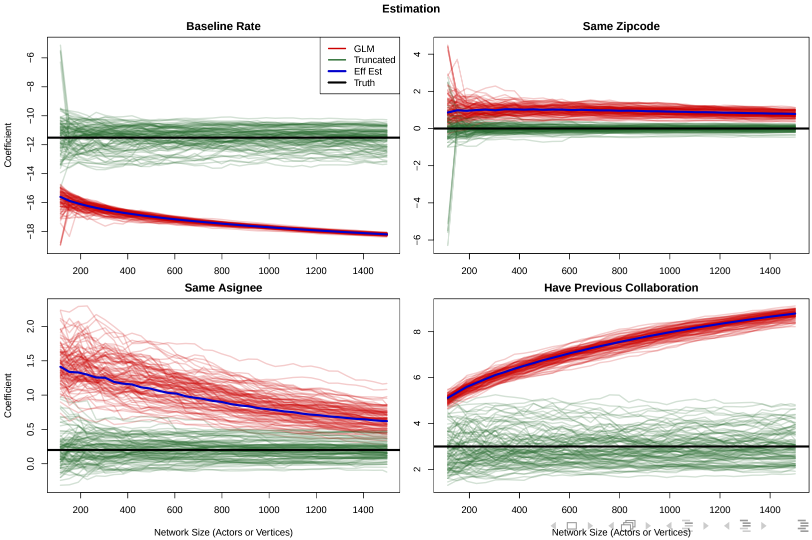
Bonus: Computation is $O(\sum_{ij} A)$.

Partial Resolution

Simulated success



Discussion

Network modeling is hard.

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Hope for network superpopulation inference?

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- More flexible global models for general questions.

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Assessing coherence of sample-wise inferences is important.

- Box: "Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful."