Week4 Report

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In this week, there are several tasks that we have done. Firstly, we update one of the conclusion we made last week about the fitness to the $\log(K)$ and $\sqrt{K}$. Secondly, we do the isotonic regression to the power curve to make it much smoother and keep non-decreasing, specifically we increase repetition times to make our results more accurate and we try to find the accuracy efficiency trade-off optimal way to do this. Thirdly, we do the analysis about scaling when we control the ratio $r_1 = \frac{Y_1}{\sigma_1}$ and $r_k = \frac{\sigma_k}{\sigma_1}$ to be stable. Finally, we change the expression of P_combined value by taking the minimum, maximum, median and mean value of P-values.

1 Critical Ratio Analysis

We have found a new finding different from last week about the fitness to the critical ratio by $\log(K)$ and $\sqrt{K}$. The 80% power related critical ratio $r_1 = \frac{Y_1}{\sigma_1}$ actually follows a **squared root K** form, and the squared root model highly explains the increase pattern, with R-squared amounting to **0.991**. Previously we controlled for the subgroup unit number size (N/K) and increase K to get a logarithm increasing form. This time we consider controlling for the total experiment unit number N rather than N/K, which seems to make more sense. In this case after changing size K the N/K will also vary, leading to the changing Yk obs and sk's distribution. The plots and results are as follow.

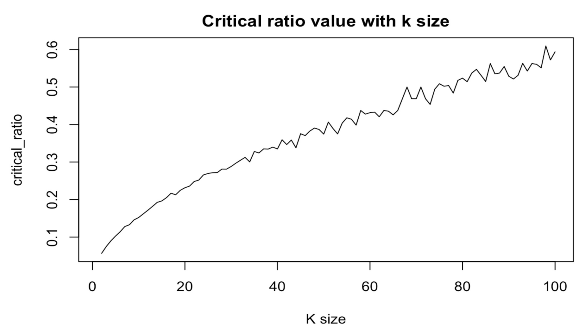
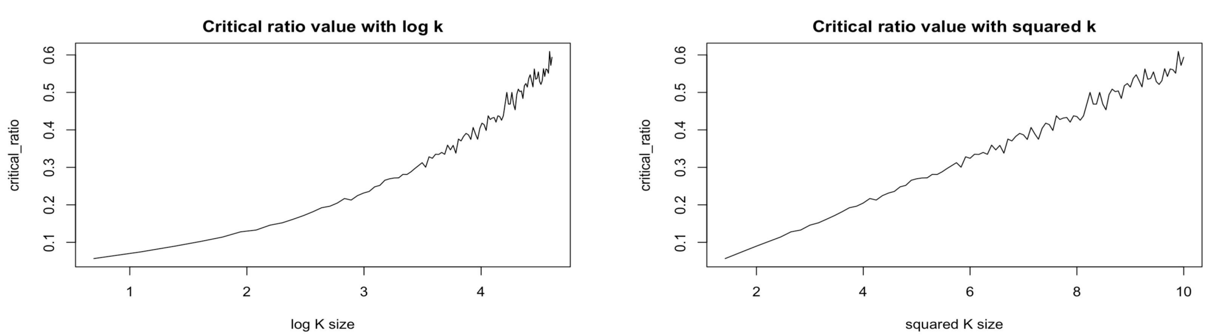


Figure 1.1: Critical Ratio for K



(a) Log K

(b) Sqrt K

Figure 1.2: Logarithm K and squared root K

We further apply the linear regression to compare $\sqrt{K}$ and $\log K$.

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Call:
lm(formula = critical_ratio ~ logk)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.05399 -0.03174 -0.01112  0.02862  0.15321 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.206102   0.017507  -11.77   <2e-16 ***
logk         0.158025   0.004642   34.05   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03936 on 97 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.9228,	Adjusted R-squared:  0.922 
F-statistic:  1159 on 1 and 97 DF,  p-value: < 2.2e-16


Call:
lm(formula = critical_ratio ~ sqrtk)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.030777 -0.005996  0.000198  0.006513  0.042270 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0426060  0.0042328  -10.07   <2e-16 ***
sqrtk        0.0615901  0.0005927  103.91   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01336 on 97 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.9911,	Adjusted R-squared:  0.991 
F-statistic: 1.08e+04 on 1 and 97 DF,  p-value: < 2.2e-16
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Figure 1.3: Regression Summary

Note the $\sqrt{K}$'s β coefficient is more statistically significant and larger, while the r^2 of log is also larger than $\log k$, this supports the reasoning that critical ratio is logarithm increase. In particular, the r squared for $\sqrt{K}$ model is high as 0.9911, indicating a potential theoretical relationship.

2 Isotonic Regression

To make the power non-decreasing, which will be a more reasonable result, we use isotonic regression to the power and get a new plot where the isotonic curve is represented by the red lines. For this plot, we hold r_k and σ to be constant while varying r_1 and Y_1.

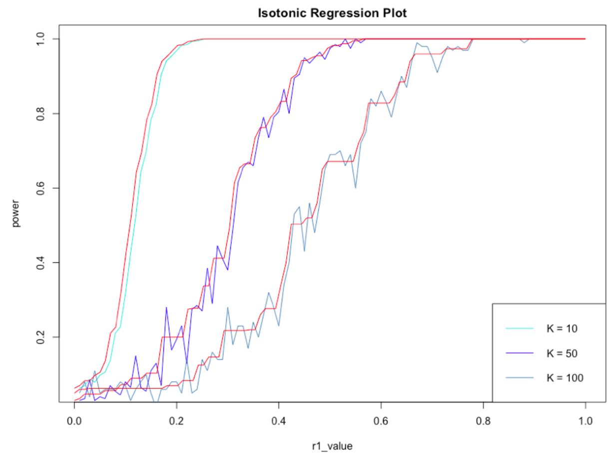
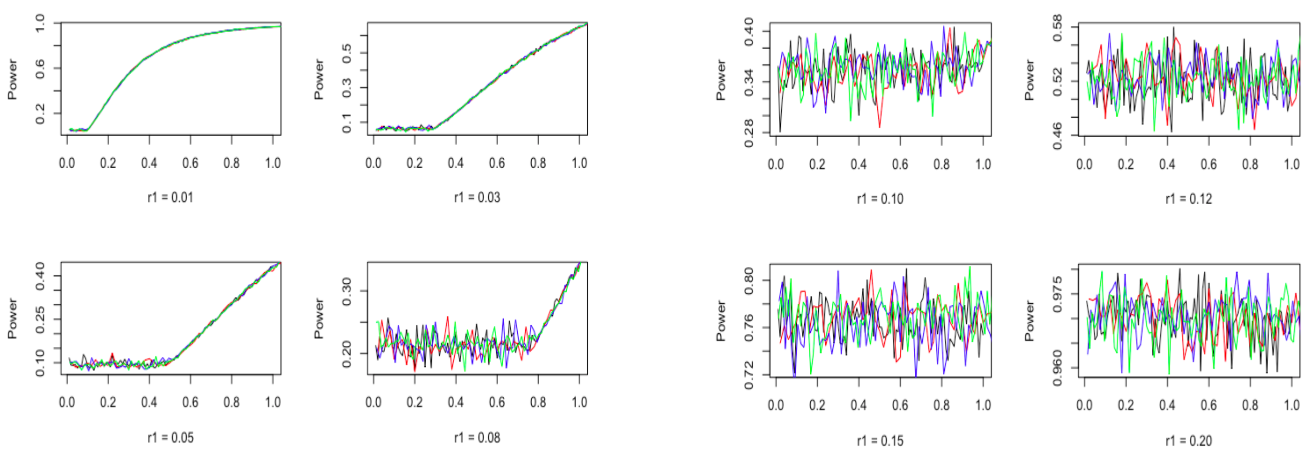


Figure 2.1: Isotonic Regression Plot

3 Scaling

For this part, we hold $r_1 = \frac{Y_1}{\sigma_1}$ and $r_k = \frac{\sigma_k}{\sigma_1}$ to be constant, which means that $\frac{Y_1}{\sqrt{\sigma_1^2+\sigma_k^2}}$ is constant. Under this condition, we try to plot Y ranging from 0 to 1 by 0.01 step, as well as 1.2Y, 1.5Y and 2Y. But at the same time, we find the the ratio r_1 also has significant impact on the power, leading to the different power pattern of the plot. So we also make different plots by r_1 ranges from 0.01 to 0.3. The first fact is that the r_1 and r_k could not determine the power, that is, the Y_1 and



(a) (b)

Figure 3.1: Scaling Plot

σ's effect could not be together accounted for as one ratio. But the good news is that when we hold r_1 and r_k constant, no matter how Y_1 changes, the power pattern seems to be the same. But from the plot we can see that when $r_1 = 0.01$, it turns out to be a "S-shape" curve; in $r_1 = 0.03$ and $r_1 = 0.05$, the horizontal part of the curve tends to oscillates more and the power, from the y-axis we can see, becomes smaller. When $r_1 = 0.08$, the plot seems all to be the horizontal oscillating part, and the power is around the smallest value, where maximum equals to around 0.3. When r_1 ranges from 0.1 to 0.2, we can see that the power all oscillates and increases closed to 1. And from Figure 3.2, we can see that all power equals to 1.

In the further discussion of r_1 (Figure 3.3), we hold Y_1 constant respectively in different level and vary r_1 from 0 to 1 to see its power pattern as follow.

We can see that it corresponds to the plot before as the power first decreases and then increases. And if Y_1 increases, the power will become more stable. Also after increasing the total number unit N, the power pattern will get closer, the vacillation effect seems to get reduced, but still there is disparity.

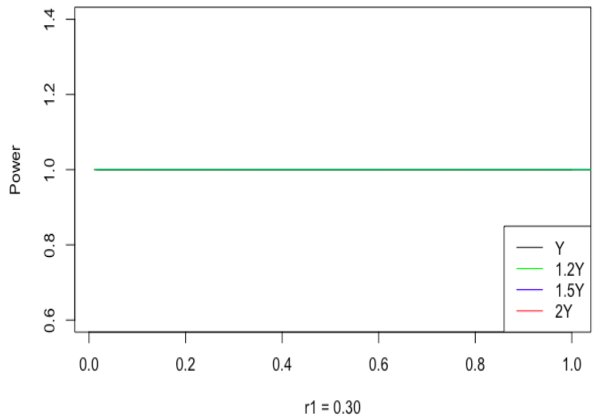
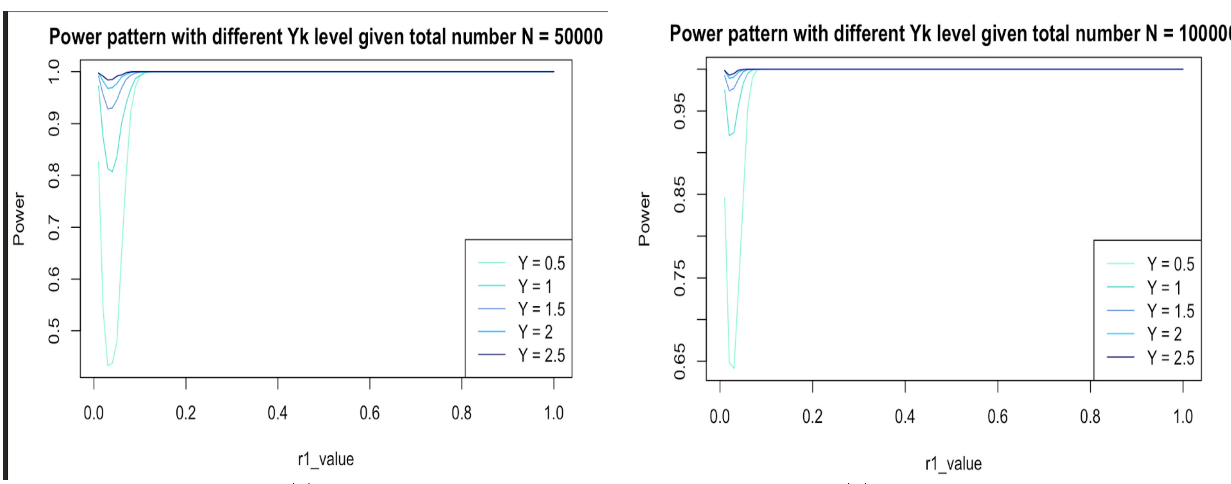


Figure 3.2: Scaling Plot

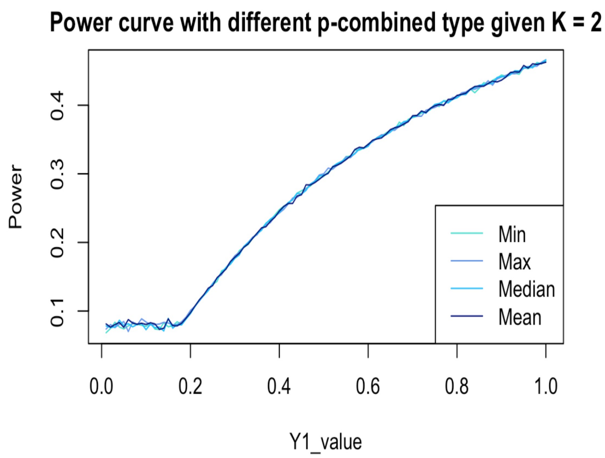


(a) (b)

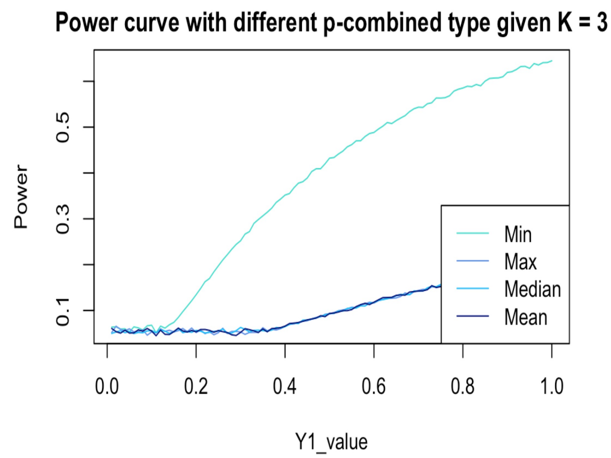
Figure 3.3: r_1 Plot with different total number N

4 P-combined Analysis

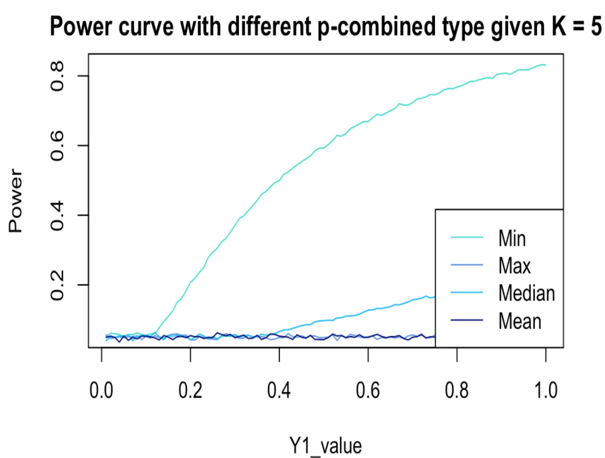
Before, we take the minimum of p-values to generate p_combined value, and this time, we take different ways to generate p_combined by taking the maximum, median and mean values. Also, we take treatment size K into consideration.



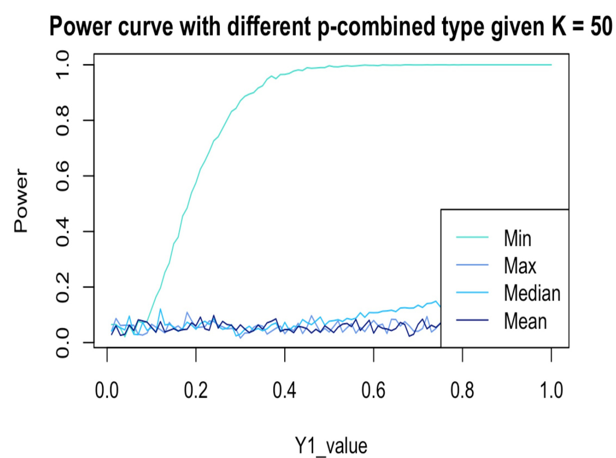
(a) K=2



(b) K=3



(c) K=5



(d) K=50

Figure 4.1: Power plot of different type of p_combined

From the plot we can see that as $K = 2$, all four types of p_combined seems to have the same pattern. But when K increases to 3, the minimum still keeps the "S-shape" curve but other three curves become lower. When $K = 5$, actually including all values bigger than 5 according to our experiments, the minimum curve still keeps its pattern; the maximum and mean curve just not increase any more and stay horizontally; the median curve has a small increasing trend, a little higher than the mean and maximum curve but much lower than the minimum curve. So we can see the the minimum curve is the most stable one.