STATISTICAL COMPUTATIONAL METHODS

Seminar Nr. 1, Random Variables and Applications

1. (Memoryless Property) The Exponential $Exp(\lambda), \lambda > 0$ and Shifted Geometric $SGeo(p), p \in (0,1)$ variables "lose memory"; in predicting the future, the past gets "forgotten", only the present matters, i.e. if $X \in Exp(\lambda)$ or $X \in SGeo(p)$,

$$P(X > x + y \mid X > y) = P(X > x),\ \forall x, y \geq 0, \forall x, y \in \mathbb{N}, \text{respectively.}$$

2. Messages arrive at an electronic message center at random times, with an average of 9 messages per hour. What is the probability of
a) receiving *exactly* 5 messages during the next hour (event A)?
b) receiving *at least* 5 messages during the next hour (event B)?

3. After a computer virus entered the system, a computer manager checks the condition of all important files. He knows that each file has probability 0.2 to be damaged by the virus, independently of other files. Find the probability that
a) at least 5 of the first 20 files checked, are damaged (event A);
b) the manager has to check at least 6 files in order to find 3 that are undamaged (event B).

4. An exciting computer game is released. Sixty percent of players complete all the levels. Thirty percent of them will then buy an advanced version of the game (only people having completed all the levels are thinking about buying the advanced version). Among 15 users,
a) what is the probability that at least two people will buy it (event A)?
b) how many people are expected to buy the new version?

5. Consider a satellite whose work is based on block A, independently backed up by a block B. The satellite performs its task until both blocks A and B fail. The lifetimes of A and B are Exponentially distributed with mean lifetime of 10 years. What is the probability that the satellite will work for more than 10 years (event E)?

6. Compilation of a computer program consists of 3 blocks that are processed sequentially, one after the other. Each block takes Exponential time with the mean of 5 minutes, independently of other blocks. Compute the probability that the entire program is compiled in less than 12 minutes (event A). Use the Gamma-Poisson formula to compute this probability two ways.

7. On the average, a computer experiences breakdowns every 5 months. The time until the first breakdown and the times between any two consecutive breakdowns are independent Exponential random variables. After the third breakdown, a computer requires a special maintenance.
a) Find the probability that a special maintenance is required within the next 9 months (event A);
b) Given that a special maintenance was not required during the first 12 months (event B), what is the probability that it will not be required within the next 4 months (event C)?

8. A computer lab has two printers. Printer I handles 40% of all the jobs. Its printing time is Exponentially distributed with mean of 2 minutes. Printer II handles the remaining jobs and its printing time is Uniformly distributed between 0 and 5 minutes. What is the probability that a document is printed in less than a minute (event A)?