An Epidemiological Mixed-Integer Nonlinear Programming Framework for Vaccine Modeling and Patient Allocation During Pandemics

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Overview

1. Preliminaries
2. Case Study: Florida
3. Model Formulation
4. Results
5. Implications
6. Conclusion

Introduction

Impact of COVID-19

- At first difficult to detect and contain
- Hospitals are overburdened, leading to unmet hospital demand
- Patients required to travel to receive healthcare
- Millions of infections and deaths

Problem Statement

- Pandemics strain hospital resources
- Optimized patient allocation can reduce unmet demand
- Transfers may increase disease spread, but are necessary to alleviate healthcare strain
- Vaccination effects must be implemented

[Lauer et al., 2020] [Florida Hospital Association, 2021]

Literature Review Map

Pandemic Allocation & Modeling
Integrates hospital-resource decisions with epidemic dynamics

Allocation Optimization
Multi-objective LP balancing ICU/non-ICU and dynamic bed forecasts

Operational Transfer Strategies
Network and queuing models for patient rerouting and surge balancing

Epidemiological Modeling
Compartmental frameworks capturing immunity, quarantine, and spatial risk

Mobility Risk Assessment
Metapopulation analyses of transfer-induced transmission risk

Study Area & Data

- **Study Period**: 155 day time horizon
- **Healthcare Facility Data**: [NIEHS, 2023]
- **Epidemiological Data**: [Abazari et al., 2024, USF, 2023, Zheng et al., 2022]

Model Component Notation

Sets

i	Regions (counties)
j	Regions (counties)
t	Time periods
t'	Decision periods

Decision Variables

$S_i^t, I_i^t, R_i^t, V_i^t$	SIRV population at time t
$u_i^{t'}$	Unmet demand
$Z_{ij}^{t'}$	Transfers $i \rightarrow j$
$\phi_i^{t'}$	Met demand
$A_{ij}^{t'}$	Transfer indicator

Parameters

$S_i^0, I_i^0, R_i^0, V_i^0$	Initial SIRV populations
N_i	County population
β_i	Infection rate
γ_i	Recovery rate
λ_i	Vaccination rate
q_i	Natural immunity loss rate
ω_i	Vaccinated immunity loss rate
ℓ_i	Leaky vaccine rate
$\alpha_i^{t'}$	Beds per infection
d_{ij}	Distance between region $i \rightarrow j$
C_i	Healthcare capacity
n	Number of counties
D	Max travel distance
M	A large number

Mathematical Model

$$\min \frac{1}{n} \sum_{i,t'} u_i^{t'}$$

Objective

subject to

$$S_i^{t+1} = S_i^t - \frac{\beta_i S_i^t I_i^t}{N_i} - \lambda_i S_i^t + \omega_i V_i^t + q_i R_i^t \qquad \forall i, t$$

$$I_i^{t+1} = I_i^t + \frac{\beta_i S_i^t I_i^t}{N_i} + \frac{\beta_i \ell_i V_i^t I_i^t}{N_i} - \gamma_i I_i^t \qquad \forall i, t$$

$$I_i^{t'+1} = I_i^{t'} + \frac{\beta_i S_i^{t'} I_i^{t'}}{N_i} + \frac{\beta_i \ell_i V_i^{t'} I_i^{t'}}{N_i} - \gamma_i I_i^{t'} + \sum_j (Z_{j,i}^{t'} - Z_{i,j}^{t'}) \qquad \forall i, t'$$

$$V_i^{t+1} = V_i^t + \lambda_i S_i^t - \omega_i V_i^t - \frac{\beta_i \ell_i V_i^t I_i^t}{N_i} \qquad \forall i, t$$

$$R_i^{t+1} = R_i^t + \gamma_i I_i^t - q_i R_i^t \qquad \forall i, t$$

SIRV dynamics

Mathematical Model (cont.)

$$u_i^{t'} = \sum_{t=t'-\psi+1}^{t'} \alpha_i^t I_i^t + \sum_{i \neq j} (Z_{j,i}^{t'} - Z_{i,j}^{t'}) - \phi_i^{t'} \qquad \forall i, t'$$

$$\phi_i^{t'} \leq \gamma_i C_i \qquad \forall i, t'$$

Unmet & satisfied demand

$$Z_{i,j}^{t'} \leq M A_{i,j}^{t'} \qquad \forall i, j, t'$$

$$Z_{i,j}^{t'} \geq A_{i,j}^{t'} \qquad \forall i, j, t'$$

$$A_{i,j}^{t'} d_{ij} \leq D \qquad \forall i, j, t'$$

Travel constraints

$$S_i^t, I_i^t, R_i^t, V_i^t \geq 0 \qquad \forall i, t$$

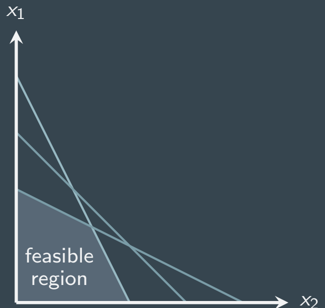
$$u_i^{t'}, \phi_i^{t'} \geq 0 \qquad \forall i, t'$$

$$Z_{i,j}^{t'} \geq 0 \qquad \forall i, j, t'$$

Non-negativity

Linearization via McCormick Envelopes

- Nonlinearity arises from $S_i^t I_i^t$ and $V_i^t I_i^t$
- We use McCormick envelopes for linearization
- Enables more efficient optimization



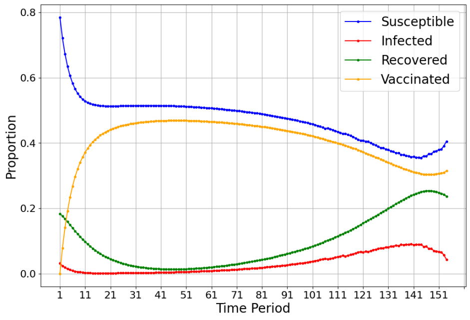
McCormick Envelopes

Let $\min z = xy$, where x and y have bounds x_l, x_u and y_l, y_u. Then:

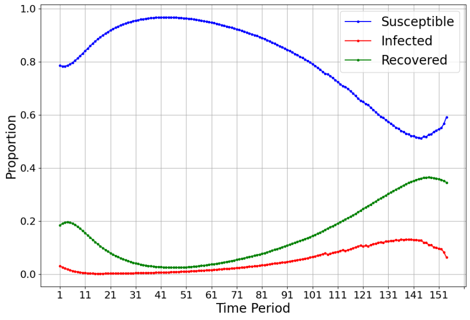
$$
\begin{aligned}
\min \quad & z \\
\text{s.t.} \quad & z \geq x_l y + x y_l - x_l y_l \\
& z \geq x_u y + x y_u - x_u y_u \\
& z \leq x_u y + x y_l - x_u y_l \\
& z \leq x y_u + x_l y - x_l y_u
\end{aligned}
$$

[McCormick, 1976]

SIRV vs SIR Dynamics



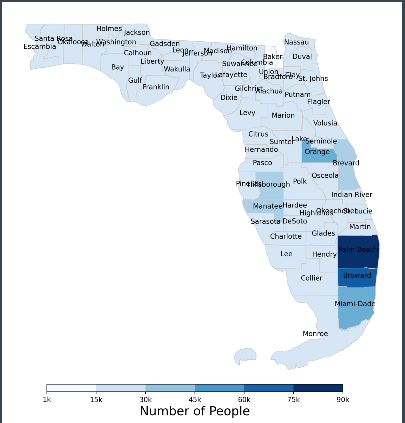
(a) Cumulative SIRV Dynamics



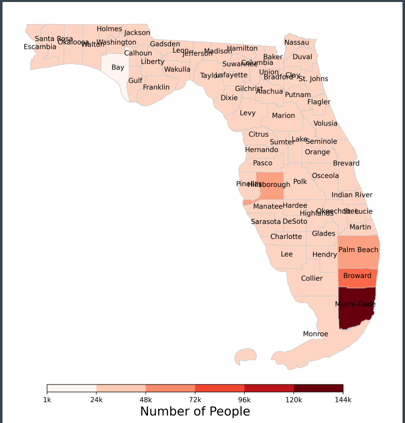
(b) Cumulative SIR Dynamics

Figure: Comparison of Cumulative SIRV and SIR Dynamics in Florida

Unmet Hospital Demand



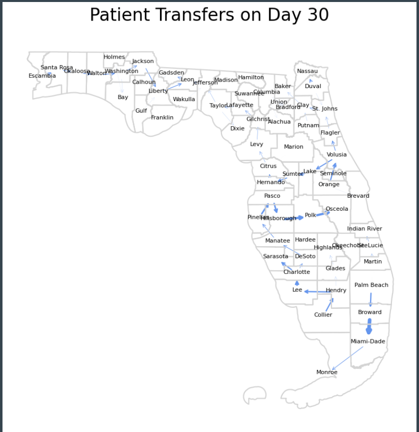
(a) Vaccinated Scenario



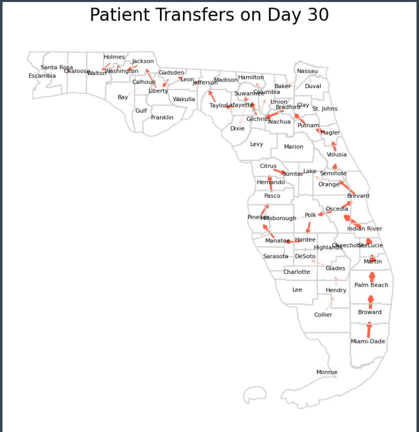
(b) Unvaccinated Scenario

Figure: Comparison of Aggregate Unmet Demand in Vaccinated vs. Unvaccinated Scenarios

Patient Transfers



(a) Vaccinated Scenario (Day 30)



(b) Unvaccinated Scenario (Day 30)

Figure: Comparison of Patient Allocation in Vaccinated vs. Unvaccinated Scenarios

Policy Recommendations

- Prioritize vaccine distribution in urban hotspots and adjust strategy as transmission shifts
- Use boosters and ongoing monitoring to maintain protection as immunity wanes
- Coordinate hospital transfers and strengthen infrastructure in high-demand areas

Key Conclusions and Future Work

Conclusions

- Coordinated vaccine and patient transfer strategies reduce hospital strain
- Urban regions tend to receive transfers; rural areas more often send patients
- Vaccination mitigates surges but waning immunity can trigger secondary waves

Future Work

- Incorporate demographics and socioeconomic factors into disease modeling
- Explicitly model vaccine supply, logistics, and effects with real data
- Refine patient transfer rules to reduce chaining and improve realism
- Dyanamic disease parameters

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Questions?