A Mixed-Integer Quadratic Programming Portfolio Optimization Model

Alexander DeLise

April 2025

Sets, Parameters, and Decision Variables

Sets

- $i \in \{1, \ldots, N\}$: Set of companies.
- $s \in S$: Set of sectors.

Parameters

- μ_i : Expected return of company i.
- Σ : Covariance matrix of returns.
- R : Minimum target return (used only in risk minimization model).
- K : Maximum number of selected companies.
- l_i : Minimum proportion of total investment in company i.
- m_i : Maximum proportion of total investment in company i.
- s_{limit} : Maximum allowable sector exposure.
- sector(i) : Sector of company i.
- maxRisk : Maximum allowable portfolio risk (used only in return maximization model).

Decision Variables

- $x_i \in [0, 1]$: Proportion of total investment in company i.
- $y_i \in \{0, 1\}$: 1 if company i is selected, 0 otherwise.

Mathematical Model

Return maximization model:

$$\max \sum_{i=1}^{N} \mu_i x_i \tag{1}$$

Risk minimization model:

$$\min x^\top \Sigma x \tag{2}$$

Subject to:

$$\sum_{i=1}^{N} x_i = 1 \tag{3}$$

$$\sum_{i=1}^{N} y_i \leq K \tag{4}$$

$$l_i y_i \leq x_i \leq m_i y_i \quad \forall i \tag{5}$$

$$\sum_{i \in s} x_i \leq s_{\text{limit}} \quad \forall s \tag{6}$$

$$x_i \leq y_i \quad \forall i \tag{7}$$

$$x_i \geq 0 \quad \forall i \tag{8}$$

Additional constraint for return maximization:

$$x^\top \Sigma x \leq \text{maxRisk}^2 \tag{9}$$

Additional constraint for risk minimization:

$$\sum_{i=1}^{N} \mu_i x_i \geq R \tag{10}$$

Model Description

The objective function either maximizes the expected return of the portfolio (1) or minimizes portfolio risk (2). The budget constraint ensures that the total investment sums to one (3). The cardinality constraint limits the number of selected companies (4). Investment bounds enforce minimum and maximum proportion limits on individual company investments (5). Sector exposure limits the total proportion invested in any single sector (6). The linking constraint ensures that no funds are allocated to unselected companies (7), and non-negativity ensures that investment proportions are non-negative (8). In the return maximization model, portfolio risk is constrained to not exceed a specified threshold (9). In the risk minimization model, the expected return of the portfolio is required to meet or exceed a minimum target (10).