Units scaling

Dr. Alessandro De Rosis

In order to use the lattice Boltzmann method (LBM) to solve a real fluid problem, we first need to convert the physical system to the LB one.

Let us consider a two-dimensional channel of length $L = 10\,\text{m}$ and height $H = 1\,\text{m}$. Let us assume the presence of a fluid of kinematic viscosity $\nu = 1 \times 10^{-6}\,\text{m}^2/\text{s}$ and density $\rho = 1000\,\text{kg}/\text{m}^3$. Moreover, let us prescribe at the left section a constant parabolic rightward velocity profile of peak $U_{\text{max}} = 1\,\text{m/s}$. Under these flow conditions, the only governing parameter is the Reynolds number that can be computed as

$$\text{Re} = \frac{U_{\text{max}} H}{\nu} = 10^6. \tag{1}$$

Now, we have to convert this system into the LB one. **First, we set the grid resolution**. In other words: how many grid points (or lattices) do we need in order to represent the problem? Thus, we choose to represent the channel height H (that is the quantity we used above in the definition of Re) by H_{LB} points. In this way, we can define the length scaling factor

$$S_{\text{l}} = \frac{H}{H_{\text{LB}}}. \tag{2}$$

Secondly, we have to **choose a value of the peak velocity in the LB system**, namely $U_{\text{max,LB}}$. To do so, we have to define the Mach number of the simulation, that is

$$\text{Ma}_{\text{LB}} = \frac{U_{\text{max,LB}}}{c_s}, \tag{3}$$

where c_s is a model parameter depending on the adopted lattice DnQm discretization. As for example, $c_s = 1/\sqrt{3}$ in the D2Q9 space. We should keep in mind that the LBM recovers the solution of the Navier-Stokes equations for incompressible flow with second-order of accuracy in the limit of very small (vanishing) Mach number. Many authors suggest that $\text{Ma}_{\text{LB}} < 0.3$. In this course, it is suggested to set $\text{Ma}_{\text{LB}} \leq 0.1$. Once Ma_{LB} is set, we have

$$U_{\text{max,LB}} = c_s\,\text{Ma}_{\text{LB}} \tag{4}$$

and we can define the scaling factor for the velocity, i.e.

$$S_{\text{v}} = \frac{U_{\text{max}}}{U_{\text{max,LB}}}. \tag{5}$$

We can now introduce the scaling factor for the density:

$$S_\rho = \frac{\rho}{\rho_{\text{LB}}}, \tag{6}$$

where $\rho_{\text{LB}} = 1$. Now, we are in the position to define any other possible scaling factor. For example, noting that forces are measured in $\text{N} = \text{kgm}/\text{s}^2$, we can write the force scaling factor as

$$S_{\text{f}} = S_\rho S_{\text{l}}^2 S_{\text{v}}^2. \tag{7}$$

Interestingly, the scaling factor for the time is

$$S_{\mathrm{t}} = \frac{S_{\mathrm{l}}}{S_{\mathrm{v}}} \tag{8}$$

and it represents the physical time step of the simulation. Finally, we can compute the viscosity in LB units by defining the corresponding scaling factor, i.e.

$$S_{\nu} = \frac{S_{\mathrm{l}}^2}{S_{\mathrm{t}}}. \tag{9}$$

Then, we obtain the viscosity in LB units as

$$\nu_{\mathrm{LB}} = \frac{\nu}{S_{\nu}}. \tag{10}$$

Eventually, the relaxation time τ is evaluated as

$$\tau = \nu_{\mathrm{LB}}\, c_s^2 + \frac{1}{2}. \tag{11}$$

One should always keep in mind that the mesh spacing Δx and the time step Δt are $\Delta x = \Delta t = 1$ in LB units.