Scaling Geometric Monitoring Over Distributed Streams

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Data Stream Systems

(Brian Babcock et al. "Models and Issues in Data Stream Systems". In: *21st ACM SIGMOD-SIGACT-SIGART*. PODS '02. 2002)

- **Data streams**: Continuous, high volume, size unbound, violative, probably distributed
- *Pull paradigm*
- Centralizing and/or polling $\rightarrow$ prohibitive in terms of communication overhead
- Examples: telecommunication, sensor networks

Motivation

Problems:

- increasing node population
- data volume
- data dimensionality
- arbitrary functions
- **communication - accuracy tradeoff**

Need for:

- scalability warranties
- tight accuracy bounds
- incremental/real-time operation
- **Minimize communication while retaining accuracy bounds**

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The Geometric Monitoring Method

(Izchak Sharfman, Assaf Schuster, and Daniel Keren. "A Geometric Approach to Monitoring Threshold Functions over Distributed Data Streams". In: *2006 ACM SIGMOD ICMD*. SIGMOD '06. 2006)

- Threshold monitoring
- Arbitrary function monitoring
- Nodes communicate when needed
- Tight accuracy bounds
- A promising framework for *distributed data stream monitoring*

Contributions

Expand the *geometric monitoring method*:

- heuristic method for violation resolution
- distance-based hierarchical node clustering (Daniel Keren, Guy Sagy, Amir Abboud, David Ben-David, et al. "Geometric Monitoring of Heterogeneous Streams." In: *IEEE TKDE* (2014))
- throughout method evaluation on synthetic and real-world datasets

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Geometric Threshold Monitoring

- Izchak Sharfman, Assaf Schuster, and Daniel Keren. "A Geometric Approach to Monitoring Threshold Functions over Distributed Data Streams". In: *2006 ACM SIGMOD ICMD*. SIGMOD '06. 2006
- **Threshold monitoring**: arbitrary function $f(\cdot)$, threshold T

$$f(\cdot) < T \text{ or } f(\cdot) > T$$

- **Idea**: decompose into local constraints at the nodes

System Architecture
Centralized Scenario

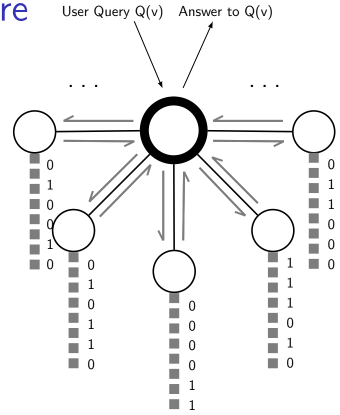


Figure: **Star-like network topology** example of the centralized scenario.The bold node represents the coordinator node. Dashed lines represent data streams and half arrows represent message exchanges.

Computational Model

Statistics vectors

- the *monitoring function* $f : \mathbb{R}^d \rightarrow \mathbb{R}$
- the *threshold* $T \in \mathbb{R}$
- the *monitoring node set* : $P = \{p_1, \ldots, p_n\}$
  with *weights* $w_1, \ldots, w_n$
- the *data streams* : $S = \{s_1, \ldots, s_n\}$
- the d-dimensional *local statistics vectors* : $\vec{v_1}(t), \ldots, \vec{v_n}(t)$
  represent each node's data stream at time t

Global statistics vector

$$\vec{v}(t) = \frac{\sum_{i=1}^{n} w_i \vec{v_i}(t)}{\sum_{i=1}^{n} w_i}$$

Computational Model

Estimate vector

Infrequent communication between nodes and coordinator:

Estimate vector

$$\vec{e}(t) = \frac{\sum_{i=1}^{n} w_i \vec{v_i}'}{\sum_{i=1}^{n} w_i}$$

- the last communicated *local statistics vector* of node p_i : $\vec{v_i}'$
- *Local statistics* divergence:$\Delta\vec{v_i}(t) = \vec{v_i}(t) - \vec{v_i}', i = 1, \ldots, n$

Centralized drift vector

$$\vec{u_i}(t) = \vec{e}(t) + \Delta\vec{v_i}(t) + \frac{\vec{\delta_i}}{w_i}$$

Computational Model

Local Constraints

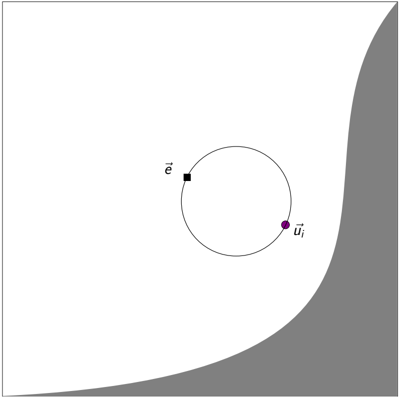


Figure: Bounding ball $B(\vec{e}, \vec{u_i}(t))$ as local constraint at node p_i

Computational Model

Balancing Process

Centralized scenario

Purpose: resolve possible false alarms

Balancing vector

$$\vec{b} = \frac{\sum_{p_i \in P'} w_i \vec{u_i}(t)}{\sum_{p_i \in P'} w_i}$$

- the *balancing set* P': a subset of nodes
- the *slack vector* at the nodes $\vec{\delta_i} = \vec{\delta_i}' + \Delta\vec{\delta_i}$, $\sum_{p_i \in P'} \Delta\vec{\delta_i} = \vec{0}$:

$$\Delta\vec{\delta_i} = w_i \vec{b} - w_i \vec{u_i}(t) \ \forall \ p_i \in P'$$

, readjusts the *drift vectors*.

Geometric Interpretation

Convexity Property & Local Constraints

Convexity Property

$$\vec{v}(t) = \frac{\sum_{i=1}^{n} w_i \vec{u_i}(t)}{\sum_{i=1}^{n} w_i}$$

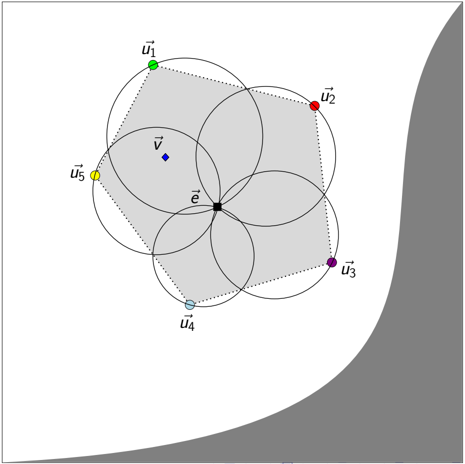


Figure: Example of a convex hull (light gray) defined by the drift vectors $\vec{u_i}, i = 1, 2, 3, 4, 5$ and bounded by spheres.

Related Work

- *Safe Zones*: optimal local constraints fitted to nodes' data distributions (Daniel Keren, Guy Sagy, Amir Abboud, David Ben-David, et al. "Safe-Zones for Monitoring Distributed Streams". In: *First International Workshop on Big Dynamic Distributed Data, Riva del Garda, Italy, 2013*)
- Ellipsoidal bounding regions, decouplement of estimate vector from bounding ball construction (D. Keren et al. "Shape Sensitive Geometric Monitoring". In: *IEEE TKDE* (2012))
- Simple shapes as local constraints, hierarchical clustering of nodes for participation to the balancing operation (Daniel Keren, Guy Sagy, Amir Abboud, David Ben-David, et al. "Geometric Monitoring of Heterogeneous Streams." In: *IEEE TKDE* (2014))
- Prediction models based on velocity and acceleration (Nikos Giatrakos et al. "Prediction-based Geometric Monitoring over Distributed Data Streams". In: *2012 ACM SIGMOD ICMD*. SIGMOD '12. 2012)

Multi-objective Optimization

- **Multiple**, possibly **conflicting** objectives to be *simultaneously* optimized
- *Pareto optimality*(*non-dominated solutions*): optimal solutions where none of the objective functions can be optimized without the simultaneous degradation of other objective functions' values.
- Let vector of m objectives $F(x) = [F_1(x), F_2(x), \ldots, F_m(x)]$:

$$\min_{x \in \mathbb{R}^n} F(x)$$
$$\text{s.t. } l \leq x \leq u$$
$$G_i = 0, i = 1, \ldots, k_e$$
$$G_j \leq 0, j = k_e + 1, \ldots, k$$

- Finding *Pareto optimal* solutions is generally **NP-hard**.

SQP

- The *Lagrangian function*: $\mathcal{L}(x,\lambda) = F(x) + \sum_{i=1}^{k} \lambda_i G_i(x)$
- *Quadratic programming subproblems*:

$$
\begin{aligned}
\min_{d \in \mathbb{R}^n} \; & \frac{1}{2} d^T H_t d + \nabla F(x_t)^T d \\
& \nabla G_i(x_t)^T d + G_i(x_t) = 0, i = 1, \ldots, k_e \\
& \nabla G_i(x_t)^T d + G_i(x_t) \leq 0, i = k_e + 1, \ldots k
\end{aligned}
$$

,where:

H_t: Hessian of the Lagrangian function at iteration t

d: search direction

The Savitzky-Golay Low-Pass Smoothing Filter

- Convolution based **smoothing**, **velocity** and **acceleration** estimation
- *Moving window averaging* paradigm: $g_i = \sum_{n=-n_L}^{n_R} c_n f_{i+n}$
- *Least-squares fit* of polynomial:

$$y_i(x) = a_0 + a_1 \frac{x - x_i}{\Delta x} + a_2 (\frac{x - x_i}{\Delta x})^2 + \cdots + a_M (\frac{x - x_i}{\Delta x})^M$$

,over window $n_L + n_R + 1$:

$$\sum_{j=i-n_L}^{i+n_R} (y_i(x_j) - f_j)^2 = \min$$

- Set g_i to the value of the fitted point x_i.

Velocity and Acceleration Estimation via SG Filtering

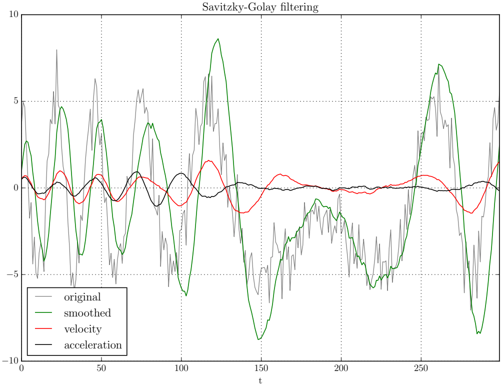
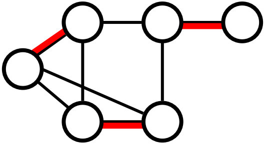


Figure: Savitzky-Golay filtering of a signal with added Gaussian noise. The smoothing window is 50 points, centered at the far right.

Maximum Weight Matching



Let $G = (V, E)$ a graph:

- *maximum weight matching* $M \subseteq E$: a subset of edges where
  - no two edges share a common vertex
  - largest possible number of edges
  - maximizes the sum of weights

Maximum Weight Matching

The Primal-Dual Method

Constraints in Primal $\iff$ Variables in Dual

Constraints in Dual $\iff$ Variables in Primal

The primal:

$$\max \sum_{(u,v)\in E} x_{u,v} w_{u,v}$$

$$\text{s.t.} \quad x_{u,v} \geq 0 \qquad , (u,v) \in E$$

$$\sum_{u\in e: e\in E} x_e \leq 1 \qquad , u \in V$$

The dual:

$$\min \sum_{u\in V} y_u$$

$$\text{s.t.} \quad y_u \geq 0 \qquad , u \in V$$

$$y_u + y_v \geq w_{u,v} \qquad , (u,v) \in E$$

Problem Formulation

Reduce the communication burden of the *Geometric Monitoring* method by:

- Optimally position *drift vectors* during the *balancing process*
- Appropriate node selection for inclusion in the *balancing set*

,in order to **increase scalability** in terms of:

- node popullation
- stream dimensionality.

The Geometric Monitoring Framework

Assumptions

- Coordinator-based scenario
- Instantaneous, loss-less, reliable communication
- Iterative operation based on time-steps
- System pause during violation resolution
- Coordinator node does not monitor a stream

The Distance-based Hierarchical Clustering

The Idea

Node selection for *violation resolution*:

- Hierarchical node clustering scheme (Daniel Keren, Guy Sagy, Amir Abboud, David Ben-David, et al. "Geometric Monitoring of Heterogeneous Streams." In: *IEEE TKDE* (2014))
- Elevate *randomness* of the initial *geometric monitoring* method (Izchak Sharfman, Assaf Schuster, and Daniel Keren. "A Geometric Approach to Monitoring Threshold Functions over Distributed Data Streams". In: *2006 ACM SIGMOD ICMD*. SIGMOD '06. 2006)
- Decouple matching from the data distribution at the nodes (Daniel Keren, Guy Sagy, Amir Abboud, David Ben-David, et al. "Geometric Monitoring of Heterogeneous Streams." In: *IEEE TKDE* (2014))
- Accurately follow *global statistics vector*
- Nodes "cancel each other out" during the *balancing process*

The Distance-based Hierarchical Clustering

The Weight Function

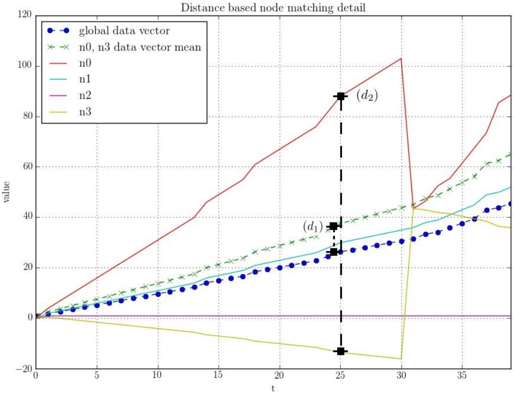
Weight function

$$w_{i,j} = \sum_{t=t_0}^{t_{end}} [(f(\vec{v}_{global}(t)) - f(\frac{\vec{v}_i(t) + \vec{v}_j(t)}{2})) + (|\vec{v}_i(t) - \vec{v}_j(t)|)]$$

The Distance-based Hierarchical Clustering

Example

Figure: Distance based node matching operating on 4 nodes ($\{n_0, n_1, n_2, n_3\}$). Distance d_1 : the distance of the data vector mean of the paired nodes n_0 and n_3 from the global mean (*global data vector*), distance d_2 : denotes the in-between distance of data vectors $\vec{v_0}(t)$ and $\vec{v_3}(t)$ of the node pair. Both distances are taking part in the edge weighting process.



The Heuristic Balancing

The Idea

In prior work:

- identical handling of nodes during the balancing process
- ignore stream idiosyncrasies and monitoring function peculiarities

The *heuristic balancing* method:

- optimally position *drift vectors in space*
- take into account stream behaviour (*velocity* and *acceleration*)

The Heuristic Balancing

The Optimizing Function

Weight function

$$\max \min \frac{(T - x_i) - accel_i(t_{lv}) * t^2}{vel_i(t_{lv})}, \forall n_i \in P'$$

where:

$$\begin{aligned} t &: \text{the variable to optimize} \\ T &: \text{monitoring threshold} \\ x_i &: \text{the maximum value of the monitoring function } f(\cdot) \text{ over the bounding ball } B(\vec{e}(t_{lv}), \vec{u_i}(t_{lv})) \\ vel_i(t_{lv}) &: \text{the estimated velocity of the maximum value of the monitoring function } f(\cdot) \\ accel_i(t_{lv}) &: \text{the estimated acceleration of the maximum value of the monitoring function } f(\cdot) \\ t_{lv} &: \text{time of Local Violation occurrence} \\ P' &: \text{the balancing set} \end{aligned}$$

The Heuristic Balancing

Example

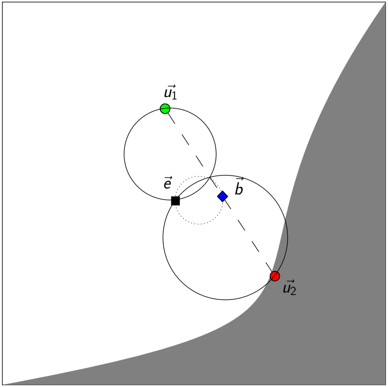


Figure: The classic balancing method.

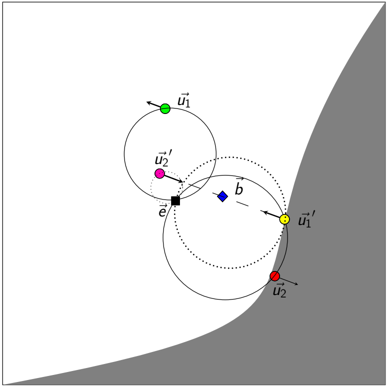


Figure: The heuristic balancing method.

Acronyms for Implemented Methods

- GM: the initial geometric monitoring method (Izchak Sharfman, Assaf Schuster, and Daniel Keren. "A Geometric Approach to Monitoring Threshold Functions over Distributed Data Streams". In: *2006 ACM SIGMOD ICMD*. SIGMOD '06. 2006)
- HM: the proposed heuristic balancing process
- DIST: the proposed distance-based hierarchical node clustering
- DISTR: the distribution-based hierarchical node clustering (Daniel Keren, Guy Sagy, Amir Abboud, David Ben-David, et al. "Geometric Monitoring of Heterogeneous Streams." In: *IEEE TKDE* (2014))

Synthetic Data

- $v_i(t_{k+1}) = v_i(t_k) + (1 - \lambda)u_k + \lambda u_{k+1}$
- 1-dimensional
- Velocities sampled from user-specified Gaussian distribution
- λ smoothing parameter
- Additive Gaussian noise
- 3 sets of datasets: *LIN*, *INT*, *NOISE*
- First 20% of data streams used as training data, when needed

Synthetic Data

Examples

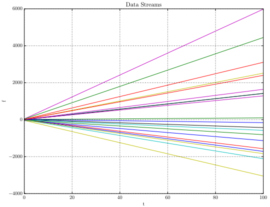


Figure: *LIN* local statistics streams of 20 nodes

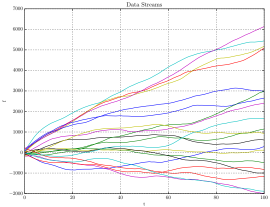


Figure: *INT* local statistics streams of 20 nodes

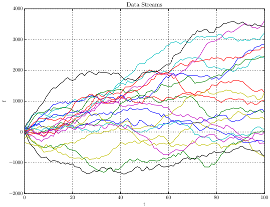


Figure: *NOISE* local statistics streams of 20 nodes

Real-world Data

- "European Environmental Agency - AQ e-Reporting" database
  (*European Environmental Agency - AQ e-Reporting.*)
- Hourly measurements of NO_2 and NO, in micro-grams per cubic meter, averaged over a window of five days for a whole year.
- Nodes correspond to randomly selected air quality measurement stations across Austria.
- First month of measurements used as training data, when needed.

Real-world Data

Examples

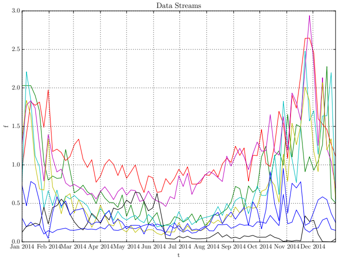


Figure: Streams of 8 nodes monitoring the ratio NO/NO_2.

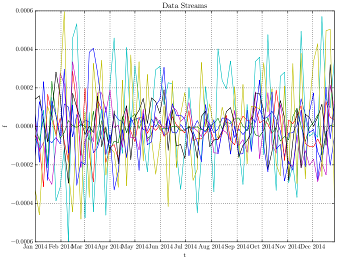


Figure: Streams of 8 nodes monitoring the variance of NO_2 air pollutant.

GM, DIST, DISTR Comparison

LIN dataset

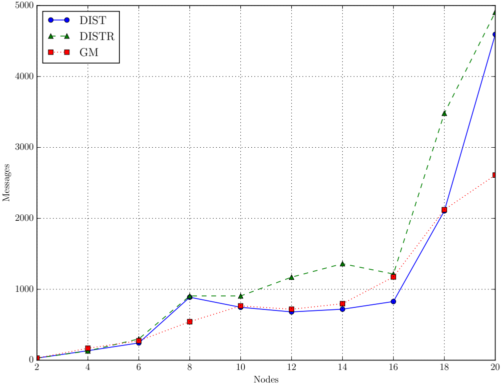


Figure: Communication costs of methods GM, DISTR and DIST for the *LIN* dataset.

GM, DIST, DISTR Comparison

INT dataset

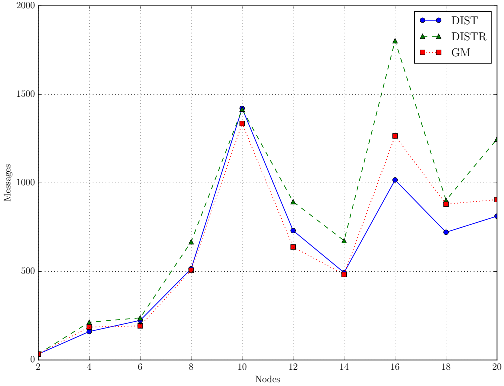


Figure: Communication costs of methods GM, DISTR and DIST for the *INT* dataset.

GM, DIST, DISTR Comparison

NOISE dataset

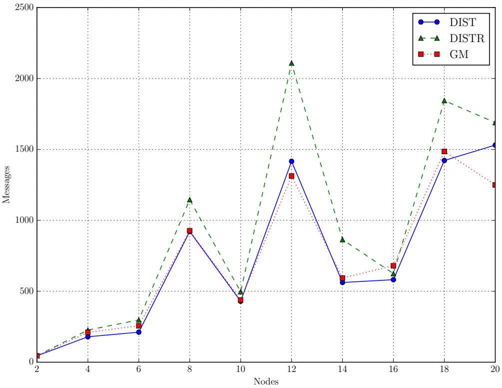


Figure: Communication costs of methods GM, DISTR and DIST for the *NOISE* dataset.

GM, DIST, DISTR Comparison

LIN dataset

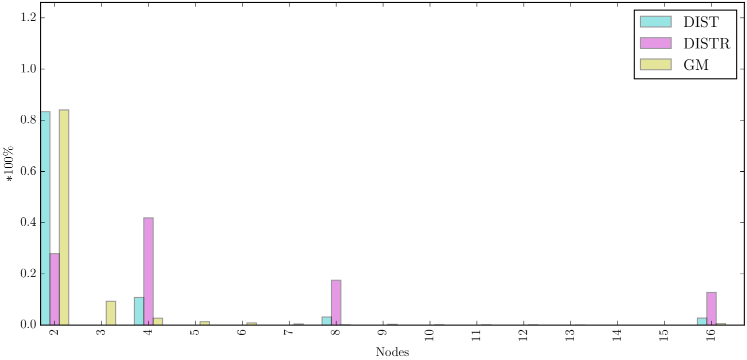
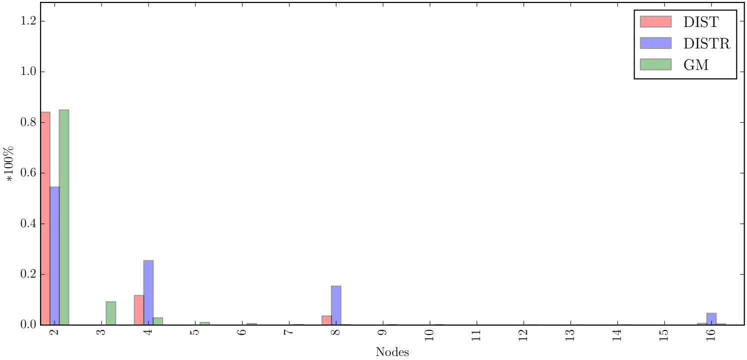


Figure: Number of nodes participating in violation resolutions as a fraction of total Local Violations, for the *LIN* dataset.

GM, DIST, DISTR Comparison

INT dataset



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GM, DIST, DISTR Comparison

NOISE dataset

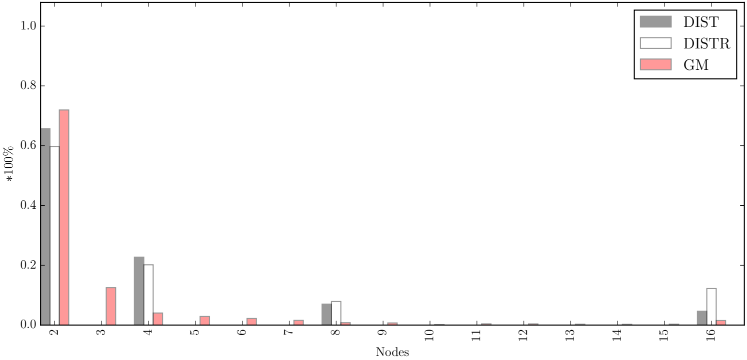

Figure: Number of nodes participating in violation resolutions as a fraction of total Local Violations, for the *NOISE* dataset.

GM, HM Comparison

Messages

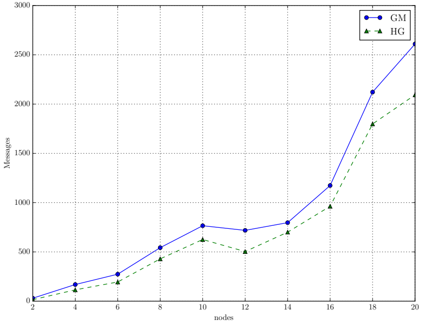

Figure: Communication cost of methods GM and HM for the *LIN* dataset.

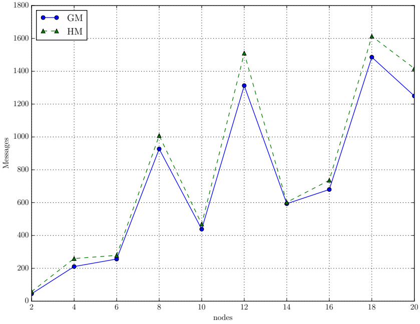

Figure: Communication cost of methods GM and HM for the *NOISE* dataset.

GM, HM Comparison

Drift vectors

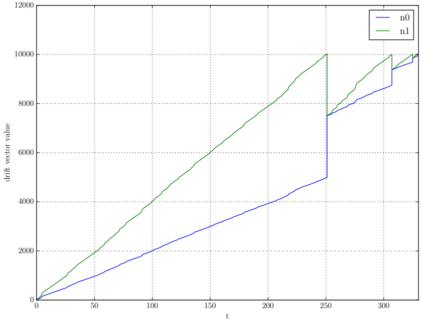

Figure: Drift vectors of 2 nodes, as formulated by the GM algorithm.

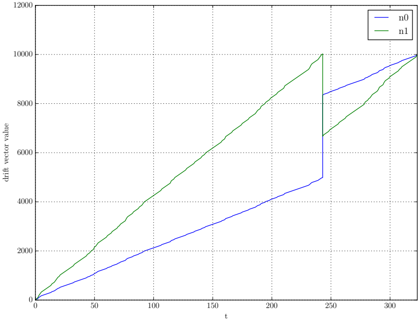

Figure: Drift vectors of 2 nodes, as formulated by the HM algorithm.

GM, HDM Comparison

LIN

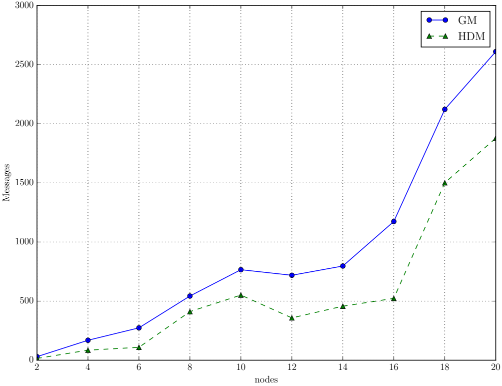

Figure: Communication cost of methods GM and HDM for the *LIN* dataset.

GM, HDM Comparison

INT

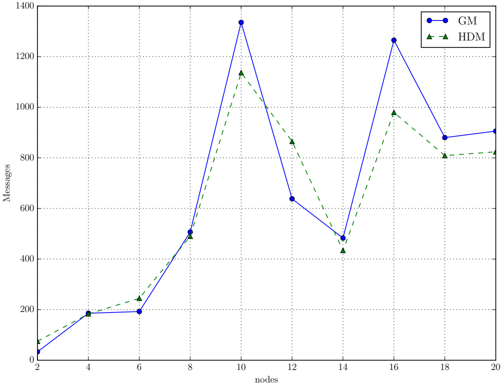

Figure: Communication cost of methods GM and HDM for the *INT* dataset.

GM, HDM Comparison

NOISE

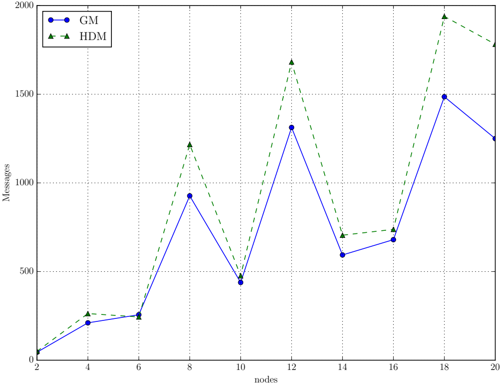

Figure: Communication cost of methods GM and HDM for the *NOISE* dataset.

GM, HDM Comparison

NOISE - window

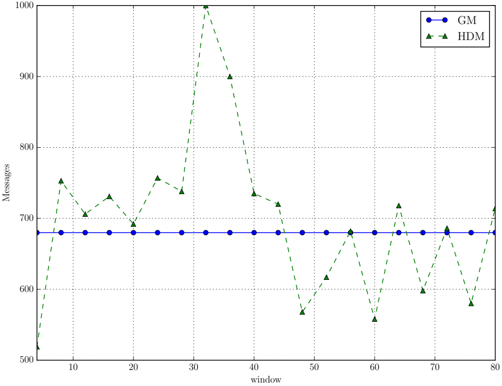

Figure: Communication cost of methods GM and HDM for the *NOISE* dataset. Approximation order is set to 1.

GM, HDM Comparison

NOISE - order

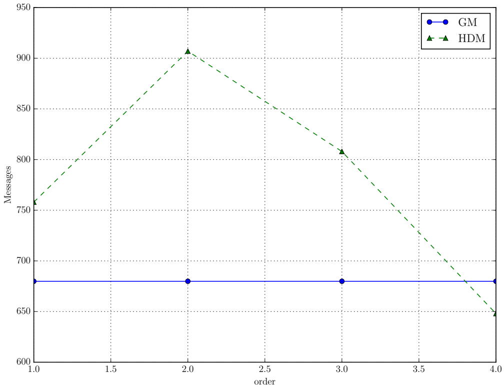

Figure: Communication cost of methods GM and HDM for the *NOISE* dataset. The Savitzky-Golay window size is set to 24.

GM, HDM Comparison

Dimensions - Quadratic Function

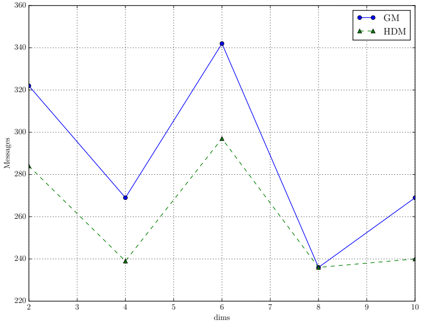

Figure: Communication cost of methods GM and HDM for the *LIN* dataset.

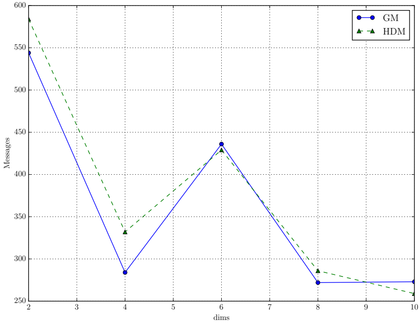

Figure: Communication cost of methods GM and HM for the *NOISE* dataset.

GM, HDM Comparison

Air Pollution Monitoring

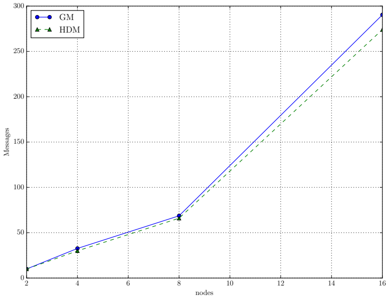

Figure: 4 to 16 nodes,variance monitoring of NO_2. The Savitzky-Golay window size is set to 6, the order is set to 2.

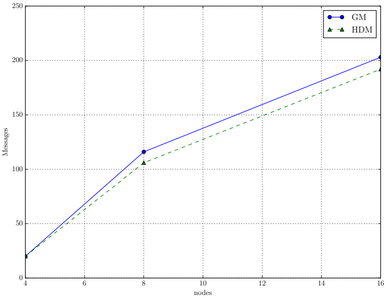

Figure: 4 to 16 nodes, when monitoring NO/NO_2. The Savitzky-Golay window size is set to 10, the order is set to 1.

Summary & Concluding Remarks

The *Geometric monitoring* method:

- An efficient framework for monitoring distributed data streams
- Scalability can be improved → reduce communication costs

Our contributions:

- Distance-based hierarchical node clustering
- Heuristic balancing method based on SQP and Savitzky-Golay Filtering
- Detailed evaluation of proposed methods

Comments:

+ Communication reduction of up to 60%
+ Methods fully compatible with the rest of the work
- Parameter tweaking for satisfactory results
- Multi-objective optimization can be computationally expensive

Future Work

- Multi-objective optimization solvers
- More elaborate optimizing fuctions
- Sophisticated prediction models (Gaussian processes)
- Parameter estimation techniques

The End

Thank you

Questions?

Assume a moving object i at point x_i, with acceleration a_i and current velocity v_i. Let v_f be the object's final velocity when it reaches a threshold point T at time t, from which it deviates by $d = T - x_i$. Let current time be $t = 0$.
Distance (or *Displacement*) in terms of velocity and acceleration is described by:

$$d = v_i t + at^2 \tag{1}$$

For which it holds:

$$\begin{aligned} d &= v_i t + a_i t^2 \leftrightarrow \\ T - x_i &= v_i t + a_i t^2 \leftrightarrow \\ t &= \frac{(T - x_i) - a_i t^2}{v_i} \end{aligned}$$

Thus, t is the expected time the moving object reaches the threshold point T.

Geometric Interpretation

Theorem (Izchak Sharfman, Assaf Schuster, and Daniel Keren. "A Geometric Approach to Monitoring Threshold Functions over Distributed Data Streams". In: *2006 ACM SIGMOD ICMD*. SIGMOD '06. 2006)

Let $\vec{x}, \vec{y_1}, \ldots, \vec{y_n} \in \mathbb{R}^d$ be a set of vectors in $\mathbb{R}^d$. Let $Conv(\vec{x}, \vec{y_1}, \ldots, \vec{y_n})$ be the convex hull of $\vec{x}, \vec{y_1}, \ldots, \vec{y_n}$. Let $B(\vec{x}, \vec{y_i})$ be a ball centered at $\frac{\vec{x}+\vec{y_i}}{2}$ and with radius of $\|\frac{\vec{x}-\vec{y_i}}{2}\|_2$ i.e., $B(\vec{x}, \vec{y_i}) = \{\vec{z} \mid \|\vec{z} - \frac{\vec{x}+\vec{y_i}}{2}\|_2 \leq \|\frac{\vec{x}-\vec{y_i}}{2}\|_2\}$, then $Conv(vecx, \vec{y_1}, \ldots, \vec{y_n}) \subset B(\vec{x}, \vec{y_i})$.

Non-linear Constraint Optimization

Primal Descent

Algorithm 1: Generic primal descent

begin

  Choose initial point $x_0 \in X$ and set $t = 0$; `/* Initialization */`

  while *maximum iteration limit OR convergence* **do** `/* Search */`

    $t = t + 1$;

    Determine search direction d_t;

    Determine step length s_t, so that $f(x_t + s_t d_t) < f(x_t)$;

    Update;

  end

end

Feasible Directions

Usable feasible direction d_t:

- a small disposition towards direction d_t does not violate any constraint i.e.,

$$d_t^T \nabla G(x_t) \leq 0$$

- a move towards d_t reduces the objective functions value i.e.,

$$d_t^T \nabla F(x_t) < 0$$

.

Savitzky-Golay Filter

$$\mathbf{J} = \begin{bmatrix} 1 & -n_L & \dots & (-n_L)^M \\ \vdots & \vdots & & \vdots \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & n_R & \dots & n_R^M \end{bmatrix}, \mathbf{a} = \begin{bmatrix} a_M \\ \vdots \\ a_1 \\ a_0 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} f_{i-n_L} \\ \vdots \\ f_i \\ \vdots \\ f_{i+n_R} \end{bmatrix}$$

The least squares fitting:

$$\|\mathbf{Ja} - \mathbf{f}\|_2 = \min$$

The *convolution coefficients* are contained in:

$$\mathbf{C} = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T$$

,and the smoothed signal can be easily computed as such:

$$g_i = (\mathbf{C}e_{M+1})^T\mathbf{f}$$

Algorithm 2: Iterative network operation

```
Data: monitoringNodes: a list of Monitoring nodes,
  coordinator: the Coordinator node
begin
    initialization;
    repeat
        foreach node ∈ monitoringNodes do
            node.DataVectorUpdate();
            node.ComputeDriftVector();
        end
        foreach node ∈ monitoringNodes do
            node.CheckForViolation();
            if localViolation then
                node.Report();
                coordinator.Balance();
            end
        end
    until globalViolation;
end
```

The Distance-based Hierarchical Clustering

The Algorithm

Algorithm 3: Recursively create Monitoring node pairs and hierarchy

Function `DistancePairer`$(nodes, i)$

  Data: $nodes = [(n_1, [\vec{v_1}(t_0), ..., \vec{v_1}(t_{end})]), ..., (n_k, [\vec{v_k}(t_0), ..., \vec{v_k}(t_{end})])]$: list of nodes with their respective data vectors, i: pair type, initial=1

  Result: $nodeHierarchy$: dictionary of $Type\text{-}k$ pairs

  if $length(nodes) = 1$ **then** `// recursion stopping condition`

    return $nodeHierarchy$;

  end

  $g = CreateCompleteGraph(nodes)$; `// complete graph with` *nodes* `as vertices`

  foreach $(n_i, n_j) \in g.Edges()$ **do** `// assign weights to edges`

    $w_{i,j} = \sum_{t=t_0}^{t_{end}} [(f(\vec{v}_{global}(t)) - f(\frac{\vec{v_i}(t)+\vec{v_j}(t)}{2})) + (|\vec{v_i}(t) - \vec{v_j}(t)|)]$;

    $g.edge(n_i, n_j).weight = w_{i,j}$;

  end

  $nodeHierarchy$(Type-i) $= g.maximalWeightMatching()$; `// node pairs of` *Type-i*

  `DistancePairer`$(nodeHierarchy(Type\text{-}i), i * 2)$;

end

The Heuristic Balancing

The Function Implementation

$$
\begin{aligned}
\min\ & -z \\
\text{s.t.}\ & z \leq g(h(\vec{e}, \vec{u_0}), vel_0, accel_0, T) \\
& z \leq g(h(\vec{e}, \vec{u_1}), vel_1, accel_1, T) \\
& \ \vdots \\
& z \leq g(h(\vec{e}, \vec{u_n}), vel_n, accel_n, T) \\
& \vec{b} = \frac{1}{\sum_{i=0}^{n} w_i} \sum_{i=0}^{n} (w_i * \vec{u_i}) \qquad\qquad , \forall n_i \in P'
\end{aligned}
$$

where:

$g : \mathbb{R}^4 \rightarrow \mathbb{R},$ the heuristic optimization function

$h : \mathbb{R}^d \rightarrow \mathbb{R},$ the max val of the monitoring function $f(\cdot)$ in $B(\vec{e}, \vec{u_i}),$

d : the data vector dimensionality

The Heuristic Balancing

The Algorithm

Algorithm 4: Heuristic Balancing

Function `RepMessageReceived`($< n_i, v_i, u_i, vel_i, accel_i >$)

  add n_i to balancing set P';

  `Balance();`

end

Function `Balance`(P')

  if $length(P') = 1$ **then**

    `RequestNode();` `// request node based on respective gathering scheme`

  end

  $\vec{b} = \sum_{P'} \frac{w_i * \vec{u_i}}{w_i}$;

  if $B(\vec{e}, \vec{b})$ *is monochromatic* **then**

    `/* heuristic optimization procedure, */`

    `/* returns the optimal drift vector positions in set` O `*/`

    $O = DriftVectorOptimizationProblem()$;

    foreach $n_i \in P'$ **do**

      $\Delta\delta_i = w_i * \vec{u_i}' - w_i * \vec{u_i}$; `//` $\vec{u_i}'$ `denotes the optimal drift vector position`

      $Send(< ADJSLK, n_i, \Delta\delta_i >)$;

    end

  end

end