Scaling Geometric Monitoring Over Distributed Streams

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Table of contents

- Introduction
- Theoretical Background
  - The Geometric Monitoring Method
  - Theoretical Tools
  - Related Work
- Problem Statement & Implementation
  - Problem Statement
  - Implementation
- Experimental Results
  - Data & Setup
  - Experiments
- Conclusions & Future Work
  - Conclusion
  - Future Work

Introduction

Theoretical Background
The Geometric Monitoring Method
Theoretical Tools
Related Work

Problem Statement & Implementation
Problem Statement
Implementation

Experimental Results
Data & Setup
Experiments

Conclusions & Future Work
Conclusion
Future Work

Data Stream Systems

- **Data streams**: Continuous, high volume, size unbound, violative, probably distributed
- *Pull paradigm*
- Centralizing and/or polling $\rightarrow$ prohibitive in terms of communication overhead
- Examples: telecommunication, sensor networks

The Geometric Monitoring Method

- Threshold monitoring
- Nodes communicate when needed
  - Local constraints
  - Violation resolution (*false alarms*)
- Arbitrary function monitoring
- Tight accuracy bounds
- A promising framework for *distributed data stream monitoring*

Motivation

Problems:

- increasing node population
- data volume
- data dimensionality
- arbitrary functions
- **communication - accuracy tradeoff**

Need for:

- scalability warranties
- tight accuracy bounds
- incremental/real-time operation
- **Minimize communication while retaining accuracy bounds**

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Contributions

Expand the *geometric monitoring method*:

- heuristic method for violation resolution
- distance-based hierarchical node clustering
- throughout method evaluation on synthetic and real-world datasets

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Geometric Threshold Monitoring

- **Threshold monitoring**: arbitrary function $f(\cdot)$, threshold T

$$f(\cdot) < T \text{ or } f(\cdot) > T$$

- **Idea**: decompose into local constraints at the nodes

System Architecture

Centralized Scenario

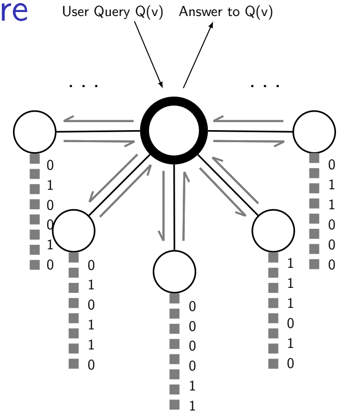


Figure: **Star-like network topology** example of the centralized scenario.The bold node represents the coordinator node. Dashed lines represent data streams and half arrows represent message exchanges.

Computational Model

Statistics vectors

- the *monitoring function* $f : \mathbb{R}^d \rightarrow \mathbb{R}$
- the *threshold* $T \in \mathbb{R}$
- the *monitoring node set* : $P = \{p_1, \ldots, p_n\}$
  with *weights* $w_1, \ldots, w_n$
- the *data streams* : $S = \{s_1, \ldots, s_n\}$
- the d-dimensional *local statistics vectors* : $\vec{v_1}(t), \ldots, \vec{v_n}(t)$
  represent each node's data stream at time t

Global statistics vector

$$\vec{v}(t) = \frac{\sum_{i=1}^{n} w_i \vec{v_i}(t)}{\sum_{i=1}^{n} w_i} \tag{1}$$

Computational Model

Estimate vector

Infrequent communication between nodes/nodes-coordinator:

Estimate vector

$$\vec{e}(t) = \frac{\sum_{i=1}^{n} w_i \vec{v}_i{}'}{\sum_{i=1}^{n} w_i} \tag{2}$$

- the last communicated *local statistics vector* of node p_i : $\vec{v}_i{}'$
- *Local statistics* divergence: $\Delta\vec{v}_i(t) = \vec{v}_i(t) - \vec{v}_i{}', i = 1, \ldots, n$

Centralized drift vector

$$\vec{u}_i(t) = \vec{e}(t) + \Delta\vec{v}_i(t) + \frac{\vec{\delta}_i}{w_i} \tag{3}$$

Computational Model

Balancing Process

Centralized scenario

Purpose: resolve possible false alarms

Balancing vector

$$\vec{b} = \frac{\sum_{p_i \in P'} w_i \vec{u_i}(t)}{\sum_{p_i \in P'} w_i} \tag{4}$$

- the *balancing set* P': a subset of nodes
- the *slack vector* at the nodes $\vec{\delta_i} = \vec{\delta_i}' + \Delta\vec{\delta_i}$, $\sum_{p_i \in P'} \Delta\vec{\delta_i} = \vec{0}$:

$$\Delta\vec{\delta_i} = w_i \vec{b} - w_i \vec{u_i}(t) \;\forall\; p_i \in P' \tag{5}$$

, readjusts the *drift vectors* (3).

Geometric Interpretation

Convexity Property

Convexity Property

$$\vec{v}(t) = \frac{\sum_{i=1}^{n} w_i \vec{u_i}(t)}{\sum_{i=1}^{n} w_i} \tag{6}$$

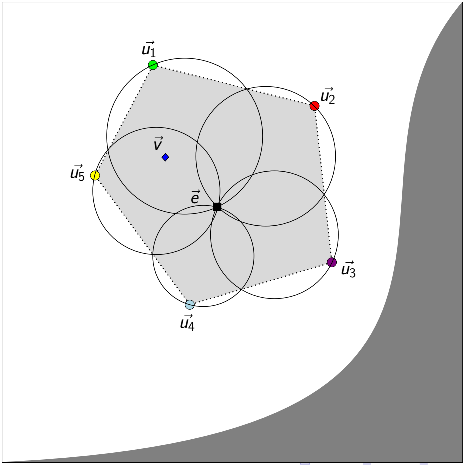
Theorem (Sharfman et al.)

Let $\vec{x}, \vec{y_1}, \ldots, \vec{y_n} \in \mathbb{R}^d$ be a set of vectors in $\mathbb{R}^d$. Let $Conv(\vec{x}, \vec{y_1}, \ldots, \vec{y_n})$ be the convex hull of $\vec{x}, \vec{y_1}, \ldots, \vec{y_n}$. Let $B(\vec{x}, \vec{y_i})$ be a ball centered at $\frac{\vec{x}+\vec{y_i}}{2}$ and with radius of $\|\frac{\vec{x}+\vec{y_i}}{2}\|_2$ i.e., $B(\vec{x}, \vec{y_i}) = \{\vec{z} \mid \|\vec{z} - \frac{\vec{x}+\vec{y_i}}{2}\|_2 \leq \|\frac{\vec{x}+\vec{y_i}}{2}\|_2\}$, then $Conv(vecx, \vec{y_1}, \ldots, \vec{y_n}) \subset B(\vec{x}, \vec{y_i})$.

Geometric Interpretation

Convexity Property & Local Constraints

Figure: Example of a convex hull (light gray) defined by the drift vectors $\vec{u_i}, i = 1, 2, 3, 4, 5$. The hull is bounded by the spheres created from the estimate vector $\vec{e}$ and the drift vectors $\vec{u_i}, i = 1, 2, 3, 4, 5$. The global statistics vector $\vec{v}$ is guaranteed to be contained in the convex hull of the drift vectors.



Protocol

Centralized Algorithm

Algorithm 1: Centralized algorithm's coordinator node operation

begin
  Wait for $< INIT, \cdot >$ messages from all monitoring nodes;
  /* Initialization */
  Compute *estimate vector* $\vec{e}(0)$;
  if *new* $< REP, \vec{v_i}(t), \vec{u_i}(t) >$ *message received* **then**
  /* Monitoring operation */
    $P' = P' \cup \{< i, \vec{v_i}(t), \vec{u_i}(t) >\}$;
    Balance(P');
  end
end
Function Balance(P') /* Balancing Process */
  Compute *balancing vector* $\vec{b}$;
  if $B(\vec{e}, \vec{b})$ *is not monochromatic* **then**
    if $P - P' \neq \emptyset$ **then**
      Send $< REQ >$ message to random node in $P - P'$ set;
    else
      Compute *estimate vector* $\vec{e}(t)$;
      Send $< NEW\text{-}EST, \vec{e}(t) >$ message to all nodes;
      return;
    end
  else
    foreach $p_i \in P'$ **do**
      Compute *slack adjustment vector* $\Delta\vec{\delta_i}$;
      Send $< ADJ\text{-}SLK, \Delta\vec{\delta_i} >$ message to node p_i;
      return;
    end
  end
end

Algorithm 2: Centralized algorithm's monitoring node operation

begin
  foreach *node* p_i **do** /* Node initialization */
    Send $< INIT, \vec{v_i}(0) >$ message to coordinator;
    $\vec{v_i}' = \vec{v_i}(0)$;
    $\vec{\delta_i} = \vec{0}$;
    Wait message from coordinator;
    if $< NEW\text{-}EST, \vec{e} >$ *message received* **then**
      Set $\vec{e}(t) = \vec{e}$;
    end
  end
  foreach *node* p_i **do** /* Main monitoring task */
    foreach *new* s_i *stream update* $\vec{v_i}(t)$ **do**
      Recalculate *drift vector* $\vec{u_i}(t)$;
      if $B(\vec{e}, \vec{u_i}(t))$ *is not monochromatic* **then**
        Send $< REP, \vec{v_i}(t), \vec{u_i}(t) >$ message to coordinator;
        Wait for $< NEW\text{-}EST, \cdot >$ or $< ADJ\text{-}SLK, \cdot >$ message from coordinator;
      end
      if *new message* $< REQ >$ *received* **then**
        Send $< REP, \vec{v_i}(t), \vec{u_i}(t) >$ message to coordinator;
        Wait for $< NEW\text{-}EST, \cdot >$ or $< ADJ\text{-}SLK, \cdot >$ message from coordinator;
      end
      if *new* $< NEW\text{-}EST, \vec{e} >$ *message received* **then**
        Set $\vec{e}(t) = \vec{e}$;
        $\vec{v_i}' = \vec{v_i}(t)$;
        $\vec{\delta_i} = \vec{0}$;
      end
      if *new* $< ADJ\text{-}SLK, \Delta\vec{\delta_i} >$ *message received* **then**
        Recompute *delta vector* $\vec{\delta_i}$;
      end
    end
  end
end

Multi-objective Optimization

- **Multiple**, possibly **conflicting** objectives to be *simultaneously* optimized
- *Pareto optimality*(*non-dominated solutions*): optimal solutions where none of the objective functions can be optimized without the simultaneous degradation of other objective functions' values.
- Let vector of m objectives $F(x) = [F_1(x), F_2(x), \ldots, F_m(x)]$:

$$\begin{aligned} &\min_{x \in \mathbb{R}^n} F(x) \\ &\text{s.t. } l \leq x \leq u \\ &\qquad G_i = 0, i = 1, \ldots, k_e \\ &\qquad G_j \leq 0, j = k_e + 1, \ldots, k \end{aligned}$$

- Finding *Pareto optimal* solutions is generally **NP-hard**.

Non-linear Constraint Optimization

Primal Descent

Algorithm 3: Generic primal descent

begin
    Choose initial point $x_0 \in X$ and set $t = 0$; `/* Initialization */`
    while *maximum iteration limit OR convergence* **do** `/* Search */`
        $t = t + 1$;
        Determine search direction d_t;
        Determine step length s_t, so that $f(x_t + s_t d_t) < f(x_t)$;
        Update;
    end
end

Feasible Directions

Usable feasible direction d_t:

- a small disposition towards direction d_t does not violate any constraint i.e.,

$$d_t^T \nabla G(x_t) \leq 0$$

- a move towards d_t reduces the objective functions value i.e.,

$$d_t^T \nabla F(x_t) < 0$$

.

SQP

- The *Lagrangian function*: $\mathcal{L}(x, \lambda) = F(x) + \sum_{i=1}^{k} \lambda_i G_i(x)$
- *Quadratic programming subproblems*:

$$\min_{d \in \mathbb{R}^n} \frac{1}{2} d^T H_t d + \nabla F(x_t)^T d$$
$$\nabla G_i(x_t)^T d + G_i(x_t) = 0, i = 1, \ldots, k_e$$
$$\nabla G_i(x_t)^T d + G_i(x_t) \leq 0, i = k_e + 1, \ldots k$$

,where:

H_t:Hessian of the Lagrangian function at iteration t

d:search direction

The Savitzky-Golay Filter

- Low-pass smoothing filter
- *Moving window averaging* paradigm: $g_i = \sum_{n=-n_L}^{n_R} c_n f_{i+n}$
- *Least-squares fit* of polynomial $y_i(x)$ over window $n_L + n_R + 1$:

$$\sum_{j=i-n_L}^{i+n_R} (y_i(x_j) - f_j)^2 = \min$$

,where:

$$y_i(x) = a_0 + a_1 \frac{x - x_i}{\Delta x} + a_2 (\frac{x - x_i}{\Delta x})^2 + \cdots + a_M (\frac{x - x_i}{\Delta x})^M$$

- Set g_i to the value of the fitted point x_i

Maximum Weight Matching

Let $G = (V, E)$ a graph:

- *maximum weight matching* $M \subseteq E$: a subset of edges where
  - no two edges share a common vertex
  - largest possible number of edges
  - maximizes the sum of weights

Maximum Weight Matching

The Primal-Dual Method

Primal-Dual Method

$$\text{Constraints in Primal} \iff \text{Variables in Dual}$$
$$\text{Constraints in Dual} \iff \text{Variables in Primal}$$

The primal:

$$\begin{array}{lll} \max & \sum_{(u,v)\in E} x_{u,v} w_{u,v} & \\ \text{s.t.} & x_{u,v} \geq 0 & ,(u,v) \in E \\ & \sum_{u\in e : e \in E} x_e \leq 1 & ,u \in V \end{array}$$

The dual:

$$\begin{array}{lll} \min & \sum_{u\in V} y_u & \\ \text{s.t.} & y_u \geq 0 & ,u \in V \\ & y_u + y_v \geq w_{u,v} & ,(u,v) \in E \end{array}$$

Related Work

- *Safe Zones*: optimal local constraints fitted to nodes' data distributions
- Ellipsoidal bounding regions, decouplement of estimate vector from bounding ball construction
- Simple shapes as local constraints, hierarchical clustering of nodes for participation to the balancing operation
- Prediction models based on velocity and acceleration

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Problem Formulation

Reduce the communication burden of the *Geometric Monitoring* method by:

- Optimally position *drift vectors* during the *balancing process*
- Appropriate node selection for inclusion in the *balancing set*

,in order to **increase scalability** in terms of:

- node popullation
- stream dimensionality.

The Geometric Monitoring Framework

Assumptions

- Coordinator-based scenarion
- Instantaneous, loss-less, reliable communication
- Iterative operation based on time-steps
- System pause during violation resolution
- Coordinator node does not monitor a stream

Algorithm 4: Iterative network operation

```
Data: monitoringNodes: a list of Monitoring nodes,
  coordinator: the Coordinator node
begin
    initialization;
    repeat
        foreach node ∈ monitoringNodes do
            node.DataVectorUpdate();
            node.ComputeDriftVector();
        end
        foreach node ∈ monitoringNodes do
            node.CheckForViolation();
            if localViolation then
                node.Report();
                coordinator.Balance();
            end
        end
    until globalViolation;
end
```

The Distance-based Hierarchical Clustering

The Idea

Node selection for *violation resolution*:

- Elevate *randomness* of the initial *geometric monitoring* method
- Hierarchical node clustering scheme
- Decouple matching from the data distribution at the nodes
- Accurately follow *global statistics vector*
- Node “cancel each other out” during the *balancing process*

The Distance-based Hierarchical Clustering

The Weight Function

Weight function

$$w_{i,j} = \sum_{t=t_0}^{t_{end}} [(f(\vec{v}_{global}(t)) - f(\frac{\vec{v}_i(t) + \vec{v}_j(t)}{2})) + (|\vec{v}_i(t) - \vec{v}_j(t)|)]$$

The Distance-based Hierarchical Clustering

The Algorithm

Algorithm 5: Recursively create Monitoring node pairs and hierarchy

Function `DistancePairer`(*nodes*,*i*)

Data: $nodes = [(n_1, [\vec{v_1}(t_0), ..., \vec{v_1}(t_{end})]), ..., (n_k, [\vec{v_k}(t_0), ..., \vec{v_k}(t_{end})])]$: list of nodes with their respective data vectors,i: pair type, initial=1

Result: *nodeHierarchy*: dictionary of *Type-k* pairs

if $length(nodes) = 1$ **then** `// recursion stopping condition`

  return *nodeHierarchy*;

end

$g = CreateCompleteGraph(nodes)$; `// complete graph with` *nodes* `as vertices`

foreach $(n_i, n_j) \in g.Edges()$ **do** `// assign weights to edges`

  $w_{i,j} = \sum_{t=t_0}^{t_{end}} [(f(\vec{v}_{global}(t)) - f(\frac{\vec{v_i}(t)+\vec{v_j}(t)}{2})) + (|\vec{v_i}(t) - \vec{v_j}(t)|)]$;

  $g.edge(n_i, n_j).weight = w_{i,j}$;

end

$nodeHierarchy(\text{Type-i}) = g.maximalWeightMatching()$; `// node pairs of` *Type-i*

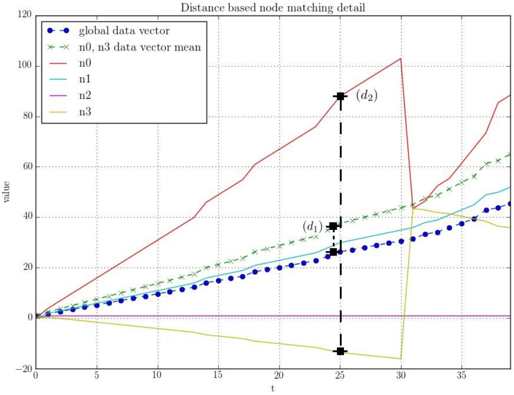
`DistancePairer`($nodeHierarchy(Type\text{-}i), i * 2$);

end

The Distance-based Hierarchical Clustering

Example

Figure: Distance based node matching operating on 4 nodes ($\{n_0, n_1, n_2, n_3\}$). Distance d_1 : the distance of the data vector mean of the paired nodes n_0 and n_3 from the global mean (*global data vector*), distance d_2 : denotes the in-between distance of data vectors $\vec{v_0}(t)$ and $\vec{v_3}(t)$ of the node pair. Both distances are taking part in the edge weighting process.



The Heuristic Balancing

The Idea

In prior work:

- identical handling of nodes during the balancing process
- ignore stream idiosyncrasies and monitoring function peculiarities

The *heuristic balancing* method:

- optimally position *drift vectors in space*
- take into account stream behaviour (*velocity* and *acceleration*)

The Heuristic Balancing

The Optimizing Function

Weight function

$$\max\min \frac{(T - x_i) - accel_i(t_{lv}) * t^2}{vel_i(t_{lv})}, \forall n_i \in P'$$

where:

$$\begin{aligned}
t &: \text{the variable to optimize} \\
T &: \text{monitoring threshold} \\
x_i &: \text{the maximum value of the monitoring function } f(\cdot) \text{ over the bounding ball } B(\vec{e}(t_{lv}), \vec{u_i}(t_{lv})) \\
vel_i(t_{lv}) &: \text{the estimated velocity of the maximum value of the monitoring function } f(\cdot) \\
accel_i(t_{lv}) &: \text{the estimated acceleration of the maximum value of the monitoring function } f(\cdot) \\
t_{lv} &: \text{time of Local Violation occurrence} \\
P' &: \text{the balancing set}
\end{aligned}$$

The Heuristic Balancing

The Algorithm

Algorithm 6: Heuristic Balancing

Function `RepMessageReceived`($< n_i, v_i, u_i, vel_i, accel_i >$)
    add n_i to balancing set P';
    `Balance();`
end
Function `Balance`(P')
    if $length(P') = 1$ **then**
        `RequestNode();` `// request node based on respective gathering scheme`
    end
    $\vec{b} = \sum_{P'} \frac{w_i * \vec{u_i}}{w_i}$;
    if $B(\vec{e}, \vec{b})$ *is monochromatic* **then**
        `/* heuristic optimization procedure, */`
        `/* returns the optimal drift vector positions in set` O `*/`
        $O = DriftVectorOptimizationProblem()$;
        foreach $n_i \in P'$ **do**
            $\Delta\delta_i = w_i * \vec{u_i}' - w_i * \vec{u_i}$; `//` $\vec{u_i}'$ `denotes the optimal drift vector position`
            $Send(< ADJSLK, n_i, \Delta\delta_i >)$;
        end
    end
end

The Heuristic Balancing

Example

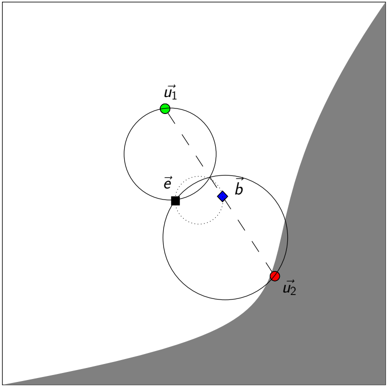


Figure: The classic balancing method.

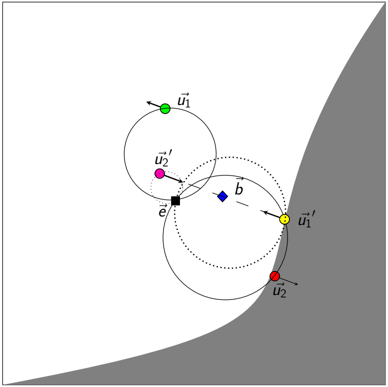


Figure: The heuristic balancing method.

Violation Detection in the Sphere

A Nested Optimization Problem

Velocity and Acceleration Estimation via SG Filtering

Implementation Challenges

Synthetic Data

Real-world Data

Notation

RAND, DIST, DISTR Comparison

GM, HM Comparison

GM, HDM Comparison

Synthetic Data Monitoring

GM, HDM Comparison

Air Pollution Monitoring

Summary & Concluding Remarks

Future Work

The end
Questions?

Appendix

Savitzky-Golay filter matrix notation

hey you